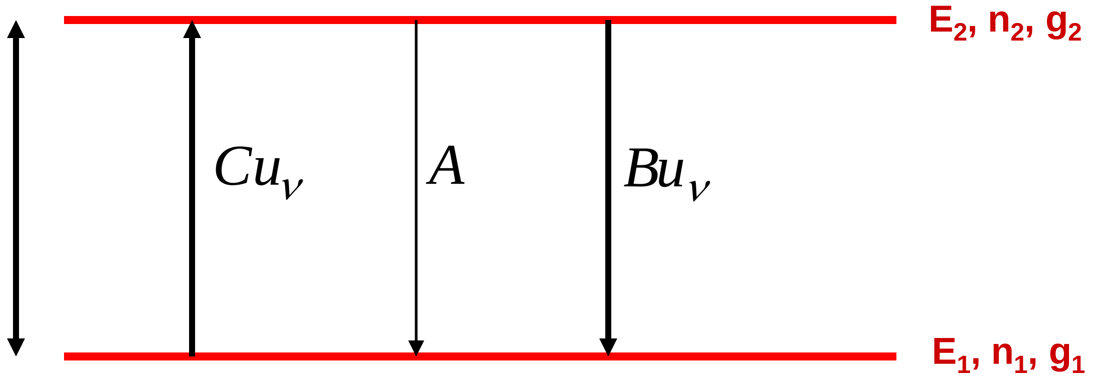
Einstein Rate Equation Model

Radiation in a two-level system

$$E_2 - E_1 = h\nu$$



u_ν **the incident radiation field (mode density)**

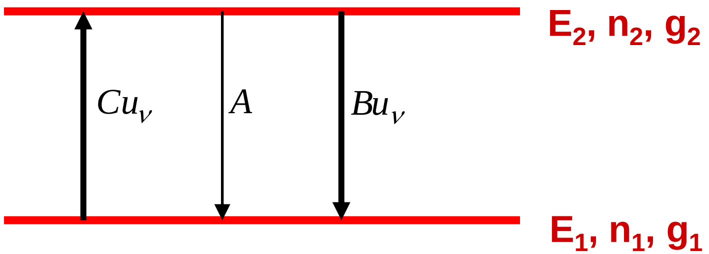
Cu_ν **1. Absorption process**

A **2. Emission process**

Bu_ν **3. Stimulated emission process**

A, B, C are the Einstein coefficients

Rate equation model



Population of states:

$$\frac{dn_2}{dt} = Cu_\nu n_1 - An_2 - Bu_\nu n_2$$

Impose an equilibrium condition

$$\frac{dn_2}{dt} = 0$$

In steady state:

$$\frac{n_1}{n_2} = \frac{A + Bu_\nu}{Cu_\nu}$$

Statistical Physics: thermal excitation

$$n(T) = \exp(-E/kT)$$

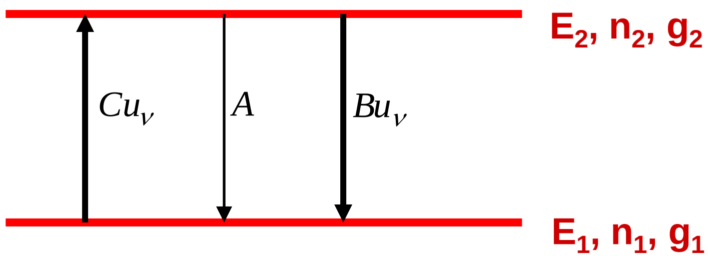
Relative population:

$$\frac{n_2}{n_1} = \frac{\exp(-E_2/kt)}{\exp(-E_1/kt)} = \exp\left[\frac{h\nu}{kT}\right]$$

Atomic two-level system in equilibrium with radiation field:

$$u_\nu = \frac{A}{C\exp[h\nu/kT] - B}$$

Results from the Einstein model



A two-level atom in equilibrium with a radiation field:

$$C = B$$

Stimulated emission is equally strong as absorption

The B constant related to the Transition dipole moment (the QM radiation strength)

