Effect of the state of coherence on the three-dimensional spectral intensity distribution near focus

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Abstract

The effect of the state of coherence on the three-dimensional spectral intensity distribution in the focal region of a system with a high Fresnel number is investigated. The structure of a Gaussian Schell-model field is studied in detail.
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1. Introduction

The classical theory of focusing, due to Debye [1,2], is valid when the field is monochromatic and the focusing system has a Fresnel number which is large compared to unity. This theory has been generalized by Wang et al. [3] to include the focusing of partially coherent light by high Fresnel number systems. They presented examples of the axial intensity distribution for focused, partially coherent cylindrical waves. Friberg et al. [4] analyzed the focal shift in partially coherent waves of various states of coherence in systems with different Fresnel numbers. However, these previous studies concerning the focusing of partially coherent light are primarily restricted to the examination of the field along the optical axis, and it is well-known, at least for fully coherent fields, that the field exhibits a complicated structure throughout the focal region. In the present paper, we examine the intensity in the entire focal region of a class of partially coherent fields focused by high Fresnel number systems, and we elucidate some general properties of the intensity distribution for such fields.

This work was stimulated by the fact that light which has traveled through the atmosphere is, in general, spatially partially coherent. It is therefore of importance to investigate the intensity distribution in the neighborhood of the geometrical focus formed by such light. Moreover, the considerable success of the use of partially coherent light in a number of practical problems such as, for example, the improvement of image resolution [5, pp. 259–260], laser-induced fusion [6], and the propagation of beams through turbulence [7,8] warrants this study.

2. Focused partially coherent wave fields

We consider a partially coherent field emerging from a circular aperture with radius a and converging towards a geometrical focus O (see Fig. 1). We have shown in [4] that the cross-spectral density of the field at two points $P(\mathbf{r}_1)$ and $P(\mathbf{r}_2)$ in the focal region is, within the paraxial approximation, given by the formula

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{\lambda^2} \int \int_{\mathscr{W}} W^{(0)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$$
$$\times \frac{e^{-ik|\mathbf{r}_1 - \mathbf{r}'_1|}}{|\mathbf{r}_1 - \mathbf{r}'_1|} \frac{e^{ik|\mathbf{r}_2 - \mathbf{r}'_2|}}{|\mathbf{r}_2 - \mathbf{r}'_2|} \, d^2 r'_1 \, d^2 r'_2. \quad (1)$$

Here $W^{(0)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ is the cross-spectral density of the incident field in the aperture. The domain of integration $\mathscr{W}$ is a reference sphere of radius f centered at O, $\mathbf{r}'_1$ and $\mathbf{r}'_2$ are position vectors of two typical points on the reference sphere, and

$$k = 2\pi/\lambda = \omega/c \quad (2)$$

is the wavenumber associated with frequency ω, c being the speed of light in vacuum and λ being the wavelength.

The *spectral intensity* of the focused field at frequency ω, at a point of observation $P(\mathbf{r})$ in the focal region, is given by the expression

$$S(\mathbf{r}, \omega) \equiv W(\mathbf{r}, \mathbf{r}, \omega). \quad (3)$$

From Eqs. (3) and (1) it follows that

$$S(\mathbf{r}, \omega) = \frac{1}{\lambda^2} \int \int_{\mathscr{W}} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$$
$$\times \frac{e^{-ik(R_1 - R_2)}}{R_1 R_2} \, d^2 r_1 \, d^2 r_2, \quad (4)$$

where
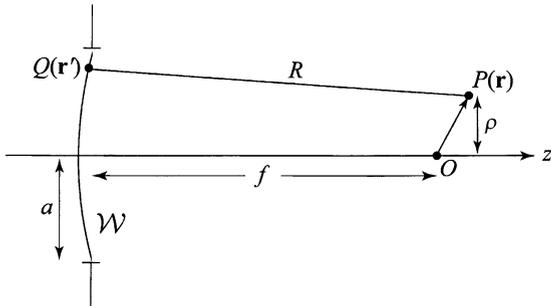
Fig. 1. Notation relating to the focusing configuration.

