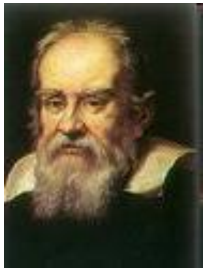
The Special Theory of Relativity

Galilean–Newtonian Relativity



Galileo Galilei

Isaac Newton

Definition of an inertial reference frame:

One in which Newton's first law is valid.

v=constant* if *F=0

Earth is rotating and therefore not an inertial reference frame,

but we can treat it as one for many purposes.

A frame moving with a constant velocity with respect to an inertial reference frame is itself inertial.

Relativity principle:

Laws of physics are the same in all inertial frames of reference

Intuitions of Galilean–Newtonian Relativity

What quantities are the same, which ones change ?

Lengths of objects are invariant as they move.

Time is absolute.

Mass of an object in invariant in for inertial system

Forces acting on a mass equal for all inertial frames

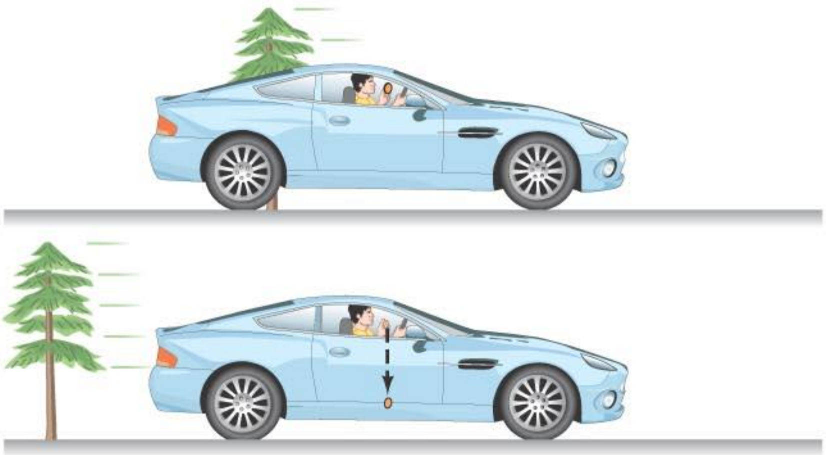
Velocities are (of course) different in inertial frames

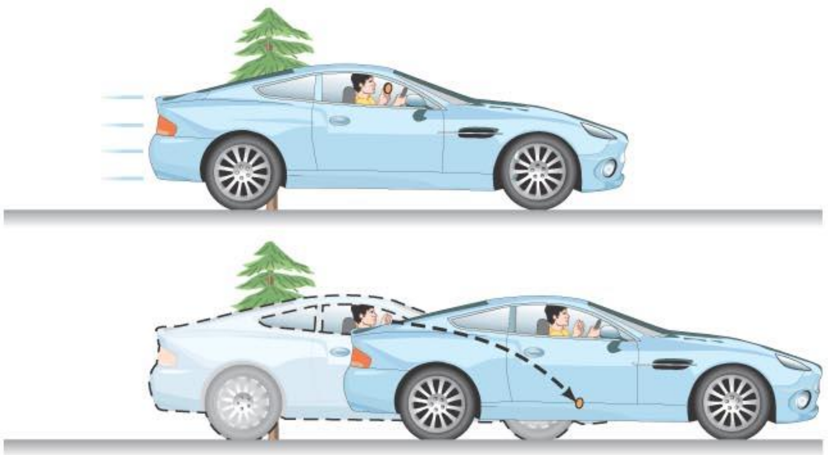
Positions of objects are different in other inertial systems (coordinate transformation)

Relativity principle:

The basic laws of physics are the same in all inertial reference frames

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$


(a)
Reference frame = car


(b)
Reference frame = Earth

Laws are the same, but paths may be different in referenceframes

Galilean–Newtonian Relativity

This principle works well for mechanical phenomena.

There seems to be a problem with electrodynamic phenomena

James Clerk Maxwell

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{with} \quad \frac{1}{\mu_0 \varepsilon_0} = c^2$$

Light is a wave with transverse polarization and speed c

Problems:

In what inertial system has light the exact velocity c

What about the other inertial systems

Waves are known to propagate in a medium; where is this “ether”

How can light propagate in vacuum ?

Laws of electrodynamics do not fit the relativity principle ?

The Michelson–Morley Experiment

Albert Michelson **Edward Williams Morley**

Nobel 1907

"for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"

Albert Abraham Michelson

Questions:

What is the absolute reference point of the Ether?

