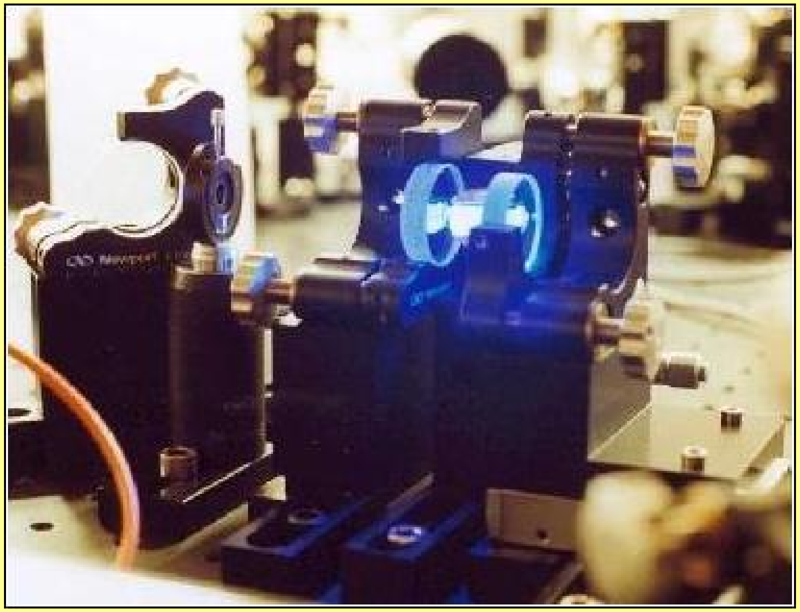
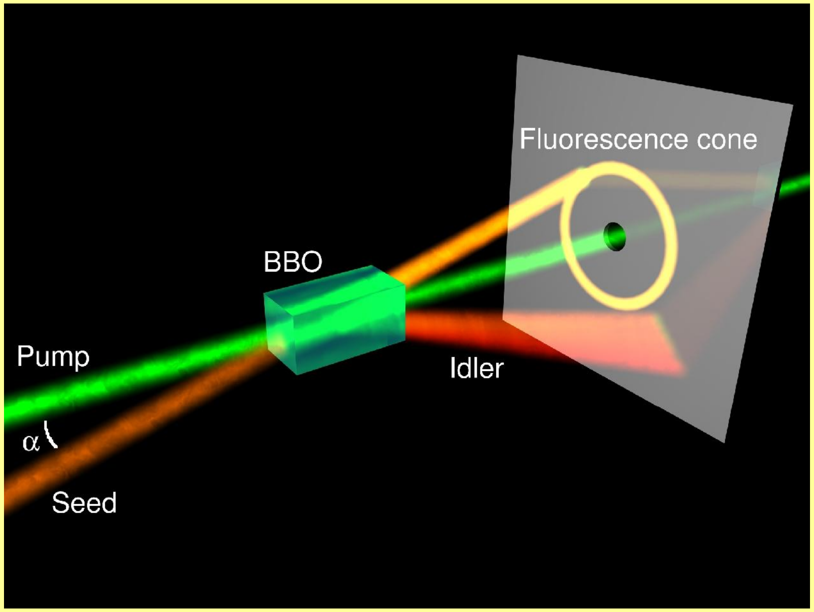
# Optical Parametric Oscillation and Amplification





# Optical Parametric Amplification

Consider a NLO-process

$$\omega_3 \to \omega_1 + \omega_2$$

Where a short-wavelength photon (pump) is converted into a photon at $\omega_1$ (signal) and a photon at $\omega_2$ (idler).

Start again from coupled wave equations:
- no absorption
- phase-matched $\Delta k=0$
- k defined

$$\kappa = d\frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}}$$

This leads to coupled amplitudes

$$\frac{dA_1}{dz} = -\frac{1}{2}i\kappa A_3 A_2^* e^{-i\Delta kz}$$

$$\frac{dA_2^*}{dz} = \frac{1}{2}i\kappa A_1 A_3^* e^{i\Delta kz}$$

Assume no depletion of the pump:

$$A_3(z) = A_3(0)$$

Define: $g = \kappa A_3(0)$

Coupled amplitudes:

$$\frac{dA_1}{dz} = -\frac{1}{2}igA_2^* \qquad \frac{dA_2^*}{dz} = \frac{1}{2}igA_1$$