$$R_1 = |\mathbf{r} - \mathbf{r}_1|, \quad (5)$$

$$R_2 = |\mathbf{r} - \mathbf{r}_2|. \quad (6)$$

For an observation point close to the geometrical focus O, we may replace, to a good approximation, the factors R_1 and R_2 in the denominator of the right-hand side of Eq. (4) by the distance f, and obtain for the spectral intensity the expression

$$S(\mathbf{r}, \omega) = \frac{1}{(\lambda f)^2} \int \int_{\mathscr{W}} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$$
$$\times e^{-ik(R_1 - R_2)} \, d^2 r_1 \, d^2 r_2. \quad (7)$$

We will study in particular the important class of fields generated by planar, secondary Gaussian Schell-model sources [5, Section 5.2.2]. Such fields are known as *Gaussian Schell-model fields*. The sources are characterized by a cross-spectral density function of the form [2]

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S^{(0)}(\omega) e^{-(\rho_2 - \rho_1)^2/2\sigma_g^2}, \quad (8)$$

where $S^{(0)}(\omega)$ is the spectral intensity on the reference sphere, σ_g denotes the effective spectral coherence length of the field in the aperture (which may or may not depend on the frequency ω) and $\boldsymbol{\rho} = (x, y)$ is the two-dimensional projection, considered as a two-dimensional vector, of the position vector $\mathbf{r}$ of the point Q on the reference sphere $\mathscr{W}$ onto the xy-plane. On introducing polar coordinates ρ, ϕ and z, with

$$\mathbf{r} = (\rho \cos \phi, \rho \sin \phi, z), \quad (9)$$

we obtain, to a good approximation, from Eqs. (7) and (8) the following expression for the spectral intensity:

$$S(\mathbf{r}, \omega) = \frac{S^{(0)}(\omega)}{(\lambda f)^2} \int_0^{2\pi} \int_0^a \int_0^{2\pi} \int_0^a$$
$$\times e^{-(\rho_1^2 + \rho_2^2)/2\sigma_g^2} e^{\rho_1 \rho_2 \cos(\phi_1 - \phi_2)/\sigma_g^2}$$
$$\times \cos[k(R_2 - R_1)] \rho_1 \rho_2 \, d\phi_1 \, d\rho_1 \, d\phi_2 \, d\rho_2. \quad (10)$$

Here we have used the relation $d^2 r_i = \rho_i \, d\phi_i \, d\rho_i$, for $i = 1, 2$, and also the fact that $S(\mathbf{r}, \omega)$ is real-valued.

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[2] Actually, we consider only Gaussian Schell-model sources for which the spectral intensity is the same at every point on the reference sphere.

In the paraxial domain the distances R_1 and R_2 can be approximated by the expressions

$$R_1 \approx f - \mathbf{q}_1 \cdot \mathbf{r}, \tag{11}$$

$$R_2 \approx f - \mathbf{q}_2 \cdot \mathbf{r}, \tag{12}$$

with $\mathbf{q}_1$ and $\mathbf{q}_2$ being unit vectors along the directions $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Here we assume that the Fresnel number of the focusing geometry $N \equiv a^2/\lambda f \gg 1$. On substituting from Eqs. (11) and (12) into Eq. (10), and using the fact that the spectral density is real-valued, it immediately follows that

$$S(\mathbf{r}, \omega) = S(-\mathbf{r}, \omega), \tag{13}$$

which implies that, just as for a monochromatic field focused by a system of high Fresnel number, *the three-dimensional spectral intensity distribution of a converging partially coherent Gaussian Schell-model field is symmetric about the geometrical focus.* Furthermore, the spectral intensity distribution is also symmetric about the focal plane.

