New effects in Young's interference experiment with partially coherent light

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We analyze the coherence properties of a partially coherent optical field emerging from two pinholes in a plane opaque screen. We show that at certain pairs of points in the region of superposition the light is fully coherent, regardless of the state of coherence of the light at the pinholes. In particular, this result also holds if each pinhole is illuminated by a different laser.

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Two hundred years after Thomas Young discussed the interference of light that passes through two pinholes,[1,2] such experiments are still sources of new insights. Recent research involving Young's experiment with partially coherent light has predicted that if two pinholes are illuminated with broadband light and, consequently, interference fringes are absent, strong spectral changes generally occur in the region of superposition.[3] These predictions have been verified experimentally.[4,5] Somewhat analogous experiments with matter waves have been carried out with neutrons beams.[6–8]

In a recent investigation of Young's interference experiment with partially coherent light,[9] expressions were derived for the cross-spectral density and the spectral density of the field in the region of superposition. In the present Letter we derive somewhat more general expressions for such a situation and show that they imply remarkable properties of the spectral degree of coherence. For example, at any pair of points in certain planes of observation the light is found to be always completely coherent, irrespective of the state of coherence of the light at the two pinholes; in particular, the light could originate from independent lasers, each illuminating only one of the pinholes.

Consider a partially coherent field propagating into the half-space $z > 0$. The cross-spectral density $W^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega)$ (Ref. 10, Chap. 2.4.4) of the field at frequency ω at any two points $\mathbf{r}_1'$ and $\mathbf{r}_2'$ in the plane $z = 0$ may be expressed in terms of the spectral density $S^{(0)}(\mathbf{r}_1', \omega), S^{(0)}(\mathbf{r}_2', \omega)$ at the two points and the spectral degree of coherence $\mu^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega)$ of the light at these points in the form (Ref. 10, Chap. 4.3.2)

$$W^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega) = [S^{(0)}(\mathbf{r}_1', \omega) S^{(0)}(\mathbf{r}_2', \omega)]^{1/2} \mu^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega). \tag{1}$$

The cross-spectral density of the field at any pair of points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ in the half-space $z > 0$ is then given by the expression [Ref. 10, Eqs. (4.4-15) and (4.4-16)]

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left(\frac{1}{2\pi}\right)^2 \iint_{z=0} W^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega) \left(ik + \frac{1}{R_1}\right) \times \left(-ik + \frac{1}{R_2}\right) \frac{\exp[ik(R_2 - R_1)]}{R_1 R_2} \times \cos\theta_1 \cos\theta_2 \, \mathrm{d}^2 r_1' \mathrm{d}^2 r_2', \tag{2}$$

where $k = \omega/c$ is the wave number associated with frequency ω, c is the speed of light *in vacuo,* $R_i = |\mathbf{r}_i - \mathbf{r}_i'|$ and θ_i is the angle between the vector $\mathbf{r}_i - \mathbf{r}_i'$ and the positive z direction ($i = 1, 2$).

Suppose now that the plane $z = 0$ is covered by an opaque screen $\mathcal{A}$ with two small pinholes at points $Q_1(\overline{\mathbf{r}}_1)$ and $Q_2(\overline{\mathbf{r}}_2)$ (see Fig. 1). For this case Eq. (2) reduces to the following (cf. the derivation as given in Ref. 10, Chap. 4.3.2, but now with inclination factors):

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left(\frac{\delta A}{2\pi}\right)^2 \{S_1(\omega) K_{11}^* K_{12} + S_2(\omega) K_{21}^* K_{22} + \sqrt{S_1(\omega) S_2(\omega)} [\mu^{(0)}(Q_1, Q_2, \omega) K_{11}^* K_{22} + \mu^{(0)*}(Q_1, Q_2, \omega) K_{12} K_{21}^*]\}, \tag{3}$$

where we used the fact that $\mu^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega) = \mu^{(0)*}(\mathbf{r}_2', \mathbf{r}_1', \omega)$. Here $S_i(\omega)$ is the spectral density at pinhole Q_i, δA is the area of each pinhole, and the factors K_{ij} are given by

$$K_{ij} = \left(-ik + \frac{1}{R_{ij}}\right) \frac{\exp(ikR_{ij})}{R_{ij}} \cos\theta_{ij}, \qquad i, j = 1, 2, \tag{4}$$

