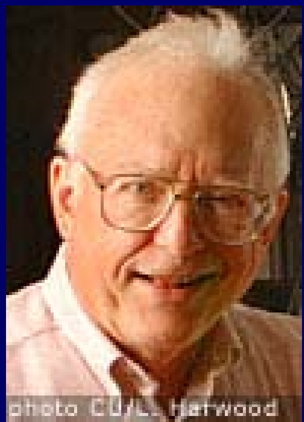
Optical frequency combs and frequency comb spectroscopy

Laser Centre
vrije Universiteit amsterdam

Frequency Combs: A revolution in measuring



J. Hall

Nobel 2005

T.W. Hänsch

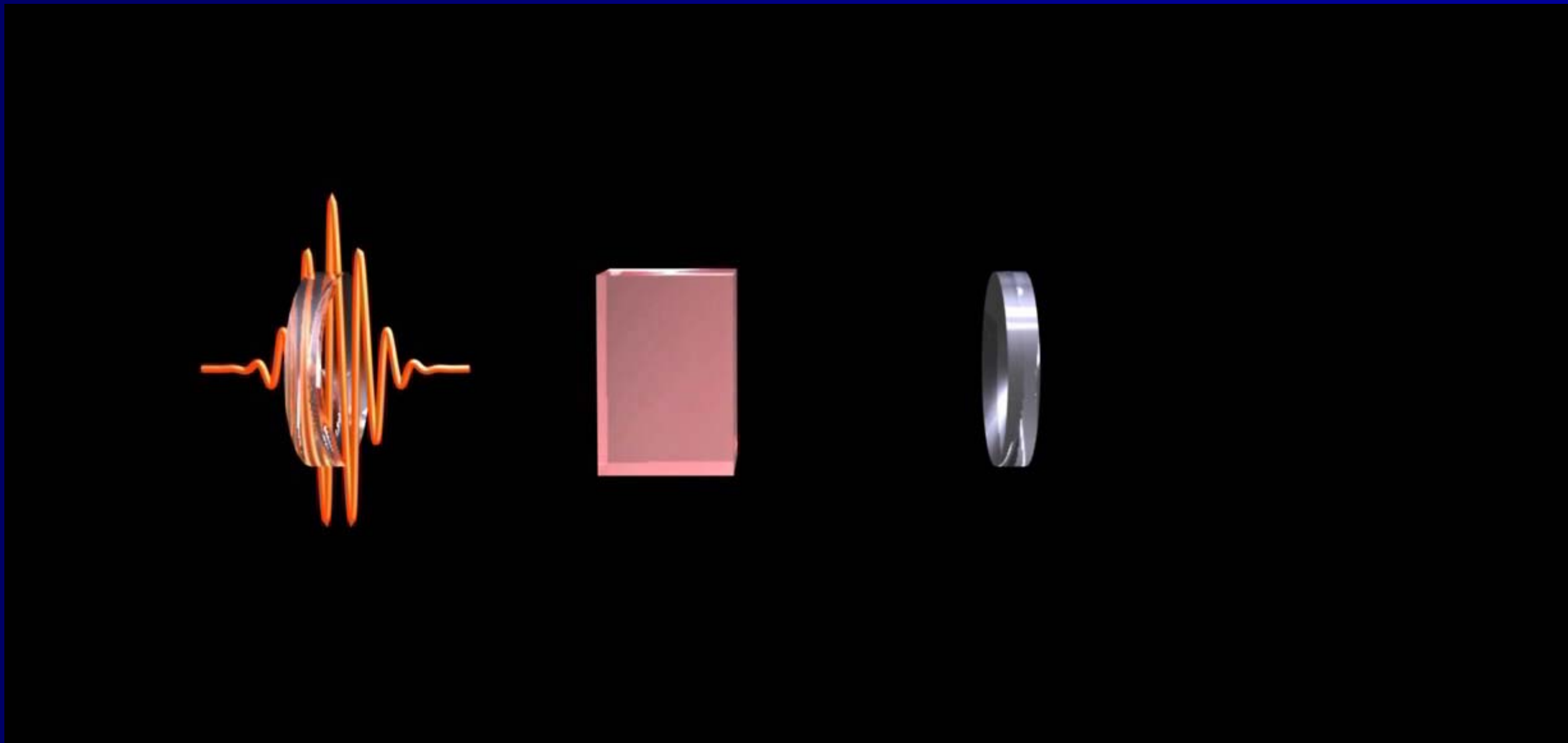
"for their contributions to the development of laser-based precision spectroscopy including the optical frequency comb technique"

Wim Ubachs

TULIP Summer School IV 2009
Noordwijk, April 15-18

On Pulsed and Continuous wave lasers

A laser consists mainly of a gain medium and an optical cavity:



Consider from time and frequency domain perspectives

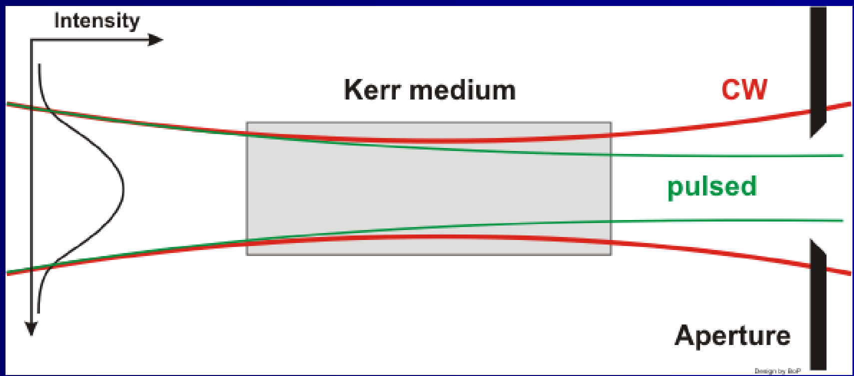
Modelocking a laser

Basic idea:

build a laser cavity that is low-loss for intense pulses,
but high-loss for low-intensity continuous beam

Solutions:

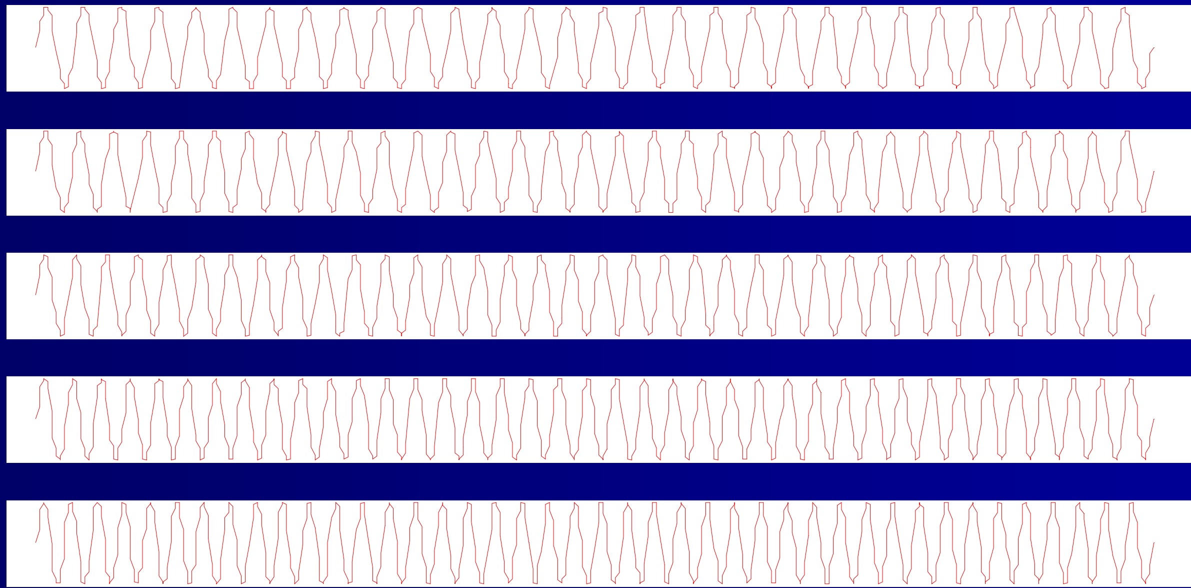
Intracavity saturable absorber, or Kerr-lensing:



- **Intensity-dependent refractive index: $n = n_0 + n_{Kerr}\, I$**
- **Gaussian transverse intensity profile leads to a refractive index gradient, resembling a lens!**

A laser running on multiple modes: a pulsed laser

lasers with “mode-locking”



