INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS A: PURE AND APPLIED OPTICS

J. Opt. A: Pure Appl. Opt. **5** (2003) 371–373 PII: S1464-4258(03)59850-6

Spectral anomalies near phase singularities in partially coherent focused wavefields

T D Visser[1,3] **and E Wolf**[2]

[1] Department of Physics and Astronomy, Free University, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
[2] Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0171, USA

E-mail: tvisser@nat.vu.nl

Received 12 February 2003, in final form 29 April 2003
Published 21 May 2003
Online at stacks.iop.org/JOptA/5/371

Abstract
The influence of the state of spatial coherence on the spectrum of light in the vicinity of phase singularities of focused wavefields is investigated. It is found that a decrease of the degree of coherence of the field reduces the spectral changes of the field in the focal region.

Keywords: Diffraction, focusing, coherence, singular optics

1. Introduction

The domain of singular optics (see, for example, [1–3]) has recently been extended to polychromatic fields. In particular, it was predicted that appreciable spectral changes may occur in the vicinity of phase singularities (i.e., points of zero intensity of certain frequency components) of focused, spatially coherent polychromatic wavefields [4, 5]. This prediction has been verified experimentally [6]. It has also been found that similar spectral changes take place under other circumstances [7–10]. These investigations were all concerned with fields which are spatially completely coherent, and, consequently, the resulting spectral changes are induced by diffraction.

The effect of the state of coherence on the *spatial* intensity distribution of focused light fields has been studied in a number of recent papers [11–14]. Spatial coherence can give rise to *spectral changes* in light propagating in free space (for a review article on this subject see [15]) and such changes may be said to be *correlation-induced*. In the present paper we examine correlation-induced spectral changes that are produced in partially coherent focused wavefields.

2. Partially coherent focused fields

In [13] the focusing of a Gaussian Schell-model field emerging from a circular aperture of radius a in a plane screen was investigated. The spectral degree of coherence of such fields in the aperture is given by the expression [16, section 4.3.2]

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \nu) = g(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \nu) = \mathrm{e}^{-(\rho_2-\rho_1)^2/2\sigma_g^2}, \qquad (1)$$

where $\boldsymbol{\rho} = (x, y)$ is the two-dimensional vector, considered as the projection of the position vector of a point Q on the wavefront $\mathcal{W}$ in the aperture, onto the xy-plane (see figure 1), and σ_g is a positive constant, assumed to be independent of the frequency. The parameter σ_g is a measure of the effective spectral coherence length of the field in the aperture. It was shown in [13] that the axial spectral intensity at frequency ν at a distance z from the geometrical focus is given by the formula

$$S(z, \nu) = S^{(0)}(\nu) k^2 \int_0^a \int_0^a \mathrm{e}^{-(\rho_2^2+\rho_1^2)/2\sigma_g^2} \mathcal{I}_0\left(\frac{\rho_1\rho_2}{\sigma_g^2}\right) \times \cos\{k[s_1(\rho_1) - s_2(\rho_2)]\} \frac{\rho_1\rho_2}{s_1(\rho_1)s_2(\rho_2)}\,\mathrm{d}\rho_1\,\mathrm{d}\rho_2. \qquad (2)$$

Here $S^{(0)}(\nu)$ is the spectral intensity of the field on the wavefront that momentarily fills the aperture. The wavenumber $k = 2\pi\nu/c$, c denoting the speed of light in vacuum. $\mathcal{I}_0$ is the modified Bessel function of order zero. Further,

$$s_i(\rho_i) = [(f + z)^2 - z\rho_i^2/f]^{1/2} \qquad (i = 1, 2), \qquad (3)$$

f denoting the radius of the reference sphere (see figure 1). Hence equation (2) may be written in the form

$$S(z, \nu) = M(z, \nu) S^{(0)}(\nu), \qquad (4)$$

[3] Author to whom any correspondence should be addressed.