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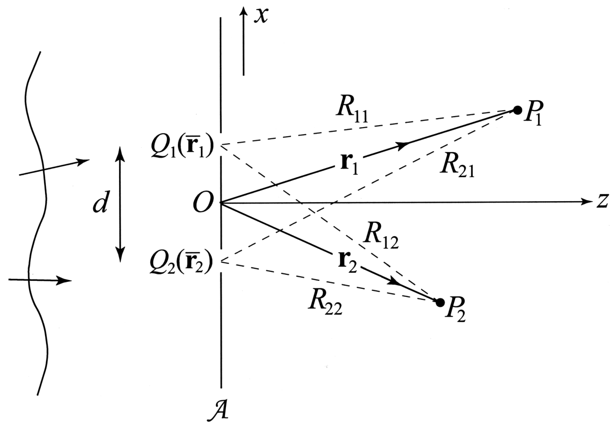

Fig. 1. Illustration of the notation relating to interference patterns formed with partially coherent light in Young's experiment.

where R_{ij} is the distance from the pinhole at $Q_i(\bar{\mathbf{r}}_i)$ to the field point $P_j(\mathbf{r}_j)$ and θ_{ij} is the angle between the line Q_iP_j and the positive z direction.

The spectral density at frequency ω of the light at a point $P_0(\mathbf{r}_0)$ in the region of superposition is given by

$$\begin{aligned} S(\mathbf{r}_0, \omega) &\equiv W(\mathbf{r}_0, \mathbf{r}_0, \omega) \\ &= \left(\frac{\delta A}{2\pi}\right)^2 \{S_1(\omega)|K_{10}|^2 + S_2(\omega)|K_{20}|^2 \\ &\quad + 2\sqrt{S_1(\omega)S_2(\omega)}\,\mathrm{Re}[\mu^{(0)}(Q_1, Q_2, \omega)K_{10}^*K_{20}]\}, \end{aligned} \tag{5}$$

where R_{10} and R_{20} denote the distances from the pinholes Q_1 and Q_2, respectively, to the point $P_0(\mathbf{r}_0)$, and Re denotes the real part.

Let us choose the coordinate system with the origin O at the midpoint between the two pinholes and with the plane $z = 0$ coinciding with the plane containing them. Let the pinholes be located symmetrically along the x axis at distance d from each other, i.e., at points with position vectors

$$\bar{\mathbf{r}}_1 = (d/2, 0, 0), \qquad \bar{\mathbf{r}}_2 = (-d/2, 0, 0), \tag{6}$$

referred to the midpoint 0. For any pair of points $P_1(\mathbf{r}_1), P_2(\mathbf{r}_2)$ in the plane $x = 0$, which we refer to as the bisecting plane Π, i.e., for points

$$\mathbf{r}_1 = (0, y_1, z_1), \qquad \mathbf{r}_2 = (0, y_2, z_2), \tag{7}$$

we have (see Fig. 1)

$$K_{11} = K_{21}, \qquad K_{12} = K_{22}. \tag{8}$$

On using Eq. (8) in Eq. (3) we obtain for the cross-spectral density

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2, \omega) &= \left(\frac{\delta A}{2\pi}\right)^2 K_{11}^*K_{12}\{S_1(\omega) + S_2(\omega) \\ &\quad + 2\sqrt{S_1(\omega)S_2(\omega)}\,\mathrm{Re}[\mu^{(0)}(Q_1, Q_2, \omega)]\}. \end{aligned} \tag{9}$$

Next we substitute from Eq. (9) into Eq. (5) and find that the spectral density

$$\begin{aligned} S(\mathbf{r}_i, \omega) &= \left(\frac{\delta A}{2\pi}\right)^2 |K_{1i}|^2\{S_1(\omega) + S_2(\omega) \\ &\quad + 2\sqrt{S_1(\omega)S_2(\omega)}\,\mathrm{Re}[\mu^{(0)}(Q_1, Q_2, \omega)]\}, \\ &(i = 1, 2). \end{aligned} \tag{10}$$

It immediately follows on using definition (1) for the spectral degree of coherence that

$$\mu(P_1, P_2, \omega) = \frac{K_{11}^*K_{12}}{|K_{11}|\,|K_{12}|}, \tag{11}$$

$$= \exp[ik(R_{12} - R_{11})] \times \frac{(ik + 1/R_{11})(-ik + 1/R_{12})}{|ik + 1/R_{11}|\,|-ik + 1/R_{12}|}, \tag{12}$$

$$= \exp[ik(R_{12} - R_{11})]\exp[i(\phi_1 - \phi_2)], \tag{13}$$

where

$$\cos\phi_i = 1/R_{1i}D_i, \qquad \sin\phi_i = k/D_i, \tag{14}$$

$$D_i = \sqrt{k^2 + 1/R_{1i}^2}. \tag{15}$$

Thus, we arrive at the conclusion that for any pair of points $P_1(\mathbf{r}_1), P_2(\mathbf{r}_2)$ that lie in the bisecting plane Π, i.e., the plane bisecting the line joining the two pinholes and perpendicular to that line, the spectral degree of coherence of the field is unimodular, i.e.,

$$|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)| = 1,$$

implying that the light at these points is mutually spatially fully coherent, irrespective of the state of coherence of the field at the two pinholes.