$f = f_a$

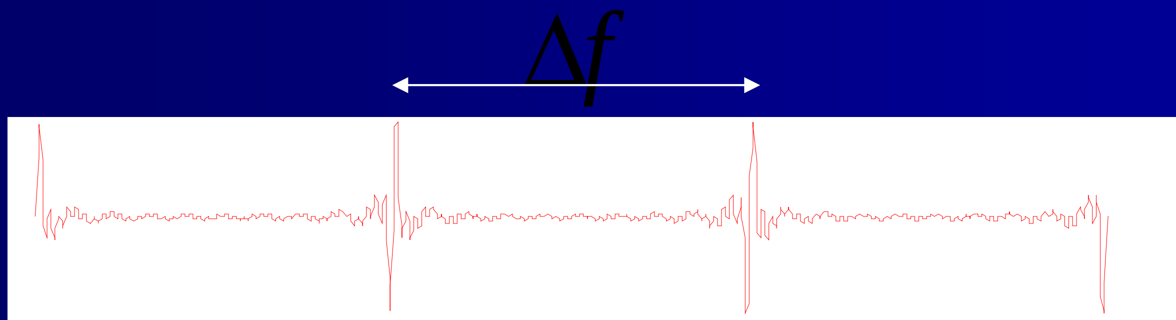
$f = f_a + \Delta f$

$f = f_a + 2\Delta f$

$f = f_a + 3\Delta f$

$f = f_a + 4\Delta f$

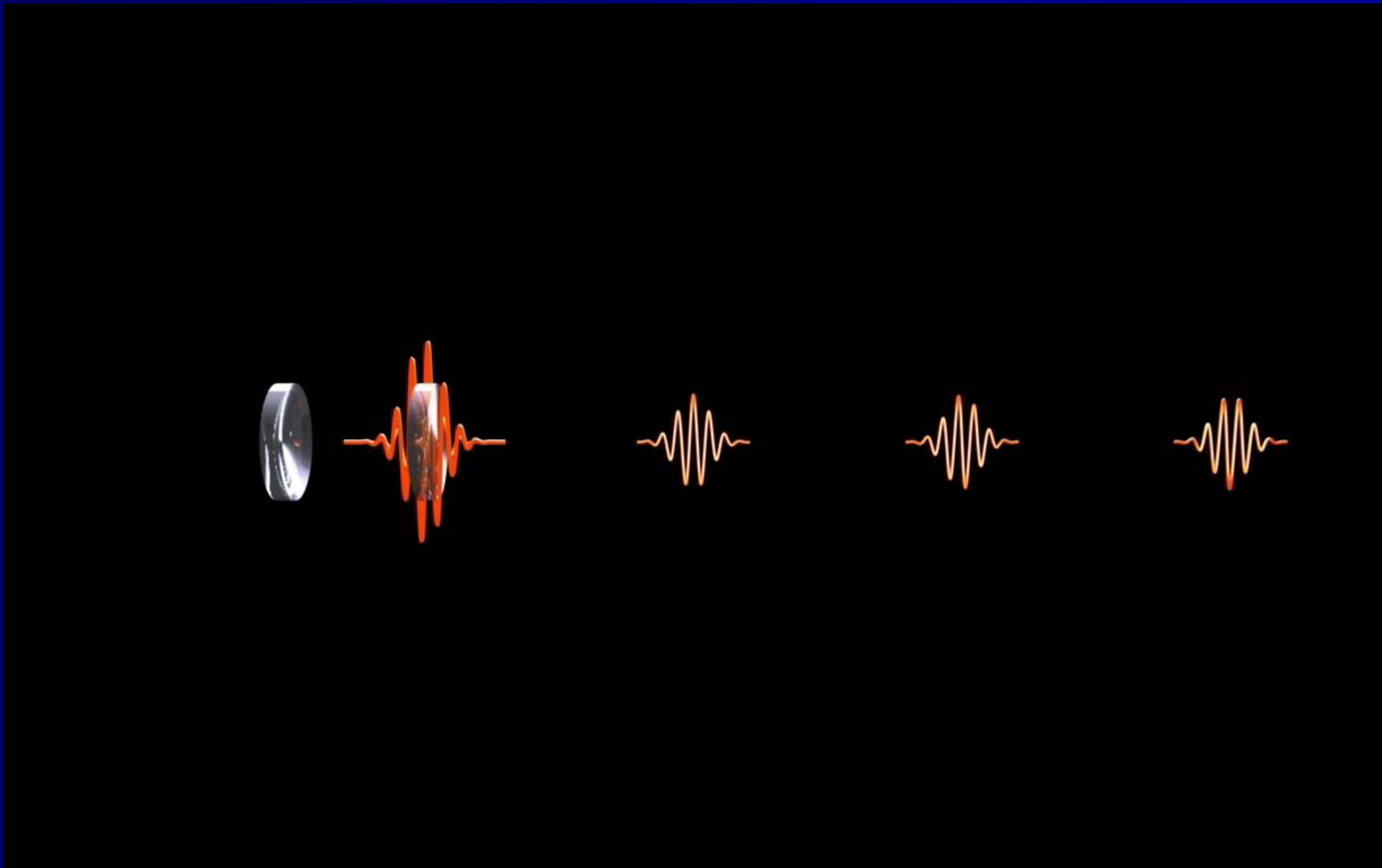
And so forth: add 30 waves:



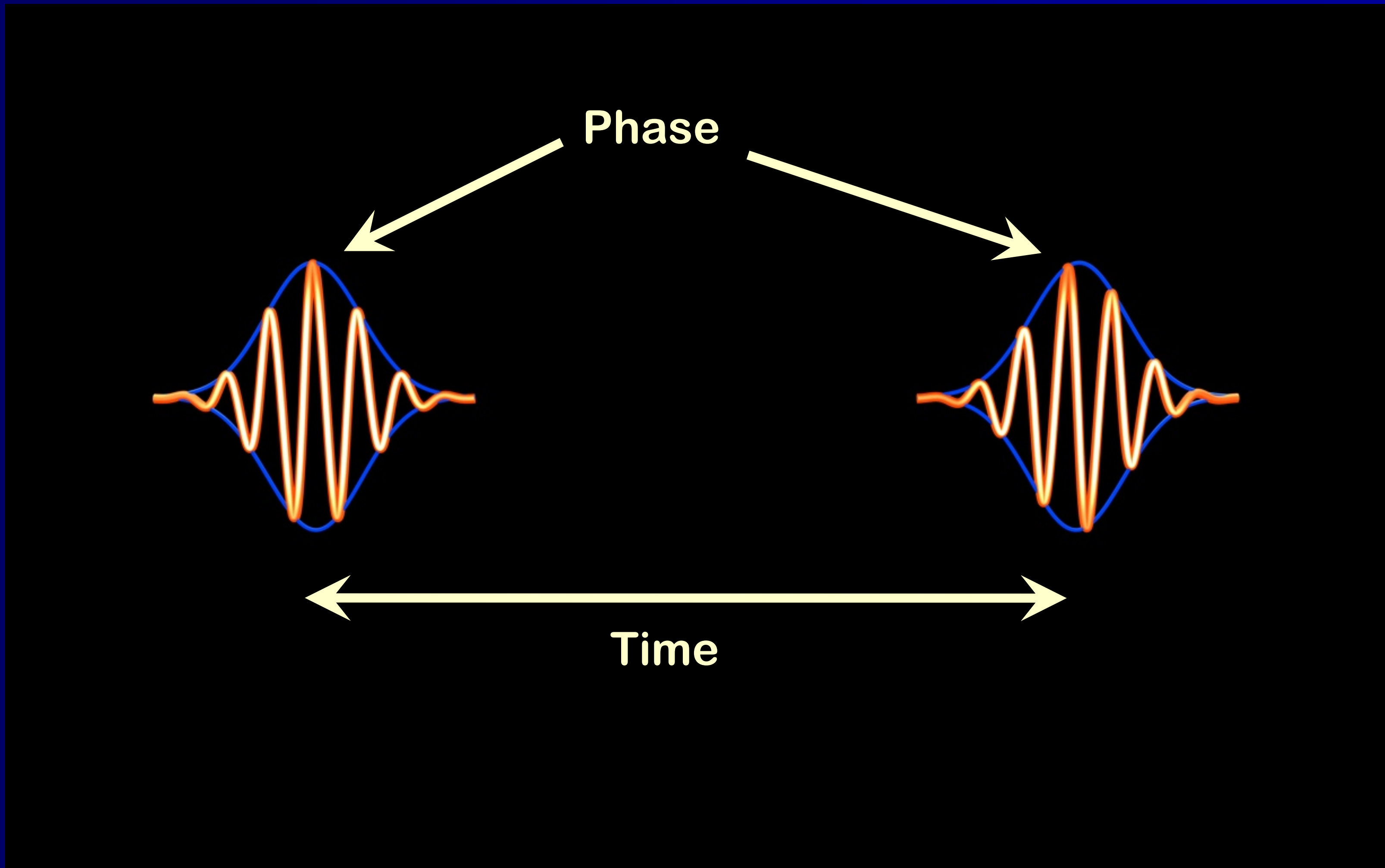
SUM

Ultrafast lasers

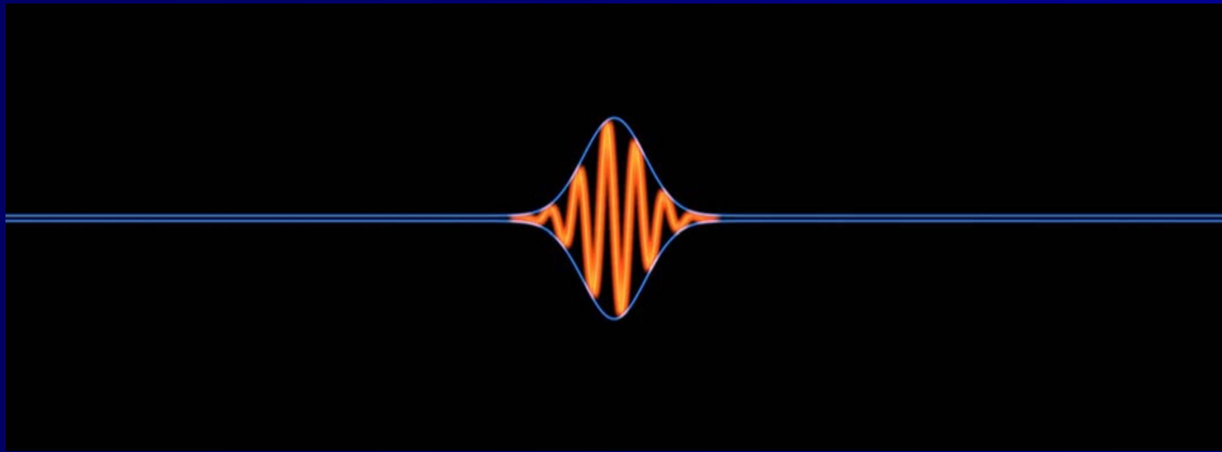
Pulsing back and forth inside the cavity



