Problem Solving Class: Van Quark tot Biomaterie

Problem Set 2: The Bohr Model of the Atom
Hand-in on paper Tuesday 11 September (during lecture 15:30 h)
Hand-in digitally, email to: m.t.talluri@vu.nl;
All documents in a single file [file: YourName-WC-Q2]
All answers in English

1) The fine structure constant α

The Bohr formula for the energy levels in the hydrogen atom can be written as:

$$E_n = -\frac{Z^2}{2n^2}\alpha^2 mc^2$$

a) Derive a formula for the fine structure constant given the presented derivation of the level energies in the H-atom.
b) What is the value of α ? And what is the dimension α ?
Hence the binding energy of electrons in the ground state of the hydrogen atom is given by:

$$E_b = -\frac{1}{2}\alpha^2 E_0$$

where E_0 is the rest mass of the electron ($E_0=mc^2$).
c) What is the reason for this universal relationship in terms of a dimensionless number α ?
In other words: What is the physical reason for the numerical value of α as calculated ?
d) Derive also, following the derivations in the notes, that the velocity of an electron in the ground state orbital in the Bohr model (n=1 orbit, for Z=1) is given by:

$$v_1 = \alpha c$$

2) Lyman-α transition in atomic hydrogen

The Lyman-α transition in the hydrogen atom is defined as the transition from quantum state n=1 to n=2.
In first order neglect the "reduced mass-effect:
a) Give an equation for the frequency and for the wavelength of this Lyman-α transition.
b) Derive a numerical value for the wavelength of this transition; a value in nanometers.
Now, in second order include the reduced mass-effect.
Consider the exotic atom "positronium" built from an electron and an anti-electron (also known as a positron).
Note that the positron is positively charged (+1e) and has a mass equal to that of the electron.
c) Derive an equation for the level energies of the positronium system.
d) What is the wavelength of the "Lyman-α" transition in positronium (in equation and numerical).