From Eq. (10) one may also determine the behavior of the spectral intensity at the geometrical focus. For $\mathbf{r} = (0, 0, 0)$, Eq. (10) reduces to the simple form (derived previously in [4]),

$$S(0, 0, 0, \omega) = \left(\frac{k}{f}\right)^2 S^{(0)}(\omega) \int_0^a \int_0^a e^{-(\rho_2^2 + \rho_1^2)/2\sigma_g^2} \mathscr{I}_0$$
$$\times \left(\frac{\rho_1 \rho_2}{\sigma_g^2}\right) \rho_1 \rho_2 \, d\rho_1 \, d\rho_2, \tag{14}$$

where $\mathscr{I}_0$ is the modified Bessel function of order zero. The behavior of this quantity as a function of the scaled coherence parameter σ_g/a is shown in Fig. 2. It can be seen that *the spectral intensity at the geometrical focus increases as the spatial coherence of the focused field increases.*

For Gaussian Schell-model fields, the maximum intensity of the focused field is always at the geometrical focus, as we now show. Noting that the cross-spectral density of a Gaussian Schell-model source $W^{(0)}$ is a non-negative quantity, and that $\cos[k(R_2 - R_1)] \leqslant 1$, it can be shown from Eq. (10) that

$$S(\mathbf{r}, \omega) < S(0, 0, 0, \omega). \tag{15}$$
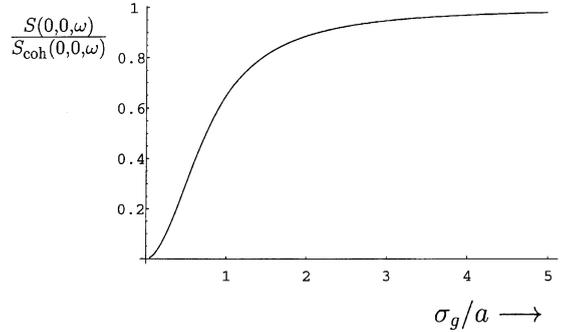
Fig. 2. The spectral intensity at the geometrical focus of a system with large Fresnel number N which focuses a Gaussian Schell-model field, as a function of the scaled coherence parameter σ_g/a. In this example $\lambda = 0.5 \times 10^{-6}$ m, $a = 0.01$ m, $f = 2$ m (Fresnel number $N = 100$).

Therefore, *the maximum spectral intensity of a converging partially coherent Gaussian Schell-model field in a system of high Fresnel number occurs at the geometrical focal point.*[3]

3. Spectral intensity distribution near the geometrical focus

Eq. (10) can be numerically integrated to determine in detail the three-dimensional spectral intensity distribution in the focal region. We will specify the position of a field point by the usual dimensionless *Lommel variables*

$$u = k \left(\frac{a}{f}\right)^2 z, \tag{16}$$

$$v = k \left(\frac{a}{f}\right) \rho = k \left(\frac{a}{f}\right) \sqrt{x^2 + y^2}. \tag{17}$$

The spectral intensity distribution in the focal region for the fully coherent case (i.e., when $\sigma_g \to \infty$) is shown in Fig. 3. This is the case that was discussed many years ago in detail by Linfoot and Wolf [2,9], amongst others. In this figure and

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[3] It should be noted that this result may not hold for other types of partially coherent fields. In particular, inequality (15) depends on the fact that the cross-spectral density of the incident field is non-negative. For other types of correlations (Bessel correlations, for example) this is not true.
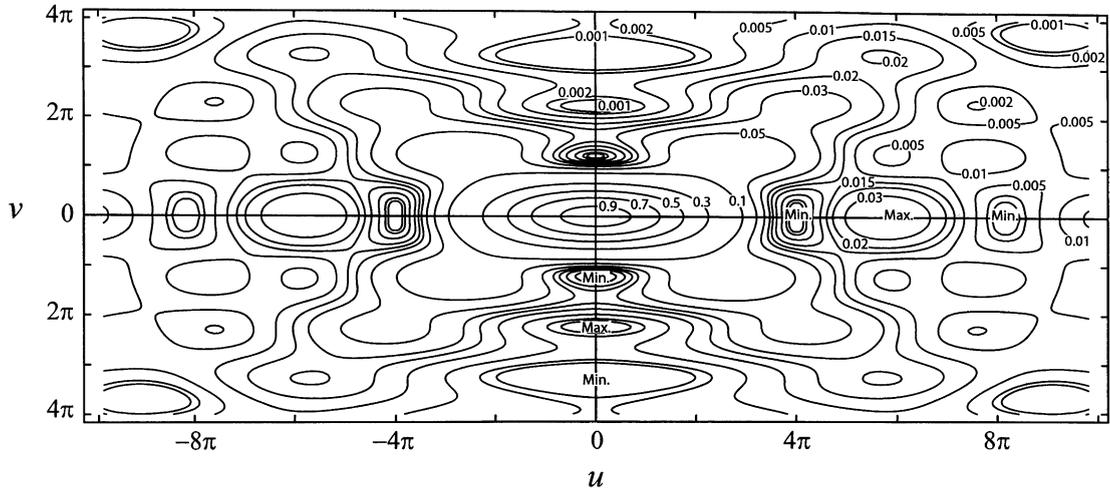
Fig. 3. Contours of constant spectral intensity of a fully coherent focused field in the neighborhood of the geometrical focus. The parameters are the same as those of Fig. 2.

