Comment on time-variation of fundamental constants

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Abstract

The possible time variation of dimensionless fundamental constants of nature, such as the fine-structure constant α, is a legitimate subject of physical enquiry. By contrast, the time variation of dimensional constants, such as $\hbar$, c, G, e, k..., which are merely human constructs whose number and values differ from one choice of units to the next, has no operational meaning. To illustrate this, we refute a recent claim of Davies et al that black holes can discriminate between two contending theories of varying α, one with varying c and the other with varying e. In Appendix A we respond to criticisms by P. Davies and two *Nature* referees. In Appendix B we respond to remarks by Magueijo and by T. Davis. In Appendix C we critique recent claims by Copi, A. Davis and Krauss to have placed constraints on $\Delta G/G$.

1 Black holes and varying constants

The claim [1] that the fine-structure constant, α-the measure of the strength of the electromagnetic interaction between photons and electrons-is slowly increasing over cosmological time scales has refuelled an old debate about varying fundamental constants of nature. In our opinion [2], however, this debate has been marred by a failure to distinguish between *dimensionless* constants such as α, which may indeed be fundamental, and *dimensional* constants such as the speed of light c, the charge on the electron e, Planck's constant $\hbar$, Newton's constant G, Boltzmann's constant k etc, which are merely human constructs whose number and values differ from one choice of units to the next and which have no intrinsic physical significance. An example of this confusion is provided by a recent paper [3], where it is claimed that "As $\alpha = e^2/\hbar c$, this would call into question which of these fundamental quantities are truly constant". By consideration of black hole thermodynamics, the authors conclude that theories with decreasing c are different from (and may be favored over) those with increasing e. Here we argue that this claim is operationally meaningless, in the sense that no experiment could tell the difference, and we replace it by a meaningful one involving just dimensionless parameters.

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Any theory of gravitation and elementary particles is characterized by a set of dimensionless parameters such as coupling constants α_i (of which the fine-structure constant is an example), mixing angles θ_i and mass ratios μ_i. To be concrete, we may take $m_i{}^2 = \mu_i{}^2 \hbar c/G$ where m_i is the mass of the i'th particle. The Standard Model has 19 such parameters, but it is hoped that some future unified theory might reduce this number. By contrast, the number and values of dimensional constants, such as $\hbar$, c, G, e, k etc, are quite arbitrary human conventions. Their job is merely to convert from one system of units to another. Moreover, the more units you introduce, the more such conversion factors you need [2].

The authors of [3] point out that the entropy S of a non-rotating black hole with charge Q and mass M is given by

$$S = \frac{k\pi G}{\hbar c}[M + \sqrt{M^2 - Q^2/G}]^2 \qquad (1)$$

They note that decreasing c increases S but increasing e, and hence Q, decreases S. It is then claimed, erroneously in our view, that black holes can discriminate between two contending theories of varying α, one with varying c and the other with varying e.

Let us define the dimensionless parameters s, μ and q by $S = sk\pi$, $M^2 = \mu^2 \hbar c/G$ and $Q^2 = q^2 \hbar c$. The mass ratio μ will depend on the fundamental dimensionless parameters of the theory α_i, θ_i and μ_i, but the details need not concern us here. In the Appendix we shall give a thought-experimental definition of s, μ and q that avoids all mention of the unit-dependent quantities G, c, $\hbar$, and e and which is valid whether or not they are changing in time. Shorn of all its irrelevant unit dependence, therefore, the entropy is given by

$$s = [\mu + \sqrt{\mu^2 - q^2}]^2 \qquad (2)$$

If we use the fact that the charge is quantized in units of e, namely $Q = ne$ with n an integer, then $q^2 = n^2 \alpha$, but we prefer not to mix up macroscopic and microscopic quantities in (1). So the correct conclusion is that such black holes might[2] discriminate between contending theories with different variations of μ and q.

The unit dependence of the claim in [3] that black holes can discriminate between varying c and varying e is now evident. In Planck units [5, 2]

$$\hbar = c = G = 1 \qquad e^2 = \alpha \qquad M^2 = \mu^2 \qquad (3)$$

[2]Since the purpose of this paper is to critique the whole school of thought, of which Ref. [3] is but an example, that believes time-dependent dimensional parameters have operational significance, we will not divert attention by getting into the question of whether black hole thermodynamics provides a good laboratory for testing time variation of dimensionless constants. Suffice it to say that that, in our view, the only sensible context in which to discuss time varying constants of nature is in theories where they are given by moduli (i.e. vacuum-expectation-values of scalar fields). The black hole entropy would then be expressed in terms of these moduli [4] whose time-dependence would have to be determined consistently by the field equations. Moreover, one would have to take into account both the entropy of the black hole and its environment.

