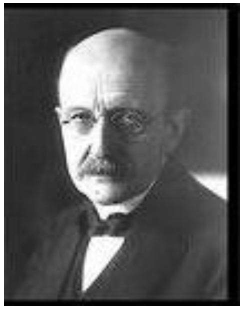
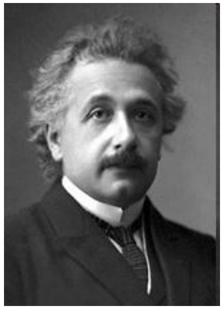
Chapter 37
The Quantum Revolution



Max Planck

The Nobel Prize in Physics 1918
"in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta"



Albert Einstein

The Nobel Prize in Physics 1921
"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"

The mystery of particles and waves

Blackbody Radiation
the classical picture

The classical radiation field: $$u_f(T) = \frac{8\pi f^2}{c^3}\langle \varepsilon \rangle$$

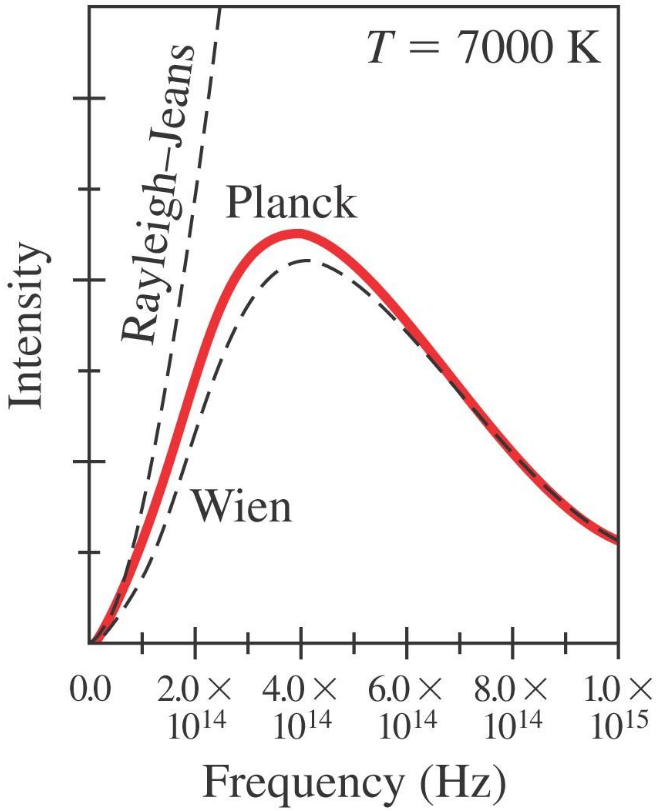
In a classical statistical theory the average energy per degree of freedom is:

$$\langle \varepsilon \rangle = k_B T$$

To the Rayleigh-Jeans law for a black body emitter

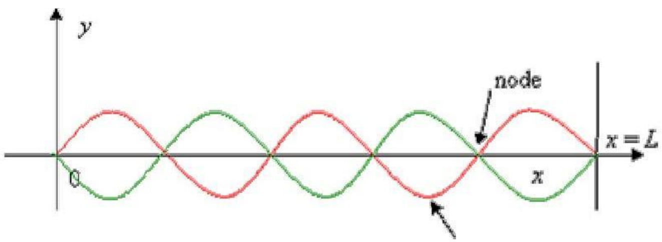
$$u_f(T) = \frac{8\pi f^2}{c^3} k_B T$$

The ultraviolet catastrophe



Extra: On the mode density of the classical radiation field: Counting standing waves

Rayleigh's method for sound waves



Possible mode of vibration of string with both ends fixed: $\lambda = 2L/5$

Allowed wavelengths on a string:
λ=2L, λ=L, λ=2L/3, ...
Frequencies:
f=c/λ=c/2L, c/L, 3c/2L, 2c/L, ...
Allowed frequencies are spaced by c/2L

Spectral density is then (in 1 dimension): Number of modes between *f* and *f*+Δ*f* → 2L/c

In three dimensions analoguous modes:

$$f = \frac{ck}{2\pi} = \frac{c}{2\pi}\sqrt{k_x^2 + k_y^2 + k_z^2}$$

The number of modes between *f* and *f*+Δ*f* is the volume in k-space in units $(\pi/L)^3$

Hence:

$$N(f)\Delta f = \frac{1}{8} \times 2 \times \frac{4\pi k^2 \Delta k}{(\pi/L)^3} = \frac{1}{4} \times \left(\frac{L}{\pi}\right)^3 \times 4\pi \left(\frac{2\pi}{c}\right)^3 f^2 \Delta f = \frac{8\pi V f^2 \Delta f}{c^3}$$

Specialties (one octant of positive k, 2 polarizations)

Radiation mode density in a closed box of Lx Lx L

$$u_m(f) = \frac{N(f)}{V} = \frac{8\pi f^2}{c^3}$$

(Note, this is irrespective of the energy per mode)