those which follow the spectral intensities are normalized to the intensity at the geometrical focus $u = v = 0$ for a spatially fully coherent wave, i.e., to

$$S_{\mathrm{coh}}(0,0,\omega) = \lim_{\sigma_g \to \infty} S(0,0,\omega) = \pi^2 N^2 S^{(0)}(\omega),$$

$$(18)$$

where we have used Eq. (14). As before, N denotes the Fresnel number of the focusing geometry.

The contours of the normalized three-dimensional spectral intensity distribution $S(u,v,\omega)/S_{\mathrm{coh}}(0,0,\omega)$ of a converging, partially coherent Gaussian Schell-model field are depicted for several values of the scaled effective coherence length σ_g/a in Figs. 4–7.
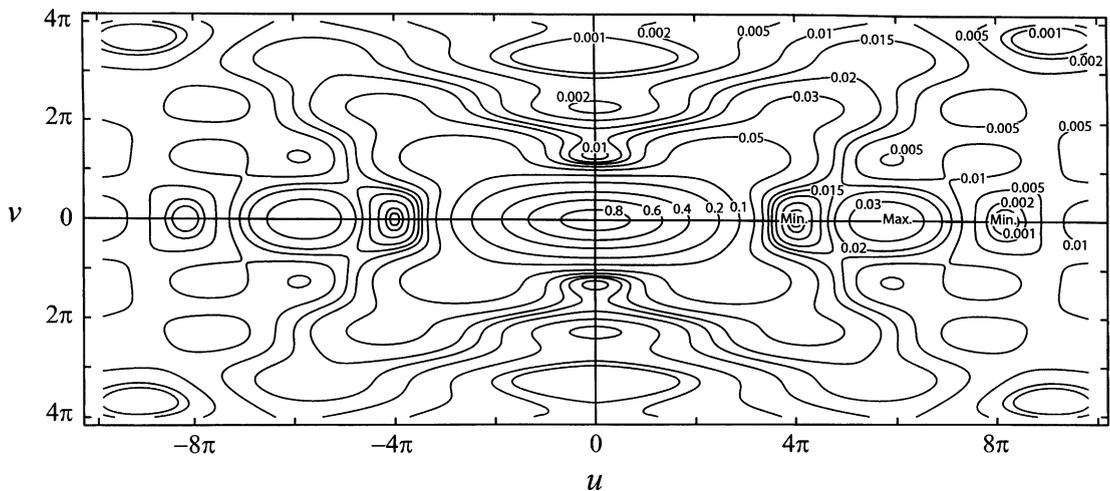
Fig. 4. Contours of constant spectral intensity of a converging partially coherent Gaussian Schell-model field with $\sigma_g/a = 2$, in the neighborhood of the geometrical focus. The normalized spectral intensity $S(u,v,\omega)/S_{\mathrm{coh}}(0,0,\omega)$ reaches a maximum value of 0.885 at the geometrical focus. The parameters are the same as those of Fig. 2.
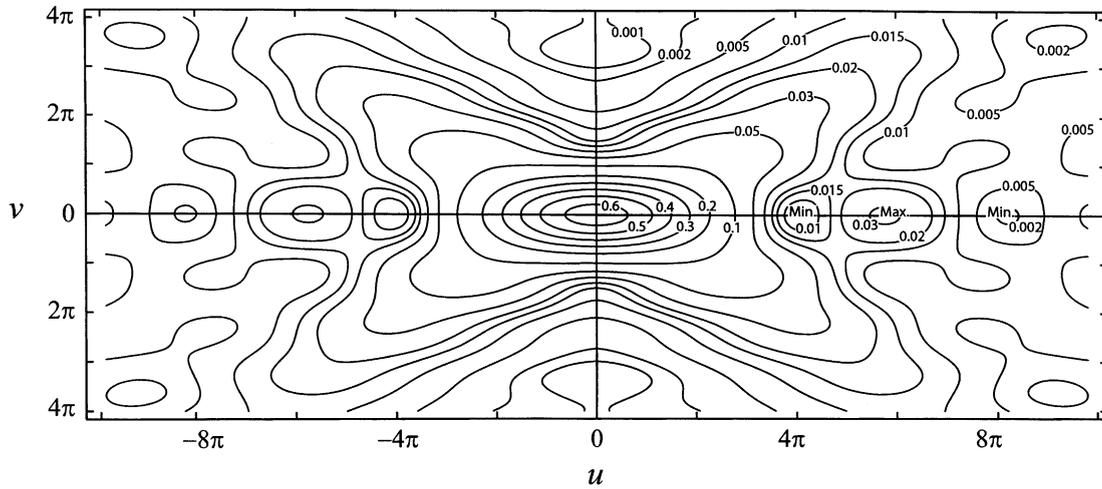
Fig. 5. Contours of constant spectral intensity of a converging partially coherent Gaussian Schell-model field with $\sigma_g/a = 1$, in the neighborhood of the geometrical focus. The normalized spectral intensity $S(u,v,\omega)/S_{\mathrm{coh}}(0,0,\omega)$ reaches a maximum value of 0.652 at the geometrical focus. The parameters are the same as those of Fig. 2.

