Problem Solving Class: Van Quark tot Biomaterie

Problem Set 8: Quantum states and transitions
Hand-in on paper Thursday 16 October
Hand-in digitally, email to: m.t.talluri@vu.nl;
All documents in a single file [file: YourName-WC-Q8]
All answers in English

1) Thermal population of quantum states

a) What is the statistical probability to find a hydrogen atom in the excited 2s state, for conditions of room temperature (with respect to ground state 1s).

b) What is the probability to find a hydrogen molecule in its v=1 vibrationally excited state (calculate for T=50 K, the typical condition for an interstellar cloud).

c) Same for rotational excitation in the J=1 state for the hydrogen molecule (for room temperature). Calculate in this case the relative population with respect to J=0.

2) Dynamics of a two level system

Consider a "two-level system" with energy levels E_2 (upper level) and E_1 (lower level) with populations $n_2(t)$ and $n_1(t)$. A rate equation including spontaneous emission with rate A, absorption with rate Bu_ν and stimulated emission with rate Bu_ν can be written as follows:

$$\frac{d}{dt}n_2(t) = Bu_\nu n_1(t) - (A + Bu_\nu)n_2(t)$$

a) Solve this differential equation for $n_2(t)$ for boundary conditions $n_1(0)=N$ and $n_2(0)=0$, where $N = n_2(t) + n_1(t)$ is the total number of atoms, independent of time.

b) Show that at short times, defined as $(A + 2Bu_\nu)t << 1$, there is a steady growth of population in the excited state.

c) How is the behaviour for $t \rightarrow \infty$?

d) Show that there is a maximum population for n_2 for all times and for all intensities u_ν. Consider even $u_\nu \rightarrow \infty$. i.e. infinitely strong intensity.

The above **should** be done analytically, including the solution to the differential equation.

You may also use Mathematica (additionally) to solve the problems. Tip: simplify the results by : "Simplify[]".

Plot $n_2(t)$ for different values of the ratio A ad Bu_ν and try to find a numerical answer to the questions stated above.

3) CO observed in radio astronomy

a) Explain why the quantized angular momentum of a rotating molecule can be expressed as:

$$E(J) = \frac{J(J+1)\hbar^2}{2I}$$

Here I is the moment of inertia (*traagheidsmoment*) of a rotating di-atomic molecule.
Use the property of the angular momentum operator $\boldsymbol{J}^2$ to have eigen values: $J(J+1)\hbar^2$.

In radio astronomy spectral Lines are observed (in absorption or emission) of the CO (carbon monoxide) molecule making a transition from state J=1 to state J=0. This transition is at f=115.271208 GHz (Gigahertz, or 10^9 Hz). (h=6.62606957 x 10^{-34} Js).

b) Calculate the separation R_e between the C-atom and the O-atom in the CO molecule. Use an expression for the reduced mass of the CO molecule (Masses M_C = 12 a.m.u; M_O = 15.994915 a.m.u., with t 1 a.m.u. (atomic mass unit)= 1.660538921 x 10^{-27} kilogram.)

c) "Usual" $^{12}C^{16}O$ molecules consist of ^{12}C and ^{16}O atoms (isotopes). However, in radio astronomy also $^{12}C^{18}O$ molecules are observed. For the heavy oxygen molecule the mass is $M_O(18)$ = 17.999161 a.m.u. Make the assumption that the chemical bond in the $^{12}C^{18}O$ molecules is the same as for the main isotopomer $^{12}C^{16}O$ molecule. So assume that R_e has the same value for the heavier system.
Now calculate the frequency of the transition from J=1 to J=0 in the $^{12}C^{18}O$ molecule.