Extra: Law of equipartition for a classical radiation field

Kinetic energy per degree of freedom

$$\langle \varepsilon_{kin} \rangle = \frac{1}{2} k_B T$$

For each sinusoidal oscillation (harmonic oscillator) the potential energy is equal to the kinetic energy

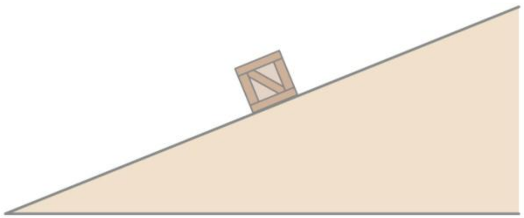
$$\langle \varepsilon_{pot} \rangle = \frac{1}{2} k_B T$$

Classical equipartition (for harmonic oscillator)

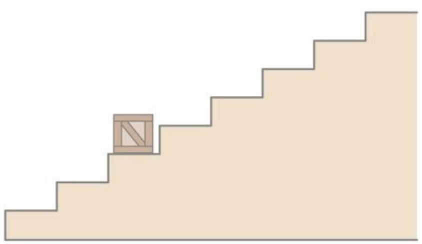
$$\langle \varepsilon \rangle = \langle \varepsilon_{kin} \rangle + \langle \varepsilon_{pot} \rangle = k_B T$$

Note, later: equipartition for quantum states of Bohr atom is different !

Blackbody Radiation
toward the Quantum Hypothesis



(a)



(b)

Planck: energy of the oscillating modes come in discrete portions $\varepsilon = nhf$

Probability that ε occurs in the energy distribution of the cavity (*Maxwell Boltzmann*)

$$p(\varepsilon) = e^{-\varepsilon / kT}$$

Mean energy

$$\langle \varepsilon \rangle = \frac{\int \varepsilon p(\varepsilon) d\varepsilon}{\int p(\varepsilon) d\varepsilon} = \frac{\sum_n \varepsilon_n p(\varepsilon_n)}{\sum_n p(\varepsilon_n)} = \frac{\sum_n nhf e^{-nhf/kT}}{\sum_n e^{-nhf/kT}} = kTx \frac{\sum_n n e^{-nx}}{\sum_n e^{-nx}}$$

With: $x = \dfrac{hf}{kT}$

Define geometrical series:

$$Z(x) = \sum e^{-nx} = 1 + e^{-x} + e^{-2x} + \cdots = \frac{1}{1 - e^{-x}}$$

$$-x \frac{d}{dx} Z(x) = -x \frac{d}{dx} \sum e^{-nx} = x \sum n e^{-nx}$$

$$\langle \varepsilon \rangle = \frac{-kTx}{Z(x)} \frac{d}{dx} Z(x) = -kTx \frac{d}{dx} \ln Z(x) = kTx \frac{d}{dx} \ln(1 - e^{-x})$$

$$\langle \varepsilon \rangle = kTx \frac{e^{-x}}{1 - e^{-x}} = \frac{kTx}{e^{x} - 1} = \frac{hf}{e^{hf/kT} - 1}$$

Planck's Quantum Hypothesis; Blackbody Radiation

Radiation density

$$u_f(T) = \frac{8\pi f^2}{c^3}\langle \varepsilon \rangle = \frac{8\pi h f^3}{c^3}\frac{1}{e^{hf/kT}-1}$$

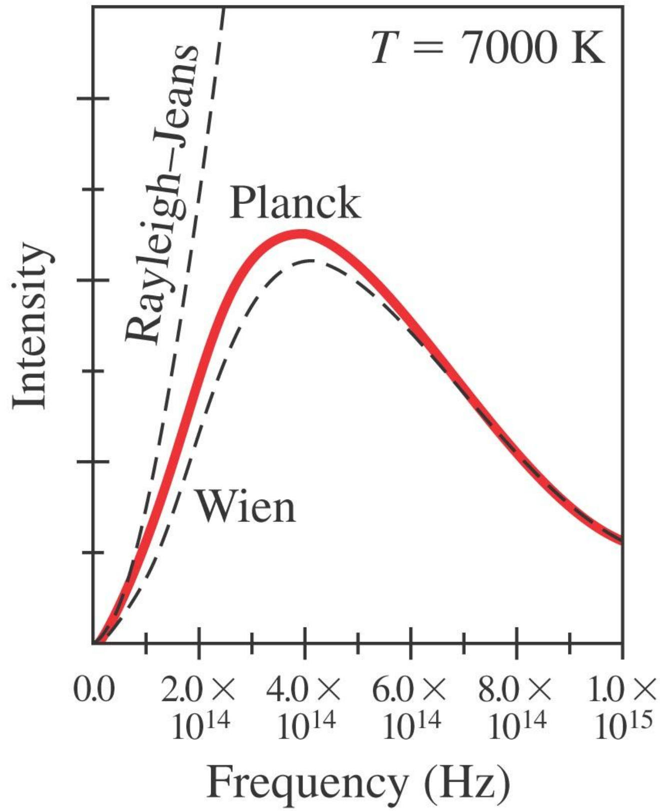
Radiation intensity

$$I_f(T) = \frac{c}{4}u_f(T) = \frac{2\pi h f^3}{c^2}\frac{1}{e^{hf/kT}-1}$$

Scaling from frequency to wavelength

$$I_\lambda(T) = \frac{c}{\lambda^2}I_f(T) = \frac{2\pi h c^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

?



