Nuclear Reactions

Fission
Fusion

Nuclear Reactions and the Transmutation of Elements

A nuclear reaction takes place when a nucleus is struck by another nucleus or particle. Compare with chemical reactions !

If the original nucleus is transformed into another, this is called transmutation.

$$ {}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H $$

α-induced

Atmospheric reaction.

$$ n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + p $$

n-induced

Deuterium production reaction.

$$ n + {}^{16}_{8}O \rightarrow {}^{15}_{7}N + {}^{2}_{1}H $$

Note: natural ↔ “artificial” radioactivity

Nuclear Reactions
and the Transmutation of Elements

Energy and momentum must be conserved in nuclear reactions.

Generic reaction:

$$a + X \rightarrow Y + b$$

The reaction energy, or Q-value, is the sum of the initial masses less the sum of the final masses, multiplied by c^2:

$$Q = (M_a + M_X - M_b - M_Y)c^2.$$

If Q is positive, the reaction is exothermic, and will occur no matter how small the initial kinetic energy is.

If Q is negative, there is a minimum initial kinetic energy that must be available before the reaction can take place (endothermic).

Chemistry: Arrhenius behaviour (barriers to reaction)

Nuclear Reactions and the Transmutation of Elements

A slow neutron reaction:

$$n + {}^{10}_{5}B \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

is observed to occur even when very slow-moving neutrons (mass M_n = 1.0087 u) strike a boron atom at rest.

Analyze this problem for: v_{He}=9.30 x 10^6 m/s;

Calculate the energy release → Q-factor

This energy must be liberated from the reactants.

(verify that this is possible from the mass equations)

Nuclear Reactions and the Transmutation of Elements

Will the reaction "go"?

$$p + {}^{13}_{6}C \rightarrow {}^{13}_{7}N + n$$

Left: M(13-C) = 13.003355
M(1-H) = 1.007825

Right: M(13-N) = 13.005739
+ M(n) = 14.014404

D(R-L)= 0.003224 u (931.5 MeV/u) = +3.00 MeV (endothermic)

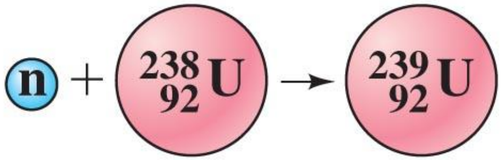
Hence bombarding by 2.0-MeV protons is insufficient

3.0 MeV is required since Q=-3.0 MeV

(actually a bit more; for momentum conservation)

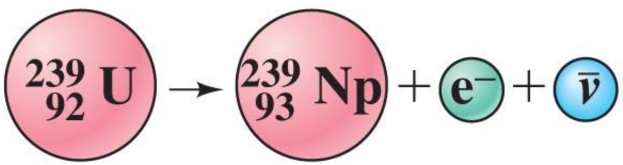
Nuclear Reactions
and the Transmutation of Elements

(a)



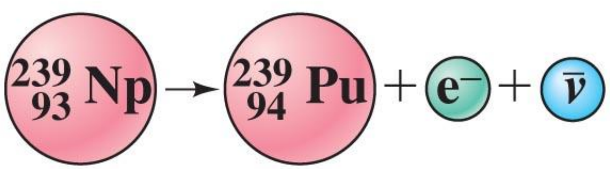
Neutron captured by $^{238}_{92}U$.

(b)



$^{239}_{92}U$ decays by β decay to neptunium-239.

(c)



$^{239}_{93}Np$ itself decays by β decay to produce plutonium-239.

Neutrons are very effective in nuclear reactions, as they have no charge and therefore are not repelled by the nucleus.

