Atomic Physics' starting point: the old Bohr model

Niels Bohr
Nobel prize 1922

Solution to radiative instability of the atom:
- atom exists in a discrete set of stationary states
- radiative transitions → quantum jumps between levels

$$h\nu = \frac{hc}{\lambda} = E_i - E_f$$

- angular momentum is quantized

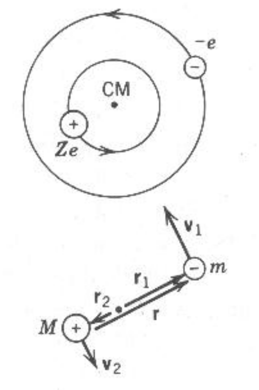
$$L = \mu \mathrm{v} r = n\hbar$$

Quantized level energies

$$E_n = -\frac{Z^2}{n^2} R_H \qquad \text{with} \qquad R_H = \frac{\mu}{m_e} R_\infty$$

Reduced mass in two-particle problem: $$\mu = \frac{m_e M}{m_e + M}$$

Reduced mass in the old Bohr model → the one particle problem



Relative coordinates:

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

Centre of Mass

$$m\vec{r}_1 + M\vec{r}_2 = 0$$

Position vectors:

$$\vec{r}_1 = \frac{M}{m+M}\vec{r}$$

$$\vec{r}_2 = -\frac{m}{m+M}\vec{r}$$

Velocity vectors:

$$\vec{v}_1 = \frac{M}{m+M}\vec{v}$$

$$\vec{v}_2 = -\frac{M}{m+M}\vec{v}$$

Relative velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Kinetic energy

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}\mu v^2$$

Angular momentum

$$L = m_1 v_1 r_1 + m_2 v_2 r_2 = \mu v r$$

With reduced mass

$$\mu = \frac{mM}{m+M}$$

Centripetal force

$$F = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{\mu v^2}{r}$$

The old Bohr model → quantisation of radial motion

Kinetic energy $K = \frac{1}{2}\mu v^2$

Potential energy $V = -\frac{Ze^2}{4\pi\varepsilon_0 r}$

Total energy

$$E = \frac{1}{2}\mu v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

System is bound by the Coulomb force

$$F_{Centripetal} = F_{Coulomb}$$

$$\frac{\mu v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

Total energy (bound system)

$$E = \left(\frac{1}{2} - 1\right)\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r}$$

Quantisation of angular momentum:

$$L = \mu v r = n\frac{h}{2\pi} = n\hbar$$

$$v^2 r^2 = \frac{Ze^2}{4\pi\varepsilon_0}\frac{r}{\mu} = \frac{n^2\hbar^2}{\mu^2}$$

Quantisation of radius in orbit:

$$r_n = \frac{n^2}{Z}\frac{4\pi\varepsilon_0\hbar^2}{e^2\mu} = \frac{n^2}{Z}\frac{m_e}{\mu}a_0$$

With Bohr radius a_0

The old Bohr model → quantisation of energy

$$E = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

$$r_n = \frac{n^2}{Z}\frac{4\pi\varepsilon_0\hbar^2}{e^2\mu} = \frac{n^2}{Z}\frac{m_e}{\mu}a_0$$

$$E_n = -\frac{Z^2}{n^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{\mu}{2\hbar^2}$$

$$E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}R_\infty$$

Rydberg constant

$$R_\infty = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{m_e}{2\hbar^2} = E_0$$

Energy levels in the Bohr model:

$$E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}R_\infty$$

Rewrite:

$$E_n = -\frac{Z^2}{n^2}\left(\frac{\mu}{m_e}\right)\frac{1}{2}\alpha^2 m_e c^2$$

dimensionless energy

$$\alpha \approx \frac{1}{137}$$

The old Bohr model → energy levels

$$E_n = -\frac{Z^2}{n^2}\left(\frac{\mu}{m_e}\right)\frac{1}{2}\alpha^2 m_e c^2$$

