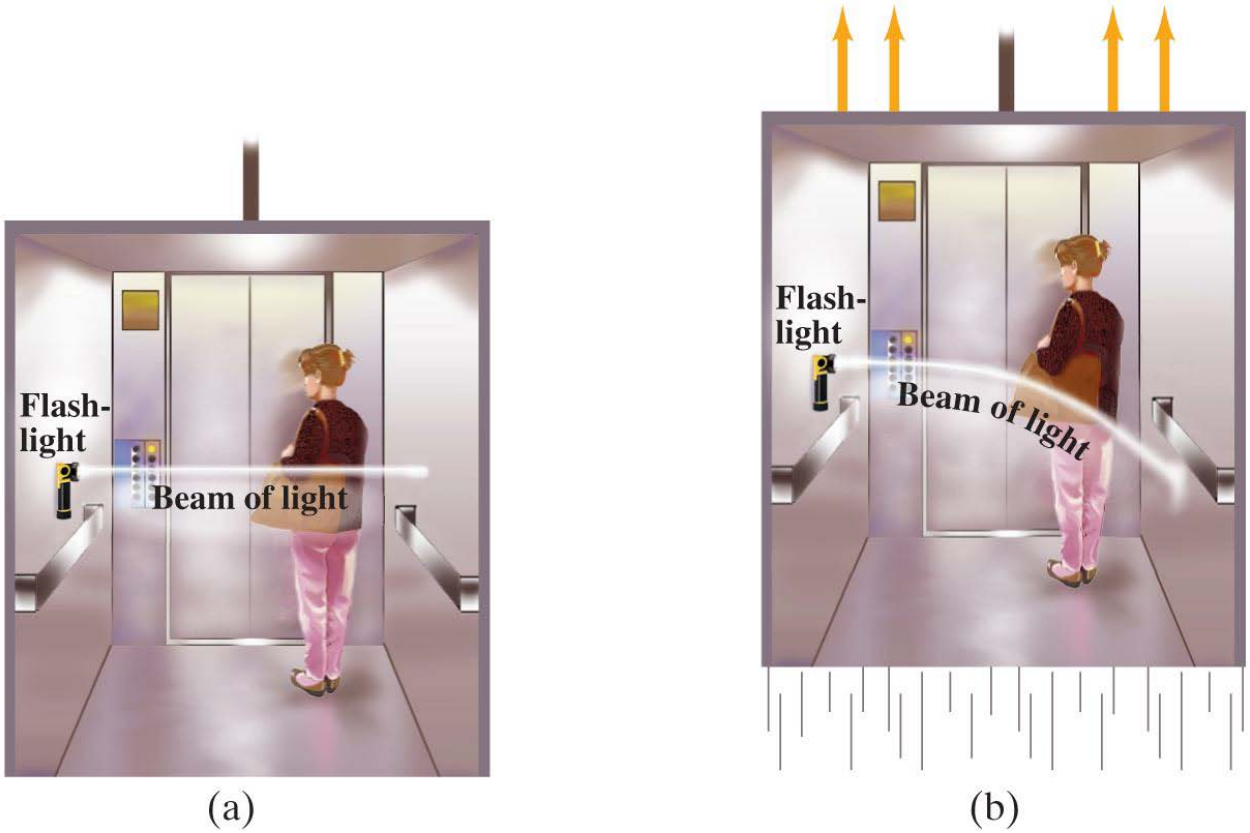
Aspects of the General Theory of Relativity

Chapter IV

1. How does gravity act
2. Cosmological redshift
3. Gravitational redshift
4. Black holes

General Relativity:
Gravity and the Curvature of Space

A light beam will be bent either by a gravitational field or by acceleration (outside observer):



Definition of a straight line; The line that a light ray follows

An outlook on General Relativity

GR deals with:

Gravitation

Acceleration

Principle of equivalence: it is impossible to distinguish a uniform gravitational field and a uniform acceleration.

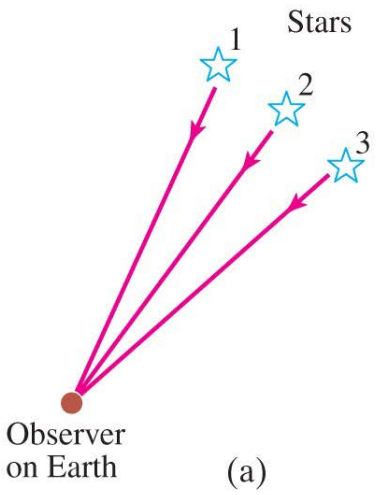
Another way to put it: mass in Newton's first law is the same as the mass in the universal law of gravitation.