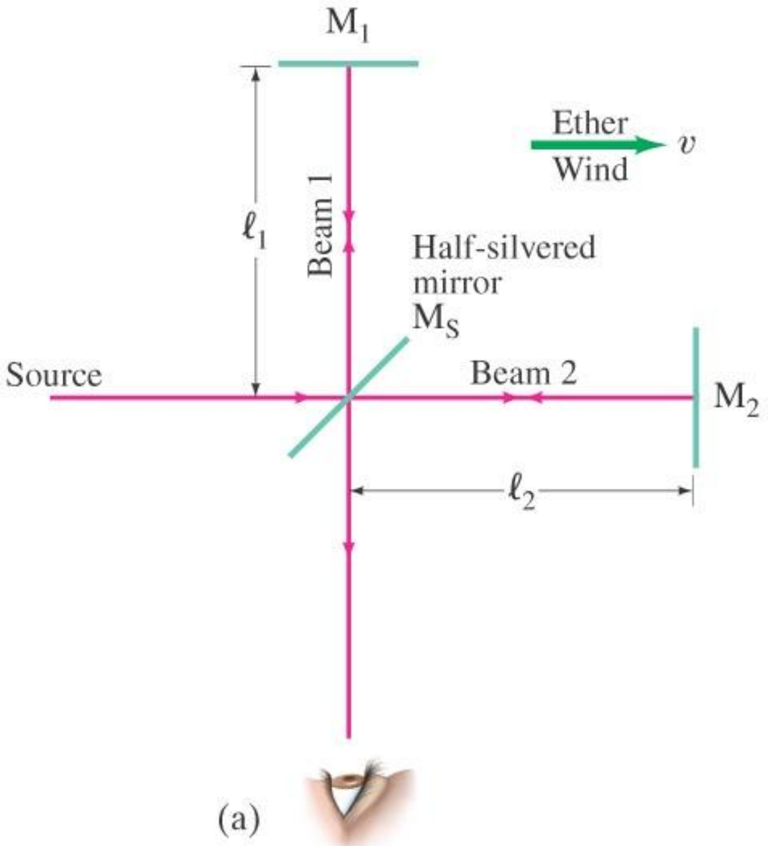
In which direction does it move ? How fast ?

Ether connected to sun (*center of the universe*) ?

$$\left.\begin{array}{l} v_{Earth} \sim 3 \cdot 10^{4} m/s \\ c \sim 3 \cdot 10^{8} m/s \end{array}\right\} \quad \frac{v}{c} \sim 10^{-4}$$

Motion of the Earth Should produce an Observable effect

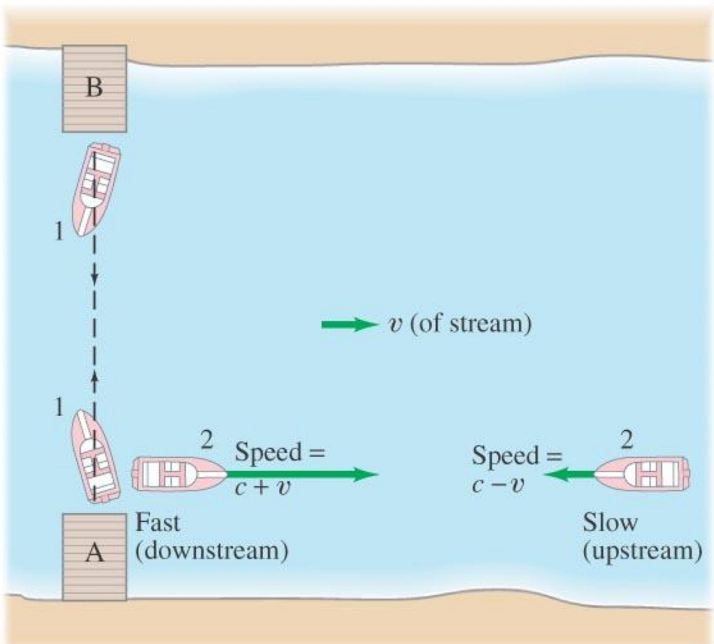
The Michelson–Morley Experiment



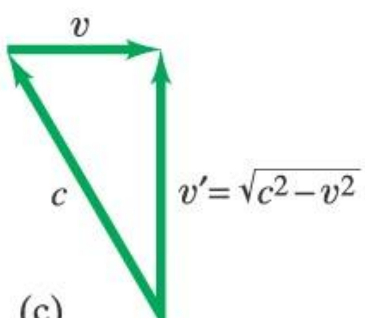
$$t_2 = \frac{\ell_2}{c+v} + \frac{\ell_2}{c-v} = \frac{2\ell_2}{c\left(1-v^2/c^2\right)}$$

Note: we adopt the classical perspective

The Michelson–Morley Experiment



(b)



(c)

axis

$$t_1 = \frac{2\ell_1}{v'} = \frac{2\ell_1}{\sqrt{c^2 - v^2}} = \frac{2\ell_1}{c\sqrt{1 - v^2 / c^2}}$$

The Michelson–Morley Experiment

$$t_2 = \frac{\ell_2}{c+v} + \frac{\ell_2}{c-v} = \frac{2\ell_2}{c\left(1-v^2/c^2\right)}$$

$$t_1 = \frac{2\ell_1}{v'} = \frac{2\ell_1}{\sqrt{c^2-v^2}} = \frac{2\ell_1}{c\sqrt{1-v^2/c^2}}$$

Interferometer: $\ell = \ell_1 = \ell_2$

$$\Delta t = t_2 - t_1 == \frac{2\ell}{c}\left(\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right)$$

If v=0, then Δt=0

no effect on interferometer

If v≠0, then Δt≠0

a phase-shift introduced

But this is not observed

(actually difficult to observe)

The Michelson–Morley Experiment

Rotate the interferometer $\ell_1 \leftrightarrow \ell_2$

$$\Delta T = \Delta t - \Delta t' =$$

$$\frac{2}{c}\left(\ell_1 + \ell_2\right)\left[\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right]$$

Approximate: $\frac{v}{c} << 1$

Then: $$\frac{1}{1-v^2/c^2} \approx 1 + \frac{v^2}{c^2}$$

$$\frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$\Delta T = \left(\ell_1 + \ell_2\right)\frac{v^2}{c^3}$$

Numbers:

- **$v \sim 3 \times 10^4$ m/s**
- **$v/c \sim 10^{-4}$**
- **$l_1 \sim l_2 \sim 11$ m**

$$\Delta T = 7 \times 10^{-16} s$$

Visible light: $\lambda \sim 550$ nm → $f \sim 5 \times 10^{14}$ Hz

Phase change (in fringes)

$$f \cdot \Delta T = 7 \times 10^{-16} \cdot 5 \times 10^{14} = 0.4$$

Should be observable !

