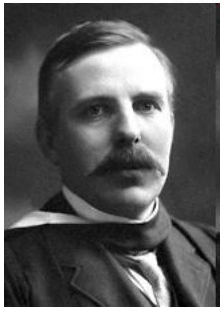
Physics of the Atom



Ernest
Rutherford

The Nobel Prize in Chemistry 1908
"for his investigations into the
disintegration of the elements,
and the chemistry of radioactive substances"

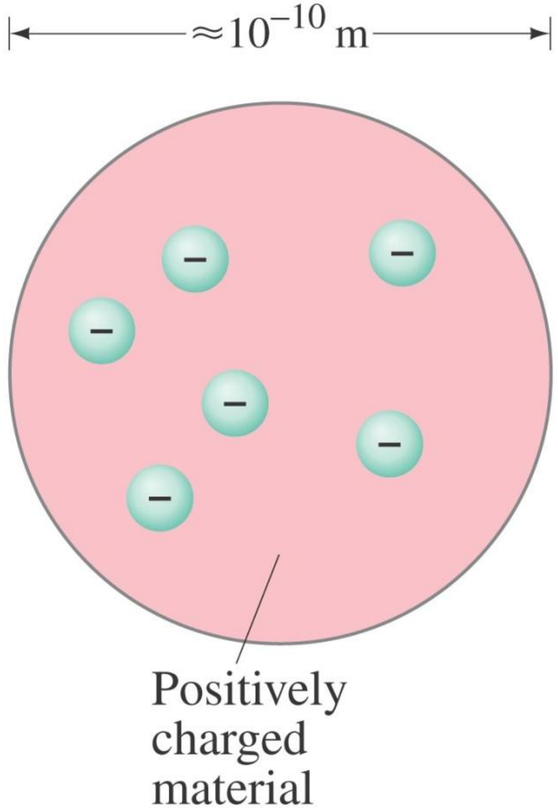
Niels Bohr

The Nobel Prize in Physics 1922
"for his services in the investigation
of the structure of atoms and
of the radiation emanating from them"

Early Models of the Atom

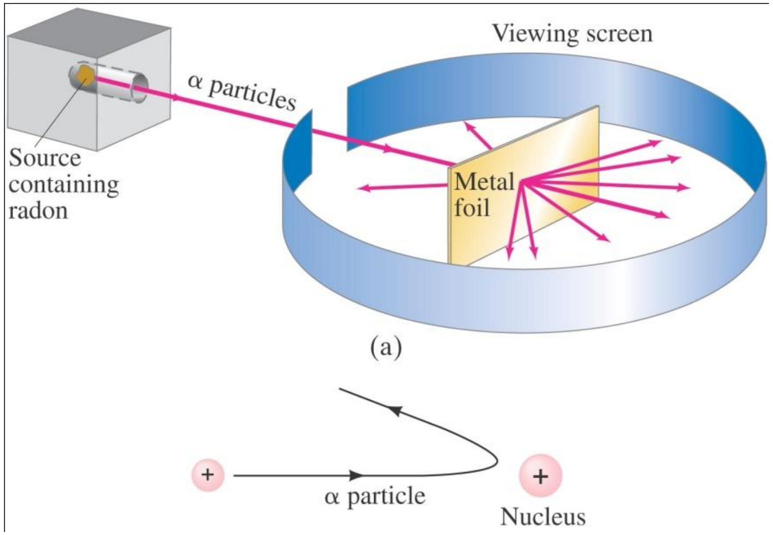
atoms : electrically neutral
they can become charged
positive and negative charges are around
and some can be removed.

popular atomic model
"plum-pudding" model:



Rutherford scattering

Rutherford did an experiment that showed that the positively charged nucleus must be extremely small compared to the rest of the atom.