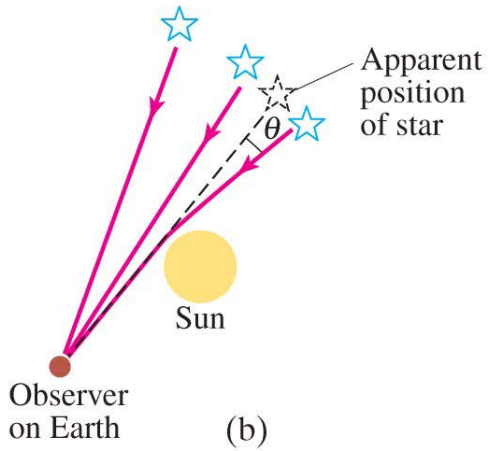
$$\vec{F} = m\vec{a} \qquad F = -G\frac{Mm}{R^2}$$

General Relativity:
Gravity and the Curvature of Space

This can make stars appear to move when we view them past a massive object:

Note:
The bending of light can in principle be explained by Newtons law (Soldner in 1801)
The difference is quantitative; a factor of 2, measured by Eddington in 1919.

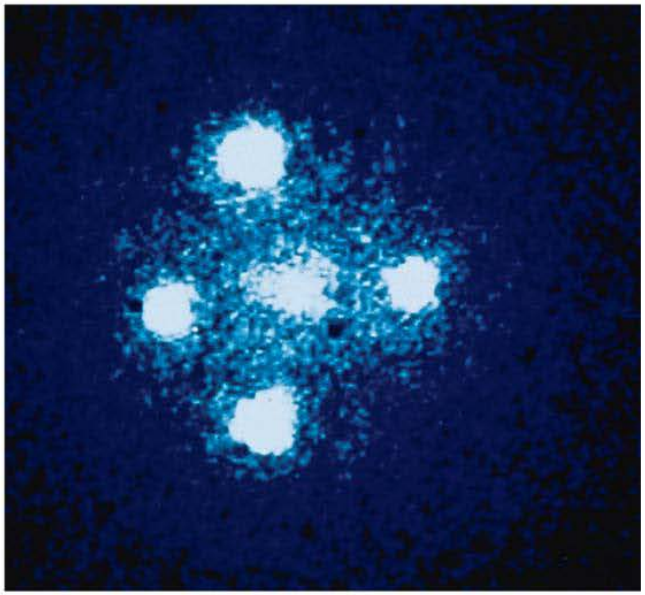




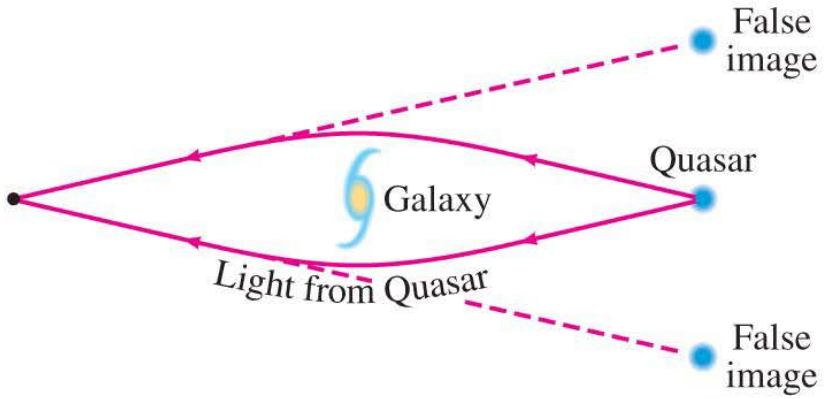
Gravity and the Curvature of Space

Gravitational lensing

This bending of light as it passes a massive object (star or galaxy) has been observed by telescopes:



(a)



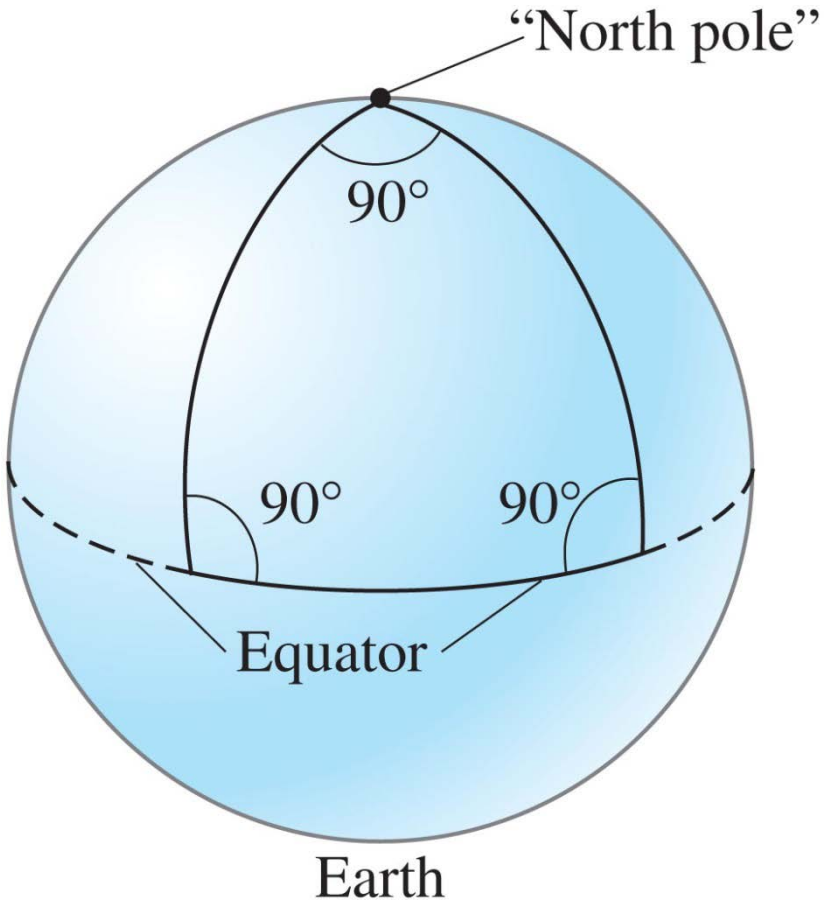
(b)

Fermat's principle in optics: light traveling between points chooses the shortest track

Gravity and the Curvature of Space

Einstein's general theory of relativity says that space itself is curved – hard to visualize in three dimensions!

