Variation of Fundamental Constants

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Dimensionless Constants

Since variation of **dimensional** constants cannot be distinguished from variation of **units**, it only makes sense to consider variation of **dimensionless** constants.

- Fine structure constant \( \alpha = \frac{e^2}{2\varepsilon_0hc} = 1/137.036 \)
- Electron or quark mass/QCD strong interaction scale, \( m_{e,q}/\Lambda_{QCD} \)
  \[ \alpha_{\text{strong}}(r) = \text{const}/\ln(r \Lambda_{QCD}/ch) \]
- Electron-to-proton mass ratio
Evidence for spatial variation of $\alpha$

Quasar spectra

Webb, King, Murphy, Flambaum, Carswell, Bainbridge, 2010

$$\alpha(x) = \alpha(0) + \alpha'(0) x + \ldots$$

$$x = r \cos(\phi), \quad r = ct - \text{distance (t - light travel time, c - speed of light)}$$

Reconciles all measurements of the variation
“Fine tuning” of fundamental constants is needed for life to exist. If fundamental constants would be even slightly different, life could not appear!

Variation of coupling constants in space provide natural explanation of the “fine tuning”: we appeared in area of the Universe where values of fundamental constants are suitable for our existence.

There are theories which suggest variation of the fundamental constants in expanding Universe.
Quasars: physics laboratories in the early universe

Lyman limit  Lyβ  Lyα  SiII  CII  SiIV  SiII  CIV

Lyα_{em}  NV_{em}  SiIV_{em}  CIV_{em}  Lyβ_{em}  Lyα_{forest}
Probing the variability of $\alpha$ with QSO absorption lines

To find dependence of atomic transition frequencies on $\alpha$ we have performed calculations of atomic transition frequencies for different values of $\alpha$.

1. Zero Approximation – Relativistic Hartree-Fock method: energies, wave functions, Green’s functions
2. Many-body perturbation theory to calculate effective Hamiltonian for valence electrons including self-energy operator and screening; perturbation

\[ V = H - H_{HF} \]

\[ \sum(r, r', E) \]

 electrons from core

3. Diagonalization of the effective Hamiltonian

Test: Energy levels in Mg II to 0.2% accuracy
Atomic transition frequencies

Use atomic calculations to find $\omega(\alpha)$.
Units cancel in the ratio of frequencies.
We use atomic units (Rydberg=1/2=const).
Dependence on $\alpha$ appears due to relativistic corrections

For $\alpha$ close to $\alpha_0$  \[ \omega = \omega_0 + q(\alpha^2/\alpha_0^2-1) \]
$q$ is found by varying $\alpha$ in computer codes:
$q = d\omega/dx = [\omega(0.1) - \omega(-0.1)]/0.2, \quad x=\alpha^2/\alpha_0^2-1$

Many-Multiplet Method 
Dzuba,Flambaum,Webb 1998
quasar spectroscopy and atomic clocks
## Results of calculations (in cm\(^{-1}\))

### Anchor lines

<table>
<thead>
<tr>
<th>Atom</th>
<th>(\omega_0)</th>
<th>(q)</th>
</tr>
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<tbody>
<tr>
<td>Mg I</td>
<td>35051.217</td>
<td>86</td>
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<tr>
<td>Mg II</td>
<td>35760.848</td>
<td>211</td>
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<td>Mg II</td>
<td>35669.298</td>
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<tr>
<td>Si II</td>
<td>55309.3365</td>
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<td>Si II</td>
<td>65500.4492</td>
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<td>Al II</td>
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<td>Al III</td>
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<td>Al III</td>
<td>53682.880</td>
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<td>Ni II</td>
<td>58493.071</td>
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</table>

### Negative shifters

<table>
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<tr>
<th>Atom</th>
<th>(\omega_0)</th>
<th>(q)</th>
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<tr>
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<td>Ni II</td>
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<td>Cr II</td>
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<td>Cr II</td>
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<tr>
<td>Fe II</td>
<td>62171.625</td>
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### Positive shifters

