The Special Theory of Relativity

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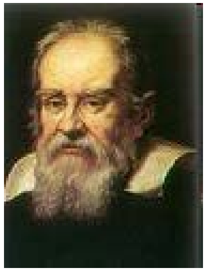
Chapter I

1. Contradictions in physics ?
2. Galilean Transformations of classical mechanics
3. The effect on Maxwell's equations – light
4. Michelson-Morley experiment
5. Einstein's postulates of relativity
6. Concepts of absolute time and simultaneity lost

Galilean–Newtonian Relativity



Galileo Galilei

Isaac Newton

Definition of an inertial reference frame:

One in which Newton's first law is valid.

v=constant* if *F=0

Earth is rotating and therefore not an inertial reference frame, but we can treat it as one for many purposes.

A frame moving with a constant velocity with respect to an inertial reference frame is itself inertial.

Relativity principle:

Laws of physics are the same in all inertial frames of reference

Intuitions of Galilean–Newtonian Relativity

What quantities are the same, which ones change ?

Lengths of objects are invariant as they move.

Time is absolute.

Mass of an object is invariant for inertial systems

Forces acting on a mass are equal for all inertial frames

Velocities are (of course) different in inertial frames
(Galileo transformations)

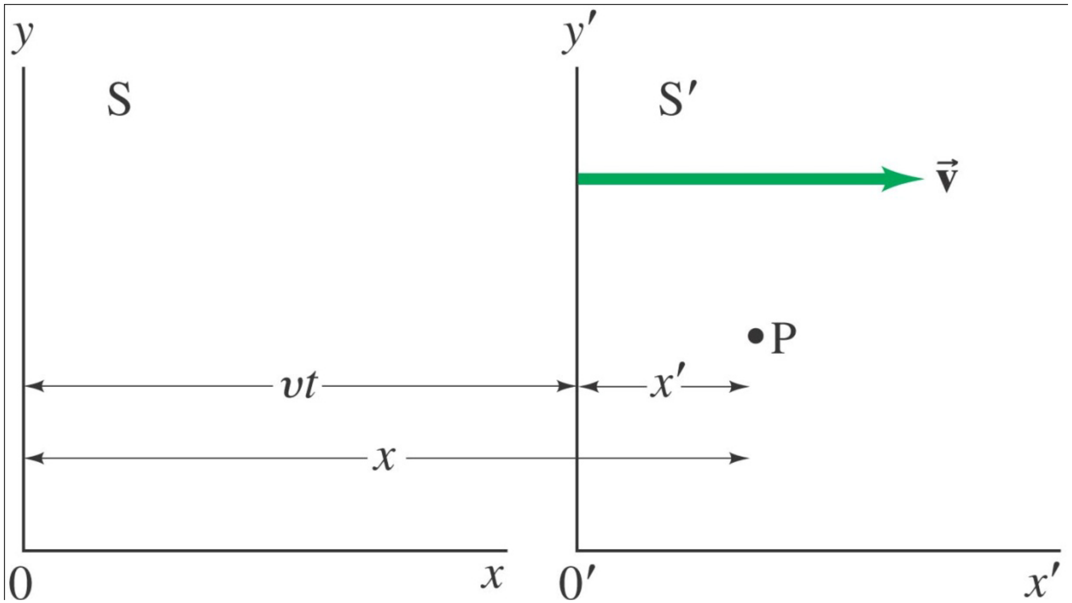
Positions of objects are different in other inertial systems
(Galileo coordinate transformation)

Galilean Transformations

A classical (Galilean) transformation between inertial reference frames:

View coordinates of point P in system S'

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$



Note;
Inverse transformation ?