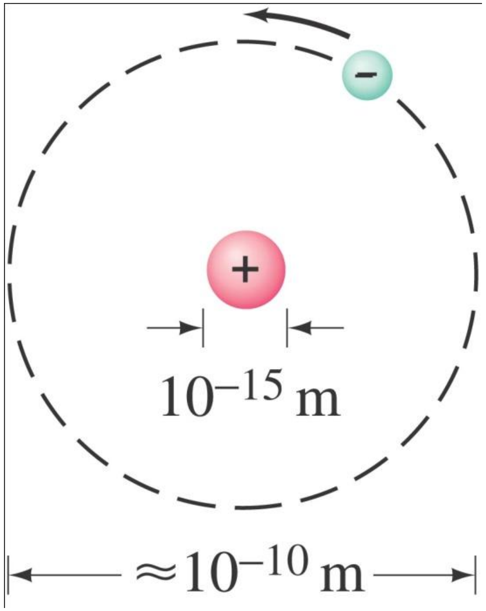


Result from Rutherford scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \frac{Zze^2}{4K} \right)^2 \frac{1}{\sin^4(\theta/2)}$$

Applet for doing the experiment:
http://www.physics.upenn.edu/courses/gladney/phys351/classes/Scattering/Rutherford_Scattering.html

Rutherford scattering
the smallness of the nucleus



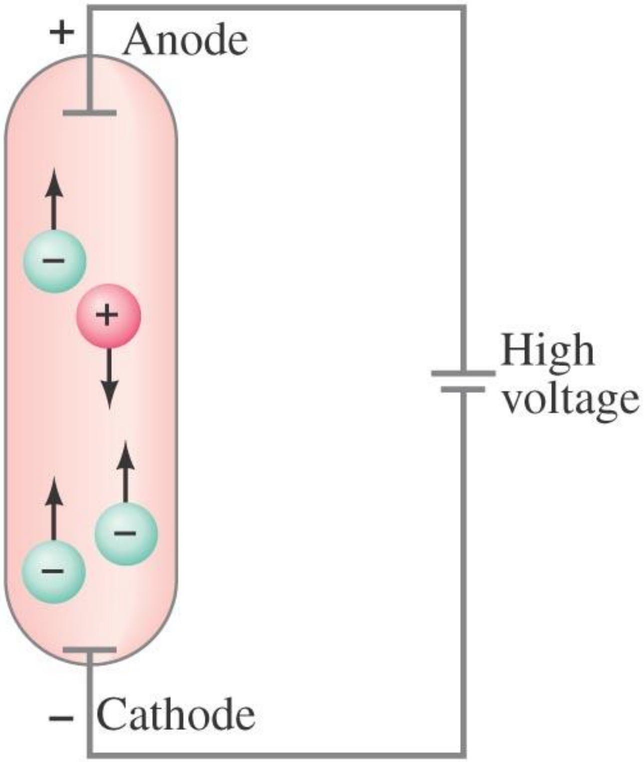
Rutherford's atomic model

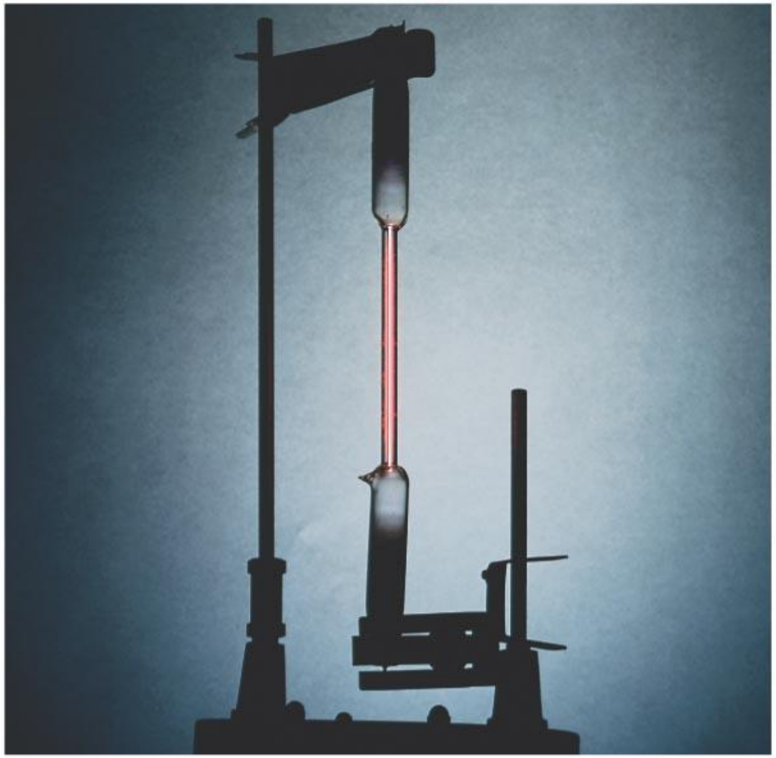
the radius of the nucleus is 1/10,000 that of the atom.