This result is illustrated in Fig. 2, in which contours of $|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ are shown. We note that the contours are not symmetric about the plane $x = 0$, even though

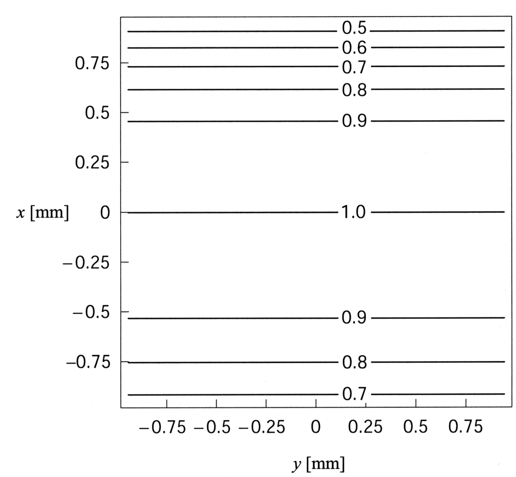

Fig. 2. Contour lines of $|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)|$, with $\mathbf{r}_1$ kept fixed at (0, 0, 1.5 m) and $\mathbf{r}_2$ varying in the plane $z = 1.5$ m. In this example $d = 1$ mm, $\omega = 10^{15}\ \mathrm{s}^{-1}$, and $\mu^{(0)}(Q_1, Q_2, \omega) = 0.2 + 0.2i$.

the geometry is. This asymmetry is due to the fact that in this particular example $\mu^{(0)}(Q_1, Q_2, \omega)$ is complex valued and $\mu^{(0)}(Q_1, Q_2, \omega) = \mu^{(0)*}(Q_2, Q_1, \omega)$.

Next consider a pair of points $\mathbf{r}_1 = (x, y, z)$, $\mathbf{r}_2 = (x, -y, z)$. In this case (see Fig. 1)

$$K_{11} = K_{12}\,, \qquad K_{21} = K_{22}\,. \tag{16}$$

On using these relations in Eq. (3) we obtain for the cross-spectral density the expression

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left(\frac{\delta A}{2\pi}\right)^2 &\{S_1(\omega)\,|K_{11}|^2 + S_2(\omega)\,|K_{21}|^2 \\ &+ 2\sqrt{S_1(\omega) S_2(\omega)} \\ &\times \mathrm{Re}[K_{11}^* K_{22} \mu^{(0)}(Q_1, Q_2, \omega)]\}\,. \end{aligned} \tag{17}$$

If we use Eqs. (16) in Eq. (5) we see that the spectral densities are given by the expressions

$$S(\mathbf{r}_1, \omega) = S(\mathbf{r}_2, \omega) \tag{18}$$

$$\begin{aligned} = \left(\frac{\delta A}{2\pi}\right)^2 &\{S_1(\omega)\,|K_{11}|^2 + S_2(\omega)\,|K_{22}|^2 \\ &+ 2\sqrt{S_1(\omega) S_2(\omega)} \\ &\times \mathrm{Re}[K_{11}^* K_{22} \mu^{(0)}(Q_1, Q_2, \omega)]\}\,, \end{aligned} \tag{19}$$

On substituting from Eqs. (17)–(19) into definition (1) for the spectral degree of coherence we find that

$$\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = 1\,. \tag{20}$$

Equation (20) shows that, for any pair of points $P_1(\mathbf{r}), P_2(\mathbf{r}_2)$ that are mirror images of each other in the plane containing the two pinholes and that is perpendicular to the screen, the spectral degree of coherence of the field is unity, irrespective of the state of coherence of the field at the two pinholes; i.e., it shows that the light is fully coherent and cophasal at such a pair of points.

It is to be noted that the light that is incident on each of the two pinholes may originate in two different sources. In particular, each pinhole might be illuminated by a different laser. Our results imply that even in such a case the light that two such independent lasers generate in the bisecting plane Π will be spatially completely coherent at every frequency contained in the spectra of both the lasers.

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