Ultrafast lasers

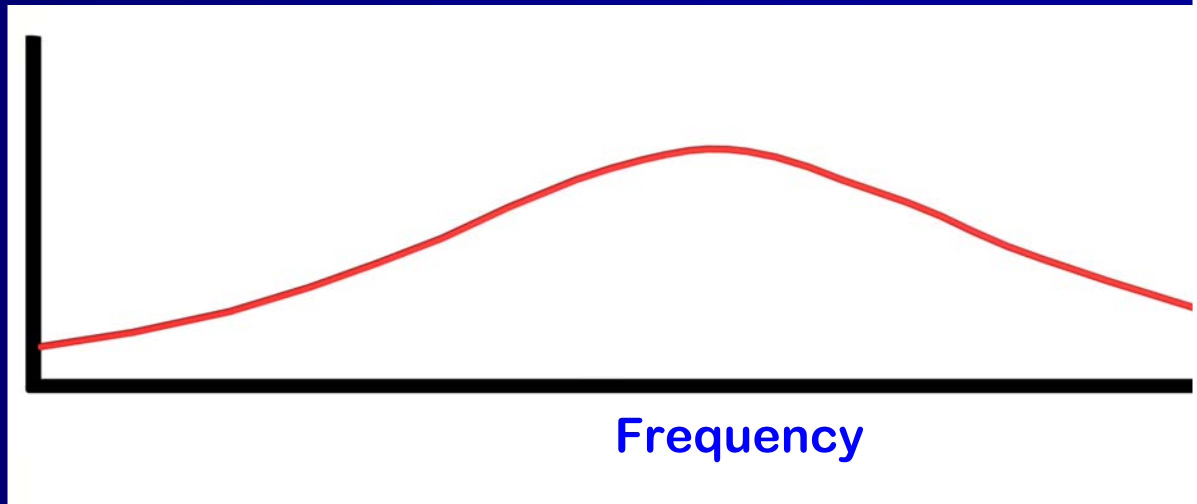


Fourier principle for short pulses

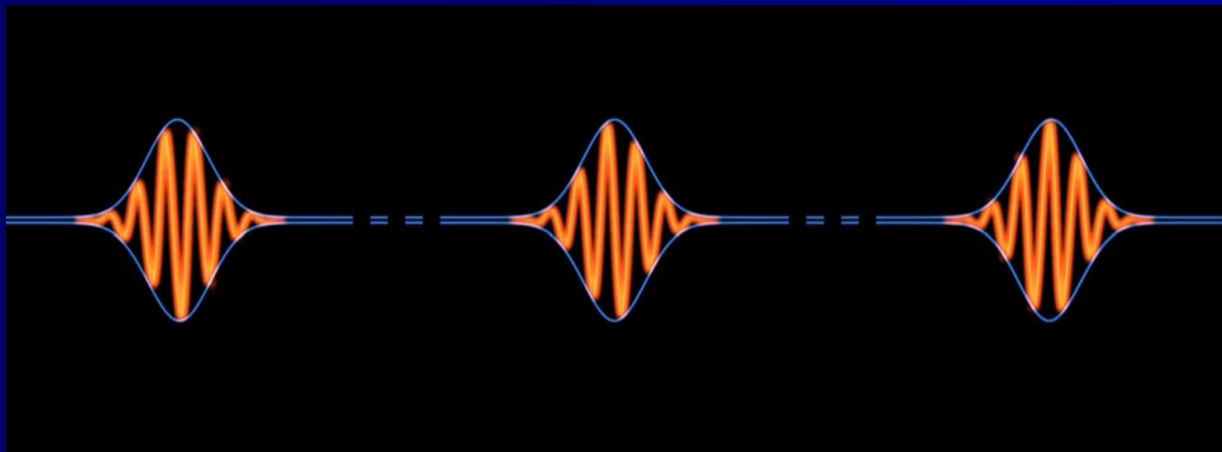


Time Domain:
Short pulse

Spectral Domain:
Wide spectrum



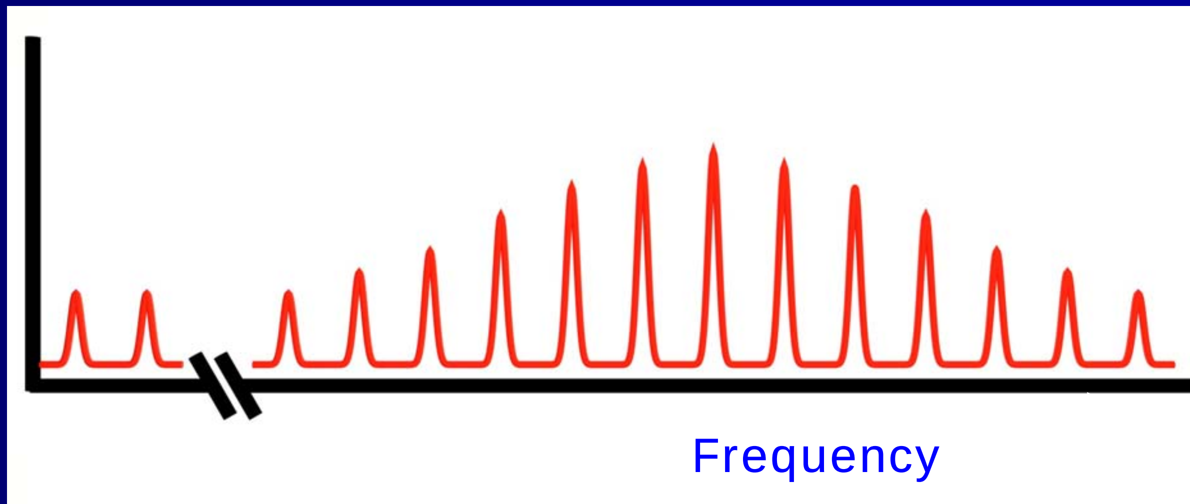
Frequency comb principle



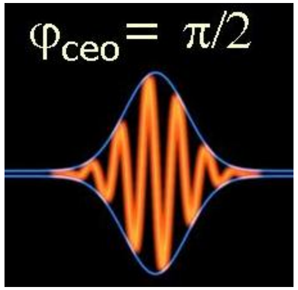
Time Domain:
Pulse train

Spectral domain:

'Comb-like' spectrum
Many narrow-band,
Well-defined frequencies



Some math: Propagation of a single pulse (described as a wave packet)



$$E(t,z)=\int_{-\infty}^{\infty}E(\omega)e^{ik(\omega)z}e^{-i\omega t}d\omega$$

Insert an inverse Fourier transform E(τ) for E(ω)

$$E(t,z)=\int_{-\infty}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}E(\tau)e^{i\omega\tau}d\tau e^{ik(\omega)z}e^{-i\omega t}d\omega$$

$$E(t,z)=\int_{-\infty}^{\infty}E(\tau)G(t-\tau,z)d\tau$$

Propagator

$$G(t-\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i(\omega(t-\tau)-k(\omega)z)}d\omega$$

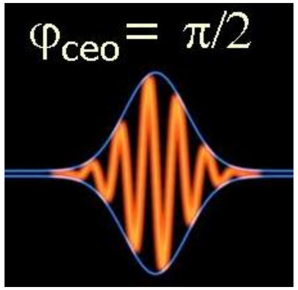
Propagation of the field

This can be used with $k(\omega)=k_0+\left.\frac{dk}{d\omega}\right|_{\omega_l}(\omega-\omega_l)+O(k^2)$

$$E(t,z)=\exp[i\omega_l(\frac{1}{v_g}-\frac{1}{v_\phi})z]E(t-\frac{z}{v_g})$$

Difference between group and phase velocity causes an extra phase → When traveling through dispersive medium The carrier/envelop phase continuously changes

Some math: Propagation of a multiple pulses in a train



$$E(t)=\sum_{n=0}^{N-1} E_{\text{single}}(t-nT)$$

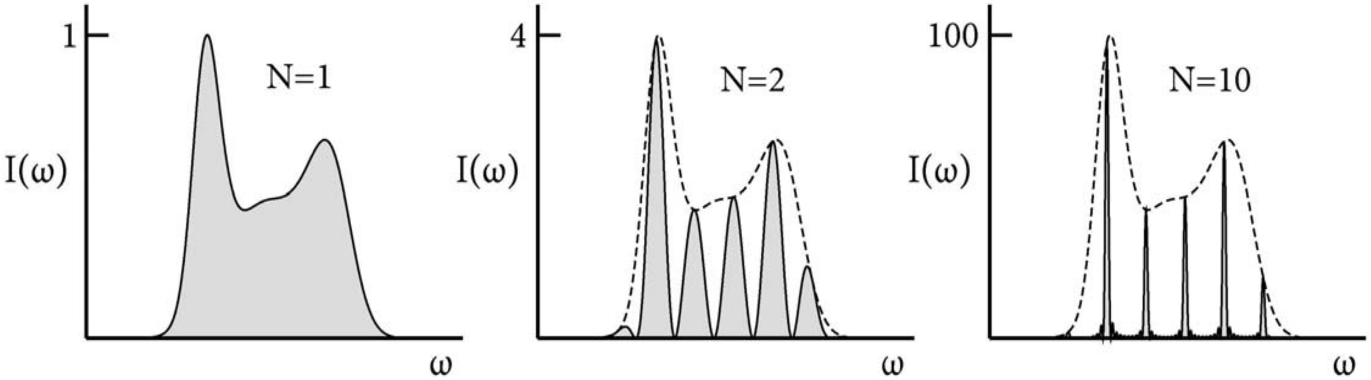
T is time delay between pulses

$$E_{\text{train}}(\omega)=E_{\text{single}}(\omega)\sum_{n=0}^{N-1} e^{-in\omega T}=E_{\text{single}}(\omega)\frac{1-e^{-iN\omega T}}{1-e^{-i\omega T}}$$

$$I_{\text{train}}(\omega)=I_{\text{single}}(\omega)\frac{\sin^2(N\omega T/2)}{\sin^2(\omega T/2)}$$

In the limit

$$I_{\text{train},\infty}(\omega)=I_{\text{single}}(\omega)\sum_{n=0}^{\infty}\delta(\omega T-2\pi n)$$

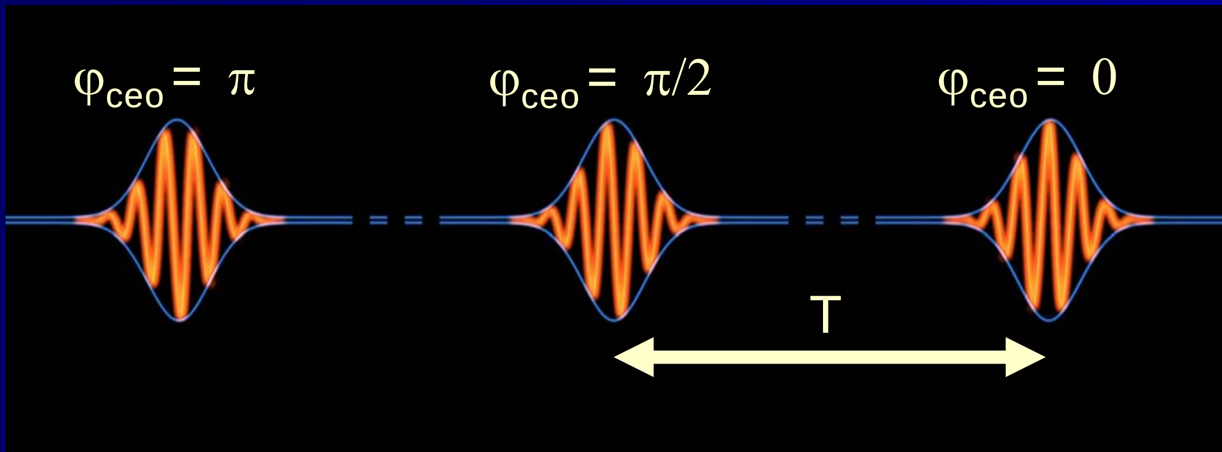


With dispersion

$$I_{\text{train},\infty}(\omega)=I_{\text{single}}(\omega)\sum_{n=0}^{\infty}\delta(\omega T-2\pi n-\phi_{CE})$$

