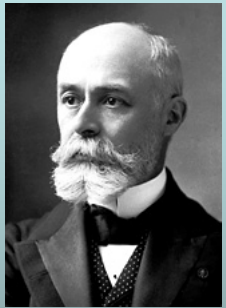
Structure of Nuclear Matter

The strong force
Protons and neutrons
The mass equation
Radioactivity
The energy of α-particles
Decay times of α-particles



Henri Becquerel

Pierre Curie

Marie Curie

Marie Curie

The Nobel Prize in Physics 1903
for their work on radioactivity

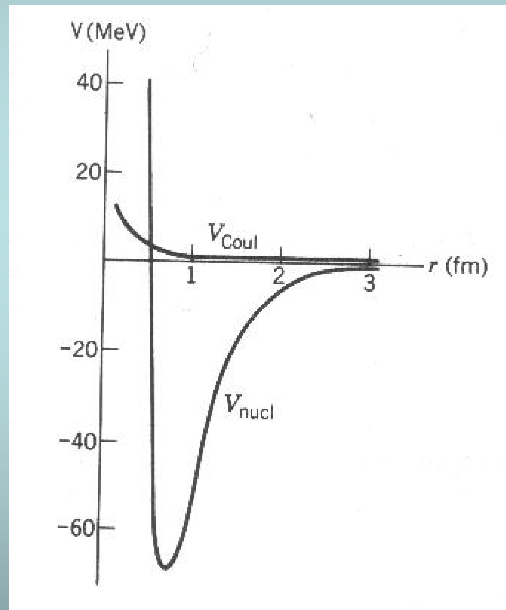
The Nobel Prize in Chemistry 1911
for her discovery of elements

The fundamental forces in nature

1. Electromagnetic
2. Gravitational
3. Strong
4. Weak

Binding Energy and Nuclear Forces

The force that binds the nucleons together is called the strong nuclear force. It is a very strong, but short-range, force. It is essentially zero if the nucleons are more than about 10^{-15} m apart. The Coulomb force is long-range; this is why extra neutrons are needed for stability in high-Z nuclei.



Calculate that at 1 fm: $V_{Coulomb} = \dfrac{e^2}{4\pi\varepsilon_0 r} = 1.44\mathrm{Mev}$

Nuclei that are unstable decay; many such decays are governed by another force called the weak nuclear force. This is related to β-radio-activity.

Structure and Properties of the Nucleus

Nuclei: protons and neutrons.

Proton has positive charge;

$$m_n = 1.67262 \times 10^{-27}\ \text{kg}$$

Neutron is electrically neutral;

$$m_n = 1.67493 \times 10^{-27}\ \text{kg}$$

Number of protons: atomic number, Z

Number of nucleons: atomic mass number, A

Neutron number: $N = A - Z$

Symbol: $${}^{A}_{Z}X$$

$$\mu = \frac{m_p}{m_e} = 1836.152\,672\,61\,(85)$$

Isotopes

No theory

Extreme density

Because of wave–particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx (1.2 \times 10^{-15}\,\mathrm{m})(A^{\frac{1}{3}}).$$

Nuclear density

~10^{15} ordinary matter

Masses scale: carbon-12 atom, 12 u
u is a unified atomic mass unit.

$$1\,\mathrm{u} = 1.6605 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV}/c^2$$

TABLE 41–1 Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

	Mass		
Object	**kg**	**u**	**MeV/c^2**
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Binding Energy and Einsteins $E=mc^2$