Planck's Quantum Hypothesis; Blackbody Radiation

Planck found the value of his constant by fitting blackbody curves to the formula

$$I(\lambda, T) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

giving

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}.$$

Planck's proposal was that the energy of an oscillation had to be an integral multiple of *hf*. This is called the quantization of energy.

Derivation of Wien's law

Radiation intensity (Planck)

$$I_\lambda(T) = \frac{c}{\lambda^2} I_f(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Define (dimensionless) $x = \frac{hc}{\lambda kT}$

$$I_\lambda(T) = \frac{2\pi (kT)^5}{h^4 c^3} \frac{x^5}{e^x - 1}$$

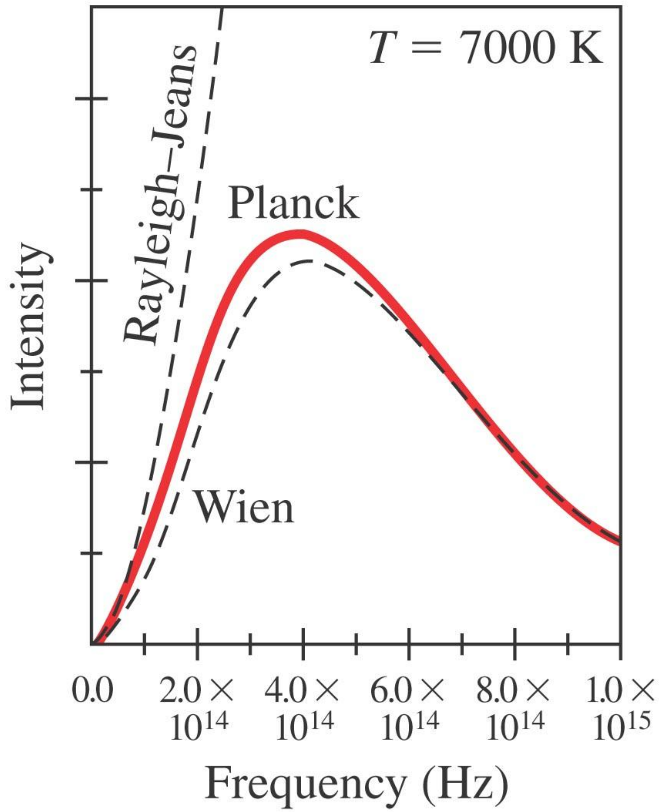
Universal shape: $g(x) = \frac{x^5}{e^x - 1}$

Maximum: $\frac{dg}{dx} = \frac{x^4}{e^x - 1}\left(5 - \frac{x}{1 - e^{-x}}\right) = 0$