This is a two-dimensional space with positive curvature:

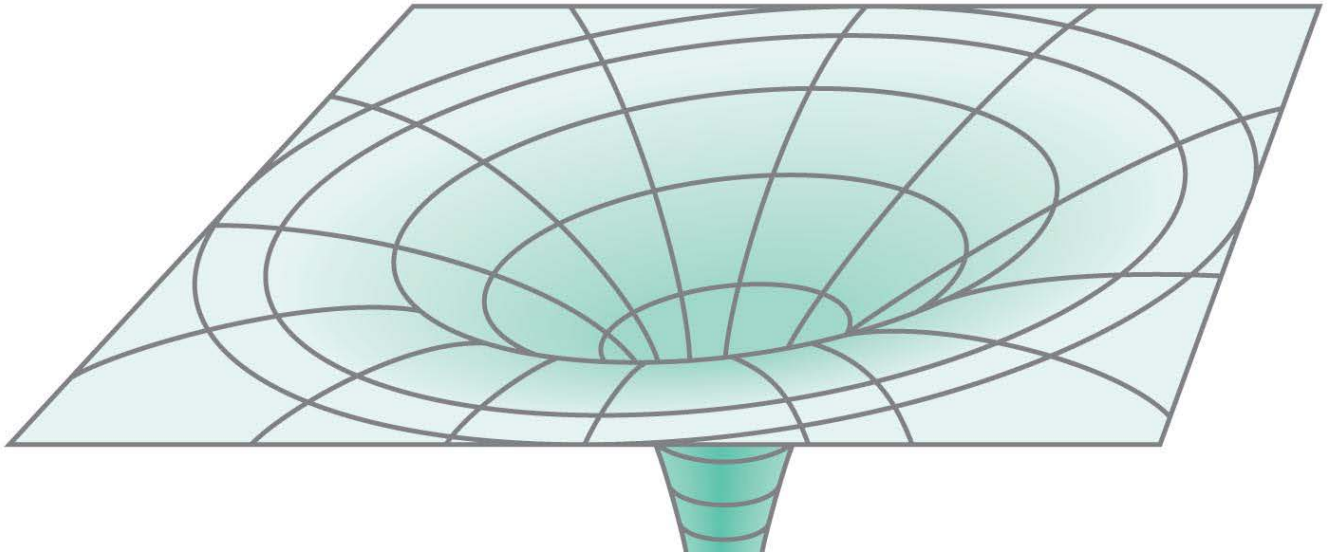


Not known what the overall curvature of the universe is (but close to zero)

NB; most curvature is local !

Gravity and the Curvature of Space

Space is curved around massive objects:



Fundamental notion of GR:

gravity is not a force but deformation of space

calculation is difficult, because of non-Euclidean geometry

Cosmological Red Shift

Edwin Hubble

$$\frac{\Delta\lambda}{\lambda^0} \equiv z \qquad \frac{\lambda}{\lambda^0} \equiv 1+z$$

Scale factor:
Cosmological redshift

Galaxies moving away from each other

Expansion of the universe

Interpretations:

- **Change of the underlying metric in expanding universe**
- **Interpretation as a Doppler shift**

