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Difference between TE and TM modal gain in amplifying waveguides: analysis and assessment of two perturbation approaches

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The commonly used confinement factor-based formula for modal gain in amplifying waveguides – $g_{mod} = \Gamma g_{mat}$, with Γ a confinement factor – is well established and accurate for TE modes. The TM case is rarely, and sometimes erroneously, described in the literature. Using a variational formulation the fundamental difference between TE and TM modal gain is illustrated. An accurate expression, correct up to first order, for the TM modal gain is then derived from a known general perturbation formula. However, as this does not lead to a true confinement factor formulation, some approximations are introduced, leading to a unified formulation of both TE and TM modal gain. A second method to calculate the modal gain, based on the analyticity of the dispersion equation, is also discussed. Simulation and comparison with modal gain values from a complex mode solver will finally illustrate the validity of the different approaches.

1. Introduction

One of the most important properties of amplifying waveguides, such as lasers or optical amplifiers, is the modal gain experienced by different modes. It has been known for a long time [1] that for TE slab modes, the modal gain is given by a confinement factor or filling factor-based expression. The modal gain can be written as a weighted sum of the confinement of the dominant TE field component in each waveguide layer. The weighing coefficients contain as the most important factor the material gain of each layer. Very often this formula is used as a perturbation approximation of the modal gain in the amplifying waveguide by using the confinement factor of the field in the waveguide without gain or loss. This avoids the need for a complex modal calculation and facilitates the simulation of the effects of different gain or loss levels in waveguide design.

As will be emphasized in this paper, TE and TM modal gain behave fundamentally differently [2]. This fact is very often not recognized in the literature. In [1] and [3] the discussion is limited to TE polarization and in [4] and [5] the TE expression, using the confinement factor for the dominant TM field component, is without motivation applied to calculate the TM modal gain. In [6] the modal gain is derived under the assumption of weak guidance, where the difference between TE and TM modes vanishes. Only in [7] and in [8] are the distinctions between both polarizations made. Furthermore in [8] a second possibility for modal gain calculation, including both polarization states, is suggested, but not applied. There it is remarked that the dispersion equation for the modal effective index is an analytic function of the refractive index profile. We will assess this approach and compare the results with mode solver and perturbation results.

In [9] it is shown that the correct modal gain formula for TM slab modes differs substantially from the TE expression. The expression derived in [9], although exact, is less useful as a starting point for a perturbation approach. Vassallo formulates in [8] a general perturbation expression for the modal gain based on a vectorial field solution for the unperturbed waveguide with arbitrary two-dimensional cross-section. Unfortunately, this expression, simplified to the case of a slab waveguide, cannot be recasted directly into a confinement factor formalism. Several approximate expressions for the TM modal gain, each of them based on a different confinement factor, will be derived and explored in this paper. It will be shown that it is possible to calculate accurately the TM modal gain using an expression which approximates Vassallo's formula and which is formally identical to the well-established TE formula and thereby retains its practical advantages. Finally, the theory of the analytic approach will be outlined and its accuracy will be proven. This approach has the same flexibility in terms of waveguide design as the perturbation formulation.

2. TE versus TM modal gain

Starting from the TE and TM wave equations for the dominant field components $e_y(x)$ and $h_y(x)$ of the one-dimensional slab waveguide with complex refractive index profile $n(x)$, and with z the propagation direction and k_0 the wavenumber in vacuum

$$\begin{gathered}\frac{\mathrm{d}^2 e_y(x)}{\mathrm{d}x^2} + (k_0^2 n^2(x) - \beta^2)e_y(x) = 0 \\ n^2(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{n^2(x)}\frac{\mathrm{d}h_y(x)}{\mathrm{d}x}\right) + (k_0^2 n^2(x) - \beta^2)h_y(x) = 0\end{gathered} \tag{1}$$

it can be shown, using a variational approach that the imaginary part of the square of the effective index satisfies [9]

$$\mathrm{Im}(n_{\mathrm{eff}}^2) = \frac{\int_{-\infty}^{+\infty} \mathrm{Im}(n^2(x))\left|e_y(x)\right|^2 \mathrm{d}x}{\int_{-\infty}^{+\infty} \left|e_y(x)\right|^2 \mathrm{d}x} \tag{2a}$$