239-Pu is fissionable material

Pros and cons of a "breeder reactor"

Cross Section

Universal concept in physics
(e.g. Lambert-Beer law for absorption or Rayleigh law for scattering):

$$I = I_0 e^{-n\sigma\ell}$$

n= density in [cm^{-3}]

σ= cross section in [cm^2]

***l* = path length in [cm^{-3}]**

Different (sub) cross sections

$$\sigma_{\mathrm{T}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{inel}} + \sigma_{\mathrm{R}}$$

***cf* light scattering (Rayleigh, Raman, Compton)**

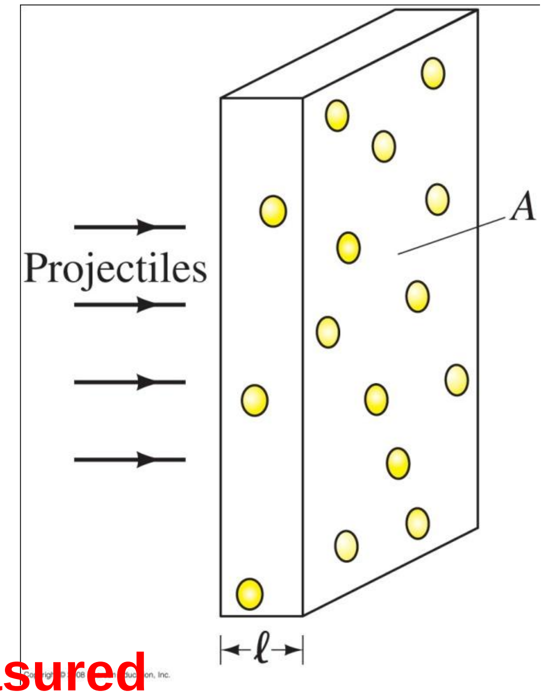
Cross Section

Effective surface $A' = n\sigma\ell A$

Number of targets $n\ell A$
Cross section for each σ

Rate of incident projectiles R_0

Rates of hits $R = R_0 \dfrac{A'}{A} = R_0 n\sigma\ell$



$$\sigma = \frac{R}{R_0 n\ell}$$

cross section can be measured

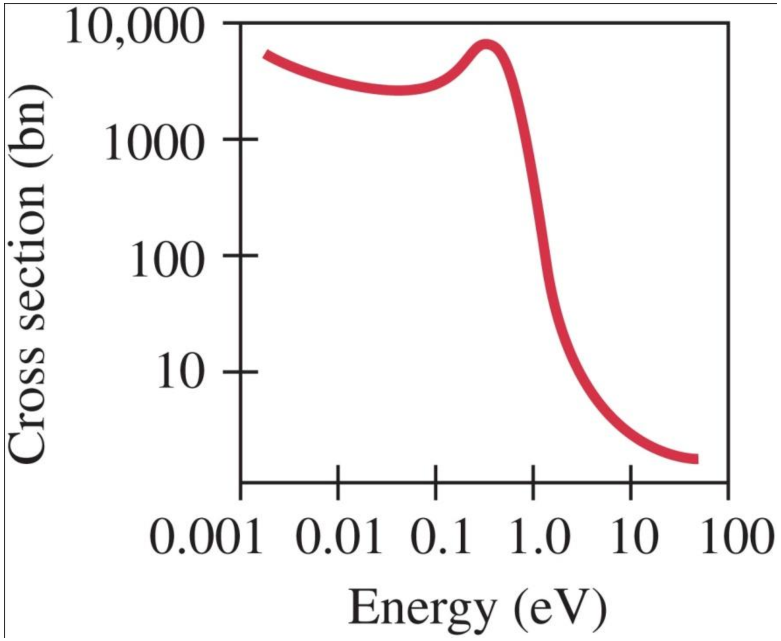
If the nucleus acted like a billiard ball, the cross section would just be the physical cross section – the size of the ball.