The A-constant follows from

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

Properties of a two-level system

Under all circumstances, i.e.
for arbitrary radiation fields u_ν

$$Cu_\nu < A + Bu_\nu$$

Emission is stronger than absorption

So if we start at

$$n_1(0) = N$$

(all population in the ground state)

It is not possible to reach:

$$n_2 > n_1$$

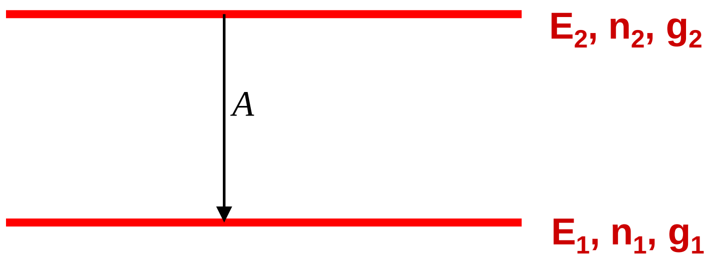
↓

Optical pumping can never result in a population inversion

↓

A two-level LASER is impossible

Decay of an excited state



In the absence of a radiation field:

$$u_\nu = 0$$

Rate equation reduces:

$$\frac{dn_2}{dt} = -An_2$$

With a boundary condition;

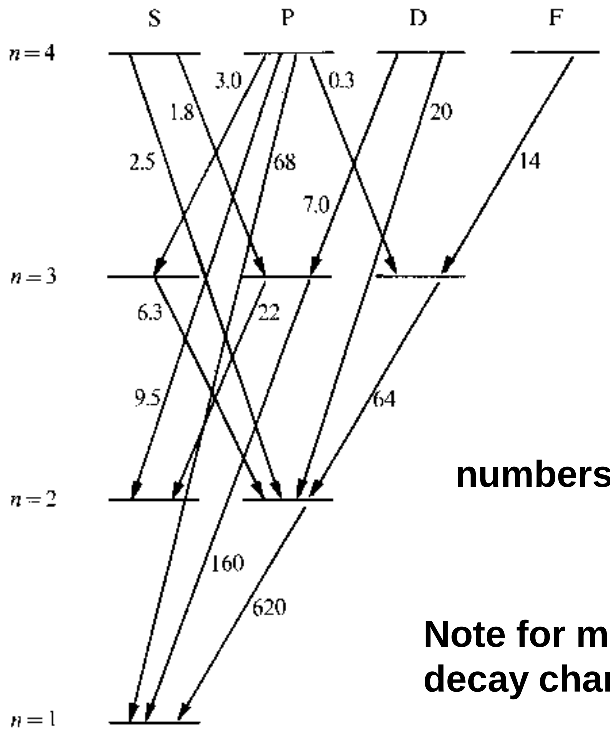
$$n_2(0) = N$$

Solution:

$$n_2(t) = Ne^{-At} = Ne^{-t/\tau}$$

Lifetime of an excited state

$$\tau = \frac{1}{A}$$



numbers in 10^6 s^{-1}

Note for multiple decay channels

$$\tau = \frac{1}{\sum_i A_i}$$

Quantum states and kinetics

Hydrogen at 20°C.

Estimate the average kinetic energy of whole hydrogen atoms (not just the electrons) at room temperature, and use the result to explain why nearly all H atoms are in the ground state at room temperature, and hence emit no light.

$$\overline{K} = \frac{3}{2} k_B T = 6.2 \times 10^{-21} J = 0.04 eV$$

Gas at elevated temperature – probability exited state is populated:

$$P_n(T) = e^{-E_n / kT}$$

Energy in gas:

$$\langle E \rangle = \frac{\sum_n E_n e^{-E_n / kT}}{\sum_n e^{-E_n / kT}}$$