the atom is mostly empty space

Atomic Spectra:
Key to the Structure of the Atom

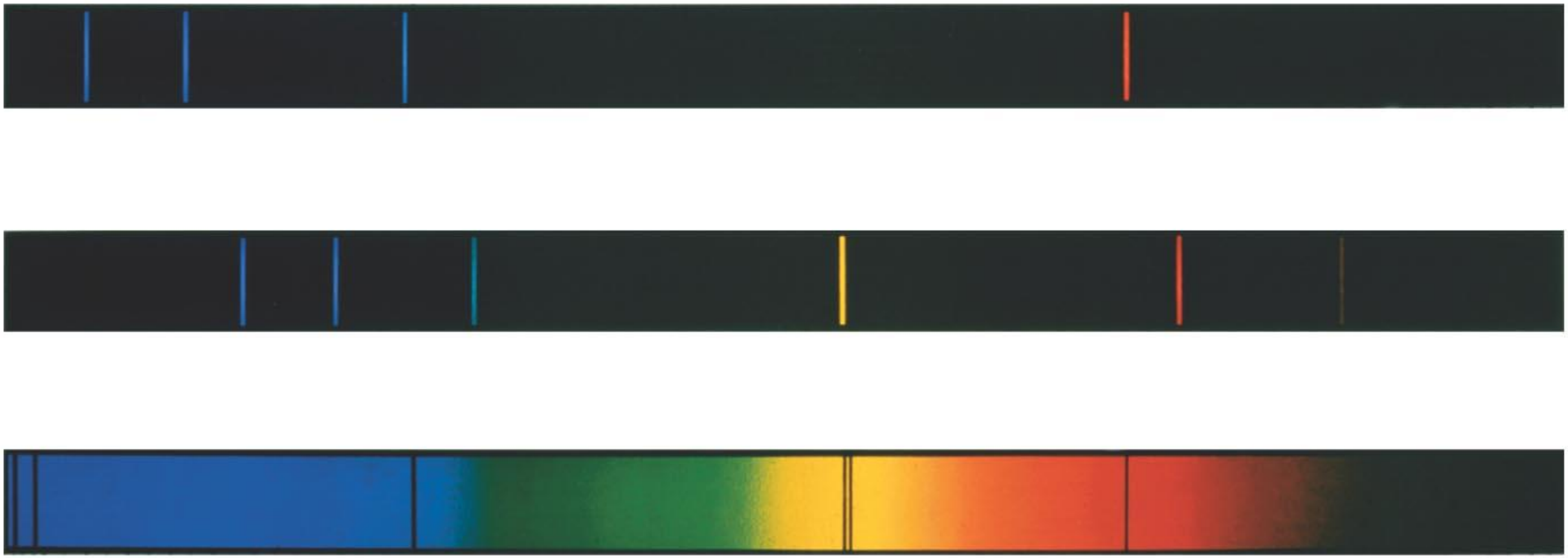
A very thin gas heated in a discharge tube emits light only at characteristic frequencies.





