Problem Solving Class: Van Quark tot Biomaterie

Problem Set 3: Quantum states in the Bohr Model
Hand-in on paper Tuesday 16 September (during lecture 15:30 h)
Hand-in digitally, email to: m.t.talluri@vu.nl;
All documents in a single file [file: YourName-WC-Q3]
All answers in English

1) The Virial Theorem

Show that for the energies in the Bohr model of the atom the general relation holds (which is different from the virial theorem for a harmonic oscillator):

$$2\langle E_{kin}\rangle = -\langle E_{pot}\rangle$$

2) Quantisation in the framework of gravity

The potential energy for electromagnetic energy is expressed by:

$$V_{EM} = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

The potential energy for a theory of gravitational energy has a similar functional form, exhibiting the same, and crucial, *1/r* dependence in the potential; consider a light particle of mass *m* rotating around a heavy particle of mass *M*:

$$V_G = -\frac{GMm}{r}$$

Treat the motion of a light body rotating around a very heavy body in the framework of Bohr's theory. Make the same assumption as Bohr did on the quantization of the angular momentum. (Consider the mass of the central body as so heavy that its relative motion can be ignored).
So postulate that:

$$L = mvr = n\hbar$$

a) Derive a formula for the fundamental radius in this quantization problem, in other words determine the "Bohr radius", defined as a_g.
Do this by performing an algebraic calculation analogous as done for the Bohr model, or by making a substitution comparing the two potentials (or forces)

b) Derive a formula for the "Rydberg constant" for this gravitational problem R_g^{∞}.

Let us from here on play with numerics. Consider the light particle as the Earth and the heavy particle to be the Sun:

m = 5.98 x 10^{24} kg (Earth)
M= 1.99 x 10^{30} kg (Sun)

(This explains why there is non eed to consider a “reduced mass”, because the heavy particle is really much heavier.)

c) Use further R_{orbit}= 150 x 10^9 m, the radius of the Erath’s orbit and G= 6.673 x 10^{-11} m^3/s^2kg, while using the known value for Planck’s constant, what is then the value of the Bohr radius for this problem ?

d) In what quantum state is the Earth when it follows the orbit it does ?

e) You may calculate the amount of energy it takes for the Earth to move from its quantum state n to a new quantum state $n+1$.

You may think about what this kind of quantization means ? First of all the quantum numbers become so large that in Bohrs “Correspondence Principle” the quantum theory approaches a classical situation. Is this the theory of quantized gravity that we are after ?