Testing the equivalence principle

the link between constants, gravitation and cosmology

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Outline

- Some words on fundamental constants
- Links between constants and gravity [Equivalence principle]
- Links with cosmology

All technical details:
JPU, Liv. Rev. Relat. 4 (2011) 2; [arXiv/1009.5514]
Physical theories involve constants

These parameters cannot be determined by the theory that introduces them.

These arbitrary parameters have to be assumed constant:
- *experimental validation*
- *no evolution equation*

By testing their constancy, we thus test the laws of physics in which they appear.

A physical measurement is always a comparison of two quantities, one can be thought as a unit
- *it only gives access to dimensionless numbers*
- *we consider variation of dimensionless combinations of constants*
Reference theoretical framework

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [General Relativity + SU(3)xSU(2)xU(1)]:

- G : Newton constant (1)
- 6 Yukawa coupling for quarks
- 3 Yukawa coupling for leptons
- mass and VEV of the Higgs boson: 2
- CKM matrix: 4 parameters
- Non-gravitational coupling constants: 3
- $\Lambda_{uv}$: 1
- c, h : 2
- cosmological constant

22 constants
19 parameters

Probably more on this in the talk of T. Mannel
Number of constants may change

This number is « time-dependent ».

**Neutrino masses**

Add 3 Yukawa couplings + 4 MNS parameters = 7 more

**Unification**

\[
\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{E}
\]

SM: \[b_i = (\frac{41}{10}, -\frac{19}{6}, -7)\]

MSSM: \[b_i = (\frac{33}{5}, 1, -3)\]

\[
\alpha^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1}
\]
Importance of unification

Unification

\[ \alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{E} \]

Variation of \( \alpha \) is accompanied by variation of other coupling constants

QCD scale

\[ \Lambda_{QCD} = E \left( \frac{m_c m_b m_t}{E^3} \right)^{2/27} \exp \left[ -\frac{2\pi}{9\alpha_s(E)} \right] \]

Variation of \( \Lambda_{QCD} \) from \( \alpha_s \) and from Yukawa coupling and Higgs VEV

Theories in which EW scale is derived by dimensional transmutation

\[ \nu \sim \exp \left[ -\frac{8\pi^2}{h_t^2} \right] \]

Variation of Yukawa and Higgs VEV are coupled

String theory

All dimensionless constants are dynamical – their variations are all correlated.

These effects cannot be ignored in realistic models.
Part I: constants and gravity

- Link with the equivalence principle
- Examples of theories with varying constants
On the basis of general relativity

The equivalence principle takes much more importance in general relativity

It is based on **Einstein equivalence principle**
- universality of free fall
- local Lorentz invariance
- local position invariance

The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

If this principle holds then gravity is a consequence of the geometry of spacetime

This principle has been a driving idea in theories of gravity from Galileo to Einstein
Underlying hypothesis

Equivalence principle
- Universality of free fall
- Local lorentz invariance
- Local position invariance

Dynamics

\[ S_{\text{matter}}(\psi, g_{\mu\nu}) \]

\[ S_{\text{grav}} = \frac{c^3}{16\pi G} \int \sqrt{-g} \, R \, d^4x \]

Relativity

\[ g_{\mu\nu} = \tilde{g}^*_{\mu\nu} \]
Equivalence principle and constants

Action of a test mass:

\[ S = - \int mc \sqrt{-g_{\mu \nu} v^\mu v^\nu} \, dt \]

with

\[ v^\mu = \frac{dx^\mu}{dt} \]
\[ u^\mu = \frac{dx^\mu}{d\tau} \]

\[ \delta S = 0 \]

\[ a^\mu \equiv u^\nu \nabla_\nu u^\mu = 0 \]

(geodesic)

\[ g_{00} = -1 + 2\Phi_N/c^2 \]

(Newtonian limit)

\[ \dot{v} = a = -\nabla \Phi_N = g_N \]
The equivalence principle in Newtonian physics

The deviation from the universality of free fall is characterized by

\[ \eta \equiv 2 \frac{|a_1 - a_2|}{|a_1 + a_2|} \]

Second law: \( F = m_1 a \)

Definition of weight \( F = m_G g \)

So that

\[ \eta = 2 \frac{|m_1^1/m_1^1 - m_2^2/m_2^2|}{m_1^1/m_1^1 + m_2^2/m_2^2} \]

Consider a pendulum of length \( L \) in a gravitational field \( g \),

\[ \ddot{\theta} + \omega^2 \theta = 0 \quad \text{où} \quad \omega \equiv \omega_0 \sqrt{\frac{m_G}{m_1}} \quad \text{et} \quad \omega_0 \equiv \sqrt{\frac{g}{L}}. \]

Then

\[ \eta \approx 2 \frac{|\omega_B - \omega_A|}{\omega_0} \]
Tests on the universality of free fall
Testing general relativity on astrophysical scales

There is a growing need to test general relativity on astrophysical scales

- dynamics of galaxies
  - and dark matter
- acceleration of the universe
  - and dark energy
  - but also theoretical motivations...