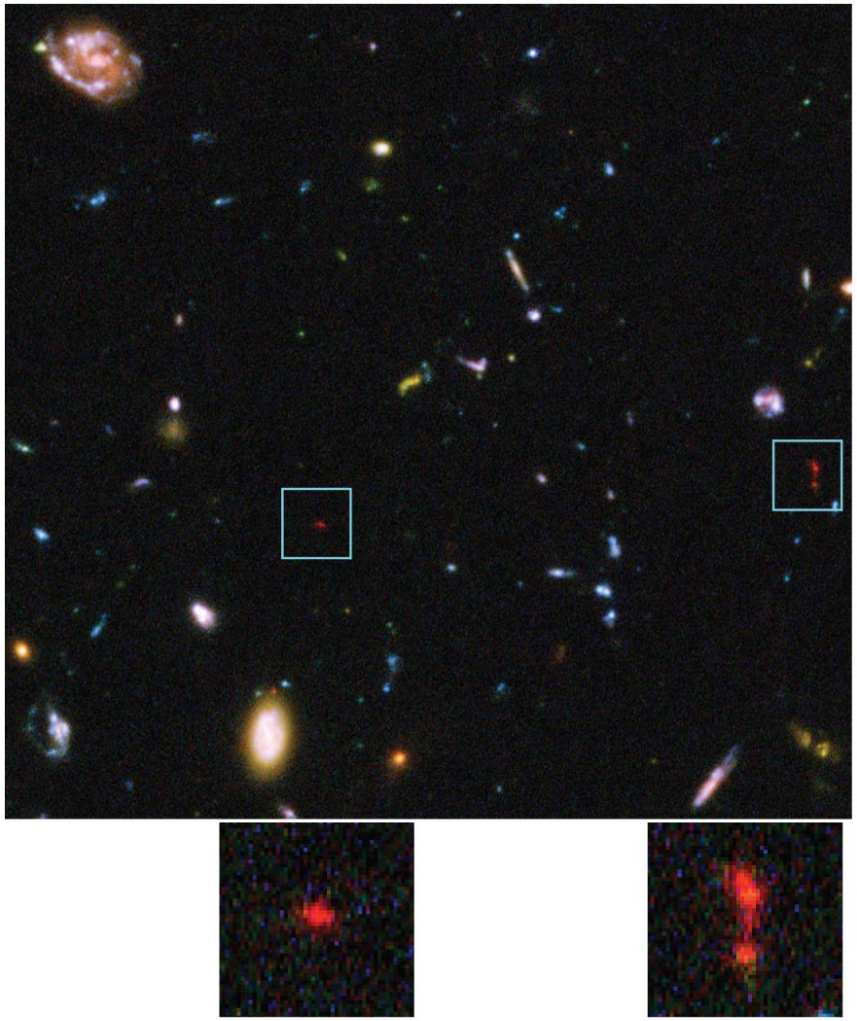
Redshift does not have dispersion

Note: $z \neq z(\lambda)$

Redshift & time

$$T = T_0\left[1 - \frac{1}{(1+z_{abs})^{3/2}}\right]$$

Expansion of the Universe



"Hubble measurements"

H = 71 km/s/Mpc = 22 km/s/Mly

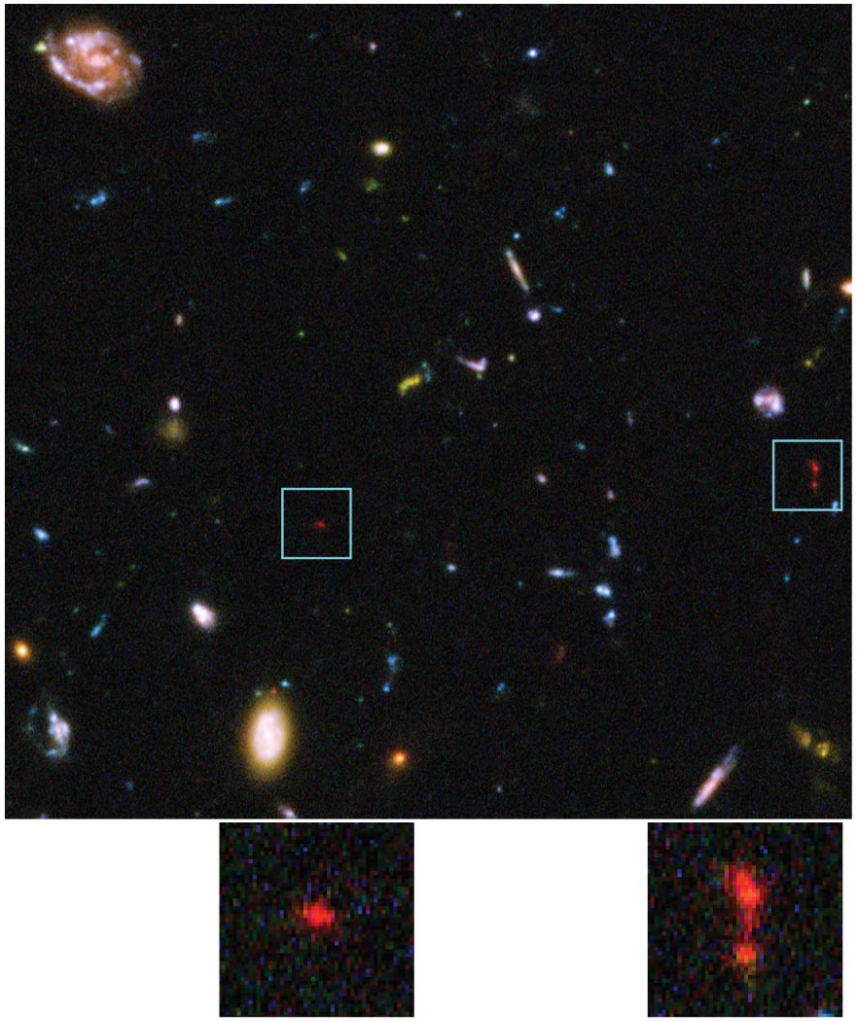
Parsec – parallax Earth-Sun is 1"

1 Light year =
$3 \times 10^8 \times 3600 \times 24 \times 365 = 9.5 \times 10^{15}$ m

$H = 22 \times 10^3 / \ 10^6 \times 9.5 \times 10^{15}\ s^{-1} =$
$2.3 \times 10^{-18}\ s^{-1}$