for the TE case, and

$$\mathrm{Im}(n_{\mathrm{eff}}^2) = \frac{\int_{-\infty}^{+\infty} \mathrm{Im}(n^2(x))\left|h_y(x)\right|^2 \mathrm{d}x}{\int_{-\infty}^{+\infty} \left|h_y(x)\right|^2 \mathrm{d}x} - \mathrm{Im}\left(\frac{\int_{-\infty}^{+\infty} h_y^*(x) \frac{\mathrm{d}h_y(x)}{\mathrm{d}x} \frac{\mathrm{d}\ln n^2(x)}{\mathrm{d}x} \mathrm{d}x}{k_0^2 \int_{-\infty}^{+\infty} \left|h_y(x)\right|^2 \mathrm{d}x} \right) \tag{2b}$$

for the TM case. It is seen that $\mathrm{Im}\left(n_{\mathrm{eff}}^2\right)$ is given by two different equations. The first term in (2b) is the direct analogue of (2a) and the second term occurs because the TM wave equation describes a magnetic field in a purely dielectric material: the derivative $\mathrm{d}h_y(x)/\mathrm{d}x$ is, unlike $\mathrm{d}e_y(x)/\mathrm{d}x$, discontinuous at a refractive index step. In typical III–V semiconductor waveguides, the dominant TE and TM field profiles are very similar. The difference between TE and TM modal gain values is therefore dominated by the second term in (2b). As we will see further on, this difference can be as large as a factor of four.

Equations 2 can be used in two ways. In the first instance they may serve as a check on the accuracy of a complex mode solver. Given the calculated mode effective index, the field profiles can be constructed and Expressions 2a and 2b should return (in an implicit way) to the effective index itself. A more practical application can be found when interpreting Equations 2 in a perturbative way. Resetting the imaginary part of the refractive index profile to zero, calculating the real modes of the real index structure and substituting the obtained mode profiles in (2a) and (2b), while taking into account the imaginary index profile, gives a first-order estimate of the imaginary part of the effective index. However, Equation 2b is in practice not very useful. Typical input data for laser modelling consist of the confinement factors of the dominant field component in the different waveguide layers. It is immediately clear that (2a) as well as the first term of (2b) lend themselves to a confinement factor formulation.

3. Confinement factor formulation of TE and TM modal gain

Consider a slab waveguide defined by a real valued refractive index profile $n_0(x)$ and suppose that a modal solution $(\mathbf{e}_0(x), \mathbf{h}_0(x))$ is known. The waveguide is now perturbed by a pure imaginary index contrast $\delta n''(x)$. Applying standard perturbation theory on Maxwell's two curl equations it is shown in [10] that the first-order correction δn_{eff} on the effective index of the mode is given by

$$\delta n_{\mathrm{eff}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{\iint\limits_{\substack{\text{cross-}\\\text{section}}} n_0(x)\delta n''(x)\left|\mathbf{e}_0(x)\right|^2 \mathrm{d}x\,\mathrm{d}y}{\iint\limits_{\substack{\text{cross-}\\\text{section}}} (\mathbf{e}_0(x) \times \mathbf{h}_0(x)) \cdot \mathbf{u}_z \,\mathrm{d}x\,\mathrm{d}y} = \delta n''_{\mathrm{eff}} \tag{3}$$

and is therefore purely imaginary. If we limit the discussion to piecewise constant refractive index profiles with layer indices $n_{0,i}$, the general perturbation result (3) can be simplified in the TE case to

$$g_{\mathrm{mod}}^{\mathrm{TE}} = \frac{1}{n_{\mathrm{eff},0}^{\mathrm{TE}}} \sum_{i=1}^{N} n_{0,i} \Gamma_i^{e_y} g_{\mathrm{mat},i} \tag{4}$$

where we introduced the modal gain $g_{\mathrm{mod}} = 2k_0\,\mathrm{Im}(n_{\mathrm{eff}}) = 2k_0\delta n''_{\mathrm{eff}}$ and the material gain of the ith layer $g_{\mathrm{mat},i} = 2k_0\,\mathrm{Im}(n_i) = 2k_0\delta n''_i$ and N, the number of layers. $\Gamma_i^{e_y}$ is the confinement factor of the dominant TE field component in layer i of the unperturbed structure

$$\Gamma_i^{e_y} = \frac{\int\limits_{\text{layer i}} \left|e_{y,0}(x)\right|^2 \, \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} \left|e_{y,0}(x)\right|^2 \, \mathrm{d}x} \tag{5}$$