Galilean Transformations

In matrix form

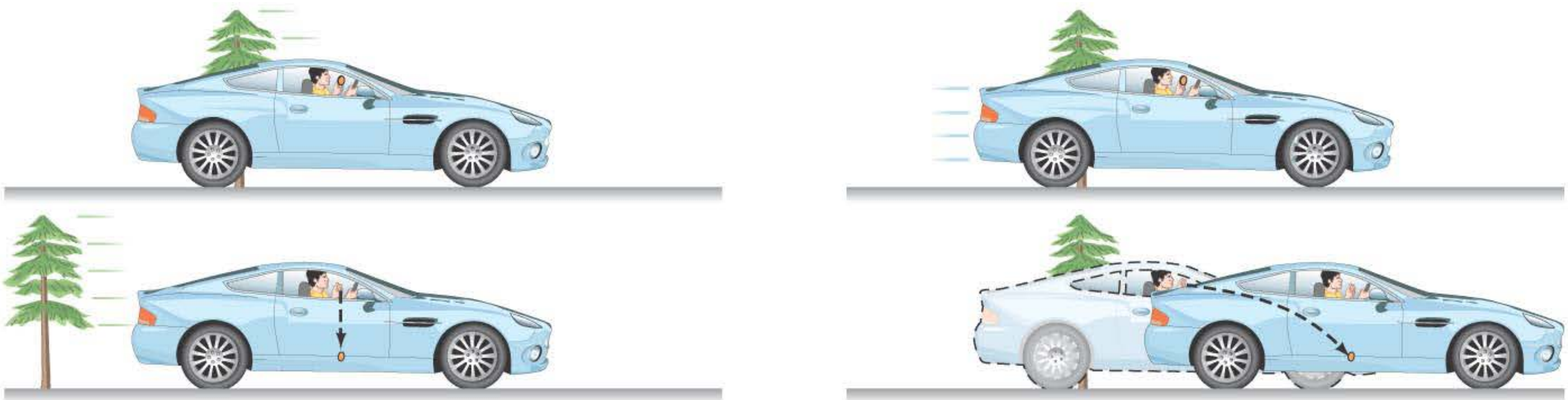
$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Relativity principle:

The basic laws of physics are the same in all inertial reference frames

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$



(a)
Reference frame = car

(b)
Reference frame = Earth

Laws are the same, but paths may be different in reference frames

The domain of electromagnetism
Maxwell's equations

	Integral form	**Differential form**
Gauss	$\oint \vec{E} \cdot dA = Q / \varepsilon_0$	$\nabla \cdot \vec{E} = \rho / \varepsilon_0$
	$\oint \vec{B} \cdot dA = 0$	$\nabla \cdot \vec{B} = 0$
Faraday	$\oint \vec{E} \cdot d\ell = -\frac{\partial \Phi_B}{\partial t}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere/ Maxwell	$\oint \vec{B} \cdot d\ell = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial \Phi_B}{\partial t}$	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Differential vector analysis for treating Maxwell's equations

in Cartesian coordinates (can be done in spherical):

Gradient
$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

Divergence
$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Curl
$$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$

Laplacian
$$\nabla^2 V = \left(\frac{\partial^2 V}{\partial^2 x}\right) + \left(\frac{\partial^2 V}{\partial^2 y}\right) + \left(\frac{\partial^2 V}{\partial^2 z}\right)$$

Product (chain) rules
$$\nabla \times (A \times B) = (B \cdot \nabla)A - (A \cdot \nabla)B + A(\nabla \cdot B) - B(\nabla \cdot A)$$

Proof theorems on second derivatives

$$\nabla \cdot \left(\nabla \times \vec{F}\right) = 0 \qquad \nabla \times \left(\nabla \times \vec{F}\right) = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F} \qquad \nabla \cdot (\nabla f) = 0$$

Derivation of the wave equations in vacuum (no charge, no current)

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Calculate:

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\left(\nabla \cdot E\right) - \nabla^2 E = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\rightarrow \qquad \nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Similarly derive: $$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

Electromagnetic wave equations

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad \nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

Note that: $$\nabla^2 f = \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$ **is in general a "wave equation"**

1855; electric and magnetic measurements $$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.107 \times 10^8 m/s$$

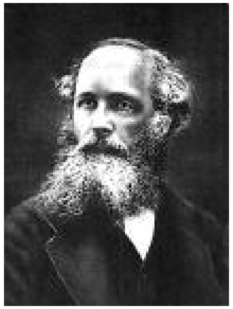
Measurement of the speed of light $$c = 3.14 \times 10^8 m/s$$ **Fizeau 1848**

$$c = 2.98 \times 10^8 m/s$$ **Foucault 1858**

History of the speed of light: http://www.speed-light.info/measure/speed_of_light_history.htm

Maxwell: $$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \qquad \nabla^2 B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}$$

Maxwell's equations



James Clerk Maxwell

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{with} \qquad \frac{1}{\mu_0 \varepsilon_0} = c^2$$

Light is a wave with transverse polarization and speed c

Problems:

In what inertial system has light the exact velocity c
What about the other inertial systems
Waves are known to propagate in a medium; where is this "ether"
How can light propagate in vacuum ?
Laws of electrodynamics do not fit the relativity principle ?