<table>
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<th>Atom</th>
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<th>(q)</th>
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<td>62065.528</td>
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<td>Fe II</td>
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<td>Fe II</td>
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<td>Fe II</td>
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<tr>
<td>Fe II</td>
<td>38660.0494</td>
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<td>Fe II</td>
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<td>Zn II</td>
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<td>2490</td>
</tr>
<tr>
<td>Zn II</td>
<td>48841.077</td>
<td>1584</td>
</tr>
</tbody>
</table>

Also, many transitions in Mn II, Ti II, Si IV, C II, C IV, N V, O I, Ca I, Ca II, Ge II, O II, Pb II, Co II,…

Different signs and magnitudes of \(q\) provides opportunity to study systematic errors!
New interpretation: Spatial variation

Northern+(new)Southern hemisphere data: Linear variation with distance along some direction, $\alpha(x)=\alpha(0)+kx$, $x=r \cos(\phi)$, $r=ct$ (Gly),

$\Delta \alpha/\alpha = 1.10(0.25) \times 10^{-6} \ r \ cos(\phi)$

gradient direction $17.6(0.6)\ h, -58(6)\ o$

$4.2 \ \sigma$ deviation from zero. Data from two largest telescopes, Keck and VLT, give consistent results. 300 systems.
4.1σ evidence for a $\Delta \alpha/\alpha$ dipole from VLT + Keck

$\Delta \alpha/\alpha = c + A \cos(\theta)$

Julian King, UNSW
Keck & VLT dipoles independently agree, p=4%
Low and high redshift cuts are consistent in direction. Effect is larger at high redshift.
Distance dependence

$\Delta \alpha/\alpha$ vs B$\cos\Theta$ for the model $\Delta \alpha/\alpha = B \cos\Theta + m$ showing the gradient in $\alpha$ along the best-fit dipole. The best-fit direction is at right ascension $17.4 \pm 0.6$ hours, declination $-62 \pm 6$ degrees, for which $B = (1.1 \pm 0.2) \times 10^{-6}$ Glyr$^{-1}$ and $m = (-1.9 \pm 0.8) \times 10^{-6}$. This dipole+monopole model is statistically preferred over a monopole-only model also at the 4.1$\sigma$ level. A cosmology with parameters $(H_0, \Omega_M, \Omega_\Lambda) = (70.5, 0.2736, 0.726)$. 

\approx 25$ absorbers per bin
Are a few high S/N outliers responsible for the signal, by chance?

- Alternative to growing error bars
- Robustness check – iterative trimming
- Adopt statistical-only errors and iteratively clip most deviant point
- How much data do we need to discard to remove the dipole?

\[ \chi^2_{\nu} = 1 \] reached when \( \sim 10\% \) clipped
- Dipole significance \( \sim 5.5\sigma \) at \( \chi^2_{\nu} = 1 \)
- Dipole significance stays above \( 3\sigma \) until \( \sim 60\% \) of data discarded
Hints that this result might be real

Three internal consistencies:

1 Keck and VLT dipoles agree. Independent samples, different data reduction procedures, different instruments and telescopes.

2 High and low redshift dipoles also agree - different species used at low and high redshift – and different transitions respond differently to the same change in $\alpha$.

3 Trimming increases significance and shows signal is present in the majority or all of the data.
Evidence for spatial variation of $\alpha$

- Webb, King, Murphy, Flambaum, Carswell, Bainbridge, arxiv:1008.3907, PRL, MNRAS
  \[ \alpha(x) = \alpha(0) + \alpha'(0) x + \ldots \]
  \[ x = r \cos(\phi), \quad r = ct - \text{distance (instead of time)} \]
  Reconciles all measurements of the variation
- Berengut, Flambaum, arxiv:1008.3957, PRL
  Manifestations on atomic clocks, Oklo, meteorites and cosmological phenomena
- Berengut, Flambaum, King, Curran, Webb, Further astronomical evidence, 1009.0591, PRD
Variation of strong interaction