It follows immediately that

$$\sum_{i=1}^{N} \Gamma_i^{e_y} = 1 \tag{6}$$

Given the definition of the real part of the longitudinal component of the Poynting vector,

$$S_z^{\mathrm{TE}}(x) = \mathrm{Re}(\mathbf{S}(x) \cdot \mathbf{u}_z) = \tfrac{1}{2}\mathrm{Re}(\mathbf{e}_0(x) \times \mathbf{h}_0^*(x)) \cdot \mathbf{u}_z = \frac{\beta_0}{2\omega\mu_0}\left|e_{y,0}(x)\right|^2 \tag{7}$$

it is seen that the TE modal gain can also be expressed by means of the power flux, as is frequently done (see for example [6]).

The result (4) is widely known and applied in practice, both for TE and TM polarization. One generally defines a confinement factor based on the dominant h_y field component in the latter case. The fact that this leads to erroneous results can be understood from (2b), as was explained in the previous section.

To express the TM modal gain the general result (3) is formulated explicitly for a TM mode of a layered slab

$$g_{\mathrm{mod}}^{\mathrm{TM}} = n_{\mathrm{eff},0}^{\mathrm{TM}} \sum_{i=1}^{N} n_{0,i} \frac{\int\limits_{\text{layer } i} \left(\left|e_{x,0}(x)\right|^2 + \left|e_{z,0}(x)\right|^2\right) \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} n_0^2(x) e_{x,0}^2(x) \, \mathrm{d}x} g_{\mathrm{mat},i} \tag{8}$$

By analogy with Equation 4, the TM ‘confinement factor’ should be defined as

$$\Gamma_i^{\mathrm{TM}} = \left(n_{\mathrm{eff},0}^{\mathrm{TM}}\right)^2 \frac{\int\limits_{\text{layer}i} \left(\left|e_{x,0}(x)\right|^2 + \left|e_{z,0}(x)\right|^2\right) \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} n_0^2(x) e_{x,0}^2(x) \, \mathrm{d}x} = \left(n_{\mathrm{eff},0}^{\mathrm{TM}}\right)^2 \frac{\int\limits_{\text{layer } i} \left|\mathbf{e}_{\mathrm{tot},0}(x)\right|^2 \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} n_0^2(x) e_{x,0}^2(x) \, \mathrm{d}x} \tag{9}$$

This expression, although exact within the perturbation approximation, has two drawbacks. First, (9) does not describe a well-defined physical quantity with a clear interpretation of field confinement or power flux confinement, as was the case for TE polarization. Second, the property (6) is not fulfilled by Γ_i^{TM}. We will now derive two approximate expressions from (9) with a physical meaning and which are true confinement factors in the sense of (6) [11].

As we are only interested in guided modes, $|k_x| \ll |k_0|$. Hence

$$n_0^2(x) = \left(n_{\mathrm{eff},0}^{\mathrm{TM}}\right)^2 + \frac{k_x^2(x)}{k_0^2} \approx \left(n_{\mathrm{eff},0}^{\mathrm{TM}}\right)^2 \tag{10}$$

which corresponds physically to the replacement of the actual refractive index profile by the average index as it is seen by the mode profile. Furthermore, it follows from the TM equations that the inequality

$$\left|\frac{e_{z,0}^2(x)}{e_{x,0}^2(x)}\right| = \left|\frac{(\mathrm{d}h_{y,0}(x)/\mathrm{d}x)^2}{\beta^2 h_{y,0}^2(x)}\right| \cong O\left(\frac{k_x^2}{\beta^2}\right) \ll 1 \tag{11}$$

holds for (well-) guided modes. Adding an extra term $\left|e_{z,0}(x)\right|^2$ in the denominator leads to the confinement factor $\Gamma_i^{\mathrm{TM},\mathbf{e}_{\mathrm{tot}}}$

$$\Gamma_i^{\mathrm{TM}} \approx \frac{\int\limits_{\text{layer } i} \left(\left|e_{x,0}(x)\right|^2 + \left|e_{z,0}(x)\right|^2\right) \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} \left(\left|e_{x,0}(x)\right|^2 + \left|e_{z,0}(x)\right|^2\right) \mathrm{d}x} = \Gamma_i^{\mathrm{TM},\mathbf{e}_{\mathrm{tot}}} \tag{12}$$