In Stoney units [6, 2]

$$c = e = G = 1 \qquad \hbar = 1/\alpha \qquad M^2 = \mu^2/\alpha \qquad (4)$$

In Schrodinger units (see Appendix)

$$\hbar = e = G = 1 \qquad c = 1/\alpha \qquad M^2 = \mu^2/\alpha \qquad (5)$$

In all three units (and indeed in any^3 units), the dimensionless entropy ratio s is the same as given by (1). To reiterate: assigning a change in α to a change in e (Planck) or a change in $\hbar$ (Stoney) or a change in c (Schrodinger) is entirely a matter of units, not physics. Just as no experiment can determine that MKS units are superior to CGS units, or that degrees Fahrenheit are superior to degrees Centigrade, so no experiment can determine that changing c is superior to changing e, contrary to the main claim of Davies et al [3].

In summary, it is operationally meaningless [2] and confusing to talk about time variation of arbitrary unit-dependent constants whose only role is to act as conversion factors. For example, aside from saying that c is finite, the statement that $c = 3 \times 10^8$ m/s, has no more content than saying how we convert from one human construct (the meter) to another (the second). Asking whether c has varied over cosmic history (a question unfortunately appearing on the front page of the *New York Times* [7], in *Physics World* [8][4], in *New Scientist* [10, 11, 12], in *Nature* [3] and on *CNN* [13]) is like asking whether the number of liters to the gallon has varied.

2 Acknowledgements

A Response to criticisms

Since, after receiving a comment from Davies and two referee reports, *Nature* rejected (a shorter version of) this paper without the opportunity to rebut the criticisms, we do so in this Appendix. In the opinion of the present author, these three commentaries contain many of the most common misconceptions regarding fundamental constants. For example:

Referee 1: It is true that if the fundamental "constants" h, c, G, k...are truly constant, then they do indeed only act as conversion factors and can e.g. be set equal to unity. However, when they are postulated (or discovered experimentally

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[3]Units in which G varies are discussed in the Appendix.
[4]To its credit, *Physics World* published the opposing view [9].

to vary) in time, then we have to take into account that varying one or the other of these constants can have significant consequences for physics[5]. Thus, varying the charge e will have different experimentally testable consequences than varying either h or c.

Response: To elaborate more on our refutation of this common fallacy, let us define Planck [5, 2] length, mass, time and charge by

$$L_P{}^2 = G\hbar/c^3$$

$$M_P^2 = \hbar c/G$$

$$T_P{}^2 = G\hbar/c^5$$

$$Q_P{}^2 = \hbar c \tag{6}$$

Note that these are independent of e, since Planck was not immediately concerned with electrodynamics in this context. Now define Stoney [6, 2] length, mass, time and charge by

$$L_S{}^2 = Ge^2/c^4 = \alpha L_P{}^2$$

$$M_S^2 = e^2/G = \alpha M_P{}^2$$

$$T_S{}^2 = Ge^2/c^6 = \alpha T_P{}^2$$

$$Q_S{}^2 = e^2 = \alpha Q_P{}^2 \tag{7}$$

Note that these are independent of $\hbar$ since Stoney knew nothing of quantum mechanics. They are obtained from Planck units by the replacement $\hbar \to \alpha\hbar$. To complete the trio we need units that take e and $\hbar$ into account but are independent of c. It seems appropriate, therefore, to call these Schrodinger length, mass, time and charge, defined by

$$L_\psi{}^2 = G\hbar^4/e^6 = L_P{}^2/\alpha^3$$

$$M_\psi^2 = e^2/G = \alpha M_P{}^2$$

$$T_\psi{}^2 = G\hbar^6/e^{10} = T_P{}^2/\alpha^5$$

$$Q_\psi{}^2 = e^2 = \alpha Q_P{}^2 \tag{8}$$

They are obtained from Planck units by making the replacement $c \to \alpha c$.