Maxwell's equations do not obey Galilei transform

Simple approach:

Consider light pulse emitted at time $t=0$**; at time** $t>0$

$$c^2t^2 = x^2 + y^2 + z^2$$ **in frame** $\{x,y,z,t\}$

So: $$-c^2t^2 + x^2 + y^2 + z^2 = 0$$

In the moving frame $\{x',y',z',t'\}$

$$-c^2t'^2 + x'^2 + y'^2 + z'^2 = 0$$

Apply Galilei transform

$$-c^2t'^2 + x'^2 + y'^2 + z'^2 = -c^2t^2 + (x - vt)^2 + y^2 + z^2 = vt(vt - 2x) \neq 0$$

Maxwell's wave equation transformed

Apply it to the wave equation in (x,t) dimensions – calculate <u>partial</u> differentials:

$$x' = x - vt \qquad t' = t \longrightarrow \frac{\partial x'}{\partial x} = 1 \quad \frac{\partial x'}{\partial t} = -v \quad \frac{\partial t'}{\partial t} = 1 \quad \frac{\partial t'}{\partial x} = 0$$

Calculate field derivatives using the "chain rule":

Spatial part

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial x} = \frac{\partial E}{\partial x'}$$

Then also second

$$\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$$

Temporal part

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial E}{\partial t'} - v\frac{\partial E}{\partial x'}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t'} - v\frac{\partial E}{\partial x'}\right) = \frac{\partial}{\partial t'}\left(\frac{\partial E}{\partial t'} - v\frac{\partial E}{\partial x'}\right)\frac{\partial t'}{\partial t} + \frac{\partial}{\partial x'}\left(\frac{\partial E}{\partial t'} - v\frac{\partial E}{\partial x'}\right)\frac{\partial x'}{\partial t}$$

$$= \frac{\partial^2 E}{\partial t'^2} - 2v\frac{\partial^2 E}{\partial x' \partial t'} + v^2\frac{\partial^2 E}{\partial x'^2}$$

Maxwell's wave equation transformed II

$$\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial t'^2} - 2v\frac{\partial^2 E}{\partial x'\partial t'} + v^2\frac{\partial^2 E}{\partial x'^2}$$

Insert in Maxwell wave equation

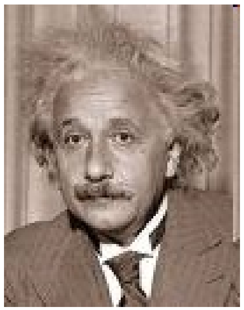
$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2} - \frac{2v}{c^2}\frac{\partial^2 E}{\partial x'\partial t'} = \left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial x'^2}$$

This is not an electromagnetic wave equation

This is what Einstein meant (see below)

Einstein and Maxwell



Albert Einstein

3. ***Zur Elektrodynamik bewegter Körper;***
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt.

The Michelson–Morley Experiment

Albert Michelson **Edward Williams Morley**

Nobel 1907

"for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"

Albert Abraham Michelson

Questions:

What is the absolute reference point of the Ether?

In which direction does it move ? How fast ?