Can we extend the test of the equivalence principle on astrophysical scales?

See talk by J. Weller
In general relativity, any test particle follow a geodesic, which does not depend on the mass or on the chemical composition.

**Imagine some constants are space-time dependent**

1. Local position invariance is violated.

2. Universality of free fall has also to be violated

Mass of test body = mass of its constituents + binding energy

In Newtonian terms, a free motion implies \( \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = 0 \)

But, now

\[
\frac{d\vec{p}}{dt} = 0 = m\ddot{\vec{a}} + \frac{dm}{d\alpha} \dot{\alpha} \dot{\vec{v}} \]

\[ m\ddot{\vec{a}}_{\text{anomalous}} \]
Equivalence principle and constants

Action of a test mass:

\[ S = - \int m_A [\alpha_i] c \sqrt{-g_{\mu\nu} v^\mu v^\nu} dt \]

with

\[ v^\mu = \frac{dx^\mu}{dt} \]
\[ u^\mu = \frac{dx^\mu}{d\tau} \]

\[ \delta S = 0 \]

\[ a_A^\mu = - \sum_i \left( \frac{\partial \ln m_A}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial x^\beta} \right) (g^{\beta\mu} + u^\beta u^\mu) \]

(NOT a geodesic)

\[ g_{00} = -1 + 2 \Phi_N / c^2 \]

(Newtonian limit)

\[ a = g_N + \delta a_A \]

Anomalous force

Composition dependent

\[ \delta a_A = -c^2 \sum_i f_{A,i} \left( \nabla \alpha_i + \dot{\alpha}_i \frac{v}{c^2} \right) \]
If a constant is varying, this implies that it has to be replaced by a dynamical field.

This has 2 consequences:

1- the equations derived with this parameter constant will be modified
   
   *one cannot just make it vary in the equations*

2- the theory will provide an equation of evolution for this new parameter

The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction, i.e. at the origin of the deviation from General Relativity.
Example of varying fine structure constant

It is a priori « easy » to design a theory with varying fundamental constants

Consider

\[ S = \int \left\{ \frac{1}{16\pi G} R - 2(\partial \mu \phi)^2 - V(\phi) - \frac{1}{4} B(\phi) F_{\mu \nu}^2 \right\} \sqrt{-g} \, d^4 x \]

But that may have dramatic implications.

\[ m_A(\phi) \supset 98.25 \alpha \frac{Z(Z-1)}{A^{1/3}} \text{MeV} \quad \rightarrow \quad f_i = \partial_\phi \ln m_i \sim 10^{-2} \frac{Z(Z-1)}{A^{4/3}} \alpha'(\phi) \]

Violation of UFF is quantified by

\[ \eta_{12} = 2 \frac{|\bar{a}_1 - \bar{a}_2|}{|\bar{a}_1 + \bar{a}_2|} = \frac{f_{\text{ext}} |f_1 - f_2|}{1 + f_{\text{ext}} (f_1 + f_2)/2} \]

It is of the order of

\[ \eta_{12} \sim 10^{-9} X_{1,2,\text{ext}}(A, Z) \times (\partial_\phi \ln B)^2_0 \]

\[ \mathcal{O}(0.1 - 10) \]

Requires to be close to the minimum
**Extra-dimensions**

Such terms arise when compatifying a higher-dimensional theories

**Example:**

\[ S = \frac{1}{12\pi^2 G_5} \int \tilde{R} \sqrt{|\tilde{g}|} \, d^5 x \]

\[ \tilde{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{M^2} \phi^2 A_\mu A_\nu & \frac{1}{M} \phi^2 A_\mu \\ \frac{1}{M} \phi^2 A_\nu & \phi^2 \end{pmatrix} \]

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g_\phi} \left( R - \frac{\phi^2}{4M^2} F_{\alpha\beta} F^{\alpha\beta} \right) \]

\[ G = \frac{3\pi G_5}{4V(5)} \quad V(5) = \int dy \]

- **5D theory**
- **Compactification**
- **4D effective theory**
- Varying fine structure constant
- Varying G
In the framework of string theory, all dimensionless constants are expected to be dynamical.