For $1 - e^{-x} = x/5$

so $\hat{x} = 4.965$

$$\lambda_{max} T = \frac{hc}{\hat{x}k}$$



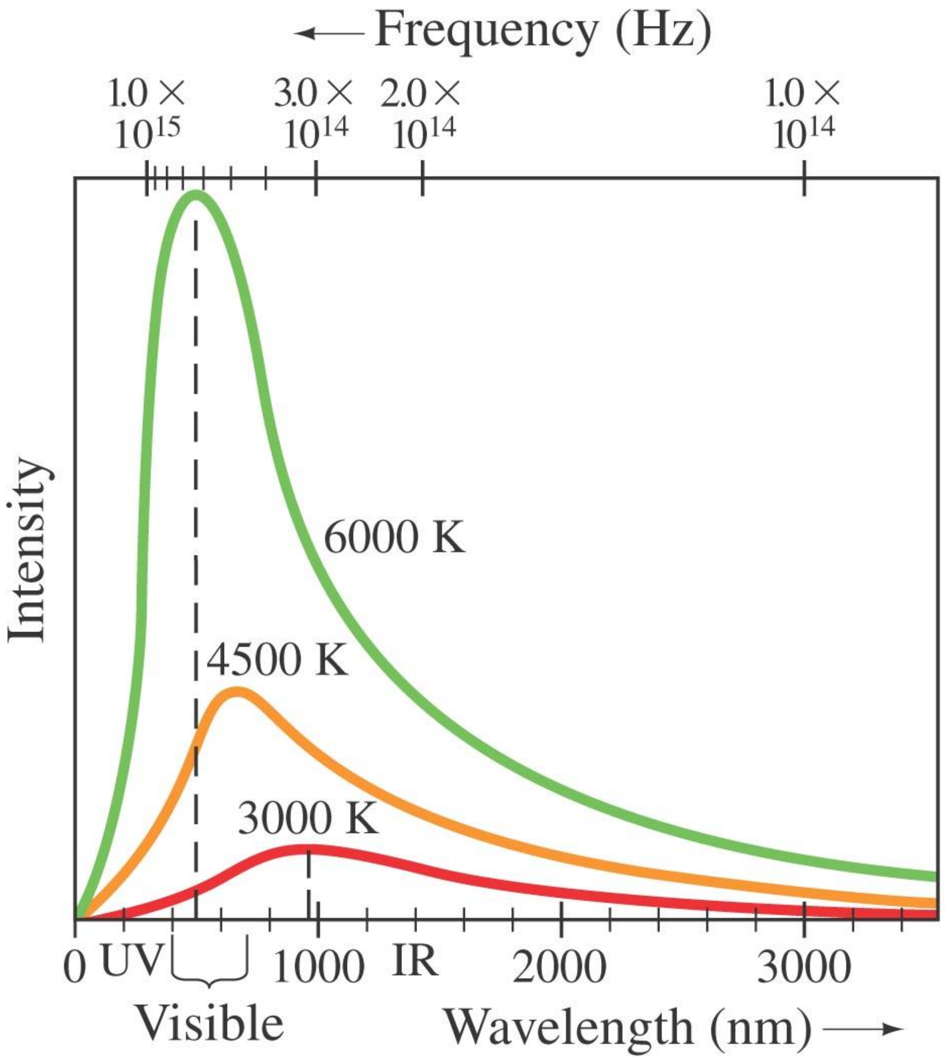
Planck's Quantum Hypothesis; leading to Wien's law

Blackbody radiation for three different temperatures

Note that frequency increases to the left.

The relationship between the temperature and peak wavelength is given by Wien's law:

$$\lambda_P T = 2.90 \times 10^{-3}\, \mathrm{m \cdot K}.$$



Planck's Quantum Hypothesis; leading to Stefan-Boltzmann's law

Radiation intensity $$I_f(T) = \frac{c}{4} u_f(T) = \frac{2\pi h f^3}{c^2} \frac{1}{e^{hf/kT} - 1}$$

Total intensity $$I(T) = \int_0^\infty \frac{2\pi h f^3}{c^2} \frac{df}{e^{hf/kT} - 1}$$

Use again: $$x = \frac{hc}{\lambda kT}$$

Then $$I(T) = \frac{2\pi h}{c^2} \left(\frac{kT}{h} \right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

Check Mathematica (or solve): $$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$$

Stefan- Boltzmann $$I(T) = \sigma T^4$$

With: $$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.676 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}$$

Interpret the Physics of this law !

Photon Theory of Light and the Photoelectric Effect

Einstein suggested that, given the success of Planck's theory, light must be emitted in small energy packets:

$$E = hf.$$

These tiny packets, or particles, are called photons.

Einstein made a step further than the assumptions of Planck who doubted the reality of the quanta

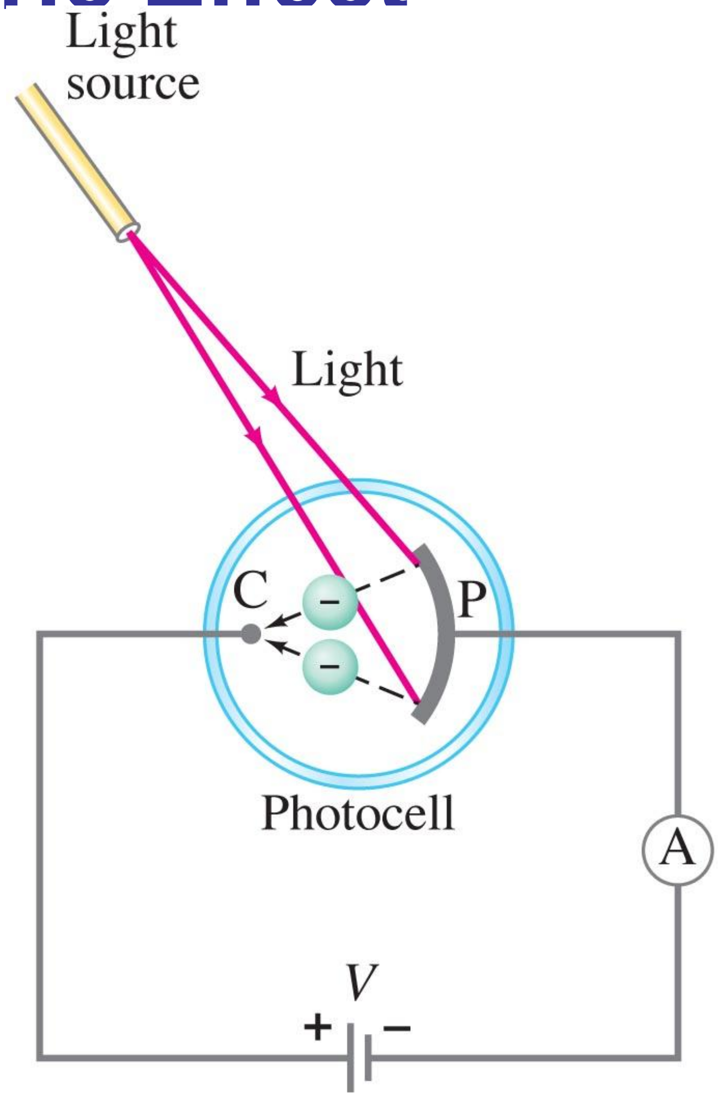
Photon Theory of Light and the Photoelectric Effect