Differential cross section $\dfrac{d\sigma}{d\Omega}$

Cross Section; energy dependence

$$n + {}^{114}_{48}Cd \rightarrow {}^{115}_{48}Cd + \gamma$$

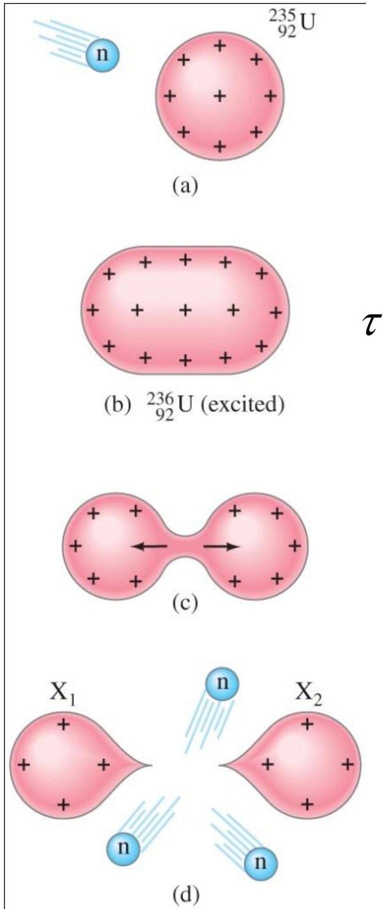
Units: 1 bn = 10^{-28} m^2.



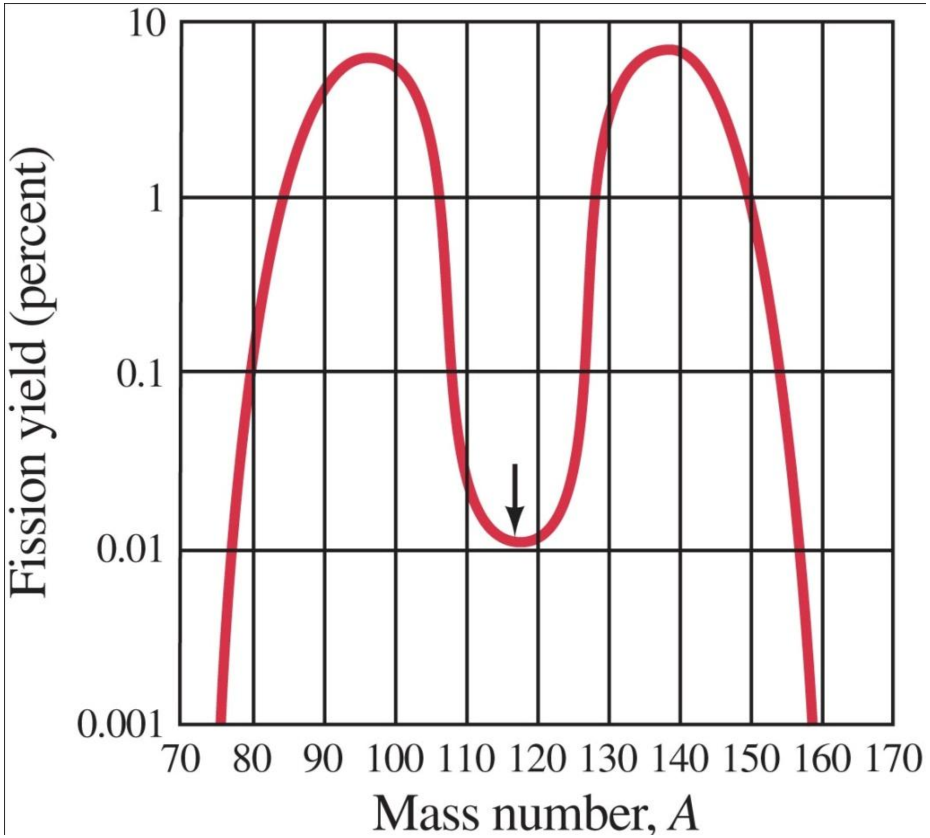
For many reactions: slow (thermal) neutrons are preferred
→ "moderator" required

Nuclear Fission; Nuclear Reactors

After absorbing a neutron, a uranium-235 nucleus will split into two roughly equal parts.