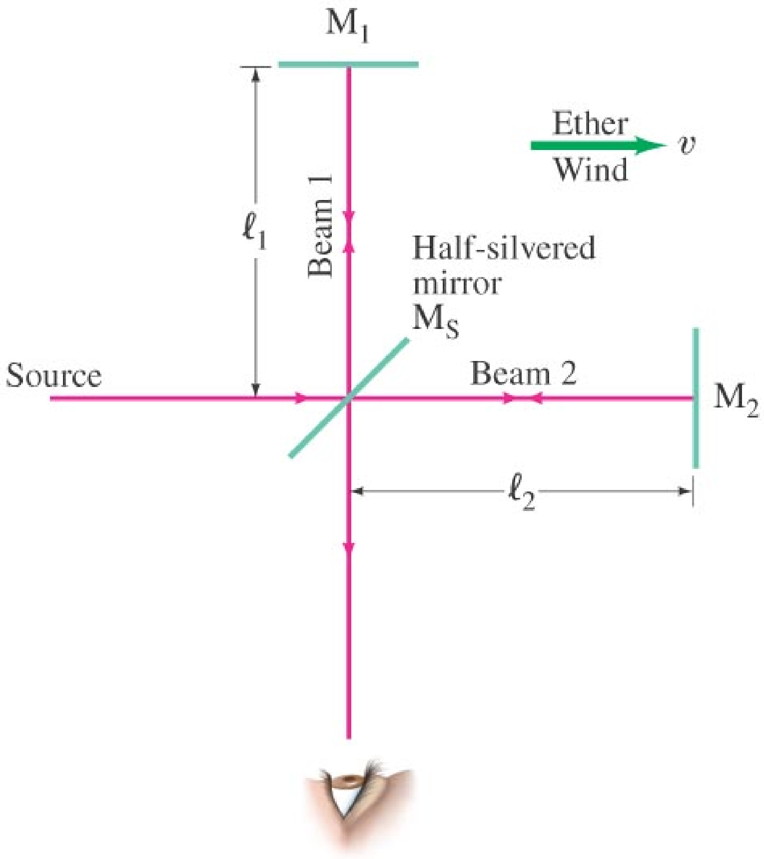
Ether connected to sun (*center of the universe*) ?

$$\left.\begin{aligned} v_{Earth} &\sim 3 \cdot 10^4 \, m/s \\ c &\sim 3 \cdot 10^8 \, m/s \end{aligned}\right\} \quad \frac{v}{c} \sim 10^{-4}$$

Motion of the Earth Should produce an Observable effect

The Michelson–Morley Experiment

Note: we adopt the classical perspective

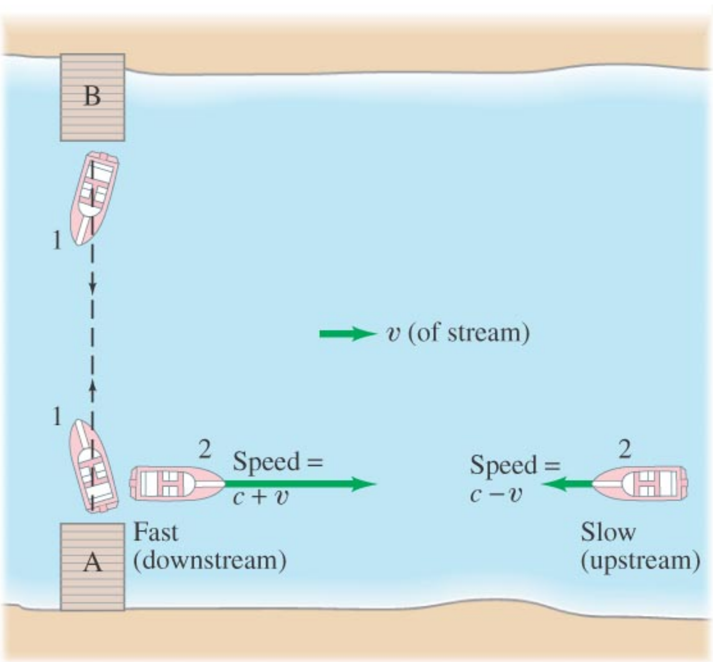


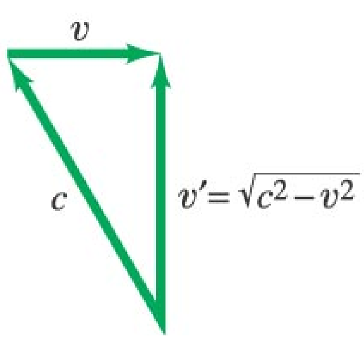
Along x-axis

$$t_2 = \frac{\ell_2}{c+v} + \frac{\ell_2}{c-v}$$

$$t_2 = \frac{\ell_2(c-v)}{c^2-v^2} + \frac{\ell_2(c+v)}{c^2-v^2} = \frac{2\ell_2}{c\left(1-v^2/c^2\right)}$$

The Michelson–Morley Experiment





Along y-axis

$$t_1 = \frac{2\ell_1}{v'} = \frac{2\ell_1}{\sqrt{c^2 - v^2}} = \frac{2\ell_1}{c\sqrt{1 - v^2/c^2}}$$

Transversal motion:
Always account for the “stream”