We now ask the question: can we give a thought-experimental (as opposed to purely mathematical) meaning to the above length, mass, time and charge units that is valid whether or not quantities are changing in time? Interestingly, the ERNBH (extreme Reissner-Nordstrom black hole) solution provides the answer. A non-rotating black hole with charge Q and mass M has Schwarzschild radius

$$R_S = \frac{GM}{c^2} + \sqrt{\frac{G^2M^2}{c^4} - \frac{GQ^2}{c^4}} \tag{9}$$

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[5]Similar objections were raised by Moffat [14].

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and Compton wavelength

$$R_C = \frac{h}{Mc}.$$ (10)

In the extreme case, moreover, we have

$$R_S = \frac{GM}{c^2} \qquad Q^2 = GM^2$$ (11)

L_P, M_P, T_P and Q_P may now be thought-experimentally defined without reference to any fundamental constants as the Schwarzschild radius, mass, characteristic time and charge of an ERNBH whose Schwarzschild radius equals its Compton wavelength divided by 2π. Thus s, μ and q count the number of times S, M and Q exceed the entropy, mass and charge of such an black hole. Similarly, L_S, M_S, T_S and Q_S are the corresponding quantities for an ERNBH whose charge is the charge on the electron. L_ψ, M_ψ, T_ψ and Q_ψ also admit a thought-experimental definition which we defer until the introduction of Bohr units below.

So even if dimensionless constants are changing in time, nothing stops us from using Planck units with $c = \hbar = 1$ and time varying e, Stoney units with $c = e = 1$ and time varying $\hbar$ or Schrodinger units with $\hbar = e = 1$ and time varying c.

Referee 1: It is conceivable that varying the charge e could lead to a theory that somehow could be re-written as a theory in which e is kept fixed and c is varied, but this would lead to a strange and very complicated revision of all of physics.

Response: On the contrary, it is nothing more than switching from Planck units to Schrodinger units.

Referee 1: He has already published his views on this issue in: M. J. Duff, L. B. Okun and G. Veneziano, JHEP 0203, 023 (2002). It is to be noted that the other two authors of this article do not appear to agree with Duff that it is "operationally meaningless" to vary dimensional constants.

Response: According to this logic, the present paper would have to be rejected whichever of the three mutually contradictory views in [2] were being put forward. I think it should be judged on its own merits.

Referee 1: Perhaps, Duff would wish to clarify his position on the issue of how the rest of physics is affected by a possible variation of dimensional constants. The issue of how physics would be affected by the experimental discovery that "constants", such as c, h, G... vary in time is clearly of fundamental importance.

Response: I believe my position is clear: physics is about dimensionless constants and is completely unaffected by the choice of units, which has no fundamental importance.

Referee 2: Physics without reference to dimensional quantities is unfortunately not a possibility. Curiously this fact only shocks the author with reference to a changing c.

Response: I beg to differ. Dimensional quantities may sometimes be useful, but from an empirical point of view, experiments measure only dimensionless quantities. From a theoretical point of view, moreover, any theory may be cast into a form in which no dimensional quantities ever appear either in the equations themselves or in their solutions (such as vacuum expectation values of scalar fields). So the

issue of whether dimensional parameters vary in time need never arise. Moreover, it should be clear from the text that my objections apply to all dimensional constants, not just c.

Referee 2: When one says that the speed of light is color dependent (as in the case for deformed dispersion relations), or that the speed of light and gravity vary with respect to each other (as for some bimetric theories of gravity), one makes dimensionless statements.

Response: Although all fundamental constants are dimensionless, the converse is not true. For example, the ratio of the Earth's radius and the Sun's radius is an accident of nature with no fundamental significance. Moreover, not all dimensionless quantities are unit independent. For example $\delta c/c$ is zero in Planck and Stoney units but non-zero in Schrodinger units.

Referee 2: Also Lorentz invariance has an operational sense and some aspects of the constancy of c are directly related to it.

Response: Let us suppose that we have a generally covariant and locally Lorentz invariant theory of gravity with scalar fields, and that time varying α is implemented by a time-dependent scalar field solution. This background will not exhibit global Lorentz invariance, but this is no different than a time-dependent Friedman-Robertson-Walker cosmology which is not Lorentz-invariant either. Alternatively, we might imagine a phase transition from one Lorentz-invariant vacuum to another in which the dimensionless constants, such as α, change abruptly. Whatever the symmetries, they will be the same whether we use varying c units or some other units. Moreover, none of this conflicts with Einstein's general covariance, contrary to certain claims in the literature and in the media.