Grand unification

\[
\frac{\Delta \left( \frac{m}{\Lambda_{QCD}} \right)}{m / \Lambda_{QCD}} = R \frac{\Delta \alpha}{\alpha}
\]

1. Proton mass \( M_p = 3\Lambda_{QCD} \), measure \( m_e / M_p \)
2. Nuclear magnetic moments

\[
\mu = g e \hbar / 4M_p c, \quad g = g \left( \frac{m_q}{\Lambda_{QCD}} \right)
\]
3. Nuclear energy levels and resonances
Dependence on quark mass

- Dimensionless parameter is $m_q/\Lambda_{QCD}$. It is convenient to assume $\Lambda_{QCD} = \text{const}$, i.e. measure $m_q$ in units of $\Lambda_{QCD}$.

- $m_\pi$ is proportional to $(m_q\Lambda_{QCD})^{1/2}$
  \[ \Delta m_\pi/m_\pi = 0.5\Delta m_q/m_q \]

- Other meson and nucleon masses remains finite for $m_q = 0$. \[ \Delta m/m = K\Delta m_q/m_q \]

Argonne: $K$ are calculated for $p,n,\rho,\omega,\sigma$.

\[ m_q = \frac{m_u + m_d}{2} \approx 4\text{MeV}, \quad \Lambda_{QCD} = 220\text{MeV} \rightarrow K = 0.02-0.06 \]

Strange quark mass $m_s = 120\text{MeV}$
Nuclear magnetic moments depends on $\pi$-meson mass $m_\pi$

Spin-spin interaction between valence and core nucleons
Nucleon magnetic moment
\[ \mu = \mu_0 (1 + am_\pi + ...) = \mu_0 (1 + b \sqrt{m_q} + ...) \]

Nucleon and meson masses
\[ M = M_0 + am_q \]

QCD calculations: lattice, chiral perturbation theory, cloudy bag model, Dyson-Schwinger and Faddeev equations, semiempirical.

Nuclear calculations: meson exchange theory of strong interaction. Nucleon mass in kinetic energy \( p^2/2M \)
Hydrogen molecule - 4 systems

\[ \Delta \left( \frac{m_e}{M_p} \right)/ \left( \frac{m_e}{M_p} \right) = 3.3(1.5) \times 10^{-6} r \cos(\phi) \]

gradient direction 16.7(1.5) h, -62(5)°

consistent with \( \alpha \) gradient direction
17.6(0.6) h, -58(6)°

If we assume the same direction
2.6(1.3) \times 10^{-6} r \cos(\phi) 4\% by chance
Big Bang nucleosynthesis:
dependence on quark mass

- Flambaum, Shuryak 2002
- Flambaum, Shuryak 2003
- Dmitriev, Flambaum 2003
- Dmitriev, Flambaum, Webb 2004
- Coc, Nunes, Olive, Uzan, Vangioni 2007
- Dent, Stern, Wetterich 2007
- Flambaum, Wiringa 2007
- Berengut, Dmitriev, Flambaum 2010
- Bedaque, Luu, Platter 2011
Deuteron binding energy is sensitive to the variation of the quark mass

- Shallow level: small variation of the potential leads to large variation of the binding energy.
- Virtual level in (n+p) is even more sensitive, and it influences the deuterium formation rate.
- BBN is exponentially sensitive to the deuteron binding energy $E$, $\exp(-E/T)$
Deuterium abundance – 7 points

Big Bang Nucleosynthesis data give direction of the gradient in the deuterium abundance consistent with the direction of the $\alpha$ gradient. However, the amplitude of the relative spatial variation 0.0045(35) is not statistically significant. This would result in relative variation of $X = m_q / \Lambda_{\text{QCD}}$

$$\Delta X/X = 0.0013(10) \, r \cos(\phi)$$

$$\Delta \alpha/\alpha = 0.003(3) \, r \cos(\phi)$$

Compare with QSO

$$\Delta \alpha/\alpha = 1.10(0.25) \times 10^{-6} \, r \cos(\phi)$$
Gradient $\alpha$ points down
Oklo natural nuclear reactor