→ Phase shift

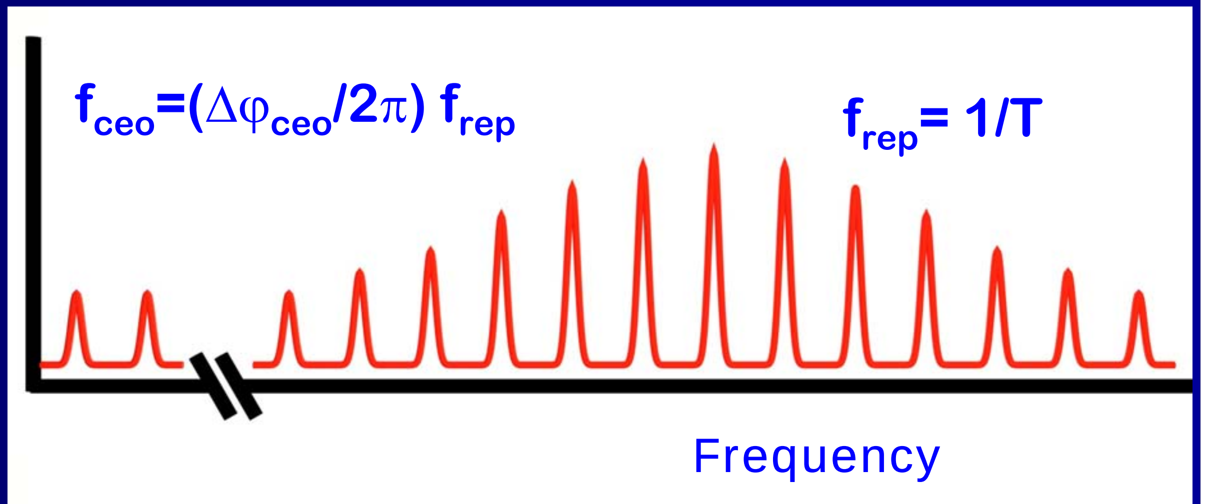
Frequency comb principle



2 RF frequencies determine the entire optical spectrum!

$$f = n\, f_{rep} + f_{ceo}$$

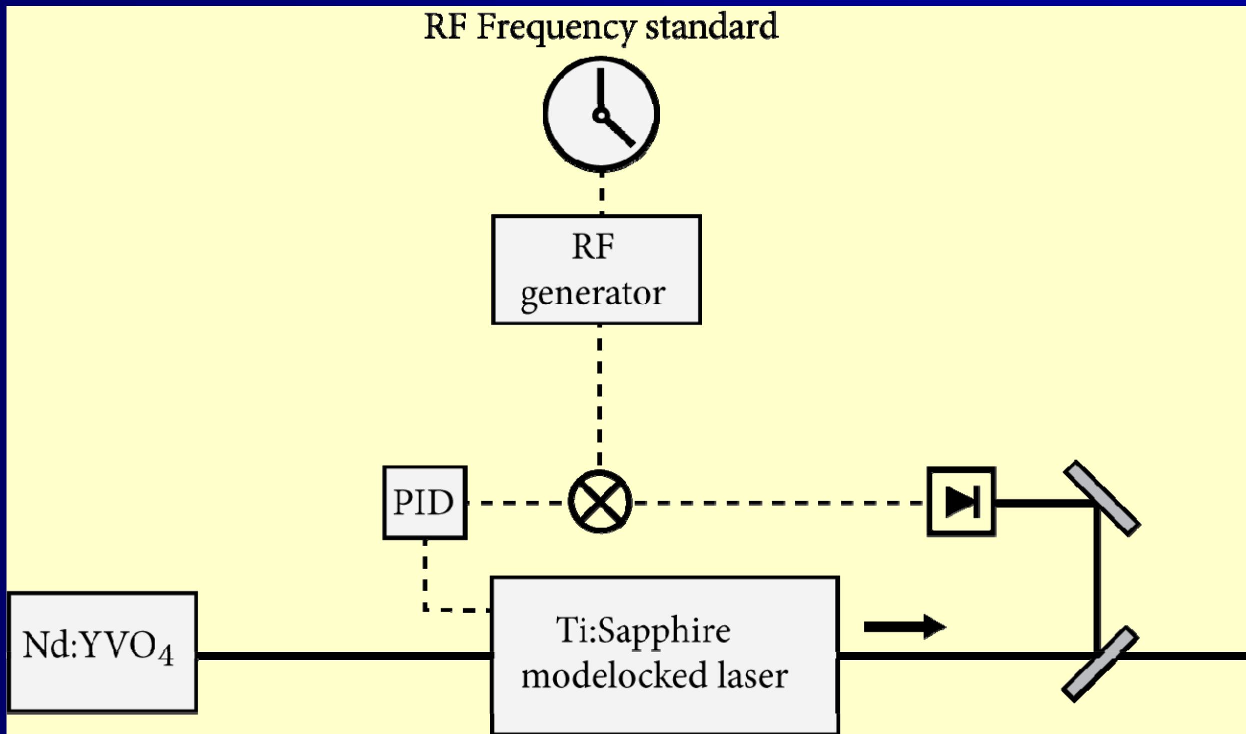
tested to $<10^{-19}$ level



Stabilization of f_{rep}

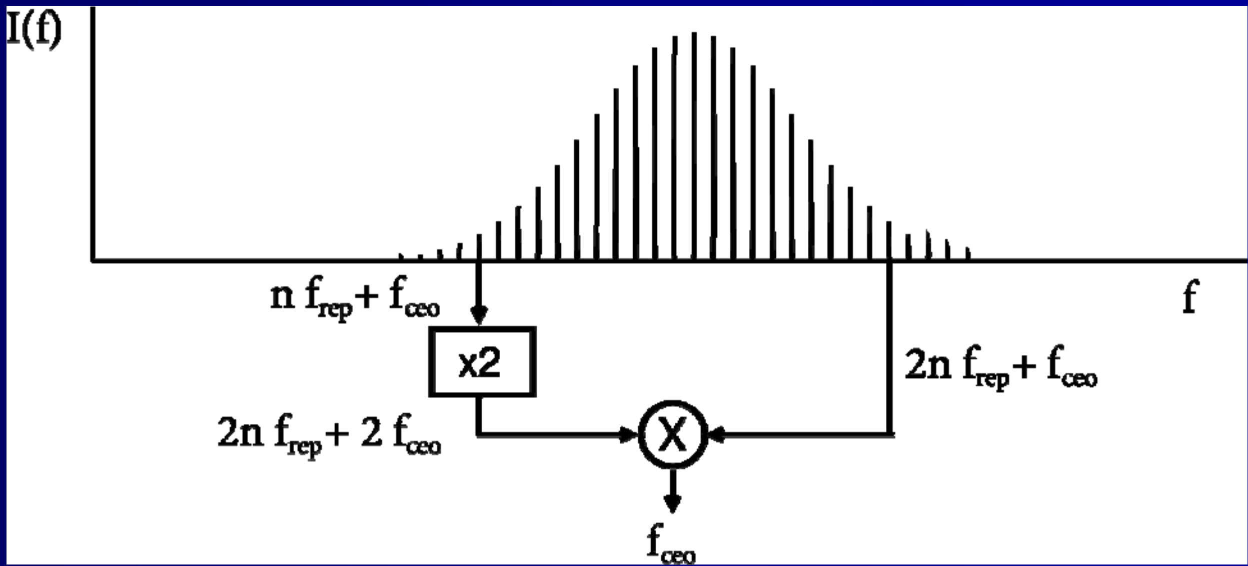
Both f_{rep} and f_{ceo} are in the radio-frequency domain
➔ can be detected using RF electronics.

Measuring f_{rep} is straightforward: **Counting**



Detection of f_{ceo}

Measuring f_{ceo} is more difficult, requires production of a beat signal between a high-f comb mode and the SHG of a low-f comb mode.



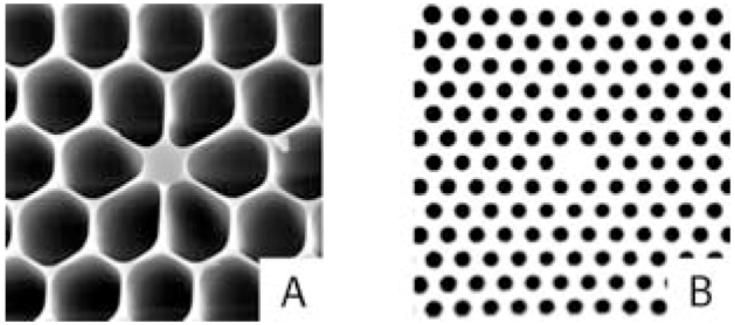
f:2f interferometer

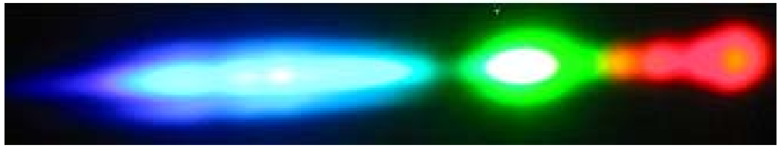
Supercontinuum generation

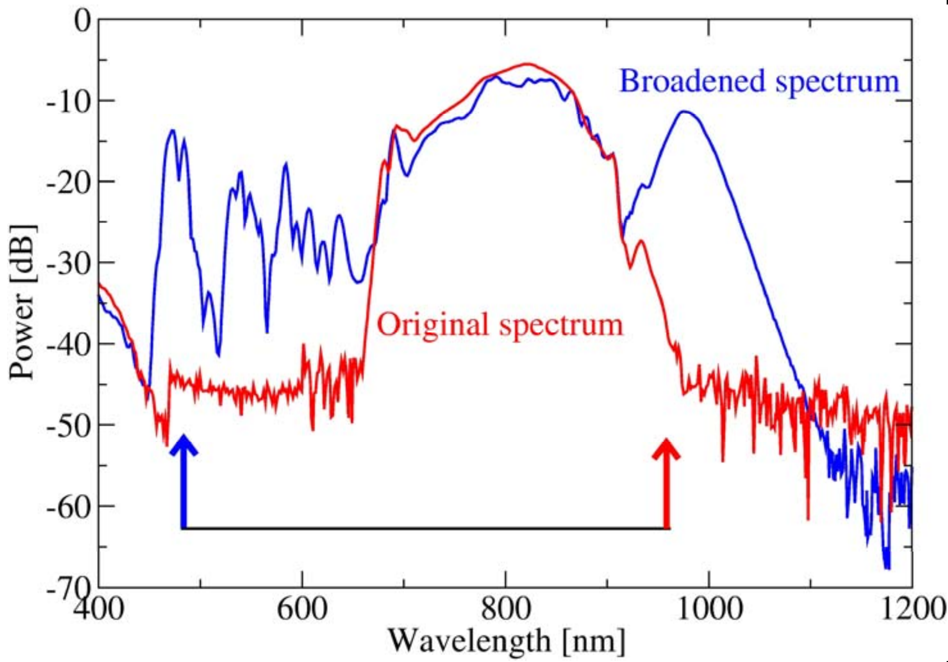
This f-to-2f detection scheme requires an octave-wide spectrum