Binding energy affects mass of composite particle

$m_{\text{H-atom}} < m_{\text{proton}} + m_{\text{electron}}$

$m_{\text{H-atom(n=1 state)}} < m_{\text{H-atom(n=2 state)}}$

$$E_b(\text{atom}) = \left[M(\text{nucleus}) + Zm_e - M(\text{atom})\right]c^2$$

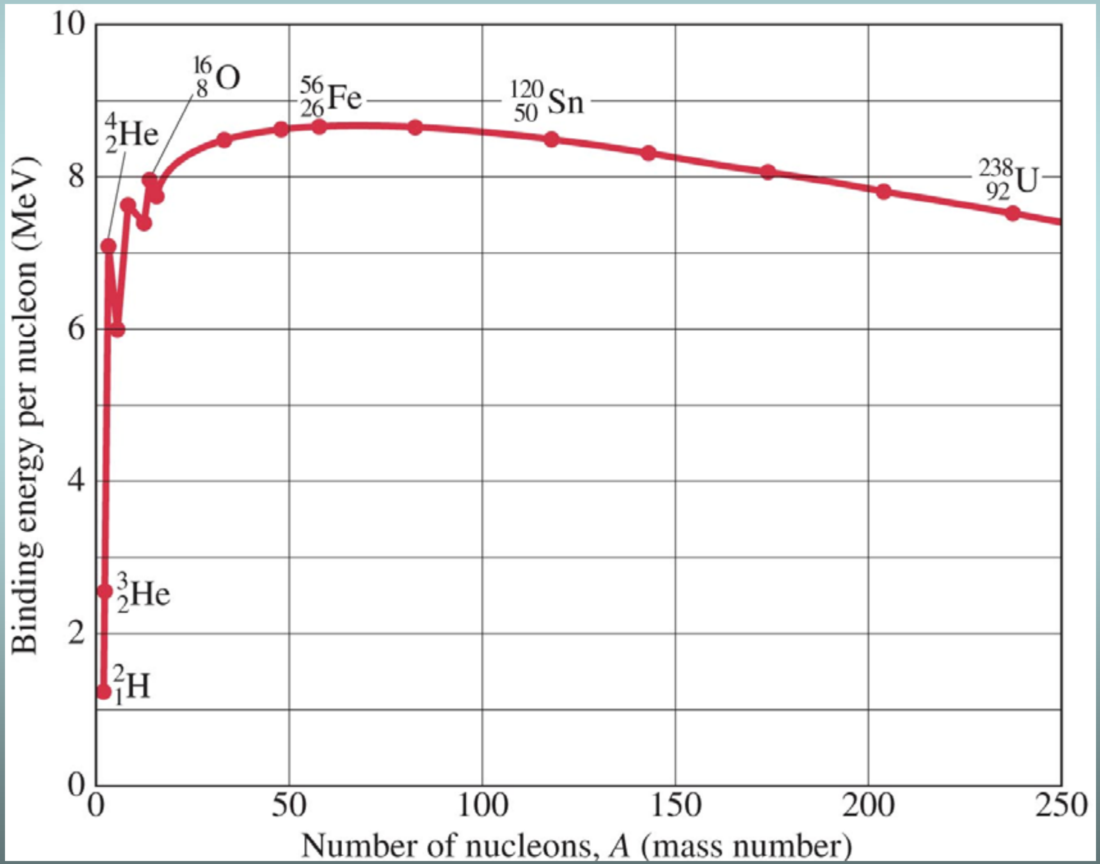
(mind the signs)

These effects are much larger in the realm of nuclei than in that of electromagnetism

$$E_b(\text{nucleus}) = \left[ZM_p + NM_n - M(\text{nucleus})\right]c^2$$

Binding Energy and Nuclear Forces

From observation: binding energy per nucleon: E_b/A



Note:
α is point of relative stability

Fusion

Fission

The semi-empirical mass formula

Von Weizsäcker
Liquid drop model

$$E_b\left({}^{A}X\right) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A/2 - Z)^2}{A} + \varepsilon_5$$

Note:

$$R \sim A^{1/3}$$

Volume term

$$\sim R^3 \sim A$$

Surface correction:
on the outer surface (4πR²) the binding is less, because There are no particles to contribute

$$\sim 4\pi R^2 \sim A^{2/3}$$

Good fit for:

$$a_1 = 15.76\text{Mev}$$
$$a_2 = 17.81\text{Mev}$$
$$a_3 = 0.7105\text{Mev}$$
$$a_4 = 94.80\text{Mev}$$
$$a_5 = 39\text{Mev}$$