$n+^{149}\text{Sm}$ capture cross section is dominated by $E_r = 0.1 \text{ eV}$ resonance. Shlyakhter-limit on $\Delta\alpha/\alpha$ two billion years ago

Our QCD/nuclear calculations

$\Delta E_r = 10 \text{MeV}\Delta X_q/X_q - 1 \text{ MeV} \Delta\alpha/\alpha$

$X_q = m_q/\Lambda_{\text{QCD}}$, enhancement $10 \text{ MeV}/0.1 \text{ eV} = 10^8$

Galaxy moves 552 km/s relative to CMB, $\cos(\phi) = 0.23$

Dipole in space: $\Delta E_r = (10 R - 1) \text{ meV}$

Fujii et al $|\Delta E_r| < 20 \text{ MeV}$

Gould et al, $-12 < \Delta E_r < 26 \text{ meV}$

Petrov et al $-73 < \Delta E_r < 62 \text{ meV}$
Consequences for atomic clocks

- Sun moves 369 km/s relative to CMB
  \[ \cos(\phi) = 0.1 \]
  This gives average laboratory variation
  \[ \frac{\Delta \alpha}{\alpha} = 1.5 \times 10^{-18} \cos(\phi) \text{ per year} \]

- Earth moves 30 km/s relative to Sun-
  \[ 1.6 \times 10^{-20} \cos(\omega t) \text{ annual modulation} \]
Atomic clocks:

Comparing rates of different clocks over long period of time can be used to study time variation of fundamental constants

Optical transitions: $\alpha$

Microwave transitions: $\alpha, (m_e, m_q)/\Lambda_{QCD}$
Calculations to link change of frequency to change of fundamental constants:

Optical transitions: atomic calculations (as for quasar absorption spectra) for many narrow lines in Al II, Ca I, Sr I, Sr II, In II, Ba II, Dy I, Yb I, Yb II, Yb III, Hg I, Hg II, Ti II, Ra II, ThIV

\[ \omega = \omega_0 + q(\alpha^2/\alpha_0^2 - 1) \]

Hg II – negative shifter, Al II – anchor
Yb II - positive and negative shifters
Microwave transitions and absolute frequency measurements

Microwave transitions: hyperfine frequency is sensitive to nuclear magnetic moments (Karshenboim) and nuclear radii

We performed atomic, nuclear and QCD calculations of powers $\kappa, \beta$ for $\text{H, D, Rb, Cd}^+, \text{Cs, Yb}^+, \text{Hg}^+$

$$V = C(\text{Ry})(\frac{m_e}{M_p})\alpha^{2+\kappa} \left(\frac{m_q}{\Lambda_{\text{QCD}}}\right)^\beta, \quad \frac{\Delta \omega}{\omega} = \frac{\Delta V}{V}$$

Primary standard Cs: $\beta=0$, absolute optical frequency measurements (optical/Cs hyperfine) sensitive to $\frac{m_e}{M_p}$! Not to proton magnetic moment.

Variation of $1/\text{Rydberg constant}$ in SI units = Cs hyperfine = $(\frac{m_e}{M_p})\alpha^{2.83}$

Future optical standard: variation of Rydberg constant determined by relativistic corrections in the optical standard frequency

Atomic units: $\text{Ry}=1/2$

Measurements of Ry variation misleading!
## Results for variation of fundamental constants