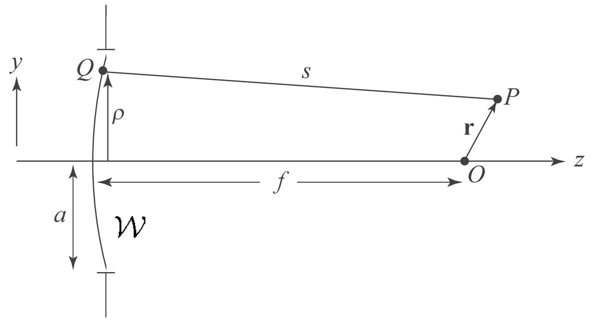

Figure 1. Illustrating the notation. The x-axis (not shown) is perpendicular to the plane of the figure.

where the factor

$$M(z,\nu) = k^2 \int_0^a \int_0^a \mathrm{e}^{-(\rho_2^2+\rho_1^2)/2\sigma_g^2} I_0\left(\frac{\rho_1\rho_2}{\sigma_g^2}\right) \times \cos\{k[s_1(\rho_1) - s_2(\rho_2)]\} \frac{\rho_1\rho_2}{s_1(\rho_1)s_2(\rho_2)}\,\mathrm{d}\rho_1\,\mathrm{d}\rho_2, \tag{5}$$

is called the *modifier function*. It follows from equation (4) that the spectrum $S(z,\nu)$ that is observed at an axial position z differs from the spectrum $S^{(0)}(\nu)$ of the field on the reference sphere in the aperture by the multiplicative factor $M(z,\nu)$. We note that this factor depends on the frequency ν and on the effective spectral coherence length σ_g of the field in the aperture.

It is convenient to introduce a normalization factor

$$N(\nu) = \lim_{\sigma_g\to\infty} M(0,\nu) \tag{6}$$

$$= k^2 \int_0^a \int_0^a \frac{\rho_1\rho_2}{f^2}\,\mathrm{d}\rho_1\,\mathrm{d}\rho_2 \tag{7}$$

$$= \frac{k^2a^4}{4f^2}. \tag{8}$$

3. Spectral changes

Suppose that the spectrum $S^{(0)}(\nu)$ of the field in the aperture has a Lorentzian shape, i.e.

$$S^{(0)}(\nu) = S_0 \frac{\Gamma^2}{(\nu-\nu_0)^2 + \Gamma^2}, \tag{9}$$

where S_0, ν_0, and Γ are positive constants. The distribution of the normalized spectral intensity $S(z,\nu)/N(\nu)$, given by equations (4) and (8), is shown in figure 2 for the fully coherent case (i.e. $\lim \sigma_g \to \infty$) at three different axial observation points. It is seen that at all three points the spectrum (solid curve) differs significantly from the spectrum of the field in the aperture (dashed curve). In (a) the observed spectrum is red-shifted with respect to the spectrum in the aperture. This particular observation point ($z = 0.25$ cm) coincides with the location of the first axial phase singularity (i.e., point of zero intensity; see [17, section 8.8, equation (26)]) of the frequency component $\nu = 6 \times 10^{14}$ Hz. In (b) the observed spectrum is seen to be split into two peaks. This second observation point coincides with the first axial phase singularity of the central frequency component $\nu_0 = 5 \times 10^{14}$ Hz. In (c) the observed spectrum is blue-shifted. This third observation point coincides with the first axial phase singularity of the frequency component $\nu = 4.29 \times 10^{14}$ Hz. Since the field is now spatially fully coherent, the spectral changes are diffraction-induced.