Referee 2: The last phrase of the paper is wrong (the same argument could be applied to variations in e or entropy, after all).

Response: The last sentence could indeed be applied to any other conversion factor but is nevertheless correct.

Davies: Where we differ substantially from Duff, and where it seems clear he is wrong, is in his claim that theories in which dimensional constants vary with time "is operationally meaningless." Such theories have existed in the literature, and specific observational tests been suggested and carried out, at least since Dirac's theory of varying G.

Response: I agree that Davies et al are the latest in a long line of authors making such claims, but Dirac was not one of them. In his seminal paper [15] he says: "The fundamental constants of physics, such as c the velocity of light, h the Planck constant, e the charge and m_e the mass of the electron, and so on, provide for us a set of absolute units for measurement of distance, time, mass, etc. There are, however, more of these constants than are necessary for this purpose, with the result that certain dimensionless numbers can be constructed from them." The phrase "more of these constants than are necessary" is crucial. Those who insist on counting the dimensional constants in a theory as well as the dimensionless ones will always have more unknowns than equations. This redundancy is nothing but the freedom to change units without changing the physics. In Einstein-Maxwell-Dirac theory, for example, one could imagine units in which (at least) five dimensional

constants, are changing in time: G, e, m_e, c, $\hbar$..., but only two dimensionless combinations are necessary: $\mu_e{}^2 = Gm_e{}^2/\hbar c$ and $\alpha = e^2/\hbar c$.

Dirac then notes that the *dimensionless* ratio of electromagnetic and gravitational forces e^2/Gm_e^2 is roughly the same order of magnitude as the *dimensionless* ratio of the present age of the universe t and the atomic unit of time e^2/m_ec^3. He makes it clear that equating these two numbers leads to a time-varying $G \sim t^{-1}$ only in these "atomic units". To be explicit, let us define Dirac [15] length, mass, time and charge by

$$L_D{}^2 = e^4/m_e{}^2c^4 = L_S^2\alpha/\mu_e{}^2$$
$$M_D^2 = m_e{}^2 = M_S^2\mu_e{}^2/\alpha$$
$$T_D{}^2 = e^4/m_e{}^2c^6 = T_S{}^2\alpha/\mu_e{}^2$$
$$Q_D{}^2 = e^2 = Q_S^2 \tag{12}$$

Note that these units are independent of G and $\hbar$. They are obtained from Stoney units by the replacement $G \to G\alpha/\mu_e{}^2$. In Dirac units

$$c = e = m_e = 1 \qquad \hbar = 1/\alpha \qquad G = \mu_e{}^2/\alpha \qquad M^2 = \mu^2/\mu_e{}^2 \tag{13}$$

Once again, the entropy is the same as given by (1). So there is no such thing as a varying G *theory*, only varying G *units*. This is familiar from string theory [16] where the string tension T is related to G via dilaton and moduli fields which may possibly vary in space and time. In Einstein units, G is fixed while T may vary, whereas in string units T is fixed while G may vary.

Davies: Some theories of fundamental physics, e.g. the Hoyle-Narlikar theory of gravitation, were explicitly designed to incorporate an additional gauge freedom (in that case, conformal invariance) to enable one to transform at will between different systems of units, without changing the physics, whilst including cosmological time variations of constants.

Response: The freedom to choose MKS units, say, over CGS units requires no symmetry of the fundamental theory but is merely one of human convention. The same is true of choosing changing c units over changing e units.

Davies: Several varying speed of light and varying electric charge theories have been published, and explicit observational predictions made. See, for example, J. Magueijo, Phys. Rev. D. 62, 103521 (2000), and "Is it e or is it c? Experimental tests of varying alpha" by J. Magueijo, J.D. Barrow and H.B. Sandvik, Phys. Rev. D, in the press (available online at arXiv: astro-ph/0202374v1, 20 February 2002).

Response: This seems a curious choice of authors to back up Davies' argument. In his paper with Albrecht [17], Magueijo says " Our conclusion that physical experiments are only sensitive to dimensionless combinations of dimensional constants is hardly a new one. This idea has been often stressed by Dicke (eg. [19]), and we believe this is not controversial." Majueijo, Barrow and Sandvik [18] say "Undoubtedly, in the sense of [2], one has to make an operationally 'meaningless' choice of which dimensional constant is to become a dynamical variable." These papers thus fall into a category whose authors are well aware that there is no experimental way of distinguishing varying e from varying c, but nevertheless choose

to label genuinely physically inequivalent theories by the names "varying c" and "varying e" merely because they find one set of units more convenient than the other. I might criticize these papers for being confusing[6], but not wrong. They provide cold comfort for Davies et al who claim that varying c and varying e are experimentally distinguishable.