$\tau < 10^{-12}\, s$



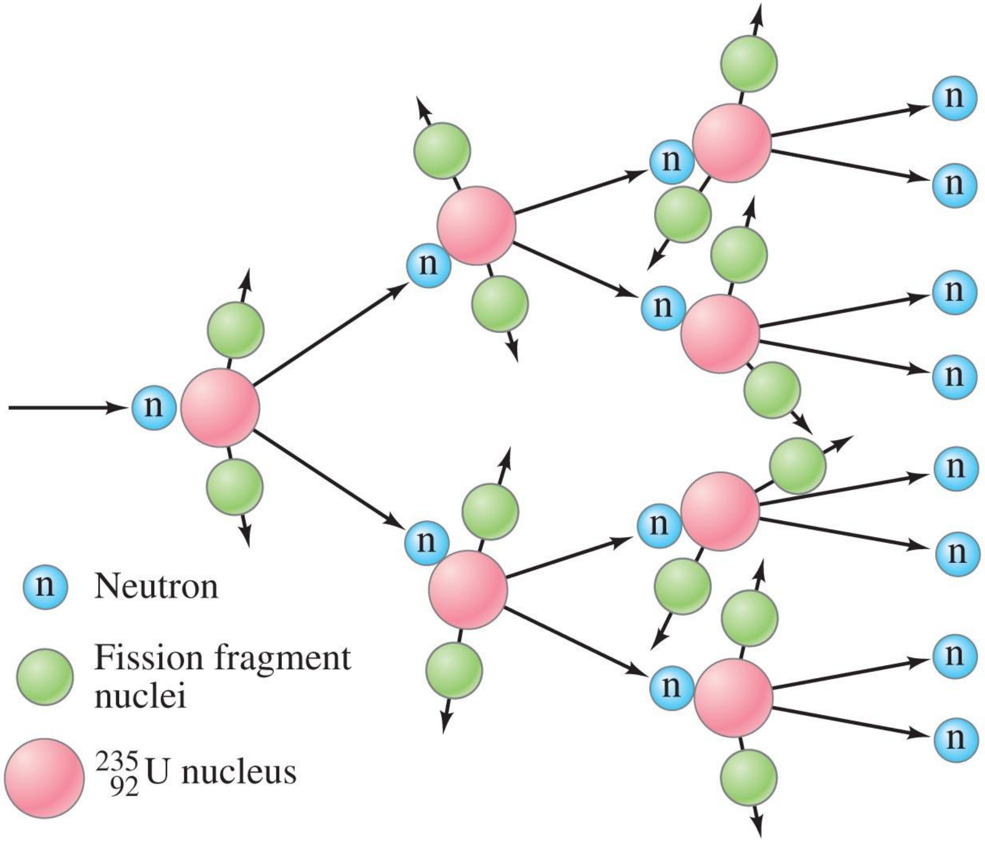
Count the nucleons

Nuclear Fission; Nuclear Reactors

Identify the element X in the fission reaction

$$n + {}^{235}_{92}U \rightarrow {}^{A}_{Z}X + {}^{93}_{38}Sr + 2n$$

Calculate the energy excess for one nucleus ~ 200 MeV



Production of n's

Chain reaction

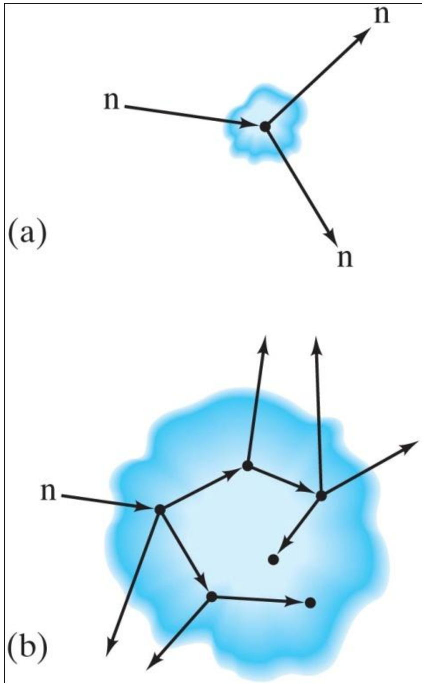
Nuclear Fission; Nuclear Reactors

Chain reaction must be self-sustained – but controlled.