Detectability: 0.01 fringe

Conclusion: The Michelson–Morley Experiment

This interferometer was able to measure interference shifts as small as 0.01 fringe, while the expected shift was 0.4 fringe.

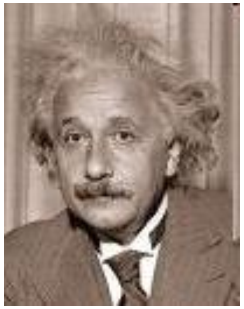
However, no shift was ever observed, no matter how the apparatus was rotated or what time of day or night the measurements were made.

The possibility that the arms of the apparatus became slightly shortened when moving against the ether was considered, but a full explanation had to wait until Einstein came into the picture.

Hendrik A Lorentz
Nobel 1902

"in recognition of the extraordinary service rendered by their researches into the influence of magnetism upon radiation phenomena"

Lorentz contraction



Albert Einstein

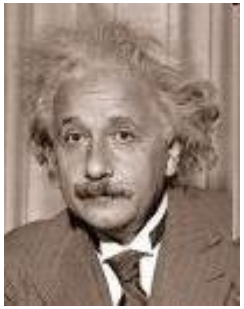
A new perspective

3. ***Zur Elektrodynamik bewegter Körper;***
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt.

Voraussetzung einführen, daß sich das Licht im leeren Raume stets mit einer bestimmten, vom Bewegungszustande des emittierenden Körpers unabhängigen Geschwindigkeit V fortpflanze.

Die Einführung eines „Lichtäthers“ wird sich insofern als überflüssig erweisen, als nach der zu entwickelnden Auffassung weder ein mit besonderen Eigenschaften ausgestatteter „absolut ruhender Raum“ eingeführt, noch einem Punkte des leeren Raumes, in welchem elektromagnetische Prozesse stattfinden, ein Geschwindigkeitsvektor zugeordnet wird.



Albert Einstein

On relativity

§ 2. Über die Relativität von Längen und Zeiten.

Die folgenden Überlegungen stützen sich auf das Relativitätsprinzip und auf das Prinzip der Konstanz der Lichtgeschwindigkeit, welche beiden Prinzipien wir folgendermaßen definieren.

1. Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichförmiger Translationsbewegung befindlichen Koordinatensystemen diese Zustandsänderungen bezogen werden.

2. Jeder Lichtstrahl bewegt sich im „ruhenden" Koordinatensystem mit der bestimmten Geschwindigkeit V, unabhängig davon, ob dieser Lichtstrahl von einem ruhenden oder bewegten Körper emittiert ist. Hierbei ist

$$\text{Geschwindigkeit} = \frac{\text{Lichtweg}}{\text{Zeitdauer}},$$

Postulates of the Special Theory of Relativity

1. **The laws of physics have the same form in all inertial reference frames**
2. **Light propagates through empty space with speed *c* independent of the speed of source or observer**

This solves the ether problem – (there is no ether)

the speed of light is the same in all inertial reference frames

Simultaneity

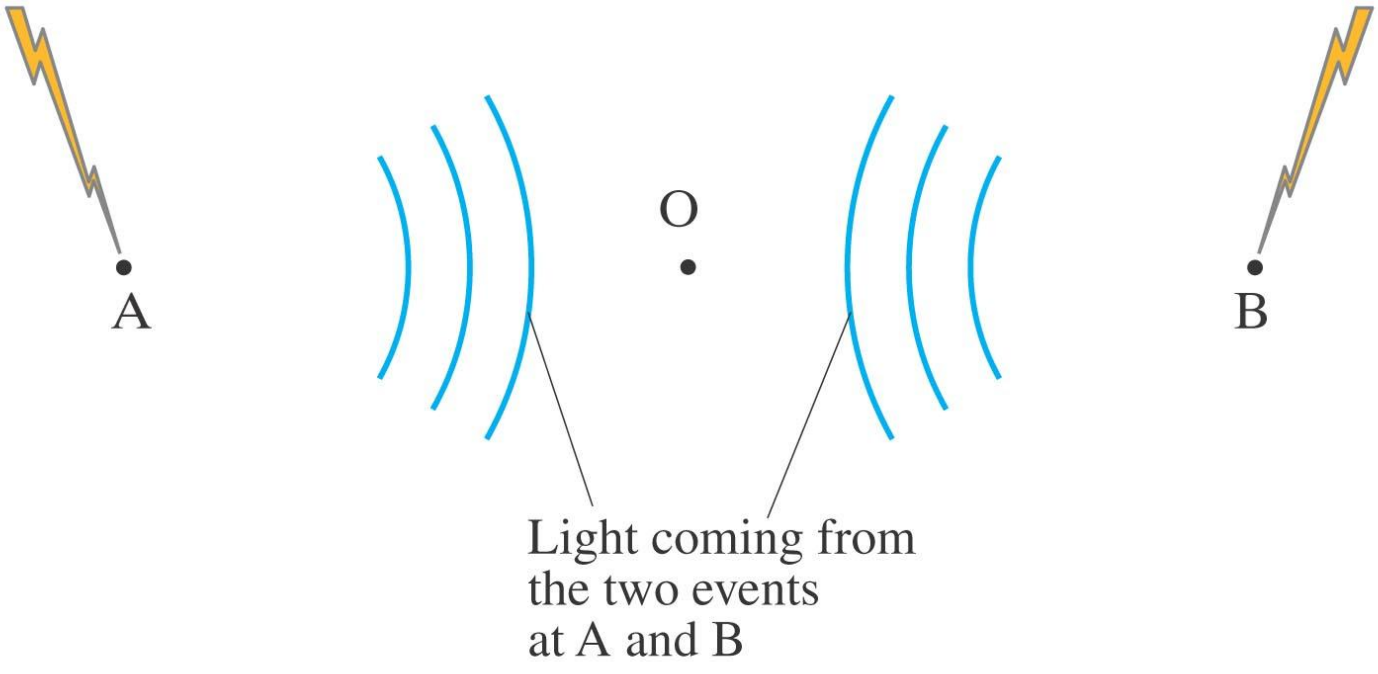
One of the implications of relativity theory is that time is not absolute. Distant observers do not necessarily agree on time intervals between events, or on whether they are simultaneous or not.