→ spectral broadening in nonlinear medium

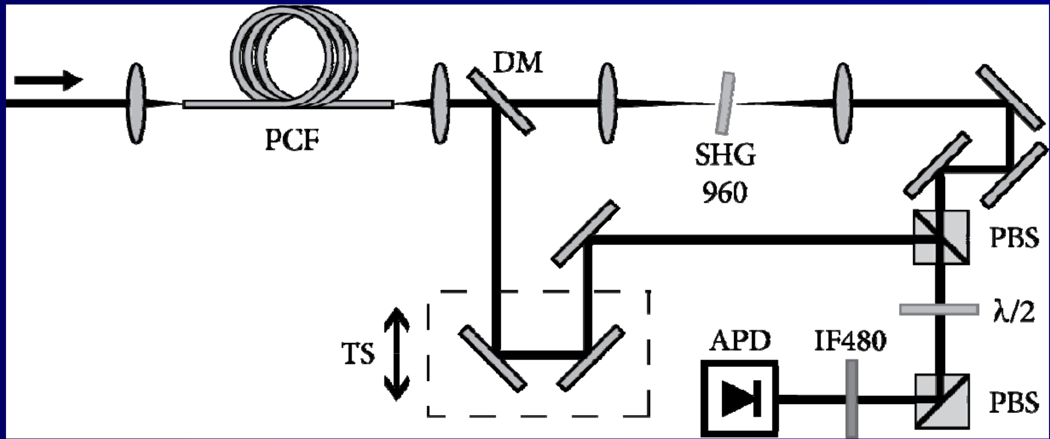
Photonic crystal fiber:



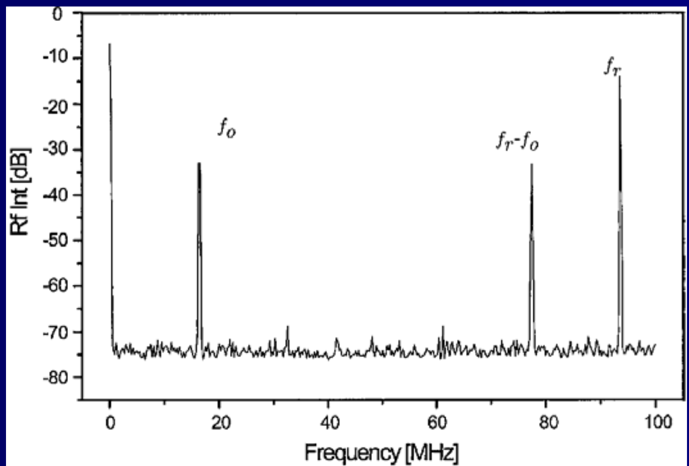




Detection of f_{ceo}



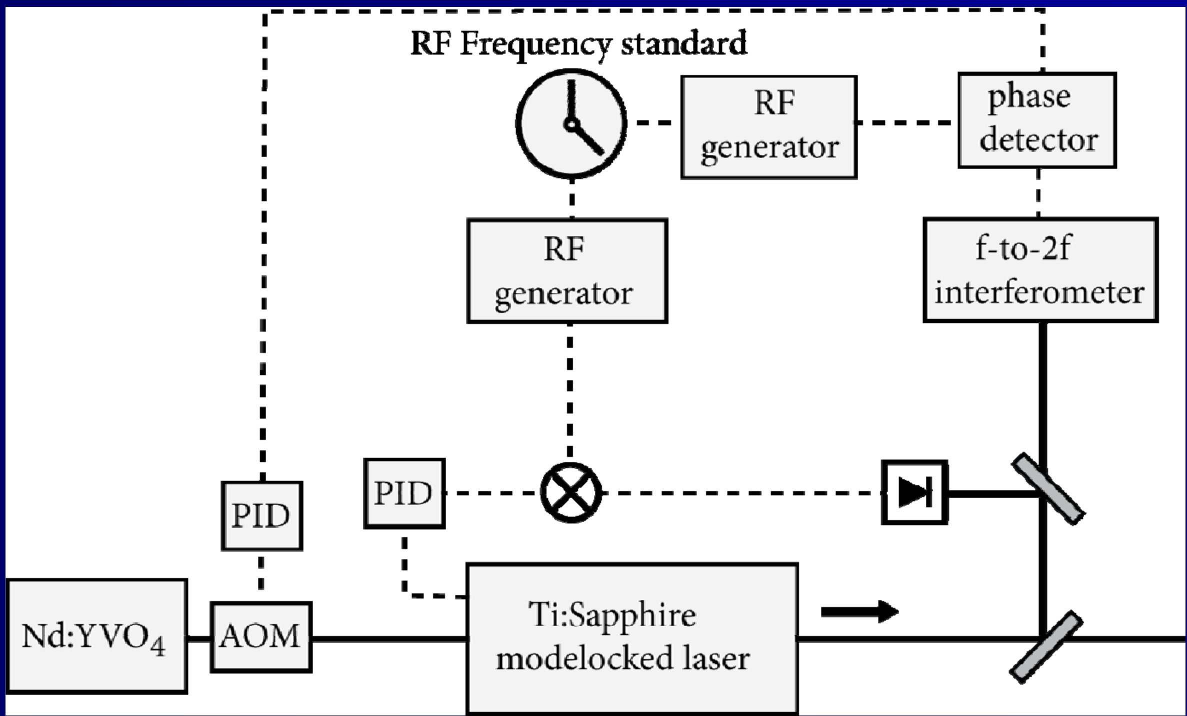
f : 2f



Beat-note measurement
(frequency counter)

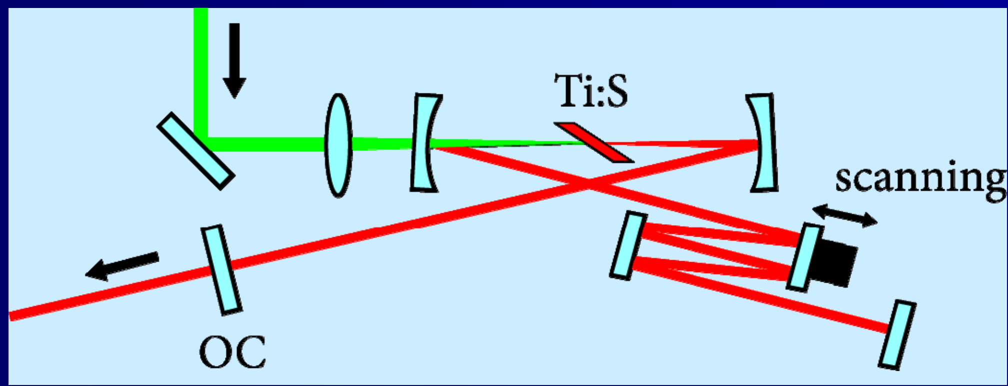
Stabilization of f_{ceo}

- The f-to-2f interferometer output is used in a feedback loop.
- An AOM controls the pump power to stabilize f_{ceo}



Scanning of f_{rep}

- Linear cavity required for long-range scanning
- Multiple reflections on single mirror to increase scan range

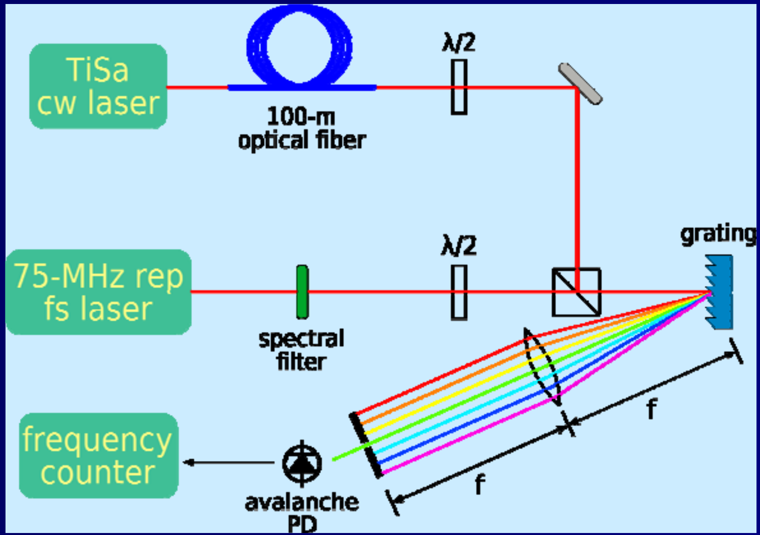


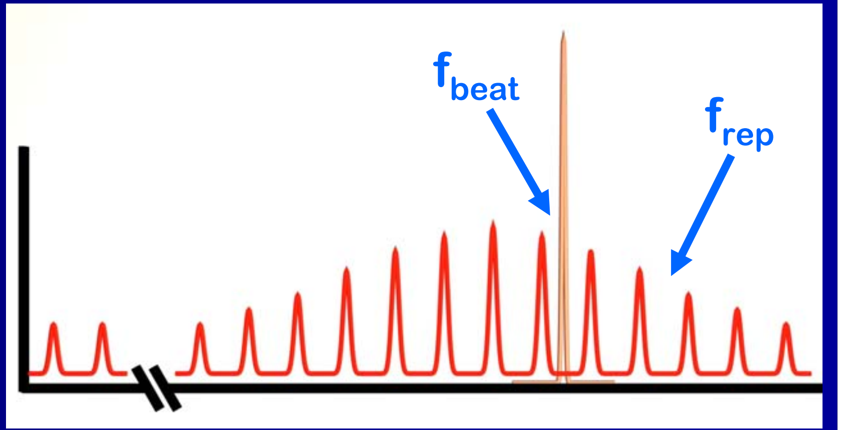
Scan range determined by:

- Cavity stability range
- Alignment sensitivity

A frequency comb as a calibration tool
for "spectroscopy laser"

The frequency of a laser can directly be determined by beating it with the nearest frequency comb mode:

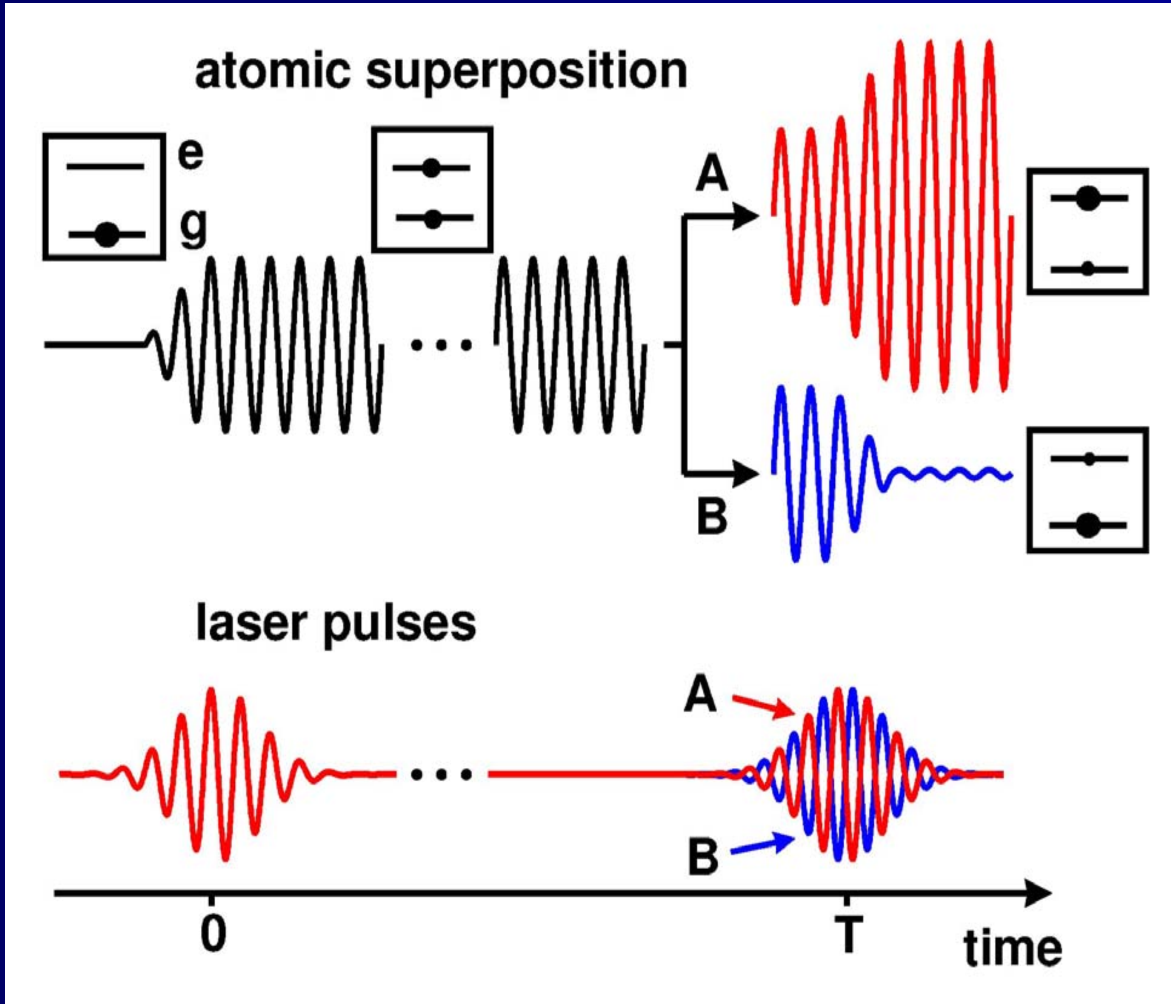




$$f_{laser} = n\, f_{rep} + f_{ceo} + f_{beat}$$

Cf: Hänsch and co-workers: atomic hydrogen

Direct frequency comb spectroscopy



Time-domain Ramsey spectroscopcy

Full control over pulse timing required

Cf :
Ramsey spectroscopy
Atomic fountain clocks

QM analysis of pulse sequences

Wave function of two-level atom: $|\psi\rangle = \begin{pmatrix} c_e \\ c_g \end{pmatrix}$

From Schrödinger equation, and some approximations (dipole, rotating wave) the upper state density can be calculated for two-pulse sequence:

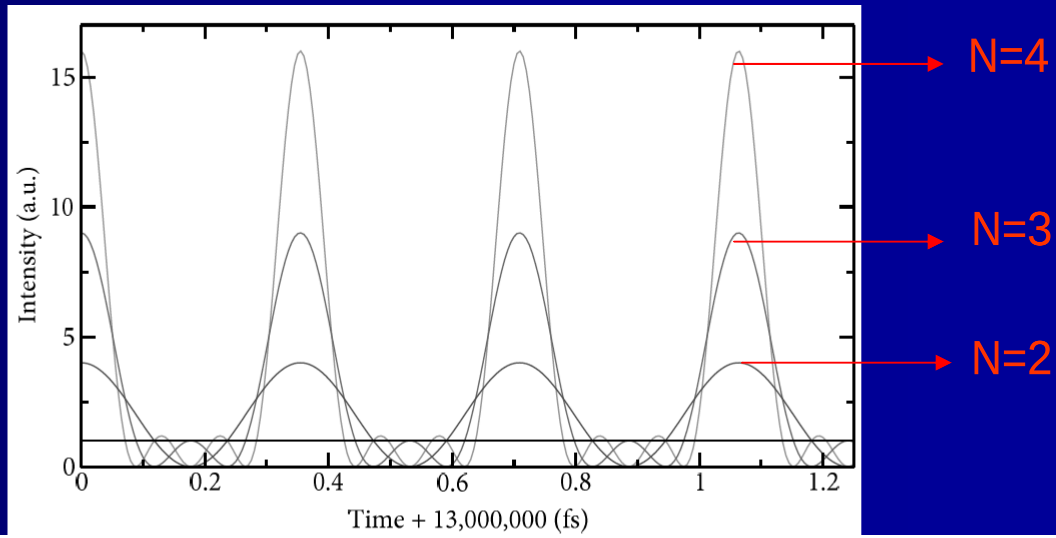
$$|c_{e2}|^2 = 4|c_{e1}|^2 \cos^2\left(\frac{\omega_0 T + \varphi}{2}\right)$$

T is time between pulses
ϕ is difference in f_{ceo} between pulses

For N pulses: $$|c_{eN}|^2 = N^2|c_{e1}|^2 \left|\sum_{n=0}^{N} e^{in(\omega_0 T+\varphi)}\right|^2$$

