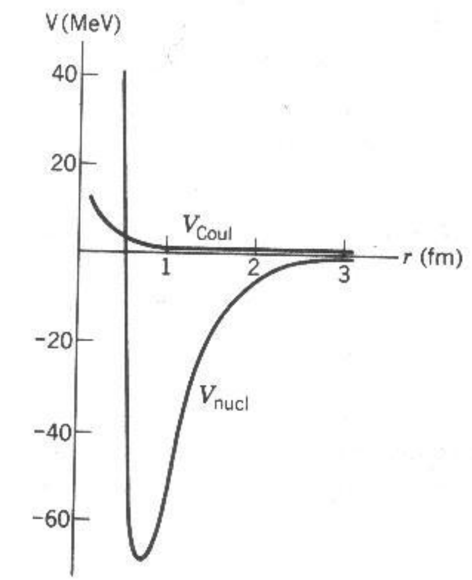
Binding Energy and Nuclear Forces

The force that binds the nucleons together is called the strong nuclear force. It is a very strong, but short-range, force. It is essentially zero if the nucleons are more than about 10^{-15} m apart. The Coulomb force is long-range; this is why extra neutrons are needed for stability in high-Z nuclei.



Calculate that at 1 fm: $$V_{Coulomb} = \frac{e^2}{4\pi\varepsilon_0 r} = 1.44\mathrm{MeV}$$

Structure and Properties of the Nucleus

Nuclei: protons and neutrons.

Proton has positive charge;

$$m_n = 1.67262 \times 10^{-27}\ \text{kg}$$

Neutron is electrically neutral;

$$m_n = 1.67493 \times 10^{-27}\ \text{kg}$$

Number of protons: atomic number, Z

Number of nucleons: atomic mass number, A

Neutron number: $N = A - Z$

Symbol: ${}^{A}_{Z}\mathrm{X}$

$$\mu = \frac{m_p}{m_e} = 1836.152\,672\,61(85)$$

Isotopes

No theory

Extreme density

Because of wave–particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx (1.2 \times 10^{-15}\,\mathrm{m})\left(A^{\frac{1}{3}}\right).$$

Masses scale: carbon-12 atom, 12 u
u is a unified atomic mass unit.

$$1\,\mathrm{u} = 1.6605 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV}/c^2$$

TABLE 41–1 Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

	Mass		
Object	**kg**	**u**	**MeV/c^2**
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Magnetic moments

For a classical point particle

$$\vec{\mu} = -g_L \mu_B \frac{\vec{L}}{\hbar} \qquad g_L = 1$$

For a relativistic spin (point)

$$\vec{\mu}_S = -g_S \mu_B \frac{\vec{S}}{\hbar} \qquad g_S = 2$$

Define *nuclear* magnetic moments (unit = the nuclear magneton)

$$\mu_N = \frac{e\hbar}{2M_p}$$

Proton magnetic moment

$$\mu_p = 2.7928\mu_N$$

Neutron magnetic moment

$$\mu_n = -1.9135\mu_N$$

Protons and neutrons are not point particles

Binding Energy and Einsteins $E=mc^2$

Binding energy affects mass of composite particle

$m_{\text{H-atom}} < m_{\text{proton}} + m_{\text{electron}}$

$m_{\text{H-atom(n=1 state)}} < m_{\text{H-atom(n=2 state)}}$

$$E_b(\text{atom}) = \left[M(\text{nucleus}) + Zm_e - M(\text{atom})\right]c^2$$

(mind the signs !)

These effects are much larger in the realm of nuclei than in that of electromagnetism

$$E_b(\text{nucleus}) = \left[ZM_p + NM_n - M(\text{nucleus})\right]c^2$$

Binding Energy and Nuclear Forces

$$E_b(\text{nucleus}) = \left[ZM_p + NM_n - M(\text{nucleus})\right]c^2$$

The total mass of a stable nucleus is always less than the sum of the masses of its separate protons and neutrons.

$$E_b(\text{nucleus}) = \left[ZM_p + NM_n + Zm_e - M(\text{atom})\right]c^2$$

Here we neglect of the binding energy inside the atom (eV).

$$E_b\left({}^{A}\text{X}\right) = \left[ZM\left({}^{1}\text{H}\right) + NM_n - M\left({}^{A}\text{X}\right)\right]c^2$$