Why not?

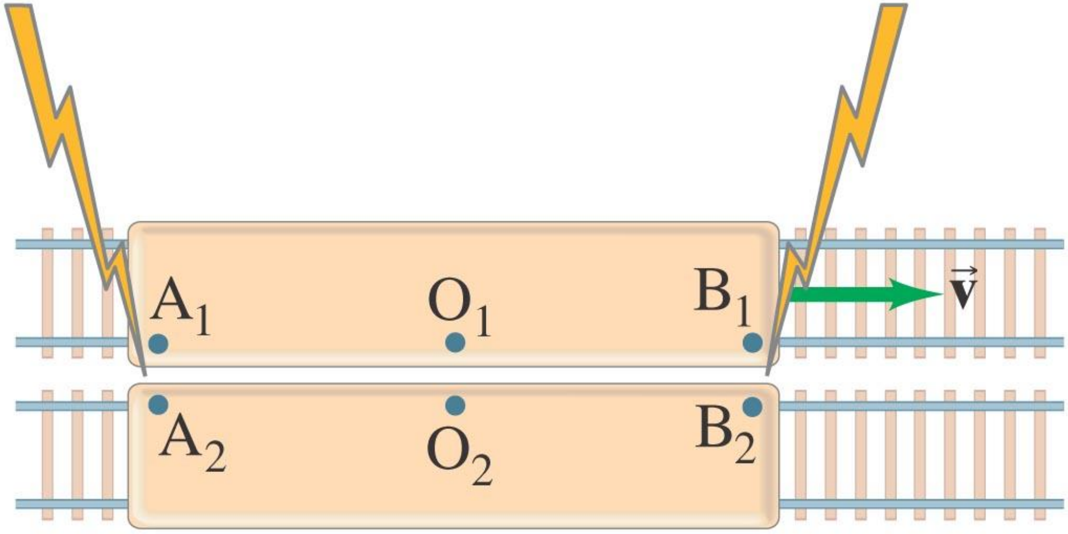
In relativity, an "event" is defined as occurring at a specific place and time. Let's see how different observers would describe a specific event.

Simultaneity

Thought experiment: lightning strikes at two separate places. One observer believes the events are simultaneous – the light has taken the same time to reach her – but another, moving with respect to the first, does not.

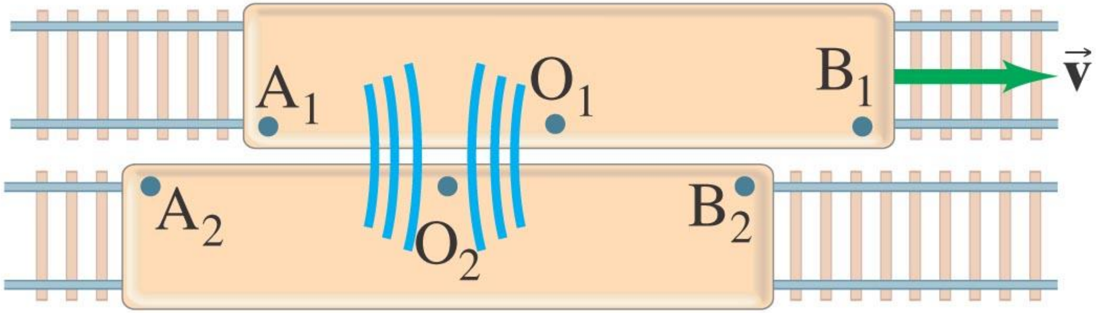


Simultaneity



(a)

From the perspective of both O_1 and O_2 they themselves see both light flashes at the same time

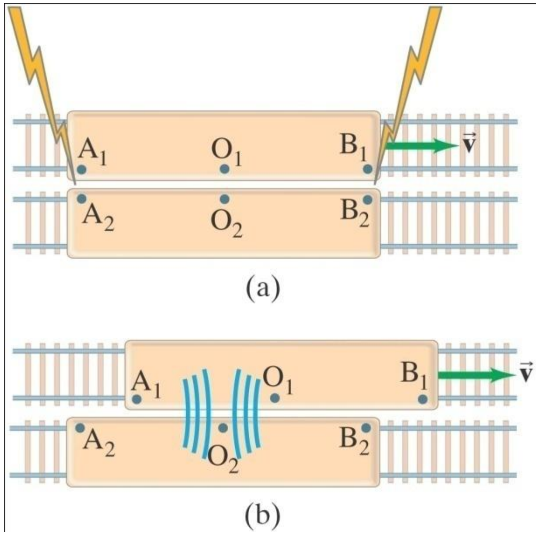


(b)

From the perspective of O_2 the observer O_1 sees the light flashes from the right (B) first.