Boundary conditions: $A_2(0)=0$ and $A_1(0)=small$

$$A_1(z) = A_1(0)\cosh\left(\frac{gz}{2}\right)$$

$$A_2^*(z) = A_1(0)\sinh\left(\frac{gz}{2}\right)$$

Approximation for $gz>0$

$$|A_1(z)|^2 = |A_2(z)|^2 \propto e^{gz}$$

Both fields grow with gain factor: $g$
**This parametric gain.**

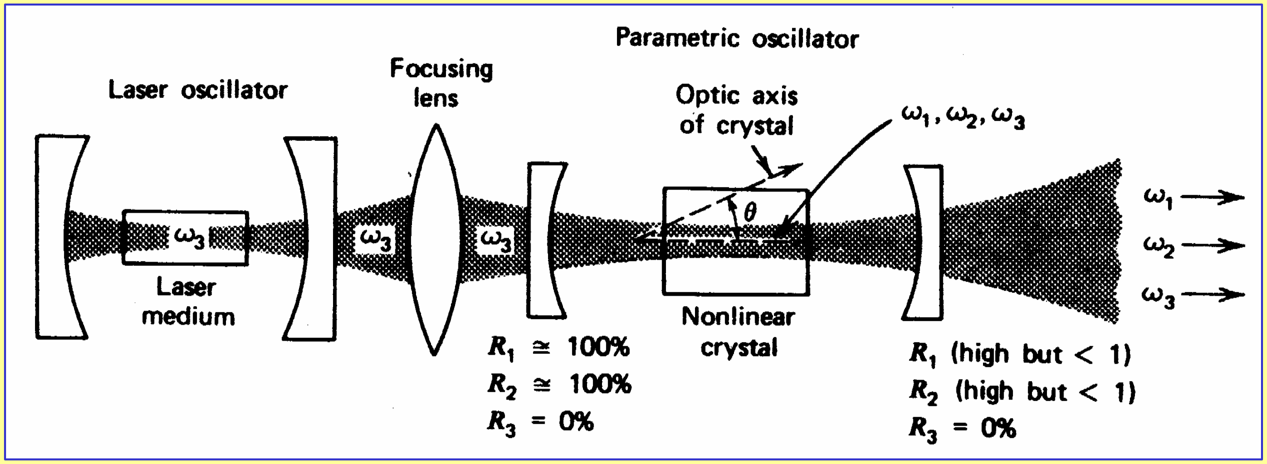
Verify that: $-\frac{dA_3A_3^*}{dz} = \frac{dA_1A_1^*}{dz} = \frac{dA_2A_2^*}{dz}$

and $-\Delta\left(\frac{P_3}{\omega_3}\right) = \Delta\left(\frac{P_1}{\omega_2}\right) = \Delta\left(\frac{P_2}{\omega_2}\right)$

Manley-Rowe equations

# Parametric oscillation

Principle: amplification starts from **noise**, as in laser




Assume again parametric gain with $\Delta k=0$

Steady state condition;

$$\frac{dA_1}{dz} = \frac{dA_2}{dz}$$

Retain losses (absorption or mirror losses)

$$-\frac{1}{2}\alpha_1 A_1 - \frac{1}{2} igA_2^* = 0$$

$$\frac{1}{2} igA_1 - \frac{1}{2}\alpha_2 A_2^* = 0$$

Nontrivial solution at threshold if:

$$g^2 = \alpha_1 \alpha_2$$

Above threshold if

$$g^2 > \alpha_1 \alpha_2$$

Gain > losses

# Tuning of an OPO

Parameter is the phase-matching condition:

$$\Delta\vec{k} = 0 \qquad \vec{k}_3 = \vec{k}_1 + \vec{k}_2$$

For **co-linear** beams this equals:

$$n_3\omega_3 = n_1\omega_1 + n_2\omega_2$$

And energy conservation

$$\omega_3 = \omega_1 + \omega_2$$

Phase-matching again in birefringent crystals
e.g., Type I

$$n_3^e(\theta_m)\omega_3 = n_1\omega_1 + n_2\omega_2$$

At each specific angle $\theta_m$ the OPO will produce a combination of two frequencies $\omega_1$ and $\omega_2$