Atomic Spectra:
Key to the Structure of the Atom

Line spectra: *absorption and emission*



The Balmer series in atomic hydrogen

Johann Jakob Balmer

The wavelengths of electrons emitted from hydrogen have a regular pattern:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad n = 3, 4, \cdots.$$

Lyman, Paschen and Rydberg series

the Lyman series:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \qquad n = 2, 3, \cdots.$$

the Paschen series:

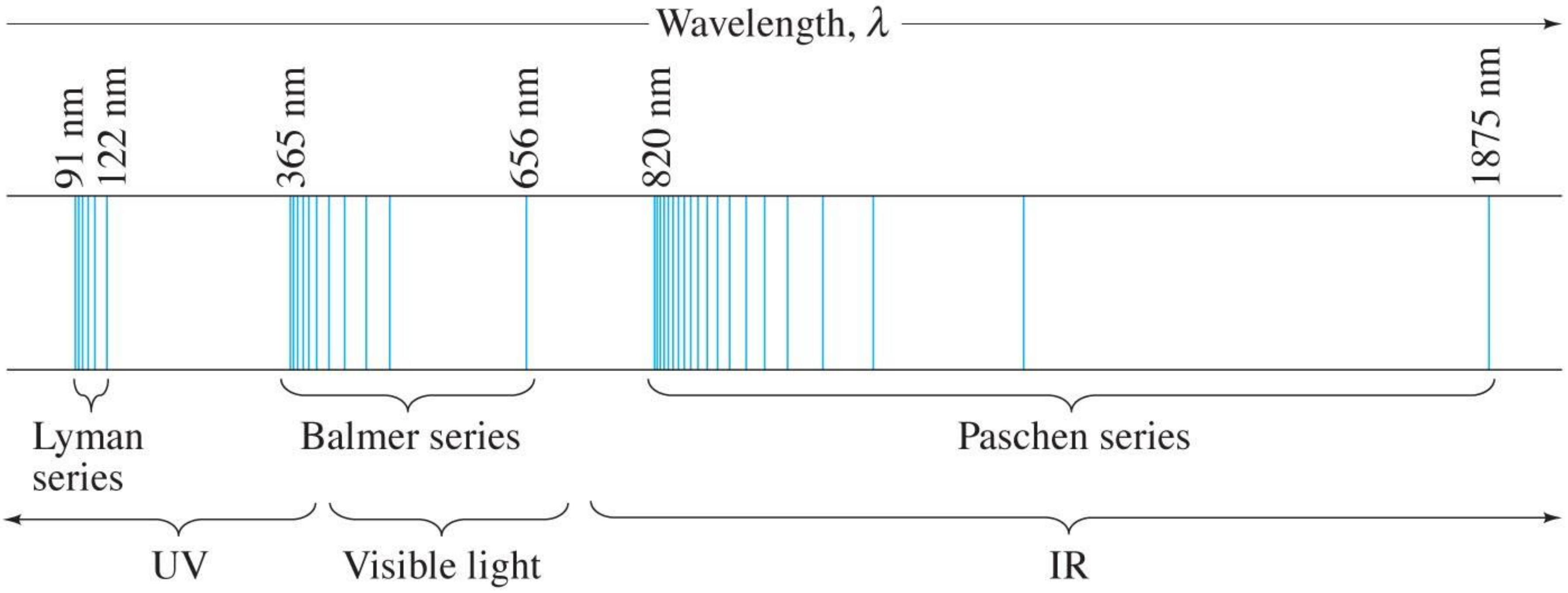
$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \qquad n = 4, 5, \cdots.$$

Janne Rydberg

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The Spectrum of the hydrogen Atom

A portion of the complete spectrum of hydrogen is shown here. The lines cannot be explained by classical atomic theory.



The Bohr Model

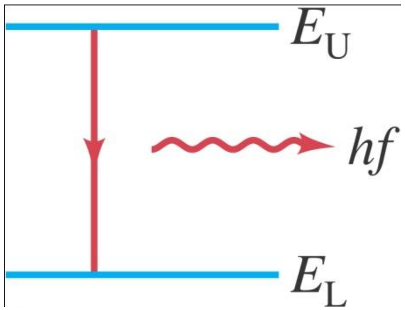
Solution to radiative instability of the atom:

- atom exists in a discrete set of stationary states

no radiation when atom is in such state

- radiative transitions → quantum jumps between levels

$$h\nu = \frac{hc}{\lambda} = E_i - E_f$$



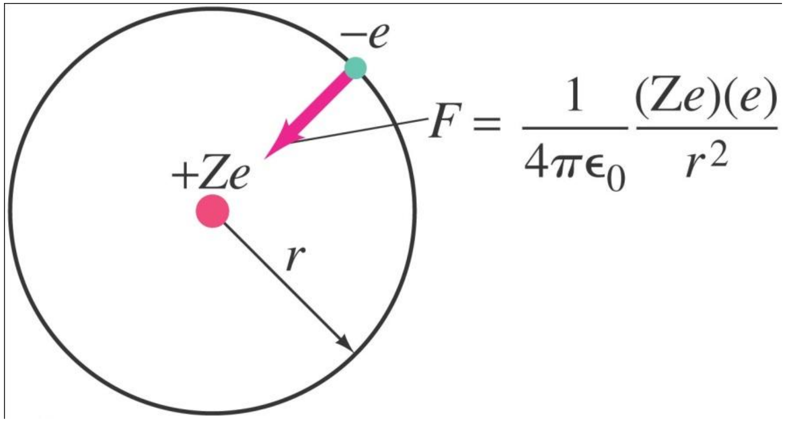
- angular momentum is quantized

$$L = mvr_n = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \cdots.$$

These are *ad hoc* hypotheses by Bohr, against intuitions of classical physics

The Bohr Model: derivation

An electron is held in orbit by the Coulomb force: (equals centripetal force)



$$F_{Centripetd} = F_{Coulomb}$$

$$\frac{mv^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

$$v^2 r^2 = \frac{Ze^2}{4\pi\varepsilon_0}\frac{r}{m} = \frac{n^2\hbar^2}{m^2}$$

Bohrs postulate:
Quantization of angular momentum

$$L = mvr = n\frac{h}{2\pi} = n\hbar$$

$$r_n = \frac{n^2h^2\epsilon_0}{\pi m Z e^2} = \frac{n^2}{Z}r_1 \qquad r_1 = \frac{h^2\epsilon_0}{\pi m e^2} = 0.529 \times 10^{-10}\,\text{m.}$$

The size of the orbit is quantized, and we know the size of an atom !

The Bohr Model: energy

Quantisation of the radius

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} = \frac{n^2}{Z} r_1$$

Quantisation of energy

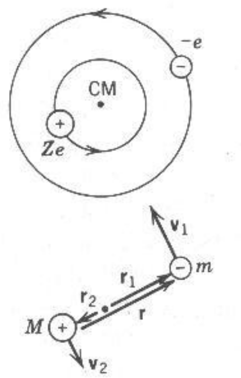
$$E_n = \frac{1}{2} m v^2 - \frac{Z e^2}{4\pi\varepsilon_0 r_n} = -\frac{Z^2}{n^2} R_\infty$$

$$R_\infty = \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{m_e}{2\hbar^2}$$

Rydberg constant

EXTRA

Reduced mass in the old Bohr model → the one particle problem



Relative coordinates:

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

Centre of Mass

$$m\vec{r}_1 + M\vec{r}_2 = 0$$

Position vectors:

$$\vec{r}_1 = \frac{M}{m+M}\vec{r}$$

$$\vec{r}_2 = -\frac{m}{m+M}\vec{r}$$

Velocity vectors:

$$\vec{v}_1 = \frac{M}{m+M}\vec{v}$$

$$\vec{v}_2 = -\frac{M}{m+M}\vec{v}$$

Relative velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Kinetic energy

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}\mu v^2$$

Angular momentum

$$L = m_1 v_1 r_1 + m_2 v_2 r_2 = \mu v r$$

With reduced mass

$$\mu = \frac{mM}{m+M}$$

Centripetal force

$$F = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{\mu v^2}{r}$$

Quantisation of angular momentum:

$$L = \mu v r = n\frac{h}{2\pi} = n\hbar$$

Problem is similar, but

m → μ

r relative coordinate

EXTRA

Reduced mass in the old Bohr model → isotope shifts

Results

Quantisation of radius in orbit:

$$r_n = \frac{n^2}{Z}\frac{4\pi\varepsilon_0\hbar^2}{e^2\mu} = \frac{n^2}{Z}\frac{m_e}{\mu}a_0$$