From a phenomenological point of view

\[ S = \int d^4x \sqrt{-g} \left( B g R - B_\phi (\partial \phi)^2 - \frac{1}{4} B F_i F_i^2 - B_\psi \psi D \bar{\psi} - V \right) \]

Little is known about these functions, the computation of which requires to go beyond tree-level.

\[ B_i = e^{-\phi} + c_0^{(i)} + c_1^{(1)} e^\phi + \ldots \]

Damour, Polyakov (1994)

For the attraction mechanism toward GR to exist, they must have a minimum at a common value.

\[ B \sim -\frac{1}{2} \kappa (\phi - \phi_m)^2 \] all deviations are proportional to \( (\phi_0 - \phi_m)^2 \)
Dirac (1937)
Numerological argument
\( G \sim 1/t \)

Kaluza (1919) – Klein (1926)
multi-dimensional theories

Teller (1948)–Gamow (1948)
Constraints on Dirac hypothesis
New formulation

Jordan (1949)
Variable constant = new dynamical field.

Lee-Yang (1955)
Dicke (1957)
Implication on the universality of free fall

Fierz (1956)
Effects on atomic spectra
Scalar-tensor theories

Scherk-Schwarz (1974)
Witten (1987)
String theory: all dimensionless constants are dynamical

Savedoff (1956)
Tests on astrophys. spectra

Oklo (1972), quasars...
Experimental constraints

Implication on the universality of free fall
Part II: Testing for constancy

JPU, Rev. Mod. Phys. 75, 403 (2003)
JPU, [astro-ph/0409424]
JPU, Liv. Rev. Relat. (in press) [arXiv/1009.5514]
JPU, B. Leclercq, De l’importance d’être une constante (Dunod, 2005)
Physical systems

- Atomic clocks
- Oklo phenomenon
- Meteorite dating
- Quasar absorption spectra

$z = 0$

$z \sim 0.2$

$z \sim 4$

$z \sim 10^3$

$z \sim 10^8$

$z = 0.14$

$z = 0.43$

Local obs

QSO obs

CMB obs

CMB

BBN
Observables and primary constraints

A given physical system gives us an observable quantity

\[ O(G_k, X) \]

External parameters: temperature, ...

Primary physical parameters

**Step 1:**

From a physical model of our system we can deduce the sensitivities to the primary physical parameters

\[ \kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k} \]

**Step 2:**

The primary physical parameters are usually not fundamental constants.

\[ \Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i \]
<table>
<thead>
<tr>
<th>System</th>
<th>Observable</th>
<th>Primary constraint</th>
<th>Other hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic clocks</td>
<td>Clock rates</td>
<td>$\alpha, \mu, g_i$</td>
<td>-</td>
</tr>
<tr>
<td>Quasar spectra</td>
<td>Atomic spectra</td>
<td>$\alpha, \mu, g_p$</td>
<td>Cloud physical properties</td>
</tr>
<tr>
<td>Oklo</td>
<td>Isotopic ratio</td>
<td>$E_r$</td>
<td>Geophysical model</td>
</tr>
<tr>
<td>Meteorite dating</td>
<td>Isotopic ratio</td>
<td>$\lambda$</td>
<td>Solar system formation</td>
</tr>
<tr>
<td>CMB</td>
<td>Temperature</td>
<td>$\alpha, \mu$</td>
<td>Cosmological model</td>
</tr>
<tr>
<td></td>
<td>anisotropies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBN</td>
<td>Light element</td>
<td>$Q, \tau_n, m_e, m_N, \alpha, B_d$</td>
<td>Cosmological model</td>
</tr>
<tr>
<td></td>
<td>abundances</td>
<td></td>
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</tr>
</tbody>
</table>
Physical systems: new and future

- Atomic clocks
- Oklo phenomenon
- Meteorite dating
- Quasar absorption spectra
- Pop III stars

[Coc, Nunes, Olive, JPU, Vangioni, 2009]

[Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni, 2009]

BBN

[Coc, Nunes, Olive, JPU, Vangioni]

Comparison: need for cosmology

**Cosmological model**: idealized description of the geometry of the universe of large scales.

**Limitations:**
- 1 universe
- Only a small part of the universe is observable (see A. Jaffe)
- Only a slice of spacetime is observable.