$1/H = 1 / 2.3 \times 10^{-18}\ s^{-1} = 4.3 \times 10^{17}$ s
$= 4.3 \times 10^{17} / 3600 \times 24 \times 365 = 13.6 \times 10^9$ yr

Expansion of the Universe



"Hubble measurements"

H = 71 km/s/Mpc = 22 km/s/Mly

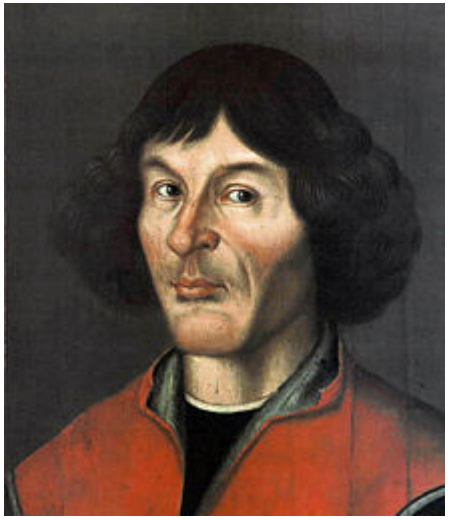
How much time did it take for galaxies to be separated at distance *d*
Assuming they depart with Hubble speed
v* = *Hd

$$t = \frac{d}{v} = \frac{d}{Hd} = \frac{1}{H}$$

This corresponds to ~13.6 billion years

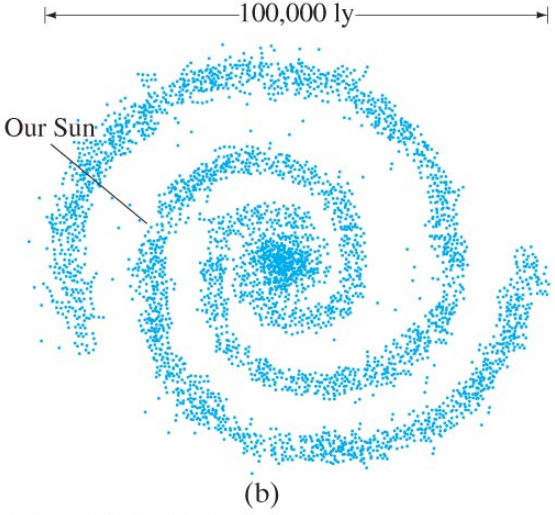
Note: Now 13.799 (21) Gyrs (Planck Collab.)
Accelerated expansion of the Universe

Copernicus and Cosmological Principle



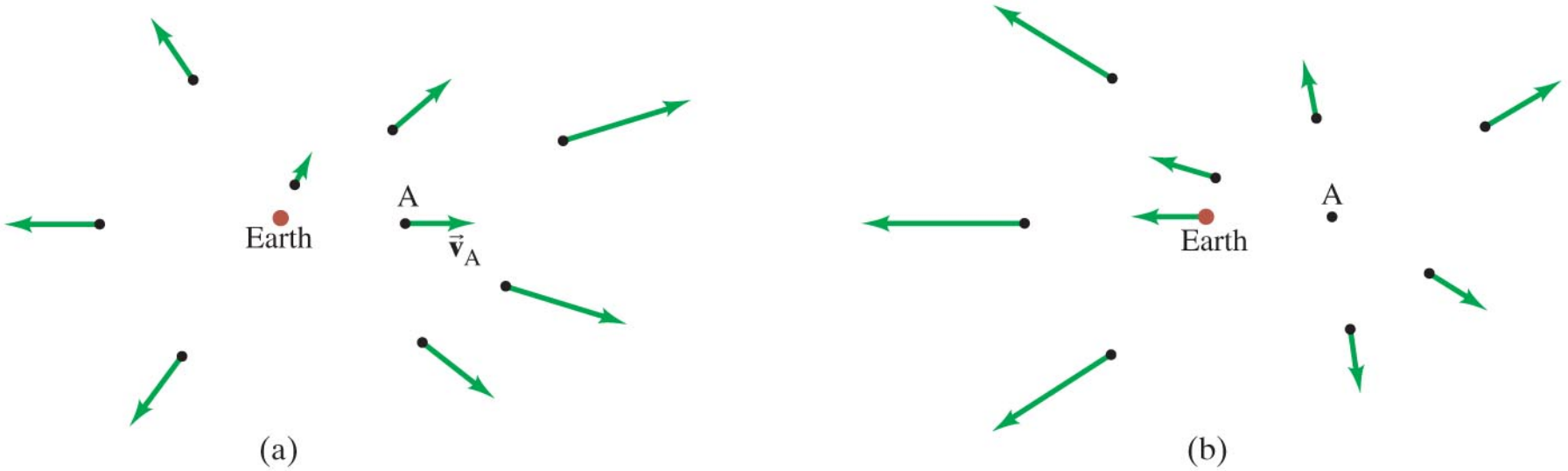
1) The Earth is not the centre of the world (solar system)

2) The Sun is not the centre of our Galaxy



3) Universe is
- isotropic (looks the same in all directions)
- homogeneous(all locations are equivalent)