Davies: The speed of light is more than an electrodynamic parameter: it describes the causal structure of spacetime, and as such is relevant to all of physics (for example, the weak and strong interactions), not just electrodynamics.

Response: What is relevant for the strong, weak and electromagnetic interactions is the special theory of relativity, i.e invariance under the Poincare group of spacetime transformations. The mathematics of the Poincare group ($x'^{\mu} = \Lambda^{\mu}{}_{\nu}x^{\nu} + a^{\mu}$) can get along just fine without c.

Davies: A variation of c cannot be mimicked in all such respects by a change in e. More obviously, one can imagine measuring the speed of light in the laboratory tomorrow and obtaining a different value from today. That is clearly operationally meaningful.

Response: This common fallacy can be eliminated by thinking carefully about how one would attempt to measure c in a world in which dimensionless constants such as α and μ are changing in time. First take a ruler with notches one Planck length apart and a clock with ticks one Planck time apart. Next measure the speed of light in vacuum[7] by counting how many notches light travels in between ticks. You will find the answer $c = 1$. You may repeat the experiment till the cows come home and you will always find $c = 1$! Repeat the experiment using Stoney length and Stoney time, and again you will find $c = 1$. But if the notches on your ruler are one Schrodinger length apart and the ticks on your clock one Schrodinger time apart, you will find $c = 1/\alpha$ and c will now have the same time dependence as $1/\alpha$. Once again we see that the time dependence of c is entirely unit-dependent. Similar remarks apply to the measurement of any other dimensional quantity.

For the sake of completeness, let us also define Bohr length, mass, time and charge, which have an obvious atomic definition as the Bohr radius etc:

$$L_B{}^2 = \hbar^4/m_e{}^2 e^4 = L_\psi^2 \alpha/\mu_e^2$$

$$M_B^2 = m_e{}^2 = M_\psi^2 \mu_e^2/\alpha$$

$$T_B{}^2 = \hbar^6/m_e{}^2 e^8 = T_\psi{}^2 \alpha/\mu_e^2$$

$$Q_B{}^2 = e^2 = Q_\psi^2 \tag{14}$$

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[6]While recognizing that time variation of dimensional quantities lacks operational definition, Carlip and Vaidya [20] nevertheless try to salvage from subjectivity the notion of changing e, for example, by saying "suitable variation of all dimensionless parameters that depend on e". But this is equally subjective: nature provides us with dimensionless parameters and humans decide where to put the c's etc. For example, which of the 19 parameters of the Standard Model depend on c? It is entirely up to you!

[7]If the experiment is performed in a medium, or a time-dependent gravitational field, one would have to factor out the effects of the refractive index, or $\sqrt{g_{xx}/g_{tt}}$. After all, light slows down when passing through a piece of glass, but no-one is suggesting that this produces an increase in α.

Note that these units are independent of G and c. They are obtained from Schrodinger units by the replacement $G \to G\alpha/\mu_e{}^2$. In Bohr units

$$\hbar = e = m_e = 1 \qquad c = 1/\alpha \qquad G = \mu_e{}^2/\alpha \qquad M^2 = \mu^2/\mu_e{}^2 \qquad (15)$$

So a thought experimental definition of Schrodinger $L_\psi{}^2$ is the Bohr $L_B{}^2$ scaled down by Dirac's large number (the ratio of electromagnetic to gravitational forces e^2/Gm_e^2) with similar definitions for M_ψ^{-2} and T_ψ^2.

Measuring the speed of light with a ruler whose notches are one Bohr length apart and a clock whose ticks are one Bohr time apart will again result in $c = 1/\alpha$. As discussed in [17], Bohr units are used when measuring c using an atomic clock, which is most sensitive to a variation of α. A pendulum clock, on the other hand, is more sensitive to the variation of μ_i. So when you think you are measuring a dimensional quantity, you are really measuring dimensionless ones.

Davies: So this is an issue of semantics and mathematical elegance, not science.