<table>
<thead>
<tr>
<th>Source</th>
<th>Clock&lt;sub&gt;1&lt;/sub&gt;/Clock&lt;sub&gt;2&lt;/sub&gt;</th>
<th>(d\alpha/dt/\alpha)(10&lt;sup&gt;-16&lt;/sup&gt; yr&lt;sup&gt;-1&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blatt et al, 2007</td>
<td>Sr(opt)/Cs(hfs)</td>
<td>-3.1(3.0)</td>
</tr>
<tr>
<td>Fortier et al 2007</td>
<td>Hg+(opt)/Cs(hfs)</td>
<td>-0.6(0.7)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Rosenband et al 2008</td>
<td>Hg+(opt)/Al+(opt)</td>
<td>-0.16(0.23)</td>
</tr>
<tr>
<td>Peik et al, 2006</td>
<td>Yb+(opt)/Cs(hfs)</td>
<td>4(7)</td>
</tr>
<tr>
<td>Bize et al, 2005</td>
<td>Rb(hfs)/Cs(hfs)</td>
<td>1(10)&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup>assuming \(m_q,e/\Lambda_{QCD} = \text{Const}\)

Combined results:

\[
\begin{align*}
\frac{d}{dt} \ln \alpha &= -1.6(2.3) \times 10^{-17} \text{ yr}^{-1} \\
\frac{d}{dt} \ln \left( \frac{m_q}{\Lambda_{QCD}} \right) &= 3(25) \times 10^{-15} \text{ yr}^{-1} \\
\frac{m_e}{M_p} \text{ or } \frac{m_e}{\Lambda_{QCD}} &= -1.9(4.0) \times 10^{-16} \text{ yr}^{-1}
\end{align*}
\]
Larger $q$ in Yb II

Transition from ground state $f^{14} 6s^2 \, ^2S_{1/2}$ to metastable state $f^{13} 6s^2 \, ^2F_{7/2}$  
$q_1 = -60,000$

For transitions from metastable state $f^{13} 6s^2 \, ^2F_{7/2}$ to higher metastable states $q_2$ are positive and large, up to 85,000.

Difference $q = q_2 - q_1$ may exceed 140,000,
so the sensitivity to alpha variation using comparison of two transitions in Yb II exceeds that in HgII/AlI comparison (measurements at NIST) 2.7 times.

Shift of frequency difference is 2.7 times larger

Dzuba, Flambaum; Porsev, Flambaum, Torgerson
Experiments: PTB, NPL,...
Largest $q$ in multiply charged ions, narrow lines

$q$ increases as $Z^2(Z_i+1)^2$

To keep frequencies in optical range we use configuration crossing as a function of $Z$

Crossing of 5f and 7s

Th IV: $q_1=-75\,300$

Crossing of 4f and 5s

Sm$^{15+}$, Pm$^{14+}$, Nd$^{13+}$

Difference $q=q_2 - q_1$ is 260 000

5 times larger than in Hg II/Al II

Relative sensitivity enhancement up to 500

Berengut, Dzuba, Flambaum, Porsev
Largest q in multiply charged ions, holes in filled shells

q increases as $Z^2 \ |I|^{1.5}$

Ionization potential $I$ is largest for filled shells.
To keep frequencies in optical range we use configuration crossing as a function of Z

Crossing of 4f and 5s in filled shells in Ir 17+
Difference $q=q_2 - q_1$ is 730 000
13 times larger than in Hg II/Al II

Transition frequency 35 000 with laser range, narrow line.
There is strong E1 transition 45 000 for trapping and cooling.

Crossing of 4s and 5p in W 7+

Berengut, Dzuba, Flambaum, Ong
Nuclear clocks

Peik, Tamm 2003: UV transition between first excited and ground state in $^{229}$Th nucleus. Energy 7.6(5) eV, width $10^{-3}$ Hz. Perfect clock!