Big Bang and Cosmological Principle



Expansion is the same from all points in the Universe

Our place is not special

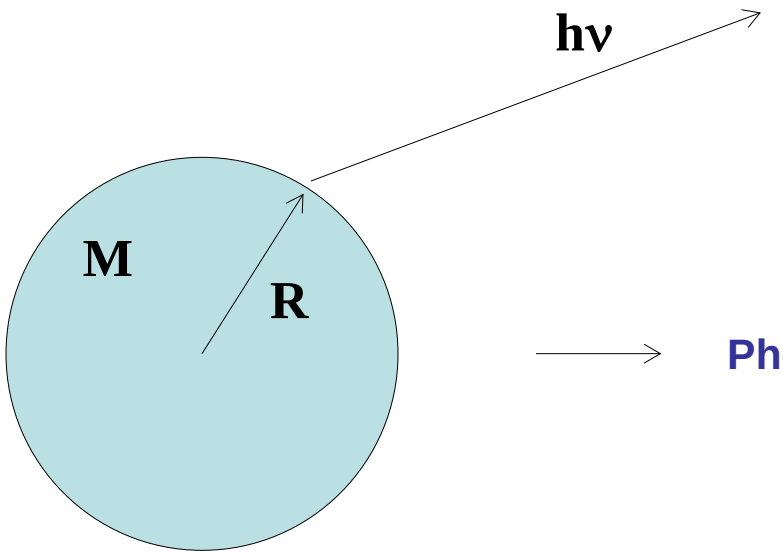
Big Bang happened “everywhere”

Gravitational Red Shift

In General Relativity it is time that depends on the gravitational dependence. This is at the heart of an explanation of gravitational redshift – it is a gravitational Time dilation. But it can be understood as an "energy loss in a gravity field".

At the surface of a heavy object $V(R) = -G\frac{Mm_{photon}}{R}$ **and** $V(\infty) = 0$

$$(g = -G\frac{M}{R^2})$$



→ **Photons loose energy when traveling "uphill"**

$$E = h\Delta\nu = m_{photon}g\Delta R$$

→ **Photons shift their energy to the red**

Red Shifts

Doppler effect

$$\lambda = \lambda_0 \sqrt{\frac{c - v}{c + v}}$$

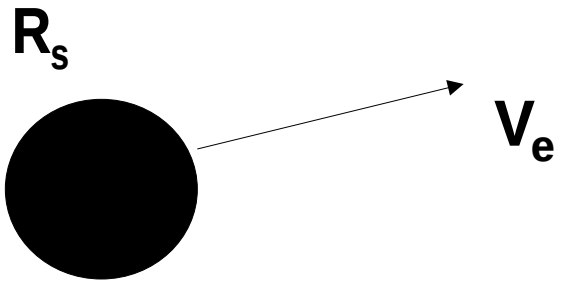
Gravitational redshift

$$\Delta \nu = \frac{m_{photon} g \Delta R}{h}$$

Cosmological redshift

$$\frac{\lambda}{\lambda^0} \equiv 1 + z$$

Simple (Newtonian) view on a Black Hole



Escape from a distance R_s with an escape velocity

Requirement:
Kinetic energy must beat the gravitational potential

$$\frac{1}{2}mv^2 = \frac{GmM}{R}$$

$$\longrightarrow \quad v = \sqrt{\frac{2GM}{R}}$$

Take c for the escape velocity
(of course not correct but some approx)