Rotation of angle near $\theta_m$ yields

$$\theta_m \to \theta_m + \Delta\theta$$

$$n_1 \to n_1 + \Delta n_1$$

$$n_2 \to n_2 + \Delta n_2$$

$$n_3 \to n_3 + \Delta n_3$$

For fixed pump this gives

$$\omega_1 \to \omega_1 + \Delta\omega_1 \qquad \omega_2 \to \omega_2 + \Delta\omega_2$$

Energy conservation; $\Delta\omega_2 = -\Delta\omega_1$

Index $n_3$ changes if it is extra-ordinary

$$\Delta n_3 = \left|\frac{\partial n_3}{\partial\theta}\right|_{\theta_m} \Delta\theta$$ angle dependence

$$\Delta n_1 = \left|\frac{\partial n_1}{\partial\omega_1}\right|_{\omega_1} \Delta\omega_1$$ dispersion

$$\Delta n_2 = \left|\frac{\partial n_2}{\partial\omega_2}\right|_{\omega_2} \Delta\omega_2$$ dispersion

New phase-matching condition:

$$(n_3 + \Delta n_3)\omega_3 =$$
$$(n_1 + \Delta n_1)(\omega_1 + \Delta\omega_1) + (n_2 + \Delta n_2)(\omega_2 + \Delta\omega_2)$$

Use: $\Delta\omega_2 = -\Delta\omega_1$ and solve:

$$\Delta\omega_1 = \frac{\omega_3\Delta n_3 - \omega_1\Delta n_1 - \omega_2\Delta n_2}{n_1 - n_2}$$

# Tuning of an OPO -2

Then:

$$\Delta\omega_1 = \frac{\omega_3\left|\frac{\partial n_3}{\partial\theta}\right|\Delta\theta - \omega_1\left|\frac{\partial n_1}{\partial\omega_1}\right|\Delta\omega_1 + \omega_2\left|\frac{\partial n_2}{\partial\omega_2}\right|\Delta\omega_1}{n_1 - n_2}$$

Solve for: $\Delta\omega_1$

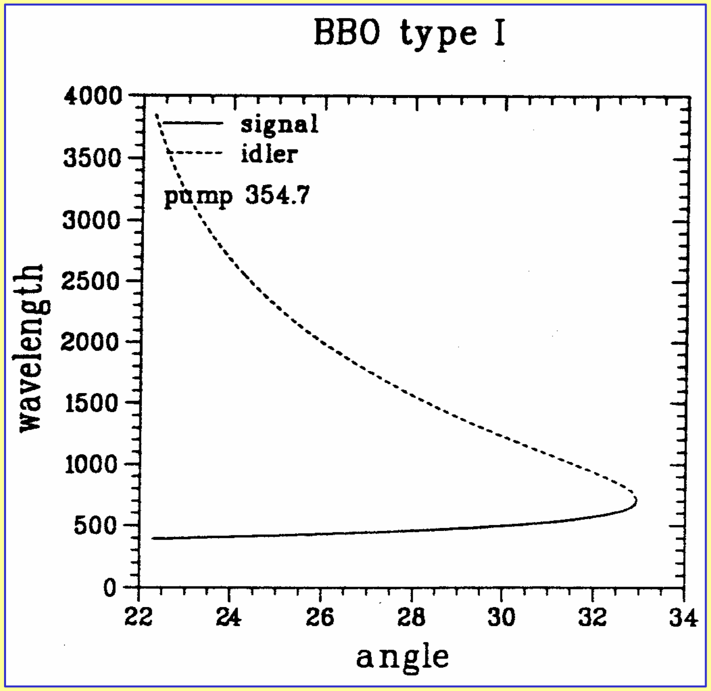
$$\frac{\Delta\omega_1}{\Delta\theta} = \frac{\omega_3\left|\frac{\partial n_3}{\partial\theta}\right|}{(n_1 - n_2) + \left[\omega_1\left|\frac{\partial n_1}{\partial\omega_1}\right| - \omega_2\left|\frac{\partial n_2}{\partial\omega_2}\right|\right]}$$

Use the result for the calculation of the opening angle obtained previously (SHG)