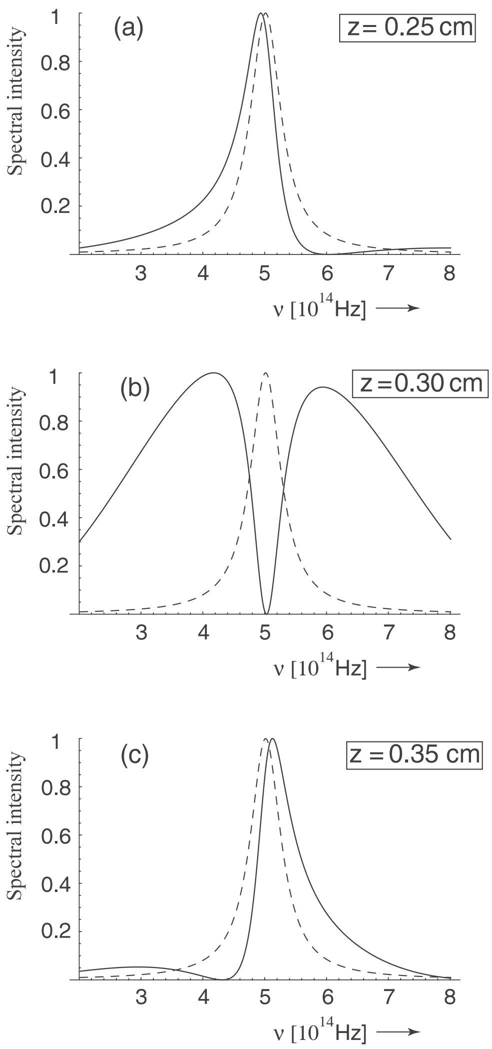

Figure 2. The spectral intensity of a converging fully coherent wave in the aperture (dashed curve) and the observed spectral intensity (solid curve) at three different axial points in the focal region, namely at $z = 0.25$, 0.30, and 0.35 cm. In this example $f = 1$ m, $a = 2$ cm, $\nu_0 = 5 \times 10^{14}$ Hz, $S_0 = 1$, and $\Gamma = 0.3 \times 10^{14}$ Hz. The Fresnel number of the focusing geometry $N = a^2/\lambda f = 667$ at the central frequency ν_0.

The dependence of the observed spectral intensity on the effective spectral coherence length σ_g is shown in figure 3 for the same three observation points as in figure 2. In this particular example the field is spatially partially coherent. This implies that the spectral changes are both diffraction-induced and correlation-induced. It is seen that the zeros in the observed spectrum disappear, and that the difference between the observed spectrum and the spectrum in the aperture decreases with decreasing σ_g. These two effects can readily be understood from the *spectral interference law* [16, section 4.3.2]. We conclude that in the focal region of a polychromatic wavefield, an increase of spatial coherence of the field in the aperture gives rise to more pronounced spectral changes.

It is useful to compare our findings with some previously reported results. Palma and Cincotti [18] discussed spectral

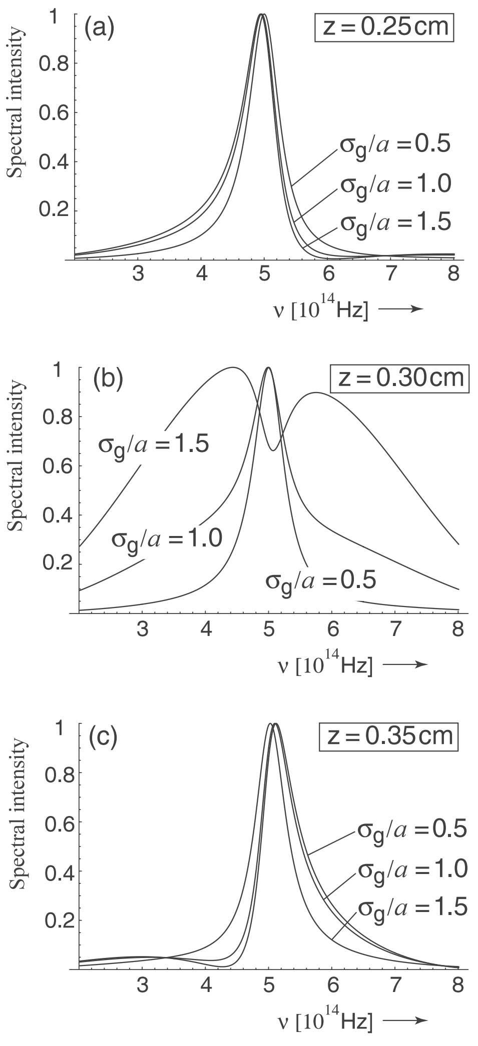

Figure 3. The spectral intensity for partially coherent focused wavefields at the same axial observation points as in figure 2 for three different scaled coherence lengths, namely $\sigma_g/a = 1.5$, 1.0, and 0.5. All other parameters are the same as in figure 2.

blue-shifts in Gaussian Schell-model beams that have passed through a lens. Because the lens was assumed to be of infinite radius, no zeros (i.e., no phase singularities) occur in the field behind the lens. The blue-shift in the observed spectrum can be attributed to the dependence of the Huygens–Fresnel integral on the wavelength [17, section 8.8.1].