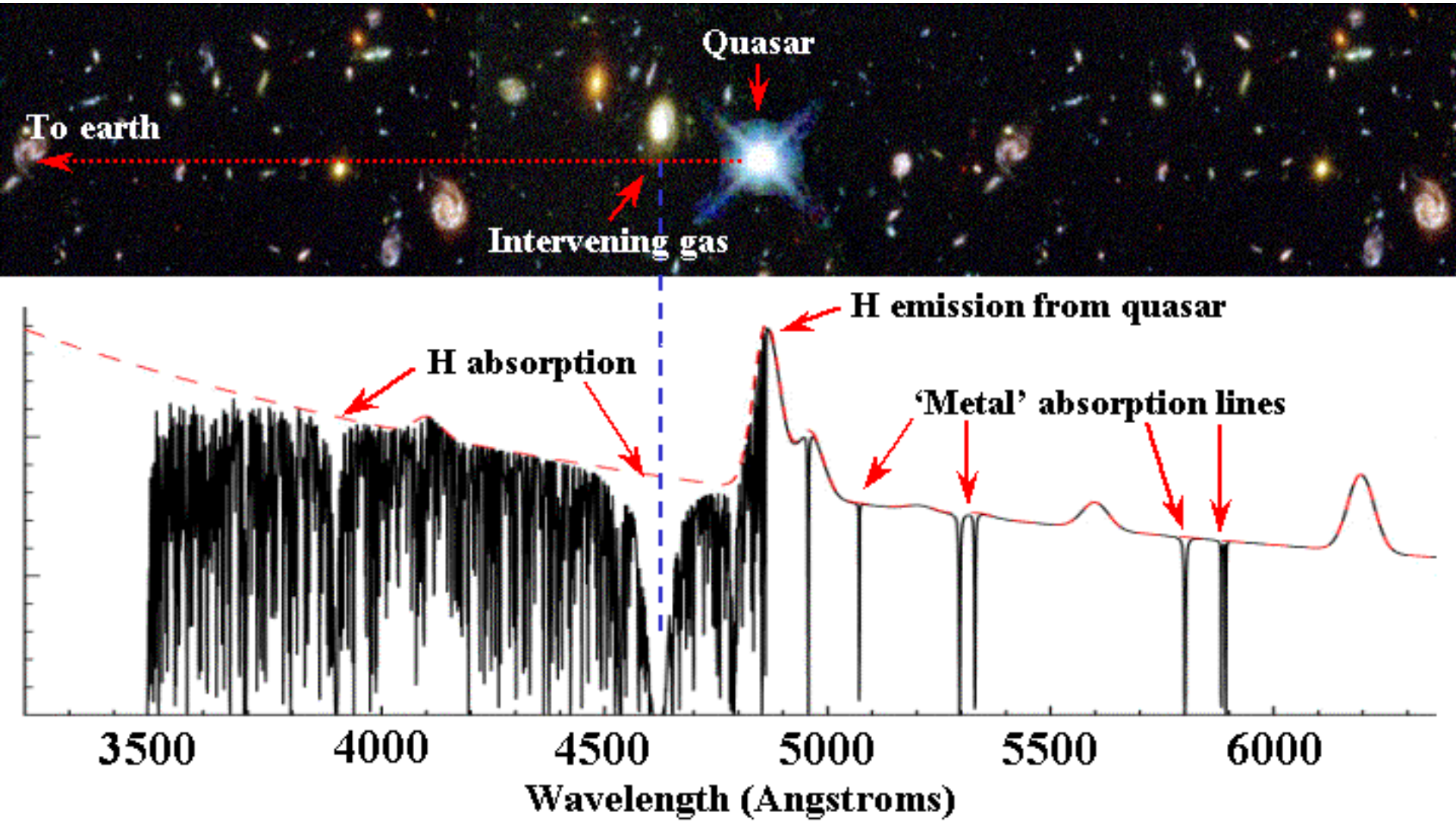
$$\longrightarrow \quad R_s = \frac{2GM}{c^2}$$

Schwarzschild radius
(also valid in GR)

Laplace (1795): "possibly the greatest luminous bodies are invisible"

Intermezzo

The spectrum of a “quasar”



Intermezzo

Search for varying constants in the early Universe

Compare the absorption spectrum of H_2 in different epochs

Each line is redshifted

$$\frac{\lambda_i^z}{\lambda_i^0} \equiv 1 + z_i$$

Lab

today

90-112 nm

QSO

12 Gyr ago

~275-350 nm

Spectral lines of a molecule depend on the fundamental constant of nature :

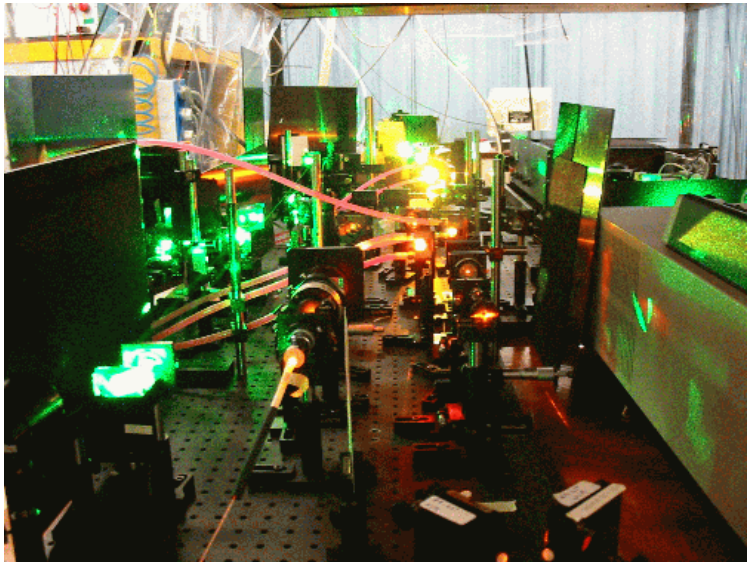
$$\mu = \frac{M_p}{m_e} = 1836.15267245(75)$$