The semi-empirical mass formula

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$$V_{rep} = \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

Coulomb energy stored in a uniform solid sphere of charge Ze and radius R

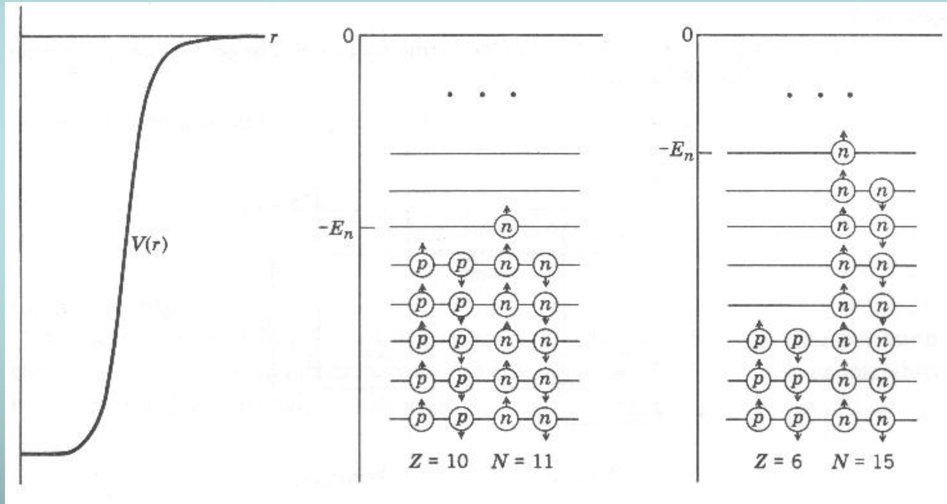
(negative binding) $\sim \dfrac{Z^2}{A^{1/3}}$

"pairing energy"

Protons and Neutrons in the Fermi-gas model

$$- a_4 \frac{(A/2 - Z)^2}{A}$$

Both protons and neutrons cannot fill the lowest “orbitals” because the have to follow the Pauli exclusion principle



“ Symmetry Energy” preference for

$$Z = N$$

Based on this concept: the “nuclear shell model”

"for the discovery concerning nuclear shell structure"