The intensity contours for the case $\sigma_g/a = 2$ are shown in Fig. 4. The incident field is still nearly fully coherent and it can be seen that the intensity maintains most of the complex structure that is present in the fully coherent case. However, the maximum normalized spectral intensity is now equal to 0.885, already signifi-cantly smaller than in the optimal (fully coherent) case.

The intensity contours for the case $\sigma_g/a = 1$ are plotted in Fig. 5. The normalized spectral intensity reaches a maximum value of 0.652. In this figure the spectral intensity distribution in the geometrical focal plane ($u = 0$) no longer
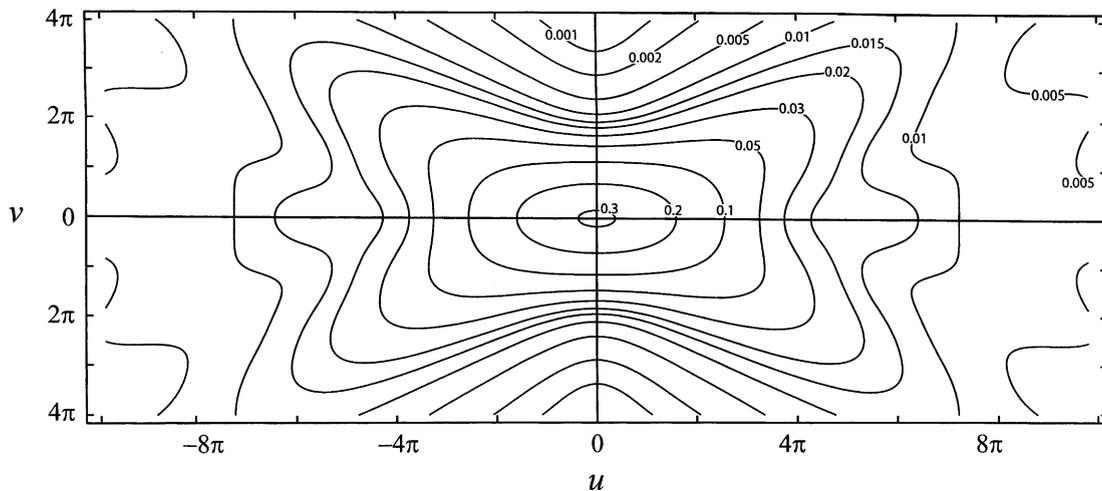
Fig. 6. Contours of constant spectral intensity of a converging partially coherent Gaussian Schell-model field with $\sigma_g/a = 0.5$, in the neighborhood of the geometrical focus. The normalized spectral intensity $S(u,v,\omega)/S_{\mathrm{coh}}(0,0,\omega)$ reaches a maximum value of 0.307 at the geometrical focus. The parameters are the same as those of Fig. 2.
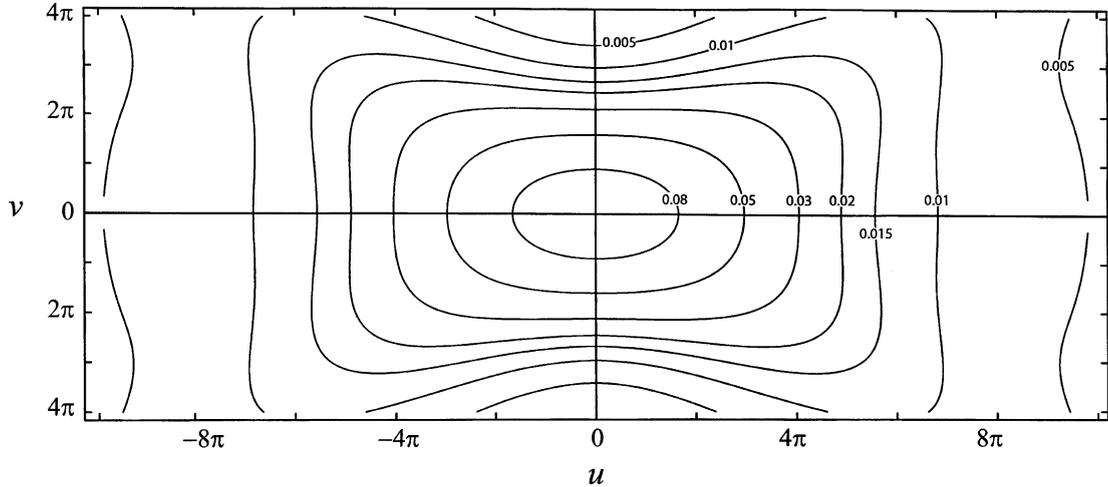
Fig. 7. Contours of constant spectral intensity of a converging partially coherent Gaussian Schell-model field with $\sigma_g/a = 0.25$, in the neighborhood of the geometrical focus. The normalized spectral intensity $S(u, v, \omega)/S_{\text{coh}}(0, 0, \omega)$ reaches a maximum value of 0.100 at the geometrical focus. The parameters are the same as those of Fig. 2.

exhibits secondary maxima or minima, but is monotonically decreasing with increasing v.

Fig. 6 presents the intensity contours for the case when $\sigma_g/a = 0.5$. The normalized spectral intensity reaches a maximum value of 0.307. The spectral intensity along the u-axis now also no longer has secondary maxima or minima, but decreases monotonically with increasing $|u|$.

Fig. 7 presents the intensity contours for the case $\sigma_g/a = 0.25$. The normalized spectral intensity reaches a maximum value of 0.100. In this case, for which the incident field is quite incoherent, interference effects are no longer evident.

Several trends can be discerned from our results: the maximum spectral intensity (i.e., the spectral intensity at the geometrical focus) decreases with decreasing spectral coherence length σ_g. It can also be seen that the size of the focal spot increases as the spectral coherence length decreases. Furthermore, with decreasing spectral coherence length the spectral intensity distribution becomes smoother in both the u and the v-directions. The secondary maxima both in the focal plane and along the optical axis gradually disappear as the spatial coherence decreases.

Acknowledgements

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