Intermezzo

Search for varying constants in the early Universe

Laboratory spectra

For z=0



Astronomical spectra

For high z

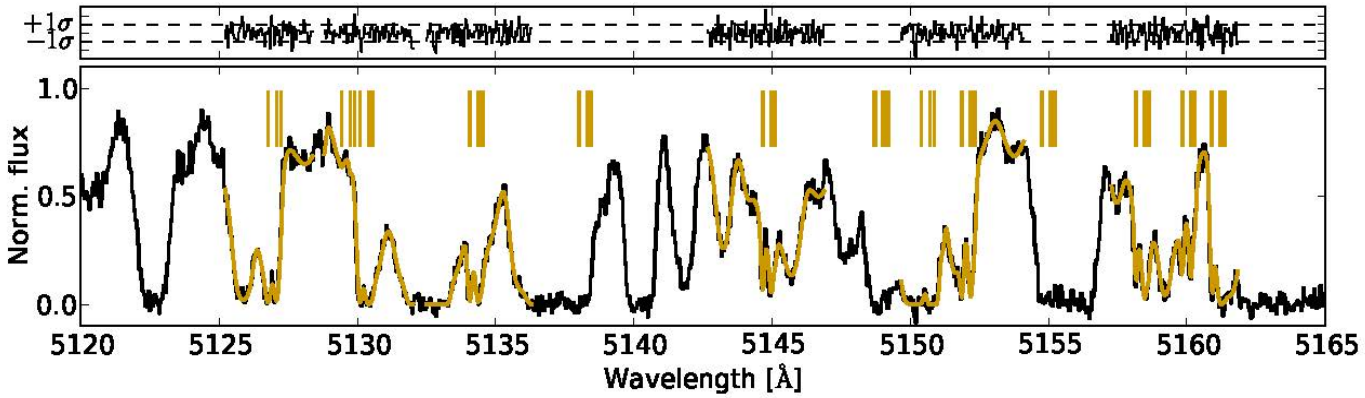
Make a comparison

Intermezzo

Quasar Q1441+272 ; the most distant

At z_{abs} = 4.22 ; 1.5 Gyrs after the Big Bang

spectrum



$$\frac{\lambda_i^z}{\lambda_i^0} \equiv \left(1+z_{abs}\right)\left(1+K_i\frac{\Delta\mu}{\mu}\right)$$

Result

$$\left|\frac{\Delta\mu}{\mu}\right| < 5\times10^{-6}$$

Important: Knowledge from Molecular Physics

K_i values different for all spectral lines

Molecules are sensitive for the fundamental constants

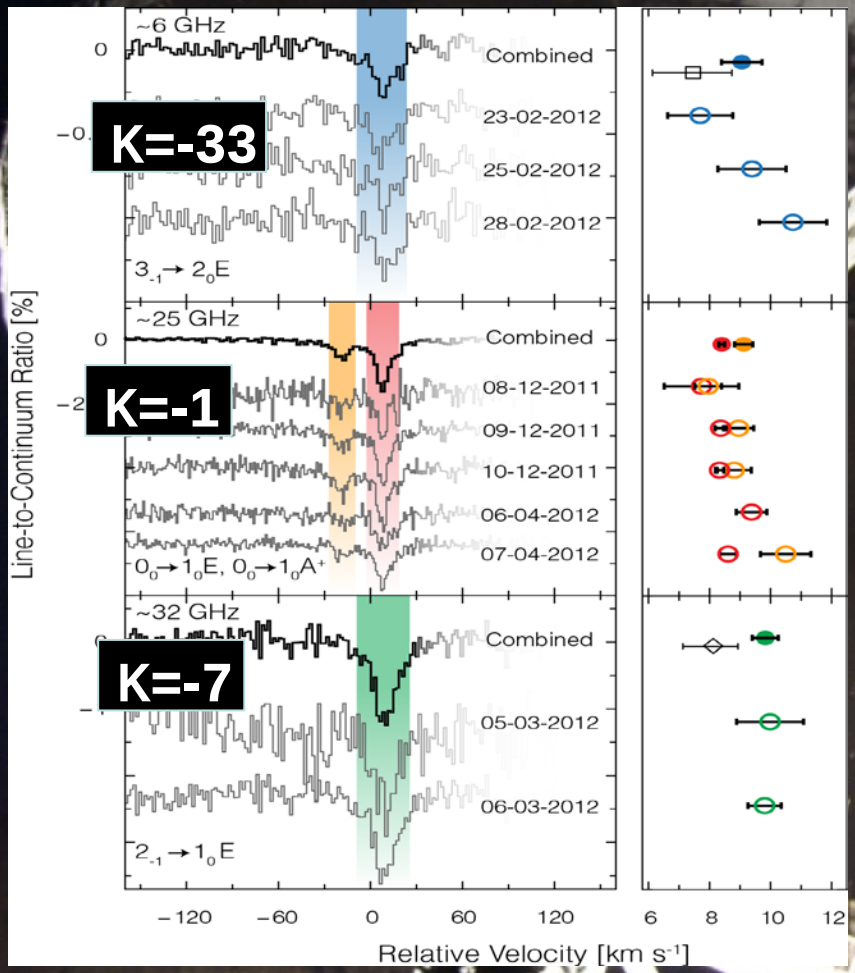
A Stringent Limit on a Drifting Proton-to-Electron Mass Ratio from Alcohol in the Early Universe

Bagdonaite, Jansen, Henkel, Bethlem, Menten, Ubachs,
Science 339 (2013) 46

Effelsberg Radio Telescope

PKS-1830-211
"molecular factory"

at z=0.88582
(7.5 Gyrs look-back)



$$\left|\frac{\Delta\mu}{\mu}\right| < 10^{-7}$$

Intermezzo

Anthropic Ideas on the Universe

$$\alpha = \frac{1}{137.035\,999\,710\,(96)}$$

$$\mu = \frac{m_p}{m_e} = 1836.152\,672\,61\,(85)$$

No theories for the values of these parameters underlying structure of matter in the universe

See the paper: Carr & Rees, Nature 278 (1979) 605
http://www.nat.vu.nl/~wimu/Varying-Constants-Papers/Carr-Rees-1979.pdf

Intermezzo

The Hoyle Resonance

Fred Hoyle