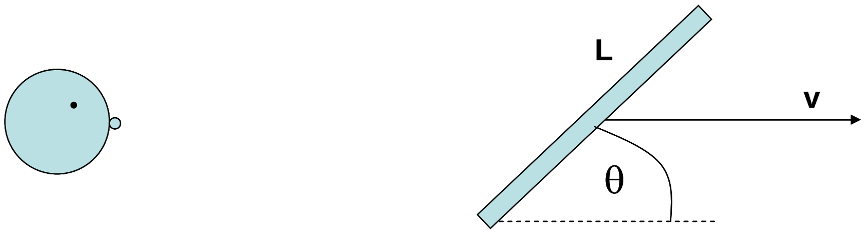
Problem Solving Class: Van Quark tot Biomaterie

Problem Set 9: Relativity
Hand-in on paper Monday 10 November (before 12:00 h)
in Mailbox Madhu Talluri (Mailboxes W&N building)
Hand-in digitally, email to: m.t.talluri@vu.nl;
All documents in a single file [file: YourName-WC-P1]
All answers in English

1) Moving stick and the transformation of angles.

All physical quantities undergo a (Lorentz) transformation describing how the quantity is observed in or from a certain reference frame.
Consider a moving stick of length **L** (in the eigen system of the stick) as drawn in the picture below.



The stick moves under an angle θ and a velocity **v** with respect to the observer.
a) Derive a formula for the length of the stick **L'** in the system of the observer.
(*Hint: use projections* L_x *and* L_y where the *x*-axis is defined along the coordinate of velocity **v**, and *y* is an axis perpendicular to that).
b) Derive an equation for the angle θ' in the system of the observer. You may express this in terms of **cosθ'** or **sinθ'**.
c) How are these results if the observer moves toward the stick (at velocity **v**) ?

2) Velocity of a relativistic particle

A particle has a momentum *p* and an energy *E*, where $p=\gamma mv$.
Show that the velocity can e written as:

$$\mathrm{v} = \frac{pc}{\sqrt{\mathrm{m}^2 c^2 + p^2}}$$

3) Velocity of particles in a particle accelerator

a) Show that for a relativistic particle (a particle moving at high speed) the velocity v can be written in terms of its deviation from the light speed c as:

$$\Delta v = c - v = \frac{c}{2}\left(\frac{m_0 c^2}{E}\right)^2$$

b) What is Δv for the electrons moving in the Soleil synchrotron accelerator in Paris (electrons at energy 3 GeV) ?

c) What is Δv for protons moving in LHC (if they reach 7 TeV for each proton).

Hint: use a Taylor expansion: $\sqrt{1-x} = 1 - \frac{1}{2}x$ for small x; or any other realistic realistic approximation.
Note: The rest mass of an electron is 511 keV/c^2 and the rest mass of a proton is 938 MeV/c^2. Here energies are expressed in eV and masses in eV/c^2. (G means Giga=10^9, T means Tera = 10^{12}).

4) Kinetics and Dynamics of elementary particles

Consider the decay of an elementary particle, the π^--meson in the reaction:

$$\pi^- \rightarrow \mu^- + \nu$$

Where μ^- is the negatively charged muon and ν a neutrino.
Note and assume the following: the neutrino is a massless particle (meaning that its rest mass is zero) and that the meson and the muon have rest masses: m_π and m_μ.
Consider now the decay process assuming that the meson decays from an initial situation of zero momentum (p=0). Use conservation of momentum and energy (before and after the reactive process) to determine the momenta of the particles after the decay process has occurred.

a) What is the kinetic energy K_ν of the neutrino ?
b) Derive an equation for the momentum of the decay products.
c) Derive an equation for the Kinetic energy of the muon (in terms of the masses of the particles).