Who is right ?

Simultaneity



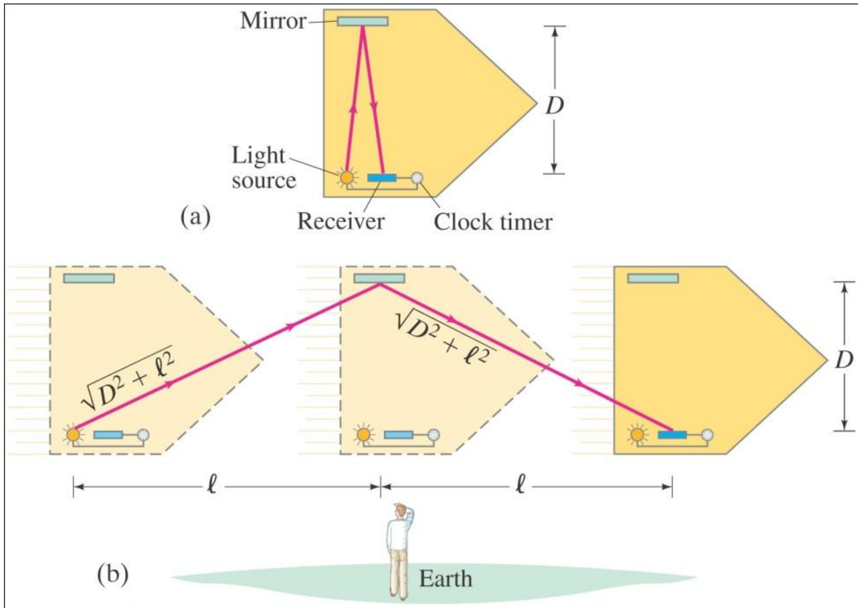
Here, it is clear that if one observer sees the events as simultaneous, the other cannot, given that the speed of light is the same for each.

Conclusions:

Simultaneity is not an absolute concept

Time is not an absolute concept

Time Dilation



a) Observer in space ship

$$\Delta t_0 = \frac{2D}{c}$$ proper time

b) Observer on Earth

speed c is the same

apparent distance longer

$$2\ell = v\Delta t$$

Light along diagonal

$$c = \frac{2\sqrt{D^2 + \ell^2}}{\Delta t} = \frac{2\sqrt{D^2 + v^2\Delta t^2/4}}{\Delta t}$$

$$\Delta t = \frac{2D}{c\sqrt{1 - v^2/c^2}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_0$$

This shows that moving observers must disagree on the passage of time.

Clocks moving relative to an observer run more slowly

Time Dilation

Calculating the difference between clock “ticks,” we find that the interval in the moving frame is related to the interval in the clock’s rest frame:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Δt_0 is ther proper time (in the co-moving frame)

It is the shortest time an observer can measure

with $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$ **then** $\Delta t = \gamma\, \Delta t_0$

On Space Travel

100 light years ~ 10^{16} m

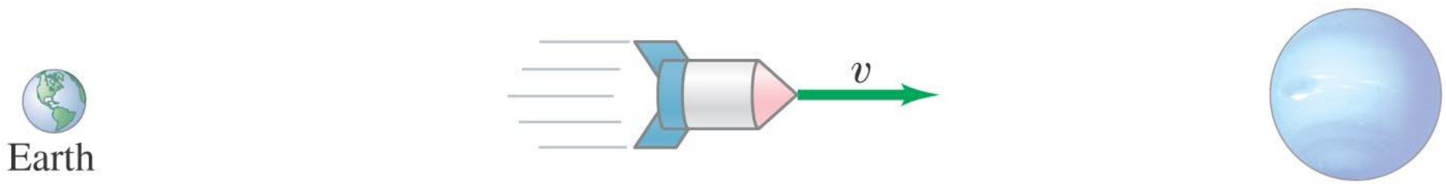
If space ship travels at v=0.999 c
then it takes ~100 years to travel.

But in the rest frame of the carrier:

$$\Delta t_0 = \Delta t \sqrt{1 - v^2 / c^2} \approx 4.5 yr$$

The higher the speed the faster you get there;
But not from our frame perspective !

