Spectral anomalies near phase singularities in partially coherent focused wavefields

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Abstract

The influence of the state of spatial coherence on the spectrum of light in the vicinity of phase singularities of focused wavefields is investigated. It is found that a decrease of the degree of coherence of the field reduces the spectral changes of the field in the focal region.

Keywords: Diffraction, focusing, coherence, singular optics

1. Introduction

The domain of singular optics (see, for example, [1–3]) has recently been extended to polychromatic fields. In particular, it was predicted that appreciable spectral changes may occur in the vicinity of phase singularities (i.e., points of zero intensity of certain frequency components) of focused, spatially coherent polychromatic wavefields \[4, 5\]. This prediction has been verified experimentally \[6\]. It has also been found that similar spectral changes take place under other circumstances \[7–10\]. These investigations were all concerned with fields which are spatially completely coherent, and, consequently, the resulting spectral changes are induced by diffraction.

The effect of the state of coherence on the spatial intensity distribution of focused light fields has been studied in a number of recent papers \[11–14\]. Spatial coherence can give rise to spectral changes in light propagating in free space (for a review article on this subject see \[15\]) and such changes may be said to be correlation-induced. In the present paper we examine correlation-induced spectral changes that are produced in partially coherent focused wavefields.

2. Partially coherent focused fields

In \[13\] the focusing of a Gaussian Schell-model field emerging from a circular aperture of radius \(a\) in a plane screen was investigated. The spectral degree of coherence of such fields in the aperture is given by the expression \[16, section 4.3.2\]
\[ \mu(\rho_1, \rho_2, \nu) = g(\rho_1 - \rho_2, \nu) = e^{-\langle \rho_1^2 - \rho_2^2 \rangle / 2\sigma_g^2}, \]
where \(\rho = (x, y)\) is the two-dimensional vector, considered as the projection of the position vector of a point \(Q\) on the wavefront \(W\) in the aperture, onto the \(xy\)-plane (see figure 1), and \(\sigma_g\) is a positive constant, assumed to be independent of the frequency. The parameter \(\sigma_g\) is a measure of the effective spectral coherence length of the field in the aperture. It was shown in \[13\] that the axial spectral intensity at frequency \(\nu\) at a distance \(z\) from the geometrical focus is given by the formula
\[ S(z, \nu) = S(0)(\nu) k^2 \left( \int_0^a \int_0^a e^{-[\rho_1^2 + \rho_2^2] / 2\sigma_g^2} J_0 \left( \frac{\rho_1 \rho_2}{\sigma_g} \right) \cos[k[s_1(\rho_1) - s_2(\rho_2)]] \frac{\rho_1 \rho_2}{s_1(\rho_1)s_2(\rho_2)} d\rho_1 d\rho_2. \]
where the factor
\[ M(z, \nu) = k^2 \int_{0}^{\infty} \int_{0}^{\infty} e^{-\left(\rho_1^2 + \rho_2^2\right)/2\sigma^2} I_0 \left( \frac{\rho_1 \rho_2}{\sigma^2} \right) \cdot \cos[k(z_1(\rho_1) - z_2(\rho_2))] \cdot \frac{\rho_1 \rho_2}{s_1(\rho_1) s_2(\rho_2)} \ d\rho_1 \ d\rho_2, \tag{5} \]
is called the **modifier function**. It follows from equation (4) that the spectrum \( S(z, \nu) \) that is observed at an axial position \( z \) differs from the spectrum \( S^{(0)}(\nu) \) of the field on the reference sphere in the aperture by the multiplicative factor \( M(z, \nu) \). We note that this factor depends on the frequency \( \nu \) and on the effective spectral coherence length \( \sigma_\epsilon \) of the field in the aperture.

It is convenient to introduce a normalization factor
\[ N(\nu) = \lim_{\sigma_\epsilon \to \infty} M(0, \nu) \]
\[ = k^2 \int_{0}^{\infty} \int_{0}^{\infty} \frac{\rho_1 \rho_2}{f^2} \ d\rho_1 \ d\rho_2 \]
\[ = \frac{k^2 a^4}{4 f^2}. \tag{8} \]

### 3. Spectral changes

Suppose that the spectrum \( S^{(0)}(\nu) \) of the field in the aperture has a Lorentzian shape, i.e.
\[ S^{(0)}(\nu) = S_0 \frac{\Gamma^2}{(\nu - \nu_0)^2 + \Gamma^2}, \tag{9} \]
where \( S_0, \nu_0 \), and \( \Gamma \) are positive constants. The distribution of the normalized spectral intensity \( S(z, \nu)/N(\nu) \), given by equations (4) and (8), is shown in figure 2 for the fully coherent case (i.e. \( \lim_{\sigma_\epsilon \to \infty} \)) at three different axial observation points. It is seen that at all three points the spectrum (solid curve) differs significantly from the spectrum of the field in the aperture (dashed curve). In (a) the observed spectrum is red-shifted with respect to the spectrum in the aperture. This particular observation point \( (z = 0.25 \text{ cm}) \) coincides with the location of the first axial phase singularity (i.e., point of zero intensity; see [17, section 8.8, equation (26)]) of the frequency component \( \nu = 6 \times 10^{14} \text{ Hz} \). In (b) the observed spectrum is seen to split into two peaks. This second observation point coincides with the first axial phase singularity of the central frequency component \( \nu_0 = 5 \times 10^{14} \text{ Hz} \). In (c) the observed spectrum is blue-shifted. This third observation point coincides with the first axial phase singularity of the frequency component \( \nu = 4.29 \times 10^{14} \text{ Hz} \). Since the field is now spatially fully coherent, the spectral changes are diffraction-induced.

The dependence of the observed spectral intensity on the effective spectral coherence length \( \sigma_\epsilon \) is shown in figure 3 for the same three observation points as in figure 2. In this particular example the field is spatially partially coherent. This implies that the spectral changes are both diffraction-induced and correlation-induced. It is seen that the zeros in the observed spectrum disappear, and that the difference between the observed spectrum and the spectrum in the aperture decreases with decreasing \( \sigma_\epsilon \). These two effects can readily be understood from the spectral interference law [16, section 4.3.2]. We conclude that in the focal region of a polychromatic wavefield, an increase of spatial coherence of the field in the aperture gives rise to more pronounced spectral changes.

It is useful to compare our findings with some previously reported results. Palma and Cincotti [18] discussed spectral
blue-shifts in Gaussian Schell-model beams that have passed through a lens. Because the lens was assumed to be of infinite radius, no zeros (i.e., no phase singularities) occur in the field behind the lens. The blue-shift in the observed spectrum can be attributed to the dependence of the Huygens–Fresnel integral on the wavelength [17, section 8.8.1].

Figure 3. The spectral intensity for partially coherent focused wavefields at the same axial observation points as in figure 2 for three different scaled coherence lengths, namely $\sigma_g/a = 1.5, 1.0,$ and $0.5$. All other parameters are the same as in figure 2.

Mention should also be made of a paper by Palma et al [19] where the changes in the spectrum of a Gaussian Schell-model beam due to its coherence properties were studied both in free space and after passage through a thin lens. However, their analysis was not concerned with the behaviour of the spectrum near phase singularities.

Pu et al [20, 21] have predicted spectral changes both in the far zone and the Fresnel zone of diffracted partially coherent wavefields. In these papers these changes are attributed to the state of coherence of the field. However, in later studies [4, 5, 7, 8] it was demonstrated that these effects are primarily 

\textit{diffraction-induced} and occur near \textit{phase singularities} of certain frequency components. In the present paper we found that a decrease in spatial coherence results in a reduction of the spectral changes in the focal region.

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