Energy levels in the Bohr model:

$$E_n = -\frac{Z^2}{n^2}\left(\frac{\mu}{m_e}\right)R_\infty$$

Rydberg constant:

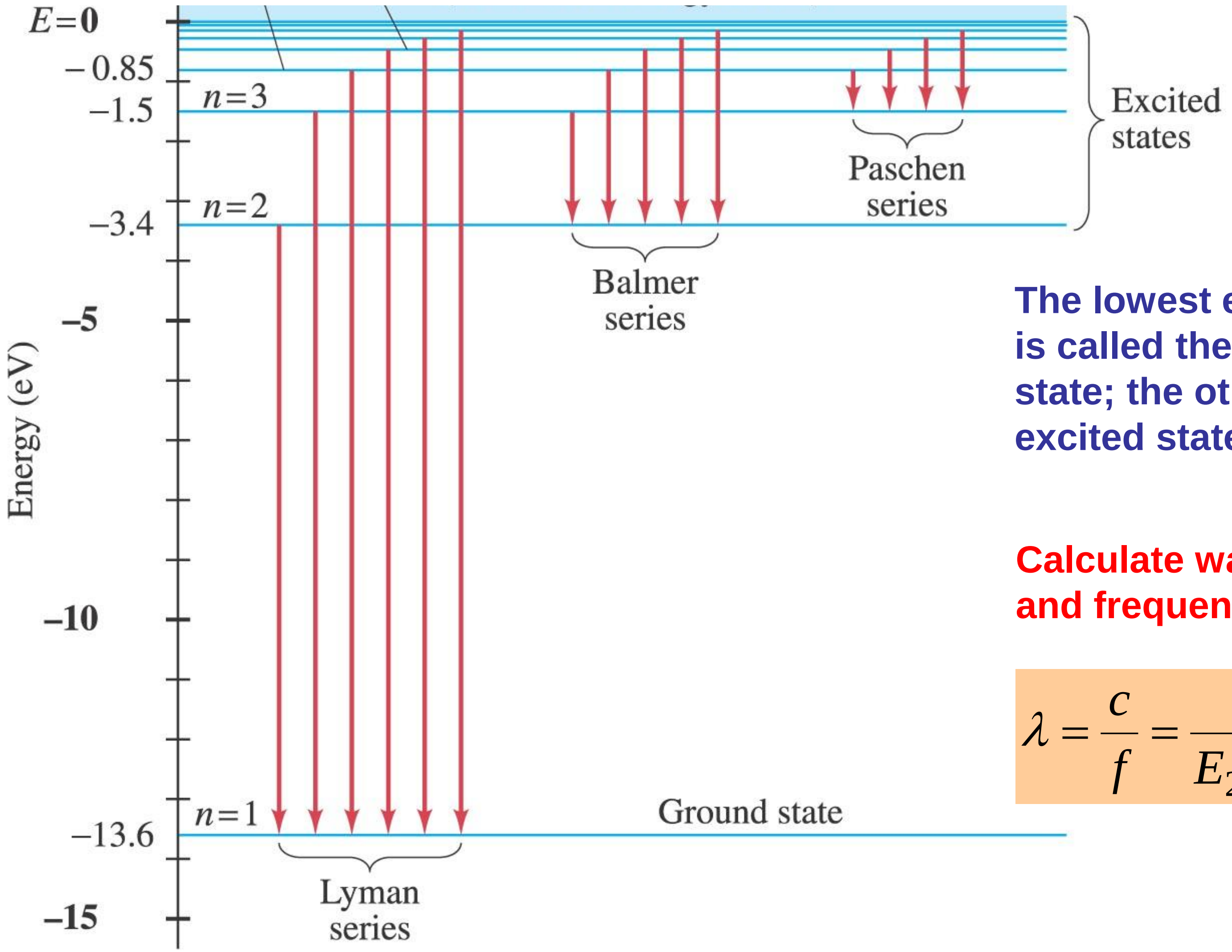
$$R_H = \left(\frac{\mu}{m_e}\right)R_\infty$$

Calculate the isotope shift on an atomic transition

e.g. In H-atom Lyman-alpha

(n=2) → (n=1)

Optical transitions in The Bohr Model



The lowest energy level is called the ground state; the others are excited states.