Length Contraction

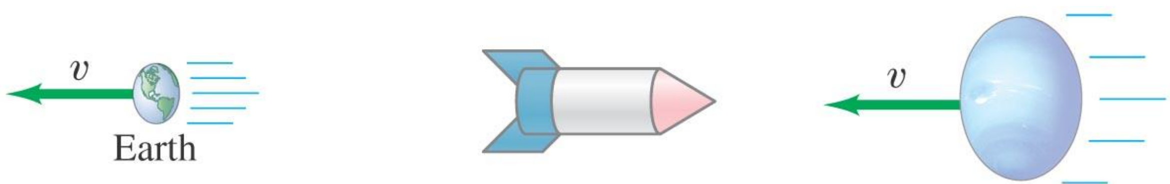


(a) Earth at rest

Distance between planets is: ℓ_0

Time for travel: $\Delta t = \dfrac{\ell_0}{v}$

Earth observer



(b) Spacecraft at rest

Time dilatation

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = \frac{\Delta t}{\gamma}$$

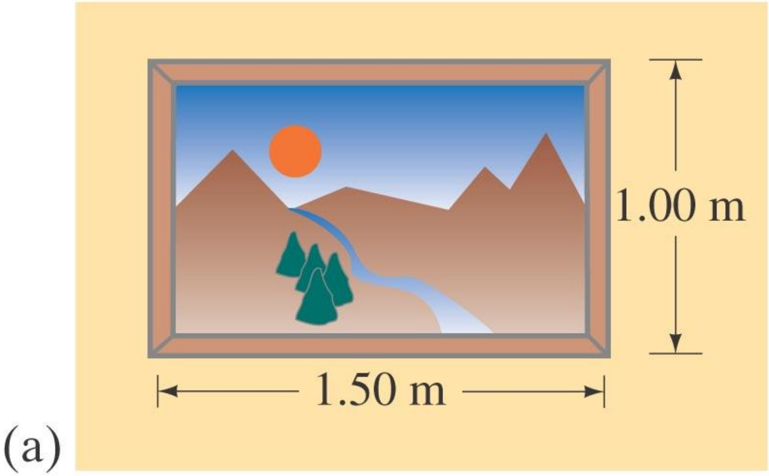
Space craft observers measure the same speed but less time

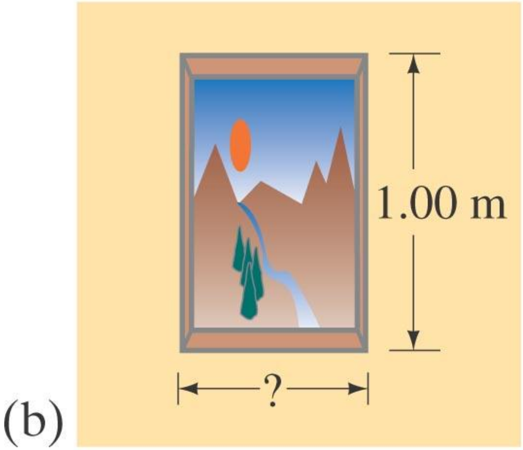
$$\ell = v\Delta t_0 \longrightarrow \ell = \frac{\ell_0}{\gamma}$$

Length Contraction

Only observed in the direction of the motion.

No contraction, or dilatation in perpendicular direction





Excercise

Fantasy supertrain

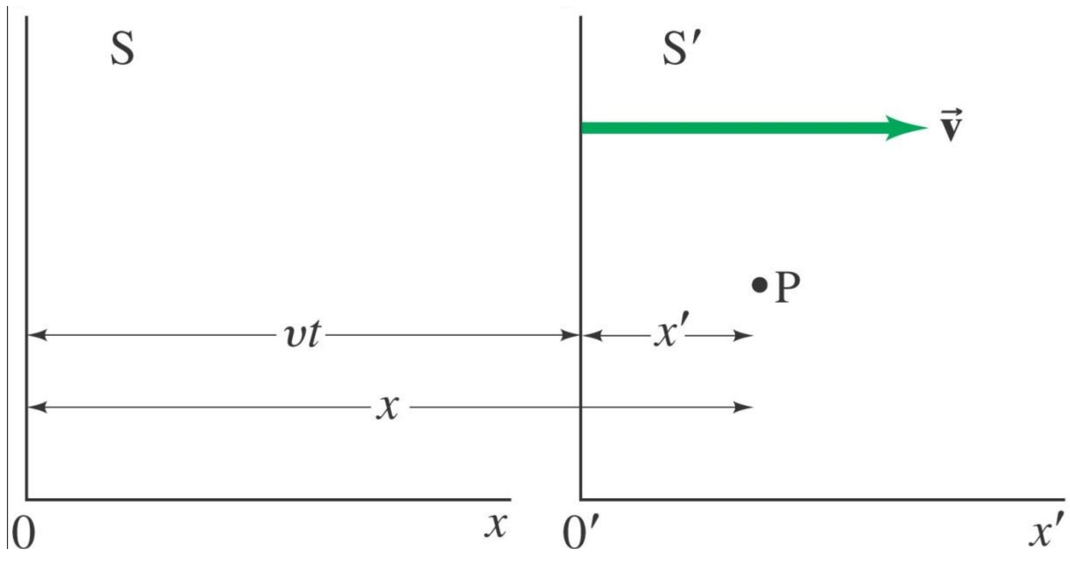
A very fast train with a *proper* length of 500 m is passing through a 200-m-long tunnel. Let us imagine the train's speed to be so great that the train fits completely within the tunnel as seen by an observer at rest on the Earth. That is, the engine is just about to emerge from one end of the tunnel at the time the last car disappears into the other end. What is the train's speed?

Galilean Transformations

A classical (Galilean) transformation between inertial reference frames:

View coordinates of point P in system S'

$$
\begin{aligned}
x &= x' + vt' \\
y &= y' \\
z &= z' \\
t &= t'.
\end{aligned}
\qquad \text{[Galilean]}
$$



Lorentz Transformations

In relativity, assume a linear transformation:

$$x = \gamma(x' + vt'), \qquad y = y', \qquad z = z'.$$

γ as a constant to be determined (γ=1 classically).
Inverse transformation with $v \rightarrow -v$

$$x' = \gamma(x - vt)$$

Consider light pulse at common origin of *S* and *S'* at *t=t'=0*
measure the distance in *x=ct* and *x'=ct'* :

$$x' \equiv ct' = \gamma(x - vt) = \gamma(ct - vt) = \gamma(c - v)t \longrightarrow t' = \gamma\frac{(c-v)}{c}t$$

$$x \equiv ct = \gamma(x' + vt') = \gamma(ct' + vt') = \gamma(c + v)t'$$

fill in

Transformation parameter $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

Lorentz Transformations

Solve further: $$x' = \gamma(x - vt) = \gamma(\gamma(x' + vt') - vt)$$

Leading to the transformations:

$$\begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right). \end{aligned}$$