Construction requires extra assumptions:

**Copernican Principle**

*how representative is our (local) observable universe from the « whole » universe*
Comparison: need for cosmology

$\eta=10^{-12}$ (blue) & $\eta=10^{-13}$ (red)

Light field not responsible for late time acceleration (dotted)

Compatible meteorite/Oklo/atomic clocks

$\eta=10^{-12}$

Light field not responsible for late time acceleration (dotted)

Different unification hypothesis

Huge sensitivity to the hypothesis of the models (coupling, unification,...)
Part III: On the value of the fundamental constants
Fine tuning

Many physical effects are sensitive to the value of the fundamental constants.

Today $\Omega_{\text{mat}} \sim 0.3$

**If $\Omega_{\text{mat}} > 10$**: Universe recollapses in a time smaller than the stellar formation time scale
*Cannot synthesize C-12, O-16 etc...*

**If $\Omega_{\text{mat}} < 0.1$**: Density inhomogeneities stop growing much before galaxies form
*No galaxies - Cannot synthesize C-12, O-16 etc...*

The worst is the tuning of the cosmological constants

$$t_0 \sim t_\Lambda \sim t_{\text{gal}}$$

Coincidence?
Many physical effects are sensitive to the value of the fundamental constants.

**Different classes of explanation:**

- *Design or consistency hypothesis*:
  includes the possibility that all parameters in the «final» physical theory are fixed by a condition of consistency or an external cause
  (e.g. necessity of mathematical consistency, structure of a meta-theory in which the low energy effective parameters are determined by a unique higher energy theory)

- *Ensemble hypothesis*:
  The universe we observe is assumed to be only a small part of the totality of physical existence.
  This structure needs not be fine-tuned and shall be sufficiently large and variegated so that it can contain as a proper part a universe like the one we observe the fine-tuning of which is then explained by an *observation selection effect* (e.g. anthropic principle).

It involves a new theoretical layers allowing for the parameters of our physical theories and cosmological model to take different values (which are all realized, hence the notion of Multiverse).
**Multiverse hypothesis**

**Low energy theory** has a set $X_a(i)$ fundamental parameters.

Each $X_a$ is mapped to a low-energy theory.

**Low energy theory** has a set $\lambda$ of fundamental parameters and $\omega$ of cosmological parameters.

$(\lambda, \omega)$ has to lie on the anthropic domain $A_A$ that can be determined from the low energy theory.

**Example:** string landscape + eternal inflation

**Multiverse hypothesis:** All $X_a$ are realized (in practice even many times)
Selection effect

We should not forget the fact that we are here (and now) to observe the universe

\[ P(\lambda = \lambda_0 | \text{obs}) \propto P_{\text{prior}}(\lambda = \lambda_0) \times P(\text{obs} | \lambda = \lambda_0) \]

Most probable value that can be observed: problem of typicality.
Proof or simple consistency check?
So what

Very speculative approach but the only existing to address the question of the value of the fundamental constants

*Many technical problems:*

- how to deal with probabilities in infinite spaces,
- notion of observers is central for the selection effect. How do we define observers?
- conclusion depends on the prior distribution. How is the multiverse populated?
- «local» variations only.

*Philosophical issue:* does not seem to be falsifiable/testable.

When one compares theories $H_1$ and $H_2$ with a set of data $d$

$$\frac{P(H_1|d)}{P(H_2|d)} = \frac{p_1}{p_2} B_{12} \quad \text{with} \quad B_{12} = \frac{P(d|H_1)}{P(d|H_2)}$$

Posterior odds can always be of order unity if $\frac{p_1}{p_2} \sim B_{12}^{-1}$
The constancy of fundamental constants is a **test of the equivalence principle**. *The variation of the constants, violation of the universality of free fall and other deviations from GR are of the same order.*

« Dynamical constants » are **generic** in most extensions of GR (extra-dimensions, string inspired model).

*Need for a stabilisation mechanism (least coupling principle/chameleon)*  
*Why are the constants so constant?*  
*Variations are expected to be larger in the past (cosmology)*  
*All constants are expected to vary (unification)*

<table>
<thead>
<tr>
<th>Newton</th>
<th>Einstein</th>
<th>String theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed spacetime</td>
<td><strong>Dynamical</strong> spacetime</td>
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<td>Fixed constants</td>
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Observational developments allow to set **strong constraints** on their variation

*New systems [Stellar physics] / new observations*

Difficulty to explain the apparent peculiarity of our universe.