Calculate wavelengths and frequencies

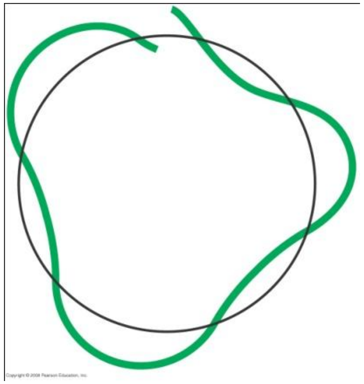
$$\lambda = \frac{c}{f} = \frac{hc}{E_2 - E_1}$$

de Broglie's Hypothesis Applied to Atoms

de Broglie relation

$$\lambda = \frac{h}{p}$$

Electron of mass m has a wave nature



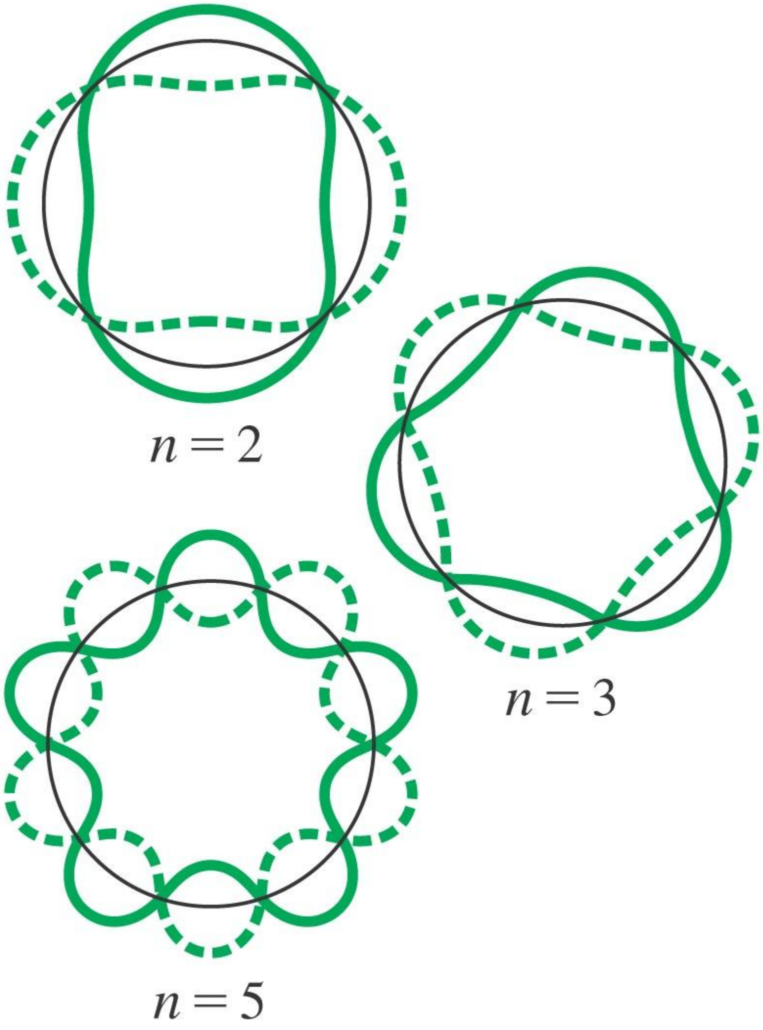
Electron in orbit: a standing wave

$$2\pi r_n = n\lambda$$

Substitution gives the quantum condition

$$L = mvr_n = \frac{nh}{2\pi}$$

de Broglie's Hypothesis Applied to Atoms



These are circular standing waves for n = 2, 3, and 5.

Standing waves do not radiate;
Interpretation:
electron does not move
(no acceleration)