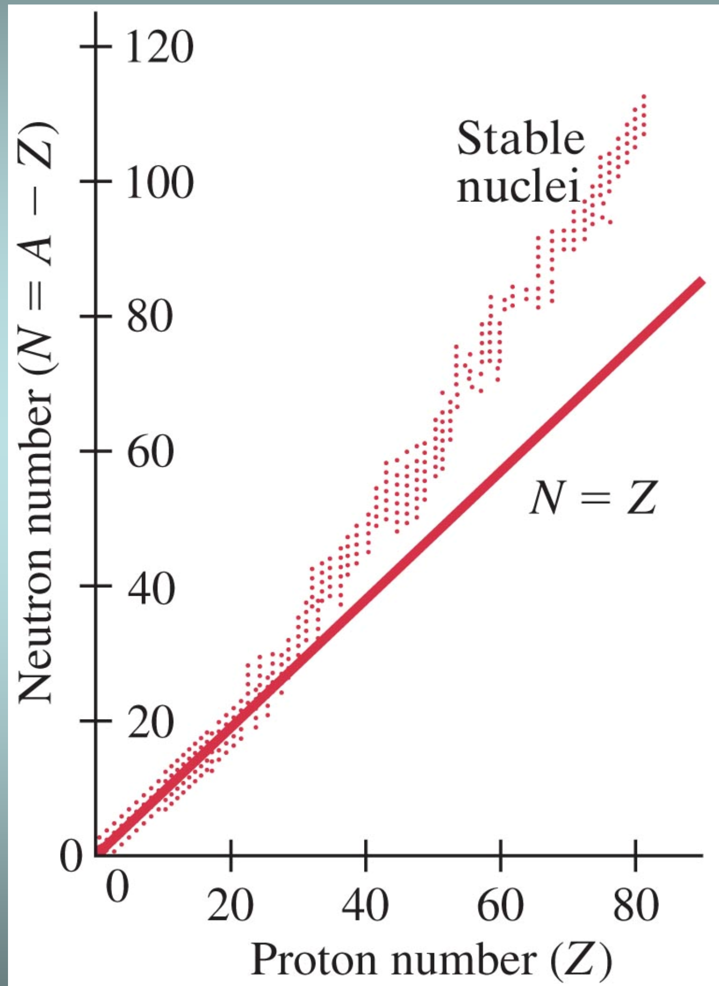
Maria Goeppert-Mayer

Binding Energy and Nuclear Forces

From observation:

The higher the binding energy per nucleon, the more stable the nucleus.

More massive nuclei require extra neutrons to overcome the Coulomb repulsion of the protons in order to be stable.



Radioactivity

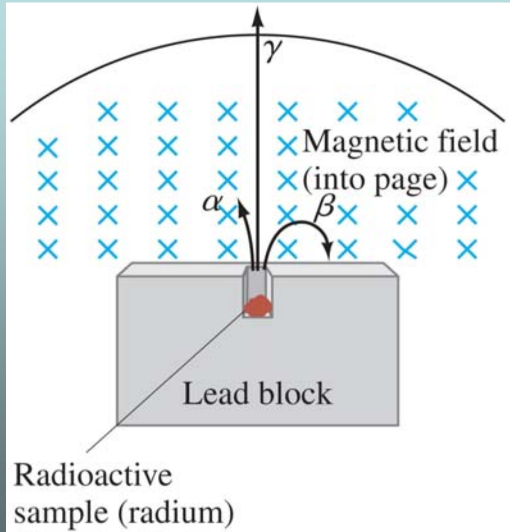
Radioactive rays were observed to be of three types:

1. Alpha rays, which could barely penetrate a piece of paper

2. Beta rays, which could penetrate 3 mm of aluminum

3. Gamma rays, which could penetrate several centimeters of lead

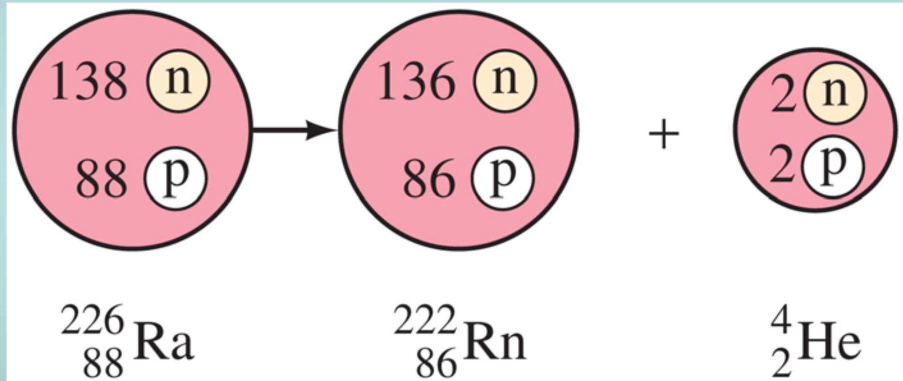
We now know that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are electromagnetic radiation.



Alpha Decay

Radium-226 will alpha decay to radon-222:

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He}$$



1. **Why do some nuclei decay ?**
2. **Why emit α's, and not protons, or neutrons ?**

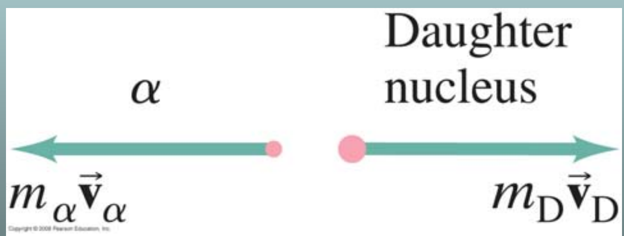
Alpha Decay

α **decay :**

$$ {}^{A}_{Z}\mathrm{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{N}' + {}^{4}_{2}\mathrm{He} $$

- strong nuclear force cannot hold a large nucleus together
- mass of the parent nucleus is greater than the sum of the masses of the daughter nucleus and the alpha particle
- this difference is called the disintegration energy.