Flambaum 2006: Nuclear/QCD estimate- Enhancement $10^5$

He, Re; Flambaum, Wiringa; Flambaum, Auerbach, Dmitriev; Hayes, Friar, Moller; Litvinova, Felmeier, Dobaczewski, Flambaum;

$\Delta \omega = 10^{19}$ Hz ($\Delta \alpha/\alpha + 10 \Delta X_q / X_q$), $X_q = m_q / \Lambda_{QCD}$,

Shift 10-100 Hz for $\Delta \alpha/\alpha = 10^{-18}$

Compare with atomic clock shift 0.001 Hz

Berengut, Dzuba, Flambaum, Porsev: Sensitivity to $\Delta \alpha/\alpha$ is expressed via isomeric shifts of $^{229}$Th atomic lines,

frequency in $^{229}$Th - frequency in $^{229}$Th*. Measure, please!
Enhancement of relative effect

Our proposal and calculations:
Dy: $4f^{10}5d6s \ E=19797.96\ldots \text{cm}^{-1}$, $q= 6000 \text{ cm}^{-1}$
$4f^95d^26s \ E=19797.96\ldots \text{cm}^{-1}$, $q= -23000 \text{ cm}^{-1}$

$\omega_0 = 10^{-4} \text{ cm}^{-1}$. Relative enhancement $\Delta\omega/\omega_0 = 10^8 \Delta\alpha/\alpha$

Measurement Berkeley $d\ln \alpha/dt = -2(3) \times 10^{-16} \text{ yr}^{-1}$

Different signs of $\omega_0$ in different isotopes: cancellation of errors!

Close narrow levels in molecules
Enhancement in molecular clocks

DeMille et al 2004, 2008 – enhancement in Cs$_2$, cancellation between electron excitation and vibration energies

Flambaum 2006 Cancellations between rotational and hyperfine intervals

\[ \Delta \omega/\omega_0 = K \Delta \alpha/\alpha \]  
Enhancement \( K = 10^2 - 10^3 \)

Flambaum, Kozlov 2007 Cancellations between fine structure and vibrations

\[ \Delta \omega/\omega_0 = K (\Delta \alpha/\alpha - 1/4 \Delta \mu/\mu) \]  
Enhancement \( K = 10^4 - 10^5 \)
Conclusions

• **Spatial gradient of alpha from quasar data**, 4.2 sigma, Keck and VLT data agree, low and high red shift data agree, no contradictions with other groups.

• It provides alpha variation for atomic clocks due to Earth motion at the level $10^{-18}$ and 1 meV shift in Oklo resonance. One-two orders of magnitude improvement in the measurement accuracy is needed. Three orders for meteorites.

• Very weak indications for the spatial variation in $\mathrm{H}_2$ quasar spectra and BBN abundance of deuterium. The same direction of the gradient!

New systems with higher absolute sensitivity include:

• **transitions between ground and metastable states in highly charged ions.** Frequencies are kept in laser spectroscopy range due to the configuration crossing phenomenon. An order of magnitude gain.

• $^{229}\mathrm{Th}$ nucleus – highest **absolute** enhancement ($10^5$ times larger shift), UV transition 7eV.

• Many systems with relative enhancement due to transition between close levels: Dy atom, a number of molecules with narrow close levels,…

• Search for anisotropy in CMB, expansion of the Universe, structure formation
Atomic parity violation

- Dominated by $Z$-boson exchange between electrons and nucleons

$$H = \frac{g}{\sqrt{2}} \left[ C_{1p} \bar{e} \gamma_\mu \gamma_5 e p \gamma^\mu p + C_{1n} \bar{e} \gamma_\mu \gamma_5 e n \gamma^\mu n \right]$$

Standard model tree-level couplings:

$$C_{1p} = \frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right); \quad C_{1n} = -\frac{1}{2}$$

- In atom with $Z$ electrons and $N$ neutrons obtain effective Hamiltonian parameterized by “nuclear weak charge” $Q_W$

$$h_{PV} = \frac{g}{2\sqrt{2}} Q_W \rho(r) \gamma_5$$

$$Q_W = 2(NC_{1n} + ZC_{1p}) \approx -N + Z(1 - 4 \sin^2 \theta_W) \approx -N$$

- APV amplitude $E_{PV} \propto Z^3$  

[Bouchiat,Bouchiat]

$Bi,Pb,Tl,Cs$ Test of standard model via atomic experiments!
Tightly constrains possible new physics, e.g. mass of extra $Z$ boson $M_{Z'} > 1$ TeV. New experiments: Ba+, 20 times enhancement in Ra+, Fr