The Michelson–Morley Experiment

$$t_2 = \frac{\ell_2}{c+v} + \frac{\ell_2}{c-v} = \frac{2\ell_2}{c\left(1 - v^2/c^2\right)}$$

$$t_1 = \frac{2\ell_1}{v'} = \frac{2\ell_1}{\sqrt{c^2 - v^2}} = \frac{2\ell_1}{c\sqrt{1 - v^2/c^2}}$$

Interferometer: $\ell = \ell_1 = \ell_2$

$$\Delta t = t_2 - t_1 == \frac{2\ell}{c}\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

If v=0, then Δt=0

no effect on interferometer

If v≠0, then Δt≠0

a phase-shift introduced

But this is not observed

(actually difficult to observe)

The Michelson–Morley Experiment

Rotate the interferometer $\ell_1 \leftrightarrow \ell_2$

$$\Delta T = \Delta t - \Delta t' =$$

$$\frac{2}{c}(\ell_1 + \ell_2)\left[\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right]$$

Approximate: $\frac{v}{c} << 1$

Then (Taylor): $\frac{1}{1-v^2/c^2} \approx 1 + \frac{v^2}{c^2}$

$$\frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$\Delta T = (\ell_1 + \ell_2)\frac{v^2}{c^3}$$

Numbers:

$v \sim 3 \times 10^4$ m/s

$v/c \sim 10^{-4}$

$l_1 \sim l_2 \sim 11$ m

$$\Delta T = 7 \times 10^{-16} s$$

Visible light: $\lambda \sim 550$ nm → $f \sim 5 \times 10^{14}$ Hz

Phase change (in fringes)

$$f \cdot \Delta T = 7 \times 10^{-16} \cdot 5 \times 10^{14} = 0.4$$

Should be observable !

Detectability: 0.01 fringe

Conclusion: The Michelson–Morley Experiment

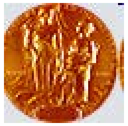
This interferometer was able to measure interference shifts as small as 0.01 fringe, while the expected shift was 0.4 fringe.

However, no shift was ever observed, no matter how the apparatus was rotated or what time of day or night the measurements were made.

The possibility that the arms of the apparatus became slightly shortened when moving against the ether was considered by Lorentz.

Hendrik A Lorentz
Nobel 1902

"in recognition of the extraordinary service rendered by their researches into the influence of magnetism upon radiation phenomena"



Lorentz contraction

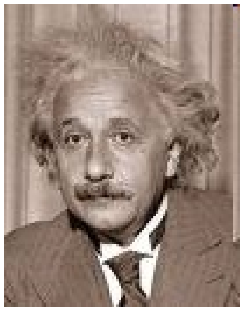
Possible solutions for the ether problem

1. The ether is rigidly attached to Earth

2. Rigid bodies contract and clocks slow down when moving through the ether

3. There is no ether

A new perspective

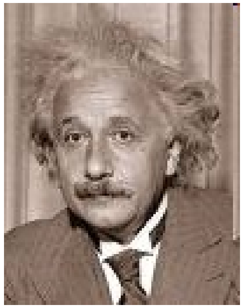


Albert Einstein

Voraussetzung einführen, daß sich das Licht im leeren Raume stets mit einer bestimmten, vom Bewegungszustande des emittierenden Körpers unabhängigen Geschwindigkeit V fortpflanze.

Die Einführung eines „Lichtäthers“ wird sich insofern als überflüssig erweisen, als nach der zu entwickelnden Auffassung weder ein mit besonderen Eigenschaften ausgestatteter „absolut ruhender Raum“ eingeführt, noch einem Punkte des leeren Raumes, in welchem elektromagnetische Prozesse stattfinden, ein Geschwindigkeitsvektor zugeordnet wird.

On relativity



Albert Einstein

§ 2. **Über die Relativität von Längen und Zeiten.**

Die folgenden Überlegungen stützen sich auf das Relativitätsprinzip und auf das Prinzip der Konstanz der Lichtgeschwindigkeit, welche beiden Prinzipien wir folgendermaßen definieren.

1. Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichförmiger Translationsbewegung befindlichen Koordinatensystemen diese Zustandsänderungen bezogen werden.