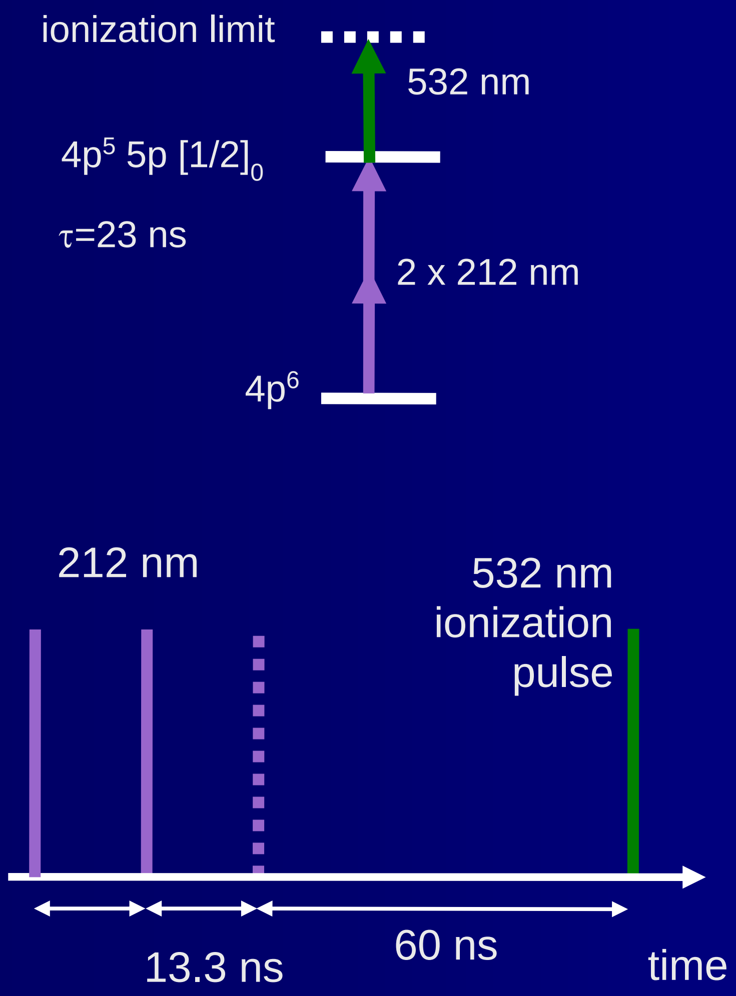
Excited state population

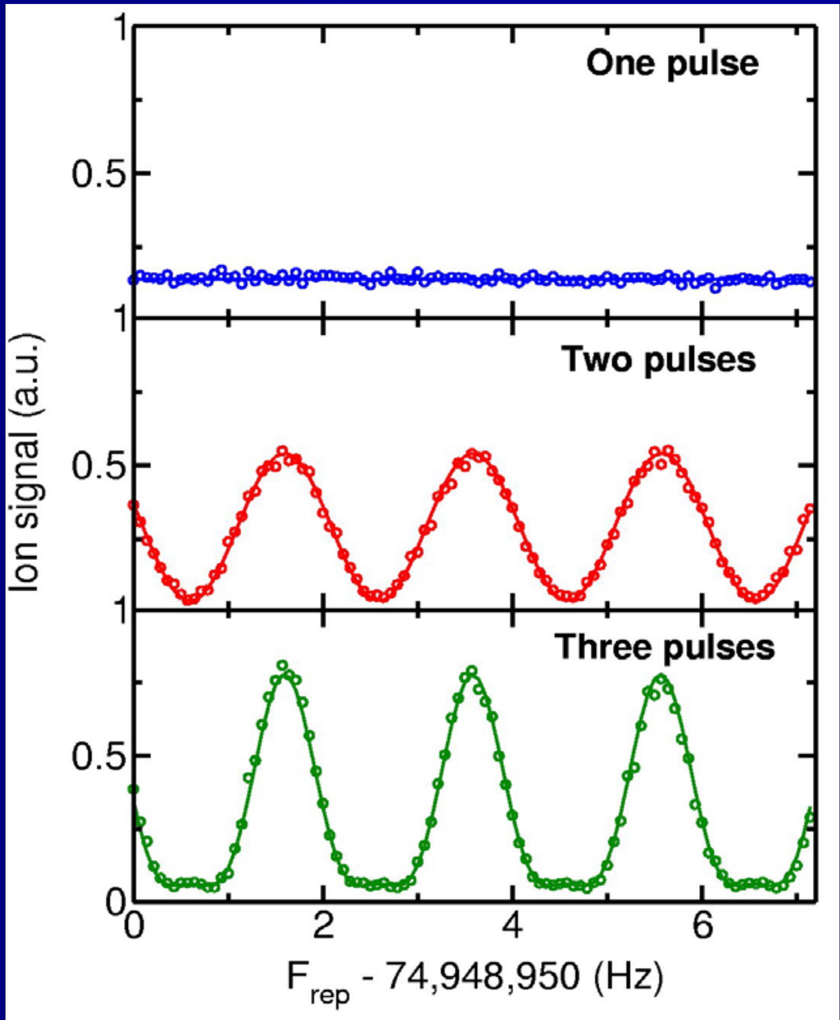
“the comb superimposed onto the atom”



Feasibility experiment in deep-UV (Kr atom)

With amplification in Titanium:Sapphire
(Amplification == Phase control)



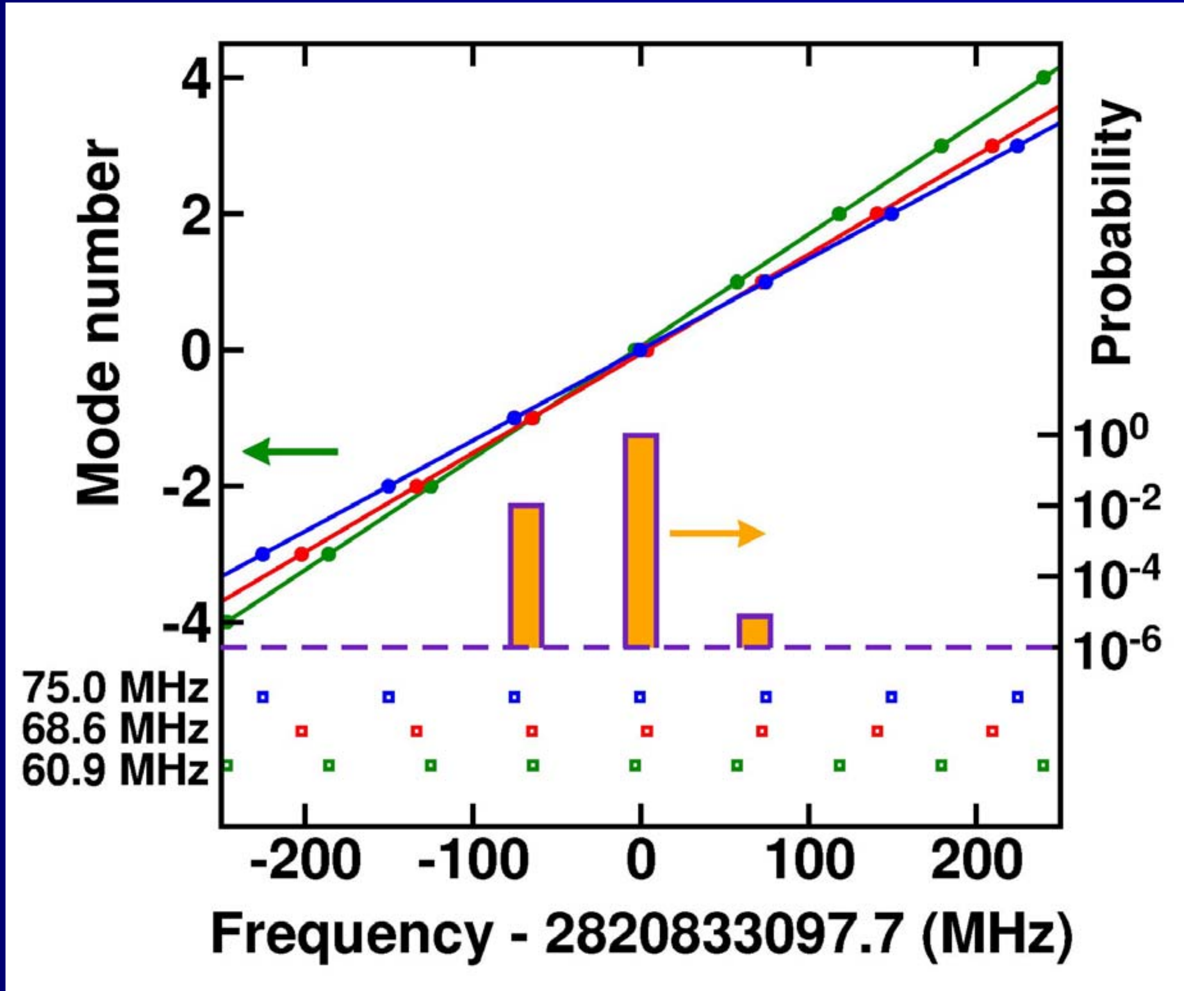


Problem with frequency comb calibration:
mode ambiguity

^{84}Kr: $4p^6$ – $4p^5$ 5p $[1/2]_0$

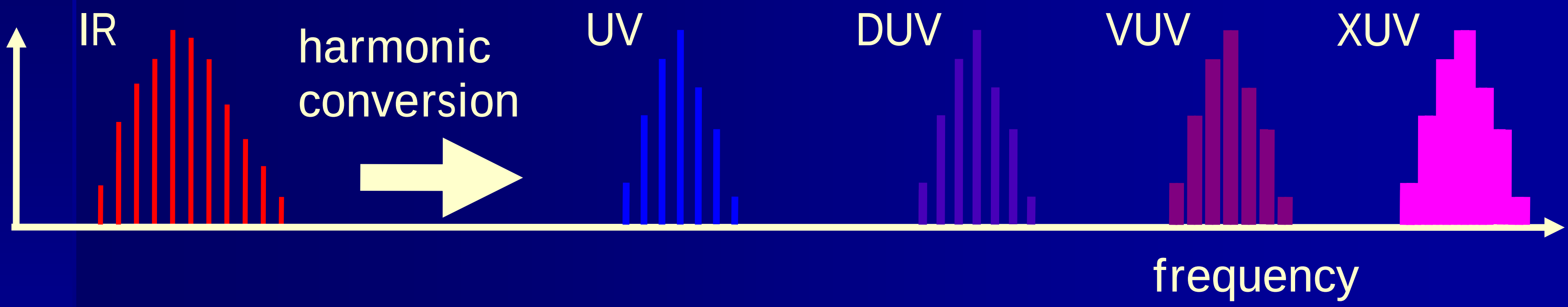
3.5 MHz accuracy with THz bandwidth laser pulses