dimensionless energy

Energy scale in the molecule

$$R_\infty = \frac{1}{2}\alpha^2 m_e c^2$$

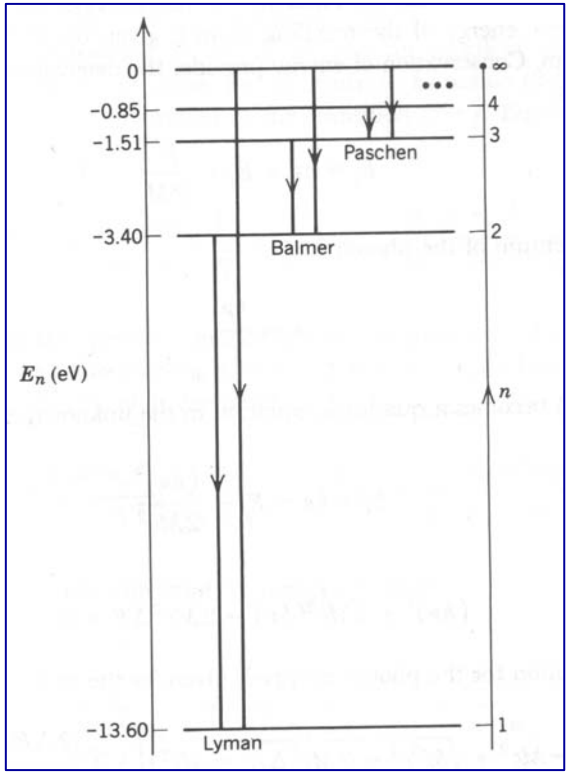
13.60 eV

109737.31534 cm^{-1}

Atomic Rydberg constant

$$R_H = \left(\frac{\mu}{m_e}\right)R_\infty$$

Energy levels and Bohr "quantum jumps"



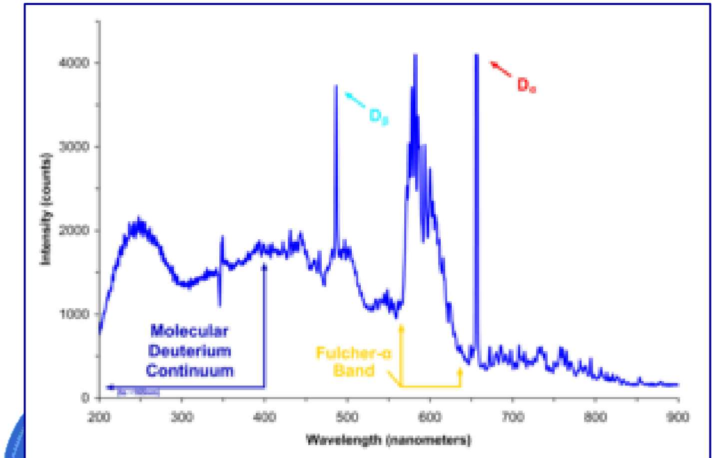
The old Bohr model : explains Balmers and Rydberg laws

Johann Jakob Balmer

$$\lambda = \frac{h\ m^2}{m^2 - n^2}$$

Janne Rydberg

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$



Succes of the Bohr model restricted to atomic hydrogen

Extension by Sommerfeld

Generalisation: isotope shifts and exotic atoms

$$E_n = -\frac{Z^2}{n^2}\left(\frac{\mu}{m_e}\right)\frac{1}{2}\alpha^2 m_e c^2$$

$$R_H = \left(\frac{\mu}{m_e}\right)R_\infty$$

$$\mu = \frac{mM}{m+M}$$

The isotope shift

For all atoms: $\mu \approx m_e$

Exotic atoms:

Muonium $(\mu^+ e^-)$

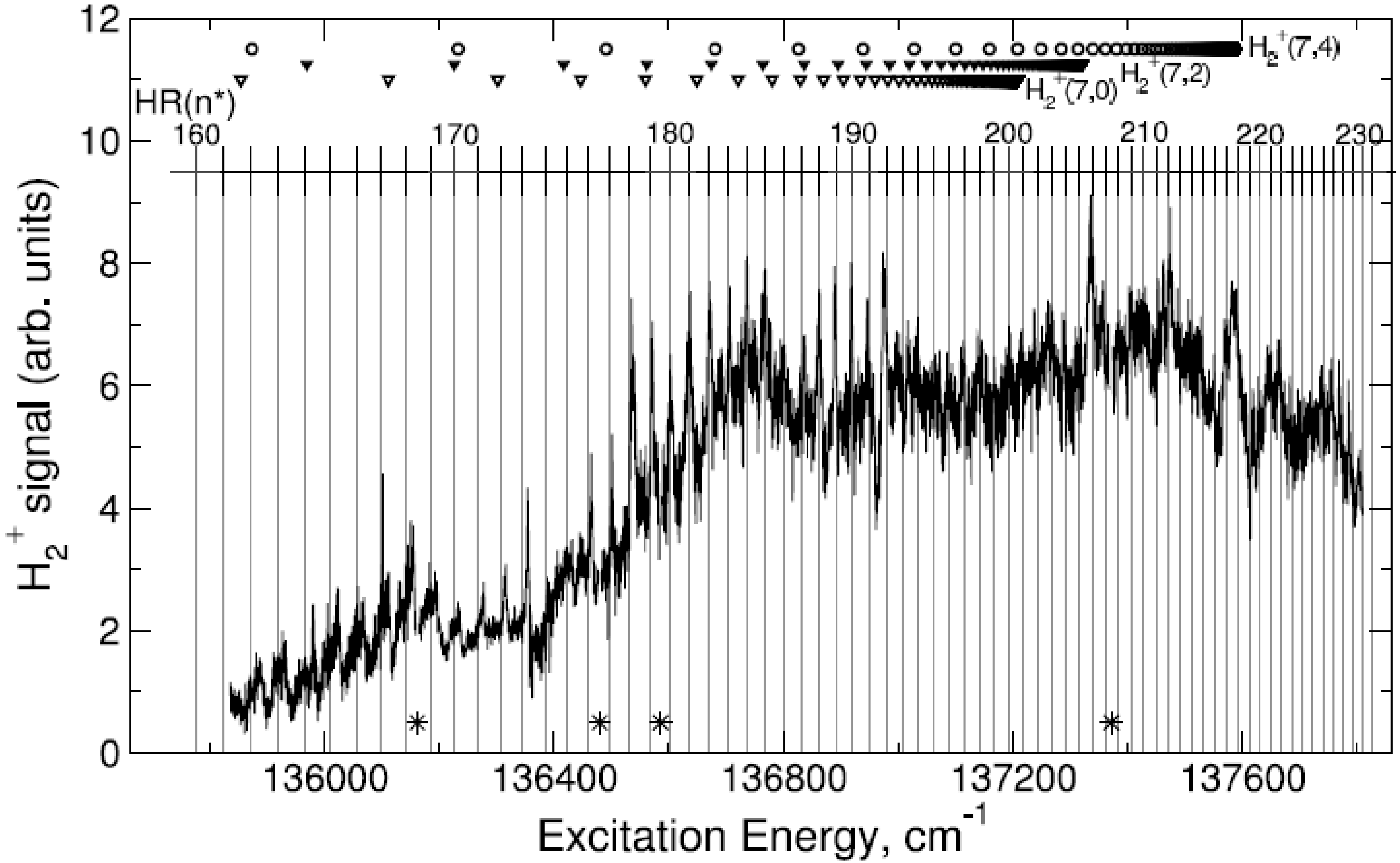
Positrinium $(e^+ e^-)$

Protonium $(p^+ \overline{p}^-)$

Ion-pair states in molecules? $(H^+ H^-)$

The heavy Bohr atom: H^+H^-

$$R_H = \left(\frac{\mu}{m_e}\right) R_\infty = 918 R_\infty$$



Generalisation: the gravitational perspective

Gravitational potential

$$V = -G\frac{M_1 M_2}{r}$$

Coorespondence

$$\frac{Ze^2}{4\pi\varepsilon_0} \leftrightarrow GM_1 M_2$$

Total energy

$$E_g = -\frac{1}{2} G M_1 M_2 \frac{1}{r}$$

Quantized radius for a gravitational system

$$r_n^g = \frac{n^2\hbar^2}{\mu}\frac{1}{GM_1 M_2} = n^2\left(\frac{M_1}{\mu}\right)a_g$$

Quantized energies for a gravitational system

$$E_n = -\frac{1}{n^2}\left(\frac{\mu}{M_1}\right)R_\infty^g$$

Gravitational Rydberg constant

$$R_\infty^g = \left(GM_1 M_2\right)^2 \frac{M_1}{2\hbar^2}$$

Gravitation, example: Sun-Earth system

Parameters

$$M_1 = 5.98 \times 10^{24} kg$$

$$M_2 = 1.989 \times 10^{30} kg$$

$$R_{orbit} = 150 \times 10^9 m$$

$$G = 6.673 \times 10^{-11} \frac{m^3}{s^2 kg}$$

$$\hbar = 1.055 \times 10^{-34} \frac{kgm^2}{s}$$

Calculate

$$GM_1 M_2 = 7.9 \times 10^{44} \frac{kgm^3}{s^2}$$

Bohr radius

$$a_g = 2 \times 10^{-138} m$$

Principal quantum number for given orbit:

$$n = 2.7 \times 10^{74}$$

Rydberg constant

$$R_\infty^g = 1.7 \times 10^{182} J$$

Energy quanta

$$\Delta E_n = \frac{2}{n^3} R_\infty^g = 2 \times 10^{-41} J$$

Gravitation, further

Calculate the Rydberg constant
for attraction between
two hydrogen atoms

$$R_{eff}^{g} = 2.62 \times 10^{-87} J = 6.60 \times 10^{-65} cm^{-1}$$

"Lyman-alpha" transition: n=1 → n=2

$$\nu = 1.48 \times 10^{-54} Hz$$

"Bohr radius"

$$a_g^H = 3.5 \times 10^{22} m = 3.7 \times 10^7 ly$$

How about a neutron in the
field of the Earth:
Space quantisation possible ?

Nesvizhevsky et al

Nature 415, 297 (2002)