Mention should also be made of a paper by Palma *et al* [19] where the changes in the spectrum of a Gaussian Schell-model beam due to its coherence properties were studied both in free space and after passage through a thin lens. However, their analysis was not concerned with the behaviour of the spectrum near phase singularities.

Pu *et al* [20, 21] have predicted spectral changes both in the far zone and the Fresnel zone of diffracted partially coherent wavefields. In these papers these changes are attributed to the state of coherence of the field. However, in later studies [4, 5, 7, 8] it was demonstrated that these effects are primarily *diffraction-induced* and occur near *phase singularities* of certain frequency components. In the present paper we found that a decrease in spatial coherence results in a reduction of the spectral changes in the focal region.

Acknowledgments

This research was supported by the US Air Force Office of Scientific Research under grant No F49620-03-1-0138, by the Engineering Research Programme of the Office of Basic Energy Sciences at the US Department of Energy under grant No DE-FG02-2ER 45992, by the Defense Advanced Research Project Agency under grant No MDA9720110043, by the Dutch Technology Foundation (STW), and by the European Union within the framework of the Future and Emerging Technologies—SLAM programme.

References

[1] Nye J F and Berry M V 1974 Dislocations in wave trains *Proc. R. Soc.* A **336** 165–90
[2] Nye J F 1999 *Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations* (Bristol: Institute of Physics Publishing)
[3] Soskin M S and Vasnetsov M V 2001 *Singular Optics (Progress in Optics vol 42)* ed E Wolf (Amsterdam: North-Holland)
[4] Gbur G, Visser T D and Wolf E 2002 Anomalous behaviour of spectra near phase singularities of focused waves *Phys. Rev. Lett.* **88** 013901
[5] Gbur G, Visser T D and Wolf E 2002 Singular behavior of the spectrum in the focal region *J. Opt. Soc. Am.* A **19** 1694–700
[6] Popescu G and Dogariu A 2002 Spectral anomalies at wave-front dislocations *Phys. Rev. Lett.* **88** 183902
[7] Ponomarenko S A and Wolf E 2002 Spectral anomalies in a Fraunhofer diffraction pattern *Opt. Lett.* **27** 1211–13
[8] Foley J T and Wolf E 2002 Phenomenon of spectral switches as a new effect in singular optics with polychromatic light *J. Opt. Soc. Am.* A **19** 2510–16
[9] Berry M V 2002 Coloured phase singularities *New J. Phys.* **4** article No 66
[10] Berry M V 2002 Exploring the colours of dark light *New J. Phys.* **4** article No 74
[11] Lü B, Zhang B and Cai B 1995 Focusing of a Gaussian Schell-model beam through a circular lens *J. Mod. Opt.* **42** 289–98
[12] Wang W, Friberg A T and Wolf E 1997 Focusing of partially coherent light in systems of large Fresnel numbers *J. Opt. Soc. Am.* A **14** 491–6
[13] Friberg A T, Visser T D, Wang W and Wolf E 2001 Focal shifts of converging diffracted waves of any state of spatial coherence *Opt. Commun.* **196** 1–7
[14] Visser T D, Gbur G and Wolf E 2002 Effect of the state of coherence on the three-dimensional spectral intensity distribution near focus *Opt. Commun.* **213** 13–19
[15] Wolf E and James D V F 1996 Correlation-induced spectral changes *Rep. Prog. Phys.* **59** 771–818
[16] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
[17] Born M and Wolf E 1999 *Principles of Optics* 7th expanded edn (Cambridge: Cambridge University Press)
[18] Palma C and Cincotti G 1997 Spectral shifts of a partially coherent field after passing through a lens *Opt. Lett.* **22** 671–2
[19] Palma C, Cincotti G and Guattari G 1998 Spectral shift of a Gaussian Schell-model beam beyond a thin lens *IEEE J. Quantum Electron.* **34** 378–83
[20] Pu J, Zhang H and Nemoto S 1999 Spectral shifts and spectral switches of partially coherent light passing through an aperture *Opt. Commun.* **162** 57–63
[21] Pu J and Nemoto S 2000 Spectral shifts and spectral switches in diffraction of partially coherent light by a circular aperture *IEEE J. Quantum Electron.* **36** 1407–11