Time dilatation and length contraction can be derived
From these Lorentz transformations

Excercise

Lorentz Transformations

Lorentz transformation → **length contraction**
→ **time dilation.**

Velocity transformations can be found by differentiating x, y, and z with respect to time.

v is the velocity between the reference frames

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$$

$$u_y = \frac{u'_y\sqrt{1 - v^2/c^2}}{1 + vu'_x/c^2}$$

$$u_z = \frac{u'_z\sqrt{1 - v^2/c^2}}{1 + vu'_x/c^2}.$$

Note that also u_y and u_z transform; this has to do with the transformation (non-absoluteness) of time

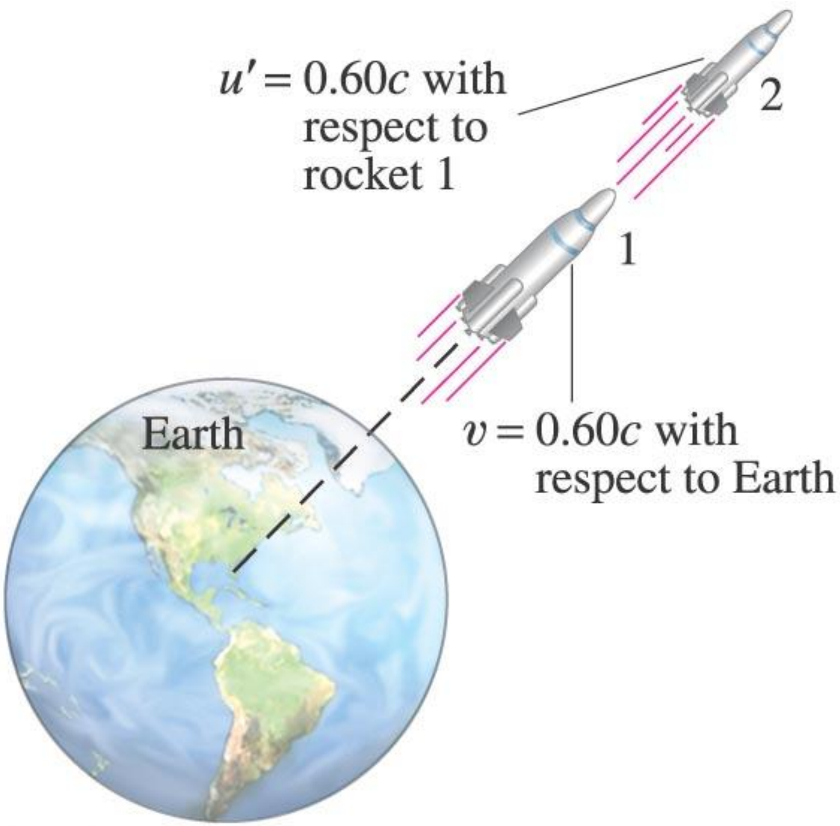
$$\frac{dt'}{dt} \neq 1$$

Verify that *c* is maximum speed

Excercise

Galilean and Lorentz Transformations

Calculate the speed of rocket 2 with respect to Earth.

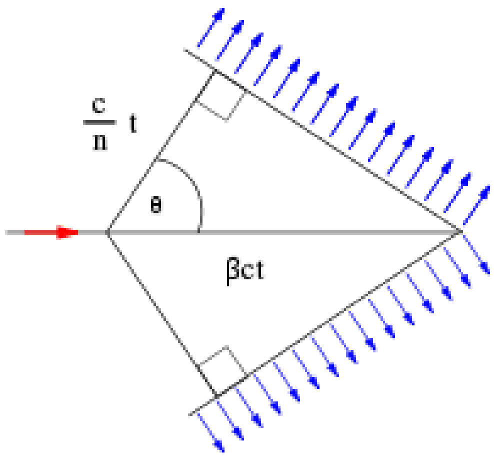


$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}} = 0.88c$$

This equation also yields as result that c is the maximum obtainable speed (in any frame).

Faster than the speed of light ? Cherenkov radiation

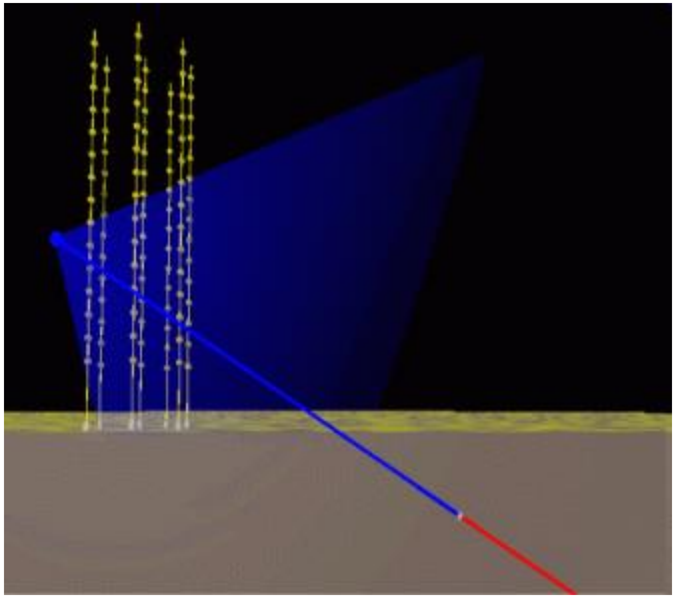
A blue light cone



Pavel Cherenkov
Nobel Prize 1958

(example of “good thinking”)