The photoelectric effect:
if light strikes a metal,
electrons are emitted.

Measurement of kinetic energy of electrons:
Stopping potential

$$K_{max} = eV_0$$

Measurements at varying ***f***



Photon Theory of Light and the Photoelectric Effect

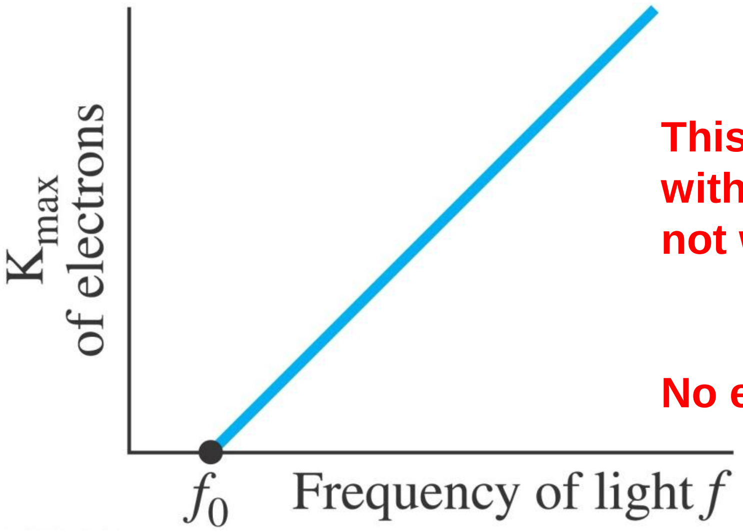
The particle theory assumes that an electron absorbs a single photon.
Plotting the kinetic energy vs. frequency:

$$hf = K + W$$

***W_0* is material property**

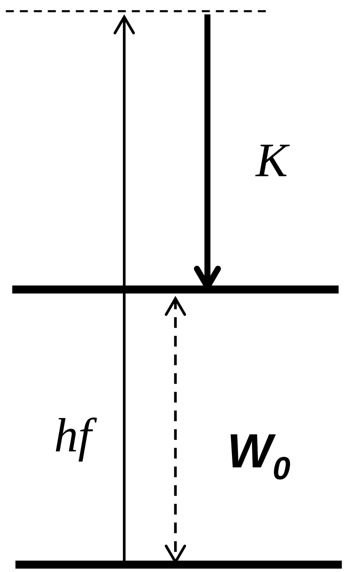
In some cases several kinetic energies measured:
Least bound electrons correspond to the work function: *W_0*
Minimum amount of energy required to release electron

$$hf = K_{\max} + W_0$$



This shows clear agreement with the photon theory, and not with wave theory:

No electrons emitted for *$f < f_0$*



Photon Theory of Light and the Photoelectric Effect

If light is a wave, theory predicts:

1. **Number of electrons and their energy should increase with intensity.**
2. **Frequency would not matter.**

If light is particles, theory predicts:

- **Increasing intensity increases number of electrons but not energy.**
- **Above a minimum energy required to break atomic bond, kinetic energy will increase linearly with frequency.**
- **There is a cutoff frequency below which no electrons will be emitted, regardless of intensity.**

Conclusion: light consists of particles with energy $E=hf$: photons

Energy, Mass, and Momentum of a Photon

Clearly, a photon must travel at the speed of light. Looking at the relativistic equation for momentum, it is clear that this can only happen if its rest mass is zero.

$$p = mv / \sqrt{1 - v^2 / c^2}$$

$$E^2 = p^2c^2 + m^2c^4$$

We already know that the energy is *hf*; we can put this in the relativistic energy-momentum relation and find the momentum:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}.$$

A photon must have directedness (and momentum) as follows from the Compton effect

Compton Effect

Compton experiments (1923)

scattered X-rays from different materials have slightly longer wavelength than the incident ones

the wavelength depends on the scattering angle:

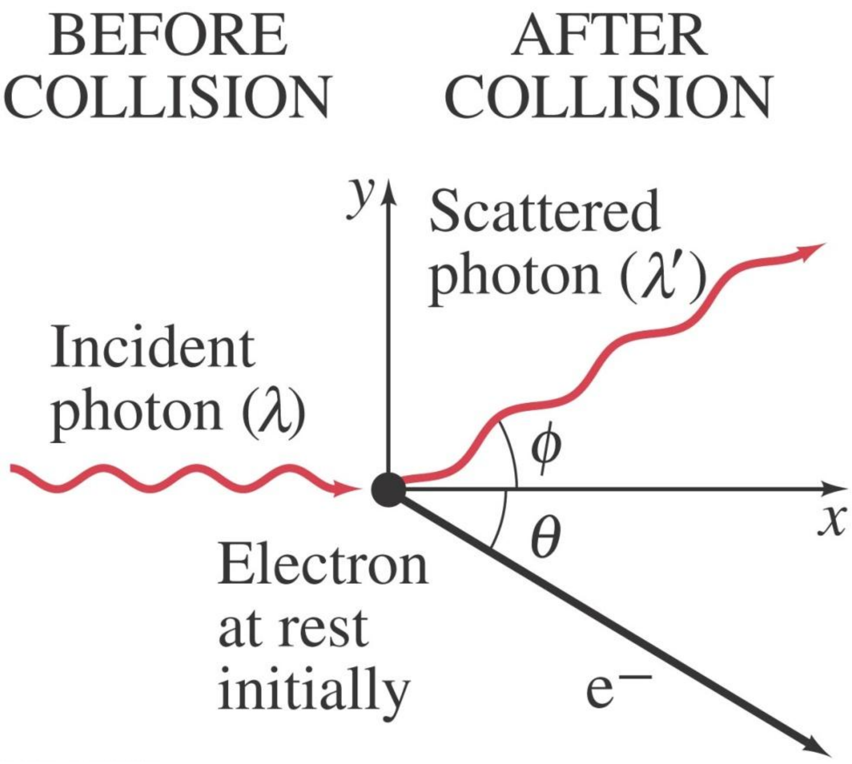
$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\phi).$$

The Nobel Prize in Physics 1927
"for his discovery of the effect named after him"

Arthur Compton

Compton Effect

This is another effect that is correctly predicted by the photon model and not by the wave model.



Before collision

photon $E = hf = \frac{hc}{\lambda}$ $p = \frac{h}{\lambda}$

electron $E_e = m_e c^2$

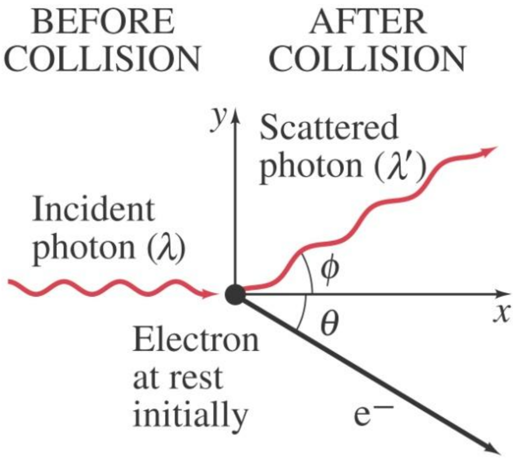
After collision

photon $E' = \frac{hc}{\lambda'}$ $p' = \frac{h}{\lambda'}$

electron $E^e_{tot} = \gamma m_e c^2$ $p_e = \gamma m_e v$

$E^e_{kin} = (\gamma - 1) m_e c^2$

Compton Effect



Conservation of energy

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$

Conservation of momentum

Along x:

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + \gamma m_e v \cos\theta$$

Along y:

$$0 = \frac{h}{\lambda'}\sin\phi - \gamma m_e v \sin\theta$$

Three equations with 3 unknowns, eliminate v and θ

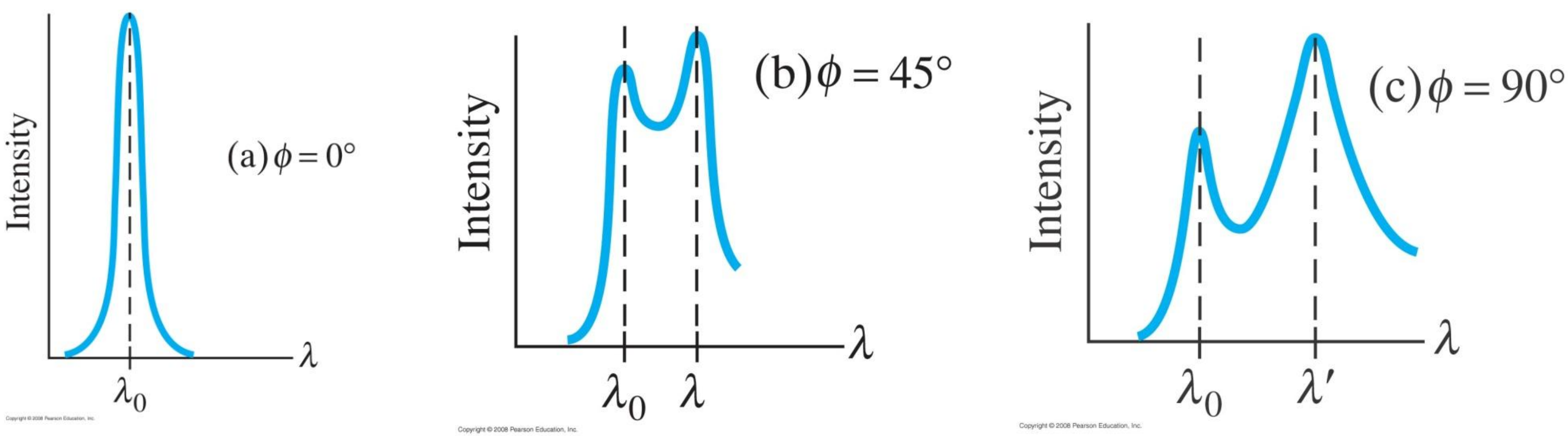
Compton scattering:

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\phi)$$

Compton Effect

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = \lambda_C(1-\cos\phi)$$

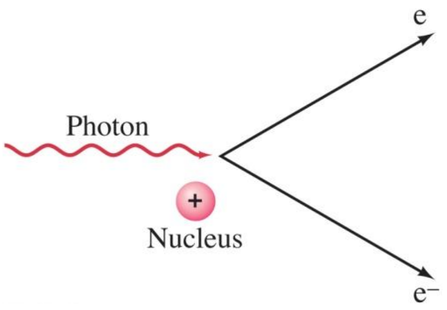
Note that λ_C ~ 0.00243 nm
So the effects is not so well visible with visible light
Compton performed his experiment with x-rays



Photon Interactions; Pair Production

Photons passing through matter can undergo the following interactions:

1. Photoelectric effect: photon is completely absorbed, electron is ejected.
2. Photon may be totally absorbed by electron, but not have enough energy to eject it; the electron moves into an excited state.
3. The photon can scatter from an atom and lose some energy.
4. The photon can produce an electron–positron pair.



Minimum energy:

$$E = \frac{hc}{\lambda} = 2m_e c^2$$

Wave Nature of Matter

Just as light sometimes behaves like a particle, matter sometimes behaves like a wave.

The wavelength of a particle of matter is

$$\lambda = \frac{h}{p}.$$

De Broglie wavelength of matter

Louis
De Broglie

The Nobel Prize in Physics 1920
"for his discovery of the
wave nature of electrons"

Wave-Particle Duality; the Principle of Complementarity

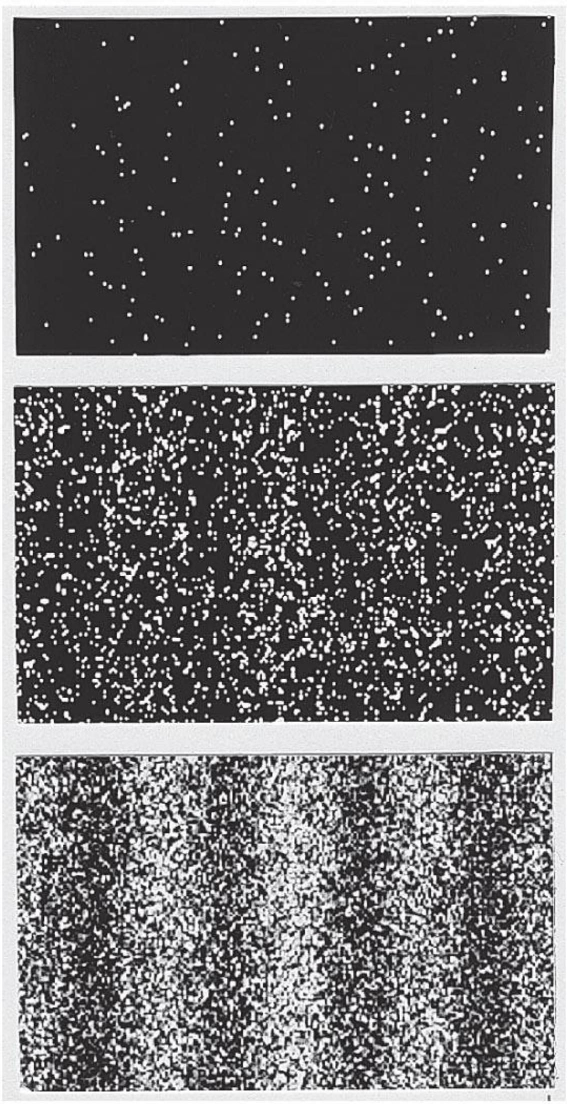
We have phenomena such as diffraction and interference that show that light is a wave, and phenomena such as the photoelectric effect and the Compton effect that show that it is a particle.

Which is it?

This question has no answer; we must accept the dual wave–particle nature of light.