Calculations [Dzuba, Flambaum, Ginges, 2002]

$$ E_{PV} = -0.897(1 \pm 0.5\%) \times 10^{-11} \, i e a_B (-Q_W/N) $$

Porsev, Beloy, Derevianko 2009 0.3%

Cs Boulder

$$ Q_W - Q_W^{SM} = 1.1 \sigma $$

$E_{PV}$ includes -0.8% shift due to strong-field QED self-energy / vertex corrections to weak matrix elements $W_{sp}$

$$ E_{PV} = \sum_p \frac{W_{sp} E1_{ps}}{E_s - E_p} $$

[Kuchiev, Flambaum; Milstein, Sushkov, Terekhov]

A complete calculation of QED corrections to PV amplitude includes also

• QED corrections to energy levels and E1 amplitudes

[Flambaum, Ginges; Shabaev, Pachuki, Tupitsyn, Yerokhin]
PV : Chain of isotopes

Dzuba, Flambaum, Khriplovich

Rare-earth atoms:
• close opposite parity levels-enhancement
• Many stable isotopes

Ratio of PV effects gives ratio of weak charges. Uncertainty in atomic calculations cancels out. Experiments:

Berkeley: Dy and Yb;
Ra, Ra\(^+\), Fr Argonne, Groningen, TRIUMF?

Test of Standard model or neutron distribution.

Brown, Derevianko, Flambaum 2008. Uncertainties in neutron distributions cancel in differences of PNC effects in isotopes of the same element. Measurements of ratios of PNC effects in isotopic chain can compete with other tests of Standard model!
Nuclear anapole moment

- Source of nuclear spin-dependent PV effects in atoms
- Nuclear magnetic multipole violating parity
- Arises due to parity violation inside the nucleus

- Interacts with atomic electrons via usual magnetic interaction (PV hyperfine interaction):

\[ h_a = e \vec{\alpha} \cdot \vec{A} \propto \kappa_a \vec{\alpha} \cdot \vec{I} \rho(r) \quad \kappa_a \propto A^{2/3} \]

[Flambaum, Khriplovich, Sushkov]

\[ E_{PV} \propto Z^2 A^{2/3} \text{ measured as difference of PV effects for transitions between hyperfine components} \]

Cs: |6s,F=3> – |7s,F=4> and |6s,F'=4> – |7s,F=3>

Probe of weak nuclear forces via atomic experiments!
Nuclear anapole moment is produced by PV nuclear forces. Measurements+our calculations give the strength constant $g$.

- Boulder Cs: $g=6(1)$ in units of Fermi constant
- Seattle Tl: $g=-2(3)$

New accurate calculations Haxton,Liu,Ramsey-Musolf; Auerbach, Brown; Dmitriev, Khriplovich,Telitsin: problem remains.

Our proposals:

$10^3$ enhancement in Ra atom due to close opposite parity state;
Dy,Yb,…(experiment in Berkeley)
Enhancement of nuclear anapole effects in molecules

$10^5$ enhancement of the nuclear anapole contribution in diatomic molecules due to mixing of close rotational levels of opposite parity. Theorem: only anapole contribution to PV is enhanced (Labzovsky; Sushkov, Flambaum). Weak charge can not mix opposite parity rotational levels and $\Lambda$–doublet.

$\Omega=1/2$ terms: $\Sigma_{1/2}, \Pi_{1/2}$. Heavy molecules, effect $Z^2 A^{2/3} R(Z\alpha)$

YbF, BaF, PbF, LuS, LuO, LaS, LaO, HgF, Cl, Br, I; BiO, BiS, PbO+, YbO+, HgO+...

PV effects $10^{-3}$, microwave or optical M1 transitions. For example, circular polarization of radiation or difference of absorption of right and left polarised radiation.

Cancellation between hyperfine and rotational intervals - enhancement.

Interval between the opposite parity levels may be reduced to zero by magnetic field – further enhancement.