Cavity length



Combs in the VUV and beyond

Harmonic conversion

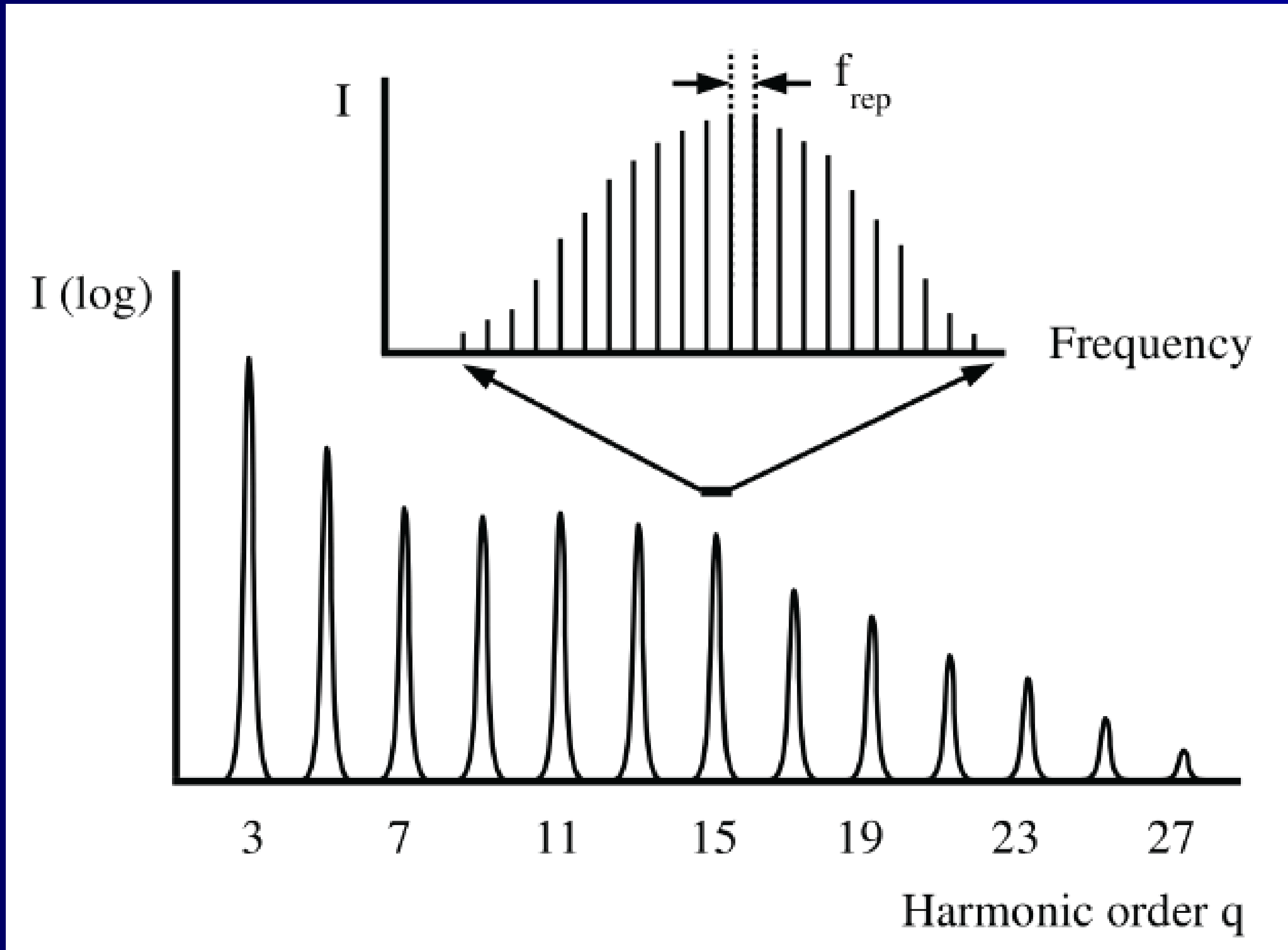


frequency comb = high power pulses = 'easy' harmonic generation

combination of high peak power and accuracy

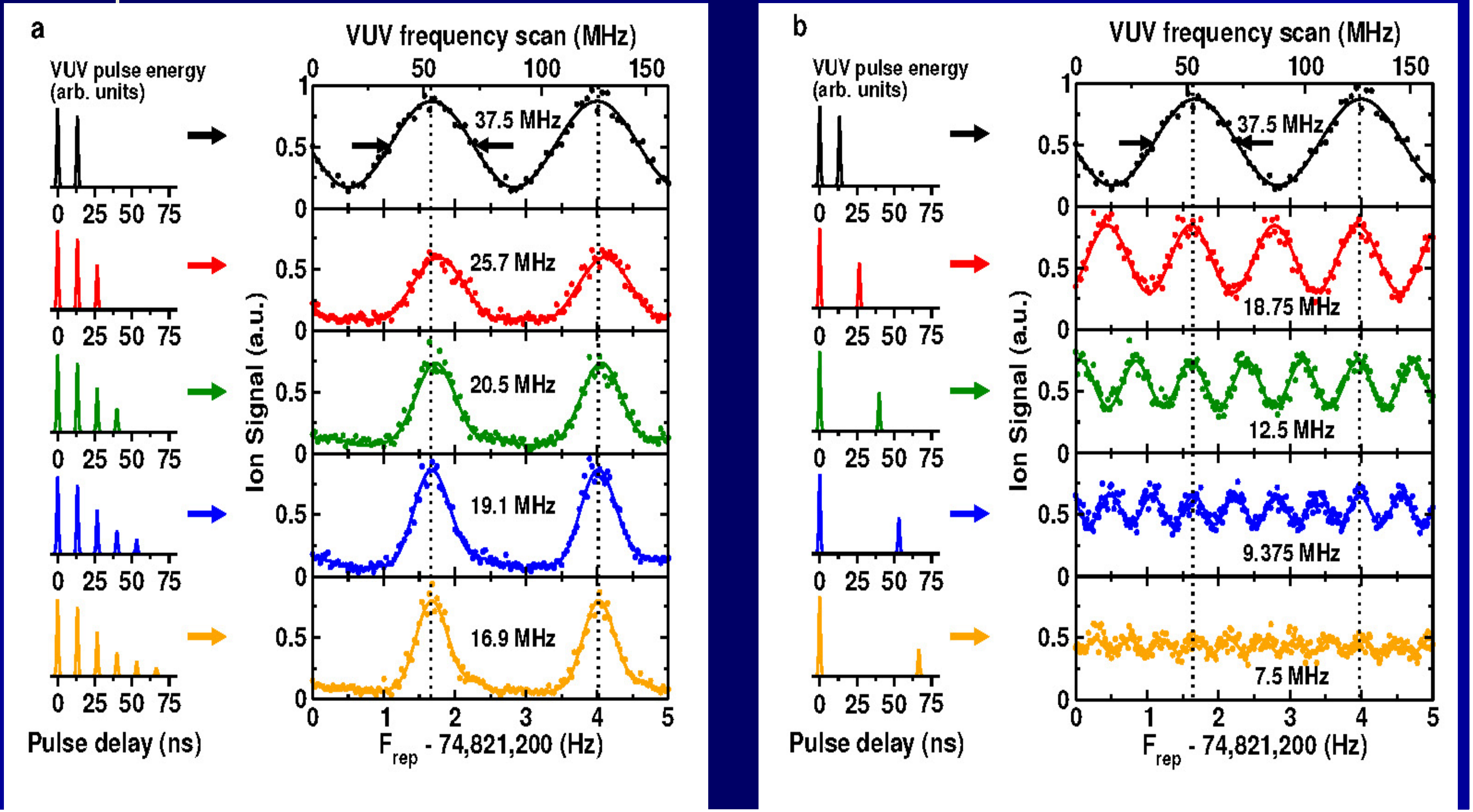
Combs in the VUV and beyond

Comb is retained in harmonics due to pulse structure
Phase control/measurement is the crucial issue

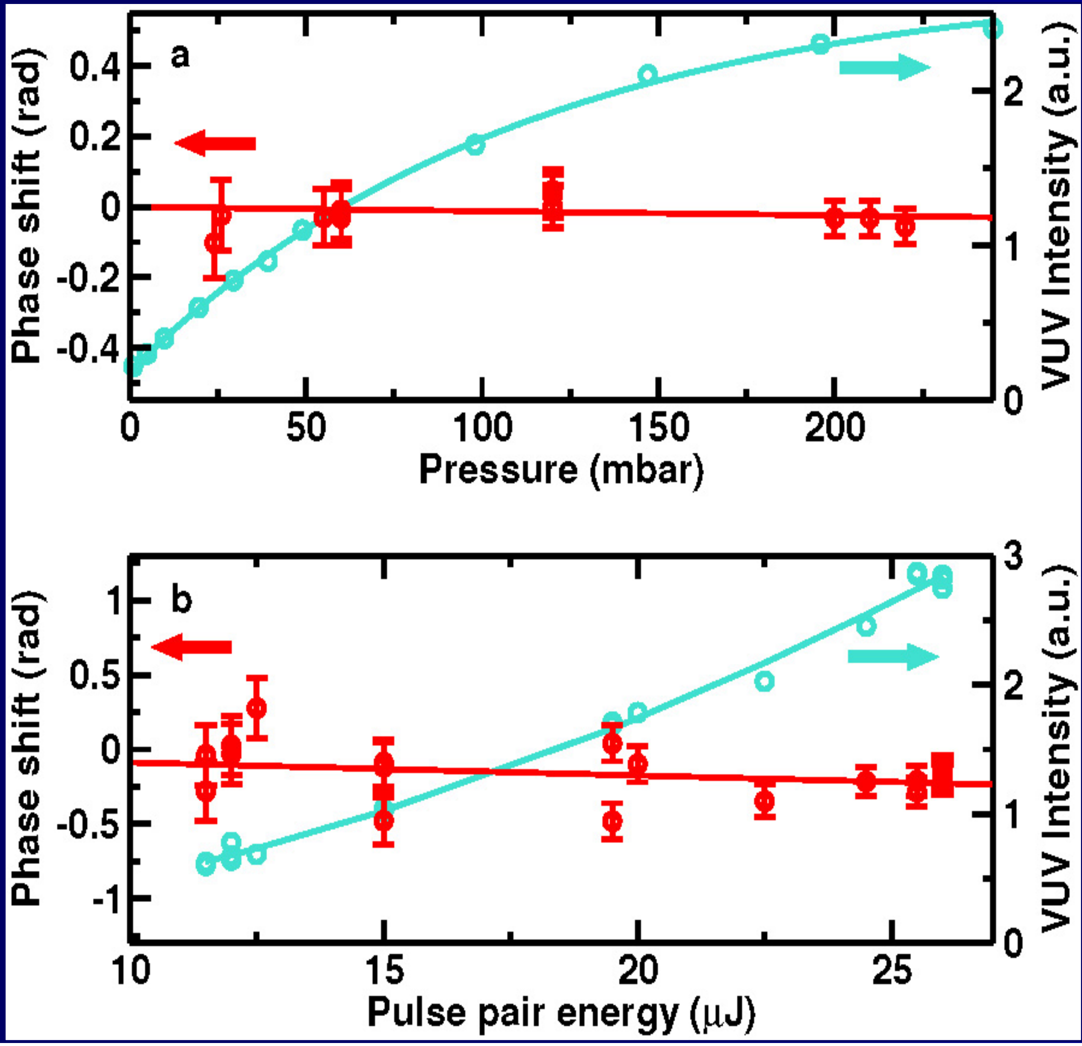


Measurements at the 7th harmonic (of Ti:Sa)

Probing Xe ($5p^6 \rightarrow 5p^5 5d$) at 125 nm (Vacuum ultraviolet frequency comb)



Phase stability (between pulses) in the VUV (effect on relative phase)



O_2 pressure dependence:
-0.12 (0.29) mrad/mbar=
-1.5(3.4) kHz/mbar

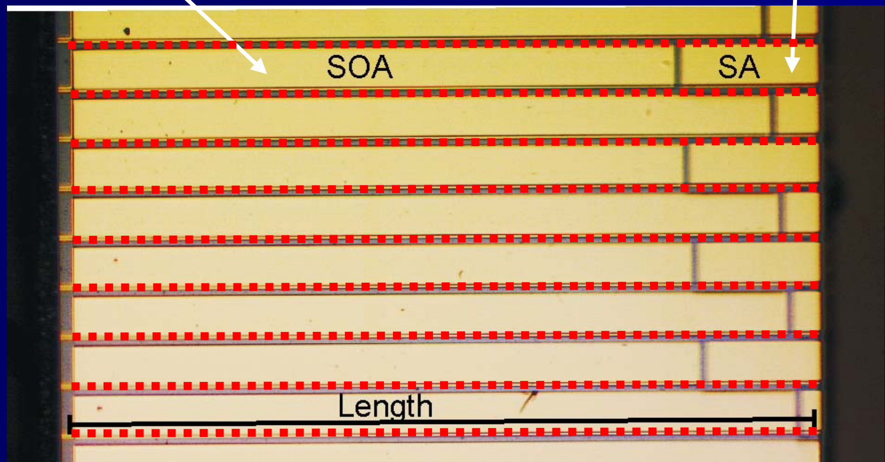
UV dependence:
-8.7(5.8) mrad/μJ =
-104(70) kHz/μJ

Novel development:
Miniaturisation of frequency comb lasers

I

Needle probes

-V



~1 cm

Mode-locked diode lasers
InP quantum dot material

Result from hybrid modelocking