Moderator is needed to slow the neutrons; for the cross section

Common moderators are **heavy water** and graphite.

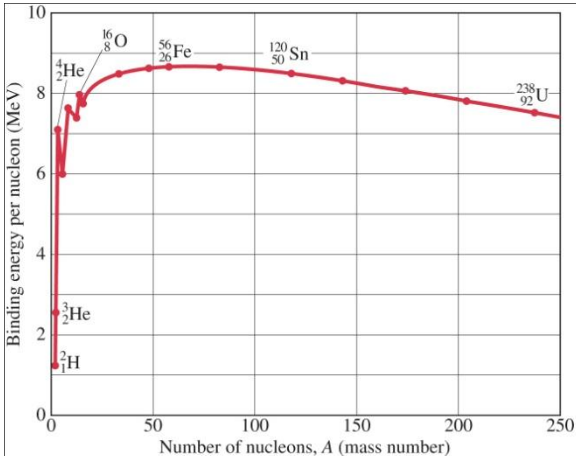
Unless the moderator is heavy water, the fraction of fissionable nuclei in natural uranium, about 0.7%, is too small to sustain a chain reaction. It needs to be enriched to about 2–3%.



Neutrons that escape from the uranium do not contribute to fission. There is a critical mass below which a chain reaction will not occur because too many neutrons escape.

Nuclear Fusion

Rationale is in the mass formula



Simplest processes, occuring in Sun

Proton-proton cycle

2x $\quad {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu \qquad (0.44\ MeV)$

2x $\quad {}^1_1H + {}^2_1H \rightarrow {}^3_2He + \gamma \qquad (5.48\ MeV)$

$${}^3_2He + {}^3_2He \rightarrow {}^4_2He + {}^1_1H + {}^1_1H \qquad (12.86\ MeV)$$

Net reaction $$4\,{}^1_1H \rightarrow {}^4_2He + 2e^+ + 2\nu + 2\gamma$$

Net energy **24.7 MeV** **(+ some extra energy from annihilation e^+)**

Nuclear Fusion

In stars hotter than the Sun, hydrogen fuses to helium primarily through the HNO cycle; the net effect is the same.

$$^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

$$^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu$$

$$^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$$

$$^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma$$

$$^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^{+} + \nu$$

$$^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$$

What is the heaviest element likely to be produced in fusion processes in stars?

Nuclear Fusion

There are three fusion reactions that are being considered for power reactors:

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H \qquad (4.00\ MeV)$$

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + n \qquad (3.23\ MeV)$$

$$^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n \qquad (17.57\ MeV)$$

These reactions use very common fuels – deuterium or tritium – and release much more energy per nucleon than fission does.

Thermonuclear bomb:

fisson for the ignition
fusion for the energy output

Edward Teller

Practicality of Nuclear Fusion

d-t fusion gives the highest energy yield

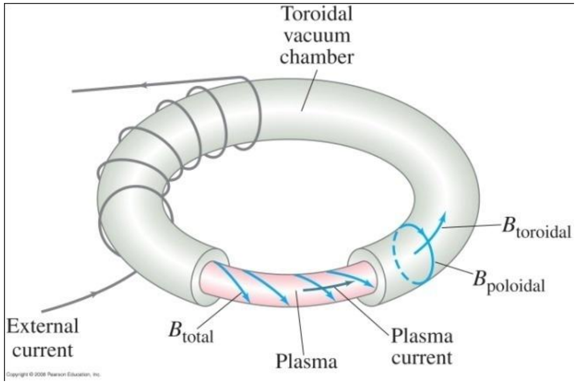
Coulomb barrier must be taken: $$V = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{(r_d + r_t)} \approx 2K \approx 0.45\text{MeV}$$

With $$\overline{K} = \frac{3}{2}kT$$

Corresponds to a temperature of $\approx 2\times10^9\,\text{K}$

Tail of kinetic energy distribution is sufficient $\approx 4\times10^8\,\text{K}$

Plasma
must be confined



Tokamak

ITER (Cadarache)