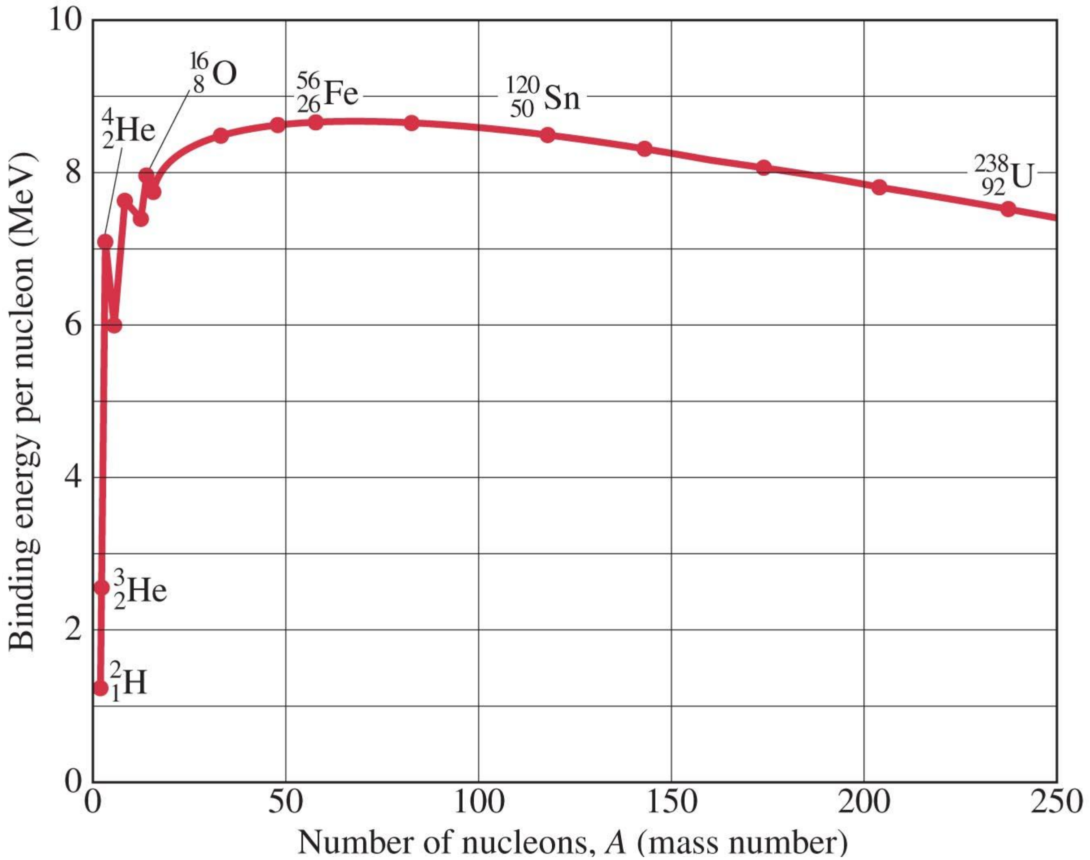
Binding of nucleus

Mass of neutral atom

This can be used to calculate E_b/A: binding energy per nucleus

Binding Energy and Nuclear Forces

From observation: binding energy per nucleon: E_b/A



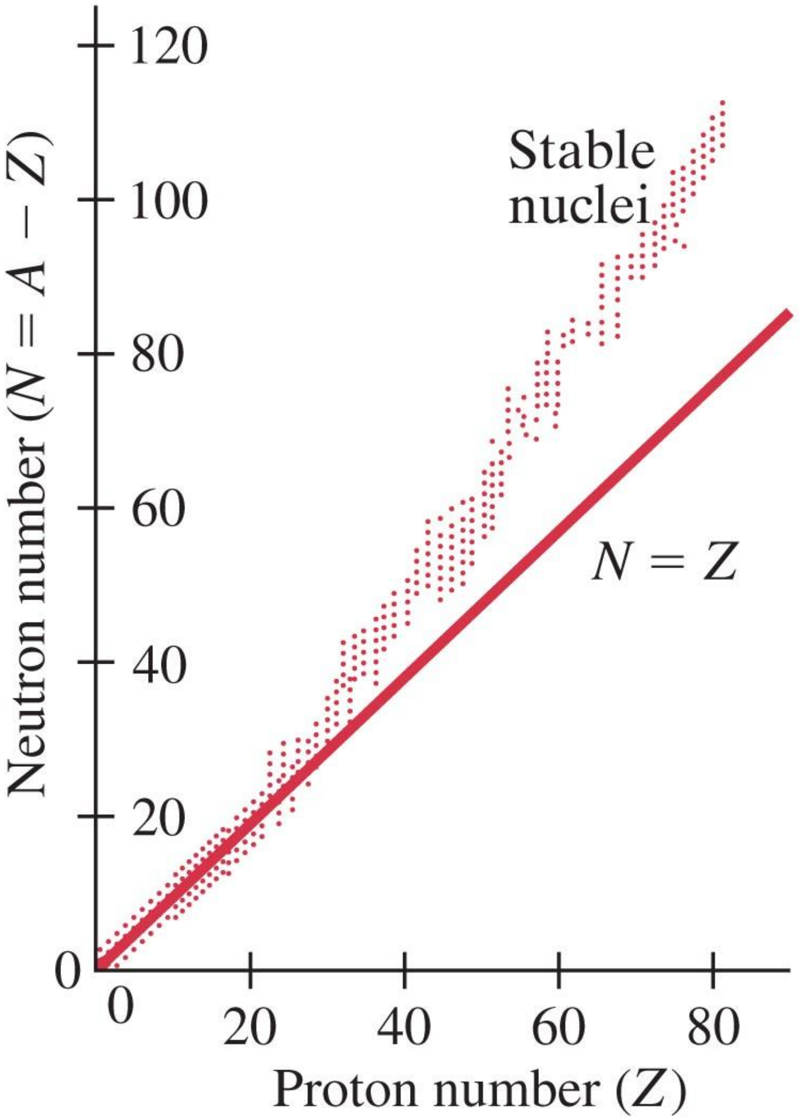
Note:
α is point of relative stability

Binding Energy and Nuclear Forces

From observation:

The higher the binding energy per nucleon, the more stable the nucleus.

More massive nuclei require extra neutrons to overcome the Coulomb repulsion of the protons in order to be stable.



Addition

The semi-empirical mass formula

Von Weizsäcker
Liquid drop model

$$E_b\left({}^{\mathrm{A}}\mathrm{X}\right) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A/2 - Z)^2}{A} + \varepsilon_5$$

Note:

$$R \sim A^{1/3}$$

Volume term

$$\sim R^3 \sim A$$

Surface correction:
on the outer surface ($4\pi R^2$) the binding is less, because There are no particles to contribute

$$\sim R^2 \sim A^{2/3}$$

Good fit for:

$$a_1 = 15.76\,\mathrm{Mev}$$
$$a_2 = 17.81\,\mathrm{Mev}$$
$$a_3 = 0.7105\,\mathrm{Mev}$$
$$a_4 = 94.80\,\mathrm{Mev}$$
$$a_5 = 39\,\mathrm{Mev}$$

Addition