Who takes the kinetic energy?



Momentum conservation: $m_\alpha v_\alpha = m_D v_D$

So: $v_\alpha = m_D v_D / m_\alpha$

Kinetic energy:

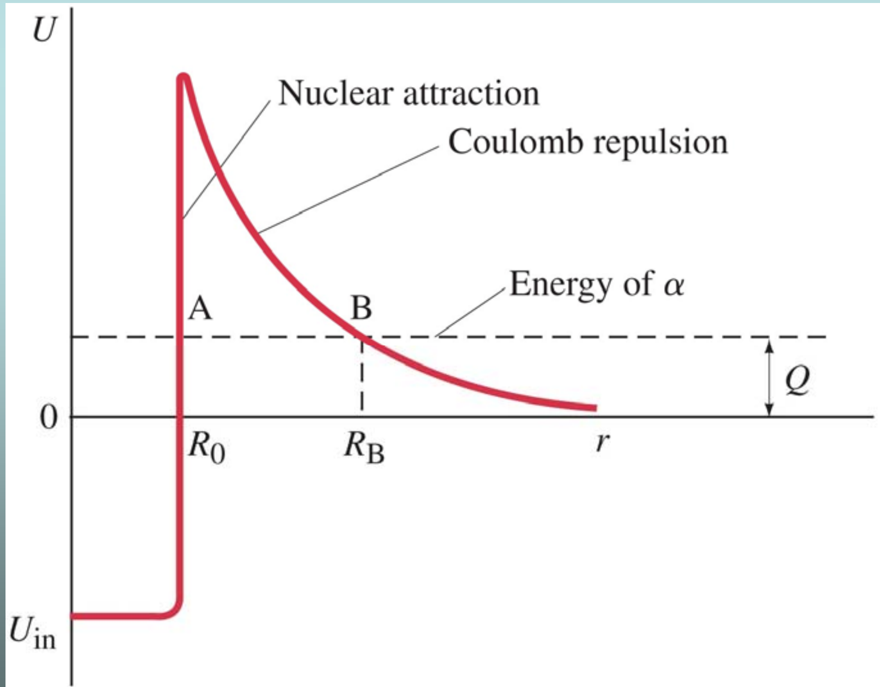
$$ K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{2} m_\alpha \left(\frac{m_D v_D}{m_\alpha} \right)^2 = \left(\frac{m_D}{m_\alpha} \right) K_D $$

$$ K_{tot} = K_\alpha + K_D $$

Alpha Decay

When a nucleus decays through alpha emission, energy is released.
Why is it that these nuclei do not decay immediately?

Although energy is released in the decay, there is still an energy barrier:



Quantum tunneling through a barrier

Heisenberg uncertainty : energy conservation can be violated as long as the violation does not last too long:

$$(\Delta E)(\Delta t) \approx \frac{h}{2\pi}$$

desintegration energy

$$Q = M_{\text{parent}}c^2 - (M_D + m_\alpha)c^2$$

Alpha Decay

Heisenberg

$$(\Delta E)(\Delta t) \approx \frac{h}{2\pi}$$

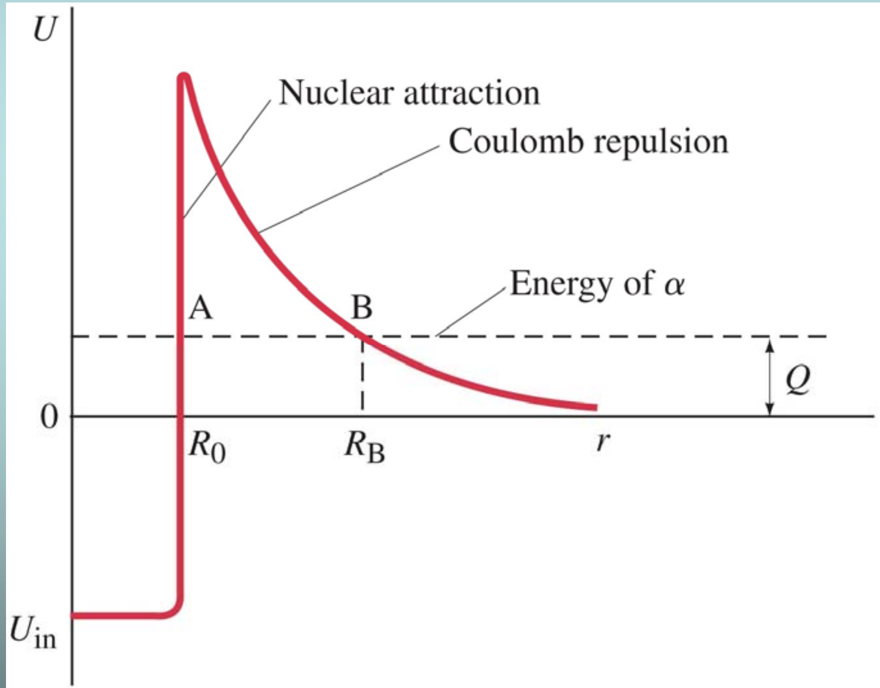
The higher the barrier, the less time the alpha particle has to get through it, and the less likely that is to happen.

quantum tunneling: quantitative

$$T \approx e^{-2\int_{r'}^{r''}\sqrt{\left(2m/\hbar^2\right)\left[V(r)-E\right]}dr}$$