2. Jeder Lichtstrahl bewegt sich im „ruhenden" Koordinatensystem mit der bestimmten Geschwindigkeit V, unabhängig davon, ob dieser Lichtstrahl von einem ruhenden oder bewegten Körper emittiert ist. Hierbei ist

$$\text{Geschwindigkeit} = \frac{\text{Lichtweg}}{\text{Zeitdauer}},$$

Postulates of the Special Theory of Relativity

1. The laws of physics have the same form in all inertial reference frames
2. Light propagates through empty space with speed c independent of the speed of source or observer

This solves the ether problem – (there is no ether)

The speed of light is the same in all inertial reference frames

Simultaneity

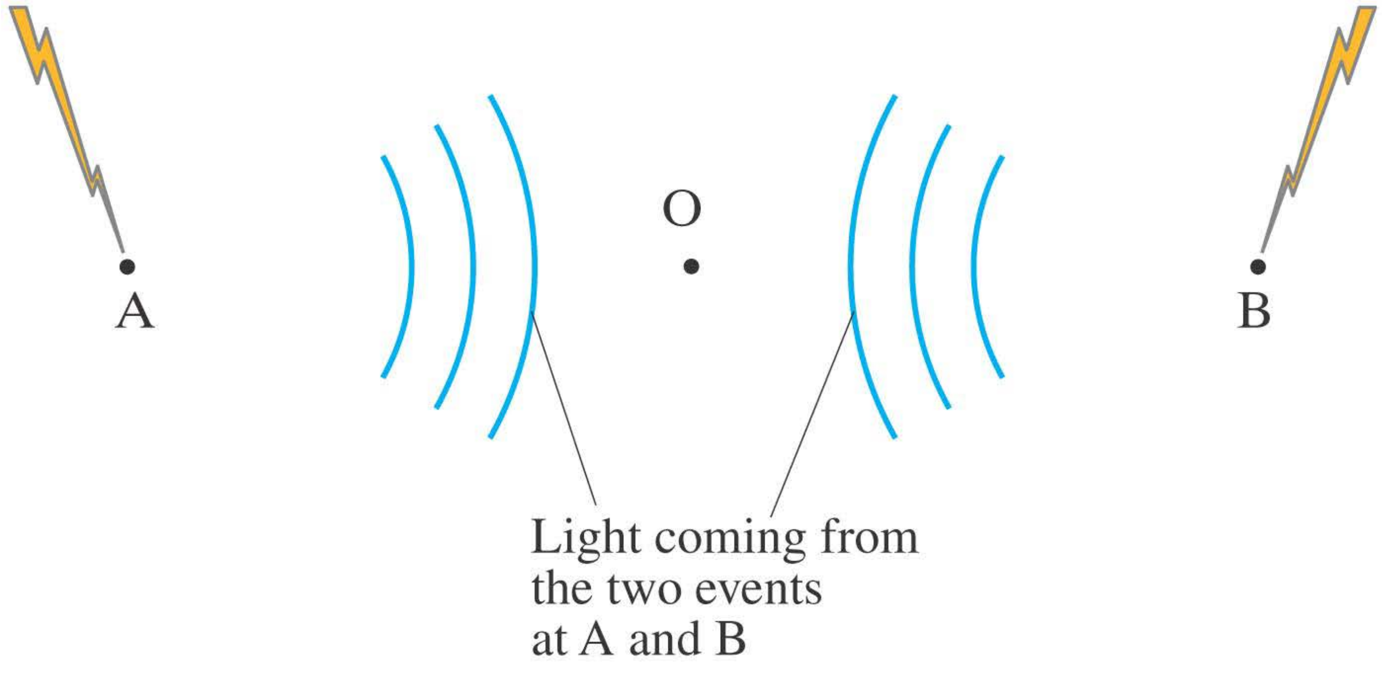
One of the implications of relativity theory is that time is not absolute. Distant observers do not necessarily agree on time intervals between events, or on whether they are simultaneous or not.

Why not?