The principle of complementarity states that both the wave and particle aspects of light are fundamental to its nature.

Particles as Waves; Waves as Particles



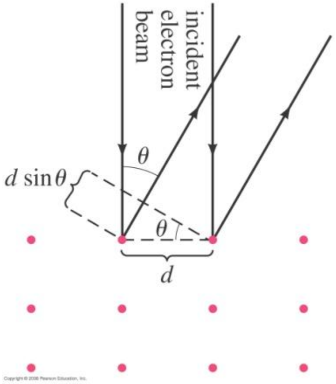
Youngs Interference experiment

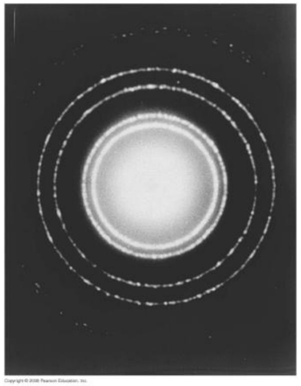
Particles as Waves

"for their experimental discovery of the diffraction of electrons by crystals"

George P Thomson

Clinton J Davisson





1961 Claus Jönsson of Tübingen
Real two-slit experiment with electrons
Dubbed: the most famous experiments

Richard Feynman on the Double Slit Paradox: Particle or Wave?

Wave–particle duality of C_{60} molecules
Markus Arndt, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw and Anton Zeilinger
Nature 401, 680-682(14 October 1999)

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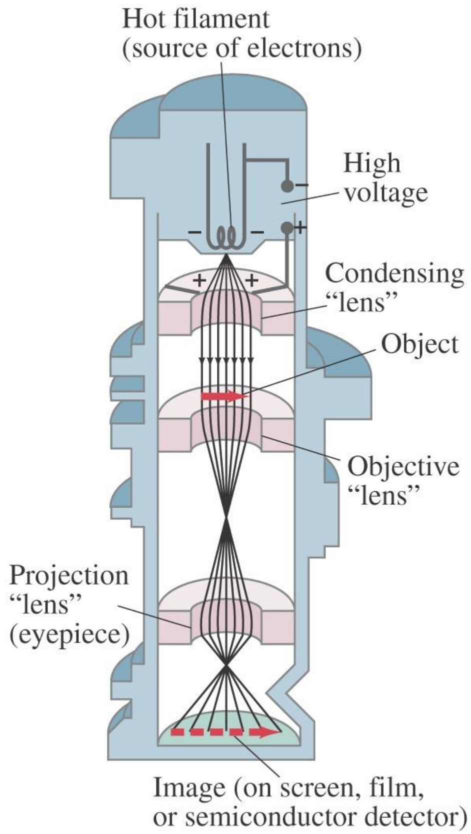
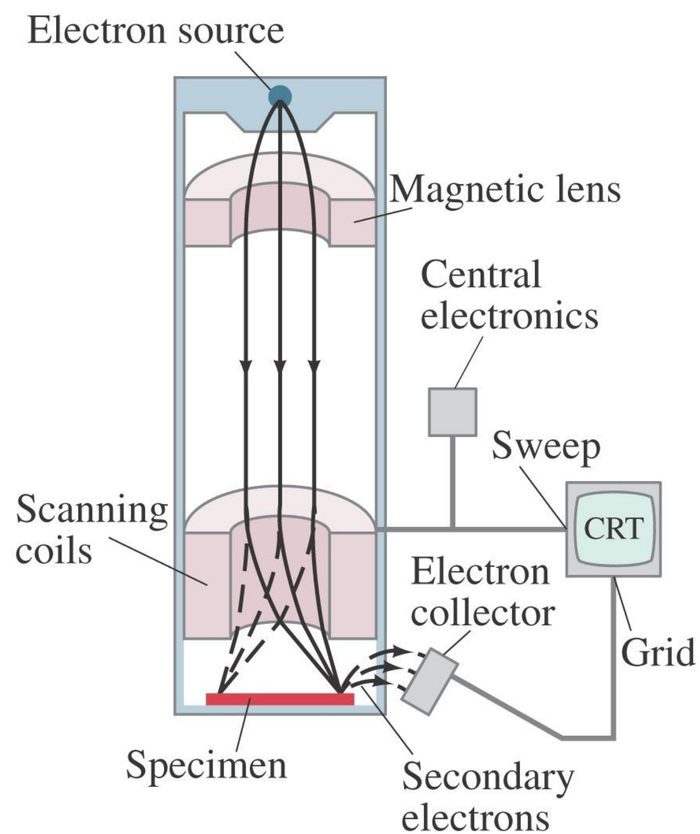
Where is the limit ?
Decoherence ??

Electron Microscopes

Electrons waves used for imaging Wavelengths of about 0.004 nm.



Transmission electron microscope – the electrons are focused by magnetic coils



Scanning electron microscope – the electron beam is scanned back and forth across the object to be imaged.