The semi-empirical mass formula

Von Weizsäcker
Liquid drop model

$$E_b\left({}^{A}X\right) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A/2 - Z)^2}{A} + \varepsilon_5$$

$$V_{rep} = \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

Coulomb energy stored in a uniform solid sphere of charge Ze and radius R

(negative binding) $\sim \frac{Z^2}{A^{1/3}}$

$\pm \frac{a_5}{A^{3/4}}$ **ee** **oo**

$= 0$ **eo** **oe**

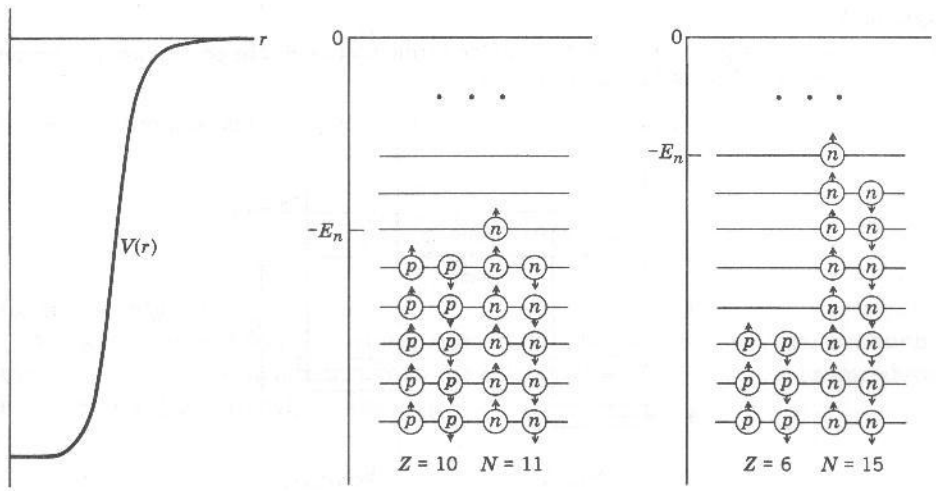
"pairing energy" (not further discussed)

Addition

Protons and Neutrons in the Fermi-gas model

$$-a_4 \frac{(A/2 - Z)^2}{A}$$

Both protons and neutrons cannot fill the lowest "orbitals" because the have to follow the Pauli exclusion principle



" Symmetry Energy" preference for

$$Z = N$$

Based on this concept: the "nuclear shell model"

"for the discovery concerning nuclear shell structure"

Maria Goeppert-Mayer

Addition

Isobars and the mass equation

Mass of nucleus:

$$M\left({}^{A}\mathrm{X}\right) = ZM\left({}^{1}\mathrm{H}\right) + (A-Z)M_n - \left[a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A/2-Z)^2}{A} + \varepsilon_5\right]/c^2$$

Isobar:

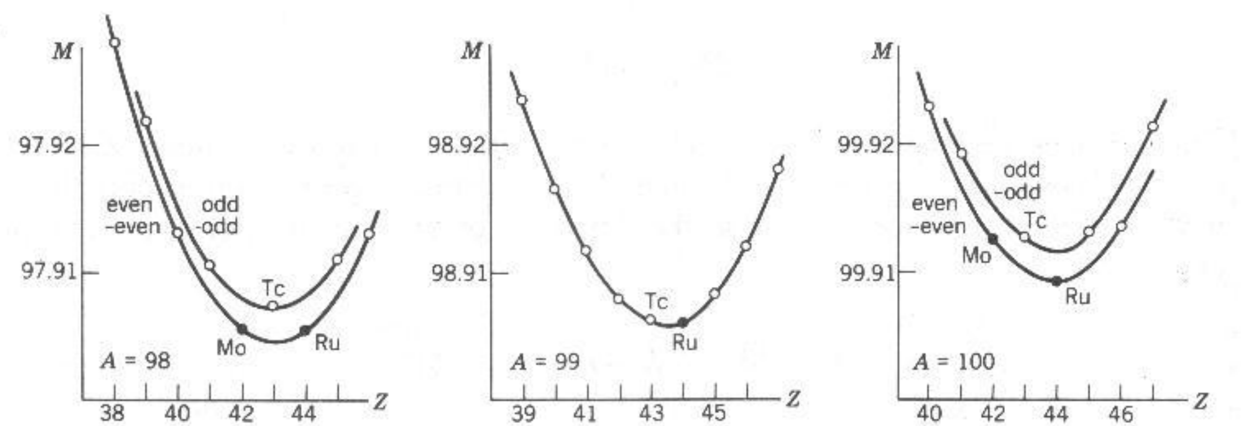
$$\frac{\partial M}{\partial Z}c^2 = \left[M\left({}^{1}\mathrm{H}\right) - M_n\right]c^2 + 2a_3 \frac{Z}{A^{1/3}} + a_4 \frac{(Z-A/2)}{A}$$

(even-odd separate due to ε_5 term)

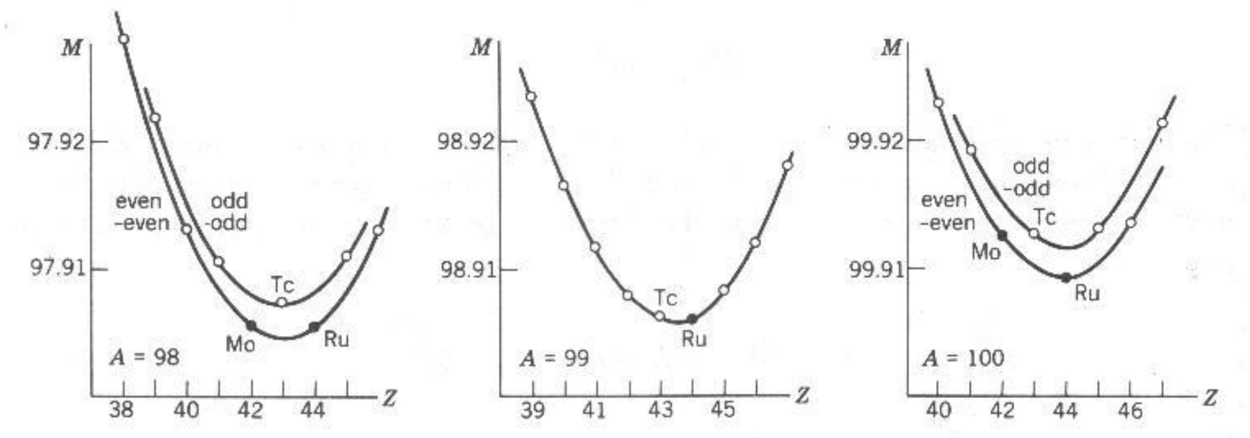
Minimum charge Z:

$$Z_A = \frac{A}{2}\,\frac{a_4 + \left[M\left({}^{1}\mathrm{H}\right) - M_n\right]c^2}{a_4 + a_3 A^{2/3}}$$

Valley of stability



Unstable elements



^{97}Tc $\tau = 2 \times 10^6$ y

^{99}Tc $\tau = 2 \times 10^5$ y

^{145}Pm $\tau = 18$ y

^{146}Pm $\tau = 5.5$ y

Stable/Meta-stable elements

^{126}Te stable, 18.95 %

^{128}Te $\tau > 5 \times 10^{24}$ y

^{130}Te $\tau = 2.5 \times 10^{21}$ y

^{233}U $\tau = 1.6 \times 10^5$ y

^{235}U $\tau = 7 \times 10^8$ y

^{238}U $\tau = 4.5 \times 10^9$ y

Why do nuclei decay → radioactivity