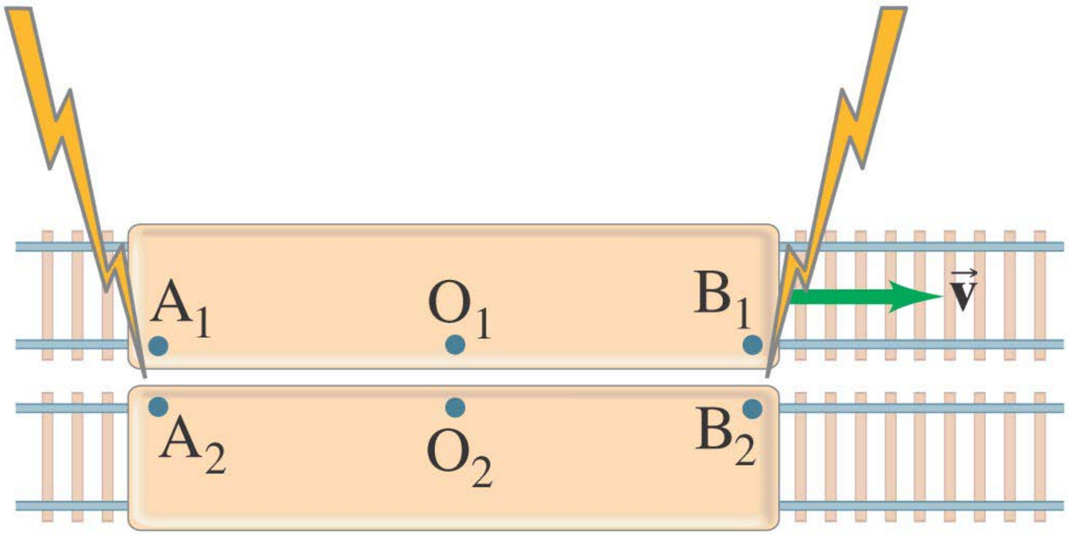
In relativity, an “event” is defined as occurring at a specific place and time. Let’s see how different observers would describe a specific event.

Simultaneity

Thought experiment: lightning strikes at two separate places. One observer believes the events are simultaneous – the light has taken the same time to reach her – but another, moving with respect to the first, does not.

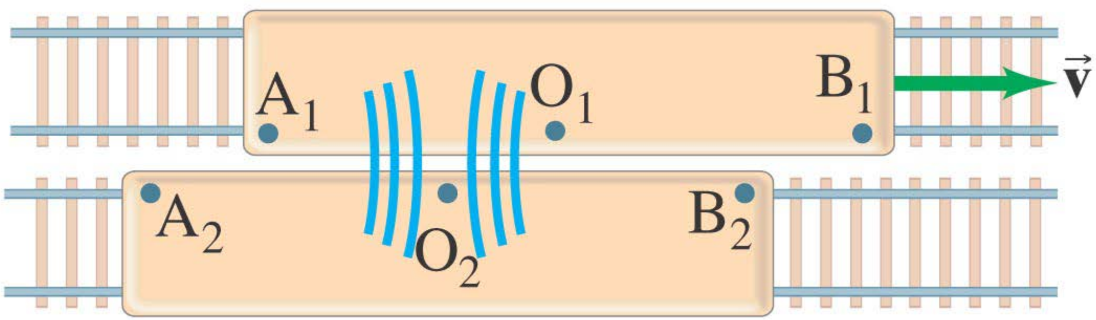


Simultaneity



(a)

From the perspective of both O_1 and O_2 they themselves see both light flashes at the same time

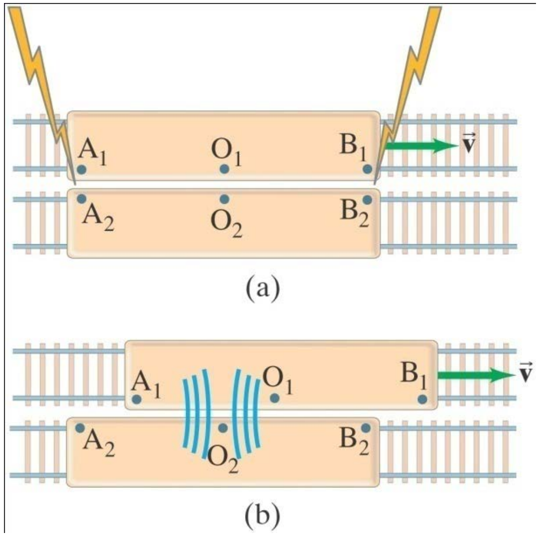


(b)

From the perspective of O_2 the observer O_1 sees the light flashes from the right (B) first.

Who is right ?

Simultaneity



Here, it is clear that if one observer sees the events as simultaneous, the other cannot, given that the speed of light is the same for each.

Conclusions:

Simultaneity is not an absolute concept

Time is not an absolute concept