AUC Course: Electrons, Waves, and Relativity 2018
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Relativity Problem set I

1) Galileian Transformation

Proof that consecutive application of Galileian transformation and the reverse transformation yield a unity transformation (so no change).

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} \qquad \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

2) Velocity transformation in classical mechanics

In Frame S an object has velocity $\vec{u} = (u_x, u_y, u_z) = (dx/dt, dy/dt, dz/dt)$.

In Frame S' the object has velocity $\vec{u}'$. Frame S' moves with speed $\vec{v}$ with respect to frame S. You may choose/define the coordinate system to have velocity $\vec{v}$ move along the *x*-axis.

a) Derive the velocity transformation, so the velocity $\vec{u}'$ expressed in terms of $\vec{u}$.
b) Proof that acceleration is the same in both frames, $\vec{a}' = \vec{a}$. What assumption is needed ?
c) Proof that Newton's second law $\vec{F} = m\vec{a}$ is the same in both frames.

3) Vector fields

a) Proof that for any vector field: $\nabla \cdot (\nabla \times A) = 0.$

b) Proof that $\nabla \times (\nabla \times A) - \nabla(\nabla \cdot A) = -\nabla^2 A$

4) Speed of light

If you were on a spaceship traveling at 0.5c away from a star, at what speed would the starlight pass you ?