Molecular experiments: Yale, Groningen
Enhancement of electron EDM

- Atoms: Tl enhancement \( d(\text{Tl}) = -585 \, d_e \)
  - Experiment – Berkeley. Our new accurate many-body calculations for Tl, Fr, Cs, …
- Molecules – close rotational levels,
  \( \Omega \) – doubling – huge enhancement of electron EDM
  (Sushkov, Flambaum 1978)
  \( \Omega = \frac{1}{2} \) \( 10^7 \) YbF London
  \( \Omega = 1 \) \( 10^{10} \) PbO, ThO Yale, Harvard
  \( \Omega = 2 \) \( 10^{13} \) HfF\(^+\) Boulder

Weak electric field is enough to polarise the molecule.
Molecular electric field is several orders of magnitude larger than external field (Sandars)
Nuclear EDM-screening: $d_N E_N$

- Schiff theorem: $E_N = 0$, neutral systems
- Extension for ions:
  Ion acceleration $a = Z_i \frac{eE}{M}$
  Nucleus acceleration $a = Z \frac{eE_N}{M}$
  \[ E_N = E \frac{Z_i}{Z} \]

In molecules screening is stronger:
  $a = Z_i \frac{eE}{(M+m)}$, $E_N = E \left( \frac{Z_i}{Z} \right) \frac{M}{(M+m)}$
Schiff: Incomplete screening in neutral atoms

- Hyperfine interaction: atomic EDM is proportional to nuclear EDM times nuclear magnetic moment
- Finite nuclear size.

Effect due to nuclear Schiff moment

We performed new calculations of both effects for all atoms of experimental interest
EDMs of atoms of experimental interest

<table>
<thead>
<tr>
<th>Z</th>
<th>Atom</th>
<th>$[S/(e \text{ fm}^3)]e \text{ cm}$</th>
<th>$[10^{-25} \eta]e \text{ cm}$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$^3\text{He}$</td>
<td>0.00008</td>
<td>0.0005</td>
<td>Seattle, Ann Arbor, Princeton, Tokyo</td>
</tr>
<tr>
<td>54</td>
<td>$^{129}\text{Xe}$</td>
<td>0.38</td>
<td>0.7</td>
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</tr>
<tr>
<td>70</td>
<td>$^{171}\text{Yb}$</td>
<td>-1.9</td>
<td>3</td>
<td>Bangalore, Kyoto</td>
</tr>
<tr>
<td>80</td>
<td>$^{199}\text{Hg}$</td>
<td>-2.8</td>
<td>4</td>
<td>Seattle</td>
</tr>
<tr>
<td>86</td>
<td>$^{223}\text{Rn}$</td>
<td>3.3</td>
<td>3300</td>
<td>TRIUMF</td>
</tr>
<tr>
<td>88</td>
<td>$^{225}\text{Ra}$</td>
<td>-8.2</td>
<td>2500</td>
<td>Argonne, KVI</td>
</tr>
<tr>
<td>88</td>
<td>$^{223}\text{Ra}$</td>
<td>-8.2</td>
<td>3400</td>
<td></td>
</tr>
</tbody>
</table>

$S$-nuclear Schiff moment; neutron $d_n = 5 \times 10^{-24} e \text{ cm } \eta$, 
Summary

• Atomic and molecular experiments are used to test unification theories of elementary particles

Parity violation
  – Weak charge: test of the standard model and search of new physics
  – Nuclear anapole, probe of weak PV nuclear forces

Time reversal
  – EDM, test of physics beyond the standard model.
  1-3 orders improvement may be enough to reject or confirm all popular models of CP violation, e.g. supersymmetric models

• A new generation of experiments with enhanced effects is underway in atoms, diatomic molecules, and solids
Publications:

- V. A. Dzuba, V. V. Flambaum, PRA 61, 034502 (2000).
- V. A. Dzuba, V. V. Flambaum, PRA, 72, 052514 (2005).
- S. G. Karshenboim et al, physics/0511180.