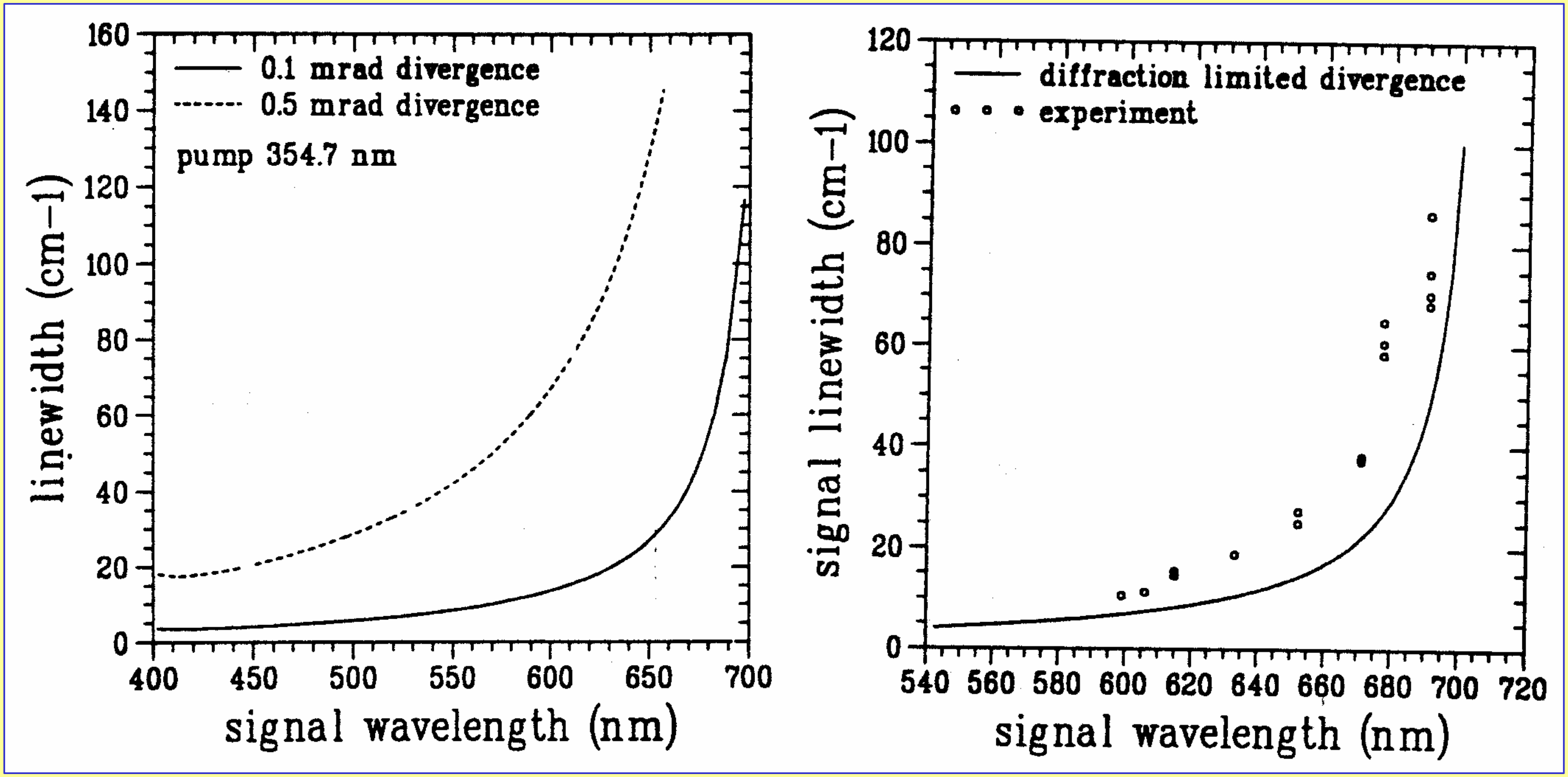
$$\left|\frac{\partial n_3}{\partial\theta}\right|_{\theta_m} = -\frac{1}{2}n_o^3\left[n_e^{-2}(\omega_3) - n_o^{-2}(\omega_3)\right]\sin 2\theta_m$$

This results in the angle tuning fucntion:

$$\frac{\partial\omega_1}{\partial\theta} = \frac{-\frac{1}{2}n_o^3\left[n_e^{-2}(\omega_3) - n_o^{-2}(\omega_3)\right]\omega_3\sin 2\theta_m}{(n_1 - n_2) + \left[\omega_1\frac{\partial n_1}{\partial\omega_1} - \omega_2\frac{\partial n_2}{\partial\omega_2}\right]}$$



# Bandwidth of an OPO



# SNLO – Public Domain Software for non-linear optics

Snlo.lnk

http://www.sandia.gov/imrl/XWEB1128/xxtal.htm