Application in the ANTARES detector



Particle travels at $x_p = \beta c t$

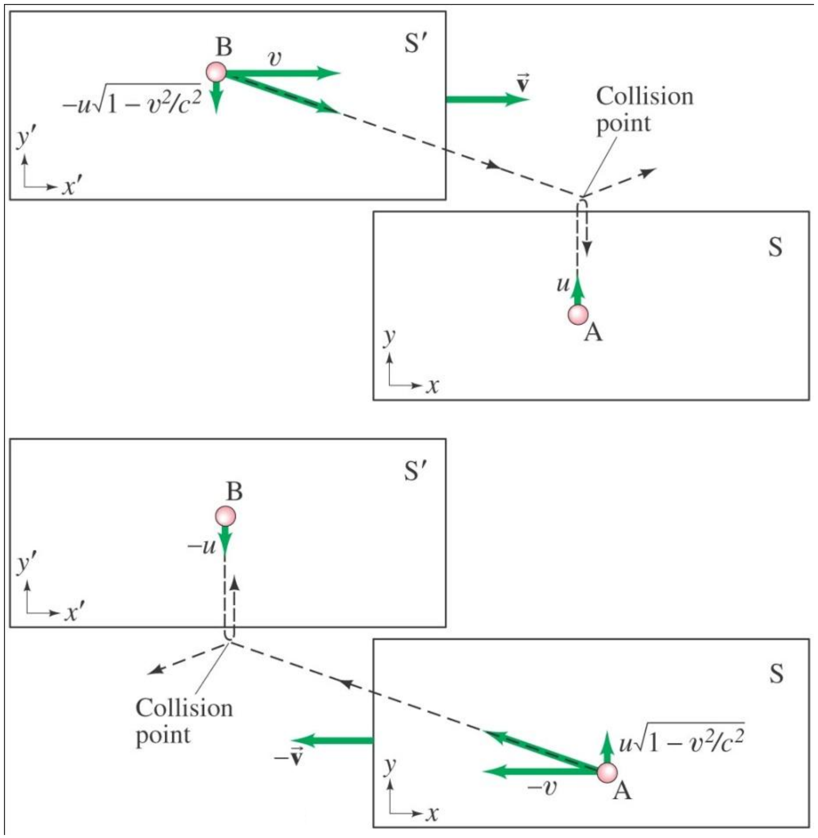
Waves emitted as (spherical) $x_e = \frac{c}{n} t$

Emittance cone: $\cos\theta = \frac{1}{n\beta}$

Excercise

Relativistic Momentum

The formula for relativistic momentum can be derived by requiring that the conservation of momentum during collisions remain valid in all inertial reference frames.



Note: that does NOT mean that the momentum is equal in different reference frames

Result

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv.$$

Go over this and derive !

Relativistic Force

Newtons second law remains valid (without proof)

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\gamma m\vec{v} = \frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right)$$

1) For every physical law it has to be established how they transform in relativity (under Lorentz transformations)
2) Quantities (like *F*) not the same in reference frames

Relativistic Mass

From the momentum:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma m v.$$

Gamma and the rest mass are combined to form the relativistic mass:

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - v^2/c^2}}.$$

Relativistic Energy

Work done to increase the speed of a particle from *v=0* (i-state) to *v=v* (f state):

$$W=\int_i^f F\,dx=\int_i^f \frac{dp}{dt}dx=\int_i^f \frac{dp}{dt}v\,dt=\int_i^f v\,dp=\int_i^f d(pv)-\int_i^f p\,dv$$

because $v\,dp=d(pv)-p\,dv$

$$\int_i^f d(pv)=pv\big|_i^f=(\gamma mv)v$$

$$-\int_i^f p\,dv=-\int_0^v \frac{mv}{\sqrt{1-v^2/c^2}}dv=mc^2\sqrt{1-v^2/c^2}\Big|_0^v=mc^2\sqrt{1-v^2/c^2}-mc^2$$

use

$$\frac{d}{dv}\left(\sqrt{1-v^2/c^2}\right)=-\left(\frac{v}{c^2}\right)/\sqrt{1-v^2/c^2}$$

So:

$$W=\gamma mv^2+\frac{mc^2}{\gamma}-mc^2=\frac{\left(\gamma^2v^2+c^2\right)}{\gamma}m-mc^2=(\gamma-1)mc^2$$

Kinetic energy of the particle is

$$K=(\gamma-1)mc^2$$

1) Amount of kinetic energy depends on inertial frame

2) Amount of kinetic energy reduces to classical value at low *v*

3) Note

$$K\neq\frac{1}{2}mv^2$$

Mass and Energy

The kinetic energy $K = (\gamma - 1)mc^2$

Can be written as the total energy: $E = \gamma mc^2 = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}}.$

Where the difference is the rest energy: $E = mc^2.$

The last equation is Einstein famous equation
implying that mass is equivalent to energy
The energy of a particle at rest.

Note that mc^2 is the same as seen from all reference frames;
It is an *invariant* upon frame transformation

Mass, Energy, Momentum

Energy $E = \gamma mc^2$ Momentum $p = \gamma mv$

Combining these relations gives

$$E^2 = p^2c^2 + m^2c^4.$$

Hence also the following
Is an invariant under Lorentz
transformations

$$E^2 - p^2c^2$$