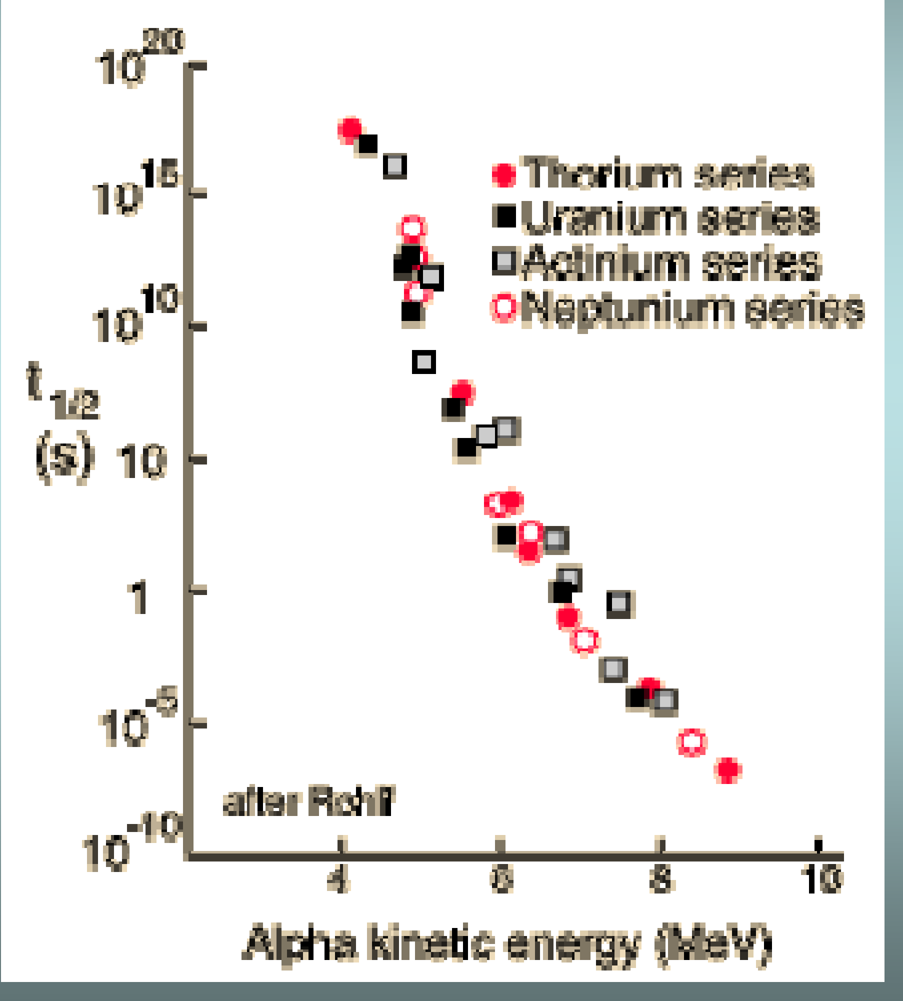
Half lives and kinetic energies of α's

$$Q = M_{\text{parent}}c^2 - \left(M_D + m_\alpha\right)c^2$$



1. **Higher kinetic energy means traversing a smaller barrier**
2. **Uni-partcile decay gives a specific energy (momentum conservation)**

Half lives and kinetic energies of α's



There is an extremely wide variety of half-lives in α-decay;
and a connection to kinetic energy.