Response: The failure to tell the difference between changing units and changing physics is more than just semantics. It brings to mind the old lady who, when asked by the TV interviewer whether she believed in global warming, responded: "If you ask me, it's all this changing from Fahrenheit to Centigrade that's causing it".

B   Response to remarks on the present paper

B.1   Remarks by Magueijo

In the abstract of a recent review article [21], Magueijo writes: "We start by discussing the physical meaning of a varying c, dispelling the myth that the constancy of c is a matter of logical consistency". The following statements appear in the text.

Magueijo: In discussing the physical meaning of a varying speed of light, I'm afraid that Eddington's religious fervor is still with us [22, 23]. "To vary the speed of light is self-contradictory" has now been transmuted into "asking whether c has varied over cosmic history is like asking whether the number of liters to the gallon has varied" [23]. The implication is that the constancy of the speed of light is a logical necessity, a definition that could not have been otherwise. This has to be naive. For centuries the constancy of the speed of light played no role in physics, and presumably physics did not start being logically consistent in 1905. Furthermore, the postulate of the constancy of c in special relativity was prompted by experiments (including those leading to Maxwell's theory) rather than issues of consistency. History alone suggests that the constancy (or otherwise) of the speed of light has to be more than a self-evident necessity.

Response: In fact my remark implies no such logical necessity. It merely means that the variation or not of dimensional numbers like c (as opposed to dimensionless numbers like the fine-structure constant) is a matter human convention, just as the variation or not in the number of liters to a gallon is a matter of human convention. In neither case is it something to be determined by experiment but rather by one's choice of units. So there is no such thing as a varying c 'theory' only varying c

'units'. For example, in units where time is measured in years and distance in light-years, $c = 1$ for ever and ever, whatever your theory!

As a matter of fact, the number of liters per gallon varies as one crosses the Atlantic. Similarly, as discussed in [2], in 1983 the Conference Generale des Poids et Mesures changed the number of meters per second, i.e the value of c. Relativity survived intact!

B.2   Remarks by T. Davis

In a recent paper [24], one of the authors of the black hole thermodynamics paper, T. Davis, responds to our criticisms. With reference to our equation (2), the following appears in the text:

T. Davis: Arguments from quantum theory suggest that it is more natural to expect that μ would remain constant than M. Under this assumption Eq. (2) suggests that any increase in α would violate the second law of thermodynamics, independent of which of e, c or $\hbar$ varies. This seems to be in contradiction to our previous result in which an increase in e decreased s but a decrease in c or $\hbar$ increased s. However, in our initial formulation we had assumed that M remained constant whereas here we are assuming μ remains constant. If we allow μ to vary such that M remains constant the result for black hole entropy is unchanged from the previous version.

Response: The variation or not of the dimensionless quantity μ is operationally meaningful, but once again, the question of whether or not the dimensional quantity M varies is a matter of human convention, not quantum theory. In Planck units (3), for example, variation of μ implies variation of M while in Stoney units (4) or Schrodinger units (5), it does not. The black hole entropy (2) and the second law of thermodynamics do not give a fig which units are chosen.

C   Comments on claims to have placed constraints on $\Delta G/G$

A recent paper by Copi, A. Davis and Krauss [25] claims to use astrophysical data to place constraints on the time variation of Newton's constant, $\Delta G/G$. Here we reiterate the point made in Appendix A that dimensionless ratios such as $\Delta G/G$, $\Delta e/e$ and $\Delta c/c$ are every bit as unit-dependent as their dimensional counterparts ΔG, Δe and Δc. An obvious example is again provided by units in which time is measured in years and distance in light-years. Here $c = 1$ and $\Delta c/c=0$, whatever your theory. Similar remarks apply to $\Delta G/G$. As discussed in Appendix A, it is guaranteed to vanish in Planck units (3), for example, but might vary in Dirac units (13). By contrast, $\Delta \alpha/\alpha$ is unit-independent.

The idea of varying G is frequently attributed to the papers of Dirac [15] and [19] Dicke, but as discussed in Appendix A, a reading of these papers reveals that both authors were aware that it is only dimensionless numbers such as μ_e and α that are operationally meaningful. The Standard Model coupled to gravity with a

cosmological constant has 20 such parameters. It is the variation of these quantities that may be contrained by the astrophysical data presented in [25], not ΔG nor even $\Delta G/G$.

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