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Reply to comment
Optimum depth of the information pit on the data surface of a compact disk

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We read the preceding comment [1] with interest and we agree that there was an algebraic error in our previous derivations [2] of the optimal pit depth. This error arises from improperly assuming that the phase difference of the focused fields within the aperture of the lens system is the same as the phase difference between the point sources on the land and on the pit surfaces of the disk, an assumption which can be shown to be incorrect by a straightforward paraxial wave analysis. However, there is an additional phase factor which influences the optimum pit depth, neglected by the authors of the preceding comment and in fact incorrectly determined in our original analysis, and we would like to address its effects.

It is worth mentioning that the original theoretical results were motivated by experimental work [3] which suggested that, under certain circumstances, the optimal pit depth is in fact $\lambda/2$, not $\lambda/4$ as is ordinarily assumed. The $\lambda/4$ result is derived by assuming that the field illuminating the CD surface is a normally incident plane wave [4]. Of course, realistic compact disk readout systems use focused fields, and it is well-known that the spacing of the equiphase surfaces of a focused wave in the region of focus differs from that of a plane wave [5], a fact that is responsible for, among other things, the so-called Gouy phase shift of focused waves ([6], p. 498). We believed that this change of spacing could be responsible in some systems for the deviation of the optimal pit depth from the usually assumed $\lambda/4$ value.

To see the effect of this 'scaling' of the spacing of phase surfaces on the optimal pit depth, we briefly rederive the results of the papers [7, 2], but incorporate the correct phase as described in [1] and show where the 'scaling' phase arises in the

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analysis. At typical points Q_1 and Q_2 on the spherical wavefronts originating from the two point sources, the field distributions can be expressed in the form

$$U_1(Q_1) = A\frac{e^{i[-kR'_1+\psi'_1]}}{R'_1}, \quad (1)$$

and

$$U_2(Q_2) = A\frac{e^{i[-kR'_2+\psi'_2]}}{R'_2}, \quad (2)$$

where

$$\psi'_1 = \psi_1 + kR_1 + kR'_1, \quad (3)$$

with a similar expression for ψ'_2. Here $\psi_1 = 0$ and $\psi_2 = -k\Delta$. Now the important point is to use diffraction theory, rather than geometrical optics, to determine the propagation of these two converging waves to the detector plane. Standard results from the theory of focusing indicate that the field in the region of focus of each of the two waves is given by the expressions ([6], section 8.8)

$$U_1(P) = Ce^{i\psi'_1}e^{i(R'_1/a)^2u_1}\int_0^1 J_0(v_1\rho)e^{-i\frac{u_1\rho^2}{2}}\rho\,d\rho, \quad (4)$$

$$U_2(P) = Ce^{i\psi'_2}e^{i(R'_2/a)^2u_2}\int_0^1 J_0(v_2\rho)e^{-i\frac{u_2\rho^2}{2}}\rho\,d\rho, \quad (5)$$

where C is a constant factor which is approximately the same for both waves, and u_i, v_i are the dimensionless Lommel variables [defined in [2], equations (3) and (4)]. The field intensity at a point P in the region of superposition is then given by the expression

$$I(P) = |U_1(P) + U_2(P)|^2 = |U_1(P)|^2 + |U_2(P)|^2 + 2|U_1(P)||U_2(P)|\cos\{\phi_1 - \phi_2\}, \quad (6)$$

where

$$\phi_i = \psi'_i + (R'_i/a)^2u_i + \delta_i, \quad (7)$$

and where δ_i $(i = 1, 2)$ is the phase of the integral in equations (4) and (5). For arbitrary u and v this latter phase is not expressible in a closed analytic form, but on the axis $(v_i = 0)$ it takes on the simple form

$$\delta_i = -\frac{1}{4}u_i. \quad (8)$$

We now consider the physical significance of each of these factors. The ψ'_i are the phase differences introduced by the path length differences of the two focused fields, now correctly given by equation (3). From the definition of u_i, it follows that the factors $(R'_i/a)^2u_i$ are simply the longitudinal phase factors kz_i. The δ_i, however, are the phase factors which were absent in our earlier work and represent the 'scaling' of the phase front spacing discussed earlier. They are proportional to the kz_i factors and thus alter the spacing of the phase fronts from the 'ideal', normally incident plane wave.

As has been stated earlier, and can be immediately seen from equation (6), the optimal pit depth is then given by the values

$$\phi_1 - \phi_2 = m\pi \quad (m = 1, 3, 5, \ldots). \tag{9}$$

Assuming that the detector plane is located directly between the geometrical focal points of the two focused waves, it follows that $u_1 \approx -u_2$ and $R'_1 \approx R'_2$, and a straightforward analysis together with the use of equations (7) of reference [2] show that

$$\phi_1 - \phi_2 \approx 2k\Delta + \frac{k\Delta'}{4}\left(\frac{a}{R'_1}\right)^2. \tag{10}$$

On substituting from this equation into equation (9), and solving for Δ, one finds that

$$\Delta = \frac{m\lambda}{2}\left[\frac{1}{2 - (a/R_1)^2}\right], \tag{11}$$

where we have used the fact that $M_T a/R'_1 = a/R_1$. If the ratio a/R_1 is small compared to unity, then the optimal delta will be an odd multiple of $\lambda/4$. If, however, this ratio is appreciable, as it can be under practical circumstances, the optimal Δ could be given by a larger value, possibly approaching $\lambda/2$. This is in agreement with the experimental evidence, and now properly confirms our earlier claim that a focused field may have a different optimal pit depth than that of a plane wave.

The basic conclusion to be drawn from this analysis is that the phase of a focused field has a non-trivial structure and that any study of the interference of such waves in the focal region must necessarily use diffraction theory and not geometrical optics as in [1], especially since the spatial separation of the sources in the problem are less than a wavelength. Because the phase spacing 'scaling' in the focal region is highly dependent on the parameters of the focusing system (through, for instance, the ratio a/R_1), it is not true that 'the optimum pit depth can in no way depend on a system parameter such as magnification', as claimed by the authors of the preceding comment.

One can, of course, argue that our model of an optical readout system is oversimplified, as it does not take into account multiple point emitters on the data surface, electromagnetic effects, and so forth. However, considering the most detailed previous theoretical study of the optimum pit depth examines only the interference of plane waves, we consider our results to be a positive step towards understanding the question at hand.

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