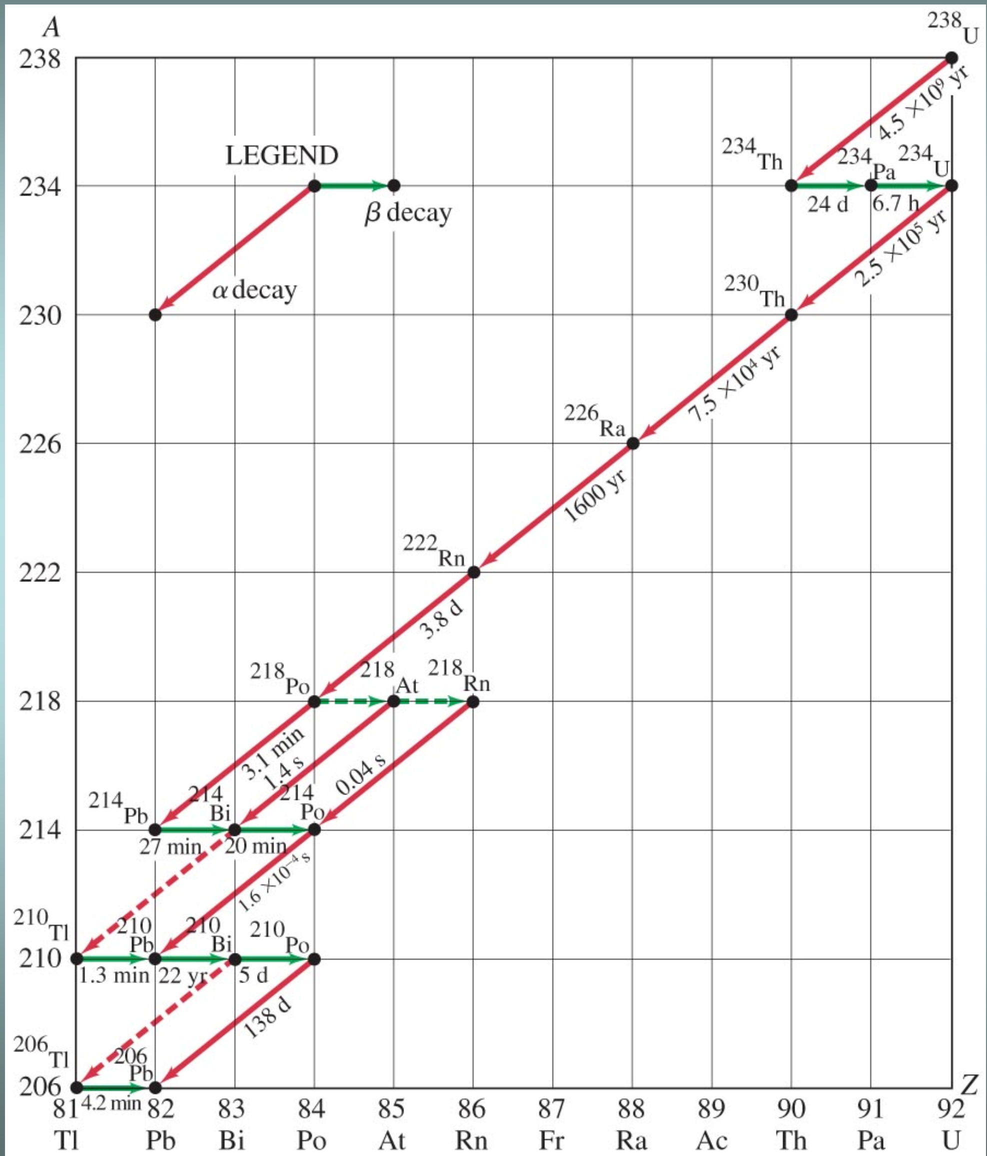
Due to the tunneling mechanism

Note: α's have a characteristic energy for each decay

Why ?

What do we mean by lifetime ?

Decay Series



A decay series occurs when one radioactive isotope decays to another radioactive isotope, which decays to another, and so on. This allows the creation of nuclei that otherwise would not exist in nature.

Pb-series

4n+2 series

Beta Decay

The electron in beta decay is not an orbital electron; it is created in the decay.

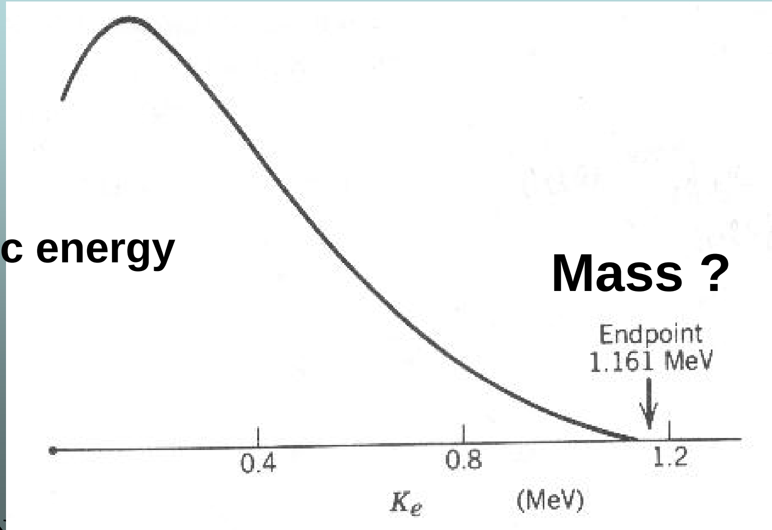
The fundamental process is a neutron decaying to a proton, electron, and neutrino:

$$n \rightarrow p + e^{-} + \text{neutrino}$$

The need for a particle such as the neutrino was discovered through analysis of energy and momentum conservation in beta decay – it could not be a two-particle decay.

1.

Electron kinetic energy In β-decay of ^{210}Bi



2.

Law of Physics; Angular momentum Conservation Particles have spin 1/2

Beta Decay

Beta decay occurs when a nucleus emits an electron. An example is the decay of carbon-14:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \text{neutrino}$$

The nucleus still has 14 nucleons, but it has one more proton and one fewer neutron.

This decay is an example of an interaction that proceeds via the weak nuclear force.

In general

$$^{A}_{Z}\mathrm{N} \rightarrow {}^{A}_{Z+1}\mathrm{N}' + e^{-} + \bar{\nu}$$

Gamma Decay

Gamma rays are very high-energy photons. They are emitted when a nucleus decays from an excited state to a lower state, just as photons are emitted by electrons returning to a lower state.