which approximates very well with the value of Γ_i^{TM}. As we only consider unperturbed problems with pure real index profile, $e_{x,0}(x)$ is also real valued and hence $e_{x,0}^2(x) = |e_{x,0}(x)|^2$. Alternatively, we can also neglect the $\left|e_{z,0}(x)\right|^2$ term in the nominator to obtain

$$\Gamma_i^{\mathrm{TM}} \approx \frac{\int\limits_{\text{layer } i} \left|e_{x,0}(x)\right|^2 \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} \left|e_{x,0}(x)\right|^2 \mathrm{d}x} = \Gamma_i^{\mathrm{TM},e_x} \tag{13}$$

the confinement factor of the $e_{x,0}(x)$ field component. This approximation gets less accurate when the ratio $\left|k_x^2/\beta^2\right|$ increases, i.e. for the higher-order guided modes of a multimode waveguide.

To summarize, we now have three perturbation formulae to calculate the TM modal gain

$$\begin{aligned} g_{\mathrm{mod}}^{\mathrm{TM}} &= \frac{1}{n_{\mathrm{eff},0}^{\mathrm{TM}}} \sum_{i=1}^{N} n_{0,i} \Gamma_i^{\mathrm{TM}} g_{\mathrm{mat},i} \\ &\approx \frac{1}{n_{\mathrm{eff},0}^{\mathrm{TM}}} \sum_{i=1}^{N} n_{0,i} \Gamma_i^{\mathrm{TM},\mathbf{e}_{\mathrm{tot}}} g_{\mathrm{mat},i} \\ &\approx \frac{1}{n_{\mathrm{eff},0}^{\mathrm{TM}}} \sum_{i=1}^{N} n_{0,i} \Gamma_i^{\mathrm{TM},e_x} g_{\mathrm{mat},i} \end{aligned} \tag{14}$$

The confinement factors of the last two equations have a clear physical meaning. The quantity $\Gamma_i^{\mathrm{TM},\mathbf{e}_{\mathrm{tot}}}$ expresses the confinement of the total electric field leading to the same formula for both TE and TM modal gain. In the case of very strong guidance, where the modal field is completely confined into the core layer of the waveguide characterized by a complex refractive index, (14) predicts that the higher-order modes (with lower n_{eff}) experience a higher modal gain. This result might seem rather surprising at first glance, but can be intuitively understood by considering the ray picture of modal propagation. Higher-order modes have a more skewed angle with respect to the waveguide axis and therefore propagate a larger distance in the core layer. They see more gain per unit distance along the waveguide axis than a paraxial propagating mode.

The quantity $\Gamma_i^{\mathrm{TM},e_x}$ is the confinement of the, for TM modes, dominant electric field component. This confinement factor has the advantage that it follows immediately from the h_y-confinement factor $\Gamma_i^{\mathrm{TM},h_y}$ as

$$\Gamma_i^{\mathrm{TM},e_x} = \frac{\Gamma_i^{\mathrm{TM},h_y} \Big/ n_{0,i}^4}{\sum_{j=1}^{N} \Gamma_j^{\mathrm{TM},h_y} \Big/ n_{0,j}^4} \tag{15}$$

For waveguides with small index contrast it is clear that $\Gamma_i^{\mathrm{TM},e_x} \approx \Gamma_i^{\mathrm{TM},h_y}$. In homogeneous space this holds even exactly. The difference between TE and TM modes vanishes in this case, so that the effective index almost equals all of the refractive indices. Hence, the ratio between $n_{0,i}$ and $n_{\mathrm{eff},0}$ can be neglected in both Equations 4 and 14, leading to the result derived by [1] (only TE). It also follows that under the conditions of extreme guidance (effective index approaches the core index and the confinement in the cladding layers can be neglected) or when the mode evolves towards cutoff (effective index approaches the cladding index and the confinement in the core can be neglected), that $\Gamma_i^{\mathrm{TM},e_x} \approx \Gamma_i^{\mathrm{TM},h_y}$. Under these circumstances, the modal gain calculated by $\Gamma_i^{\mathrm{TM},h_y}$ will give an accurate result. This will also be the case for the confinement of the power flux $\Gamma_i^{\mathrm{TM},S_z}$.

4. Analyticity of the dispersion relation

In the previous section we focused on a perturbation approach to calculate the influence of a small imaginary perturbation of the refractive index profile of a slab waveguide on the modal effective index. In this section we will explore an alternative solution. It is known that the dispersion relation for the effective index as a function of the waveguide geometry and index profile is an analytic function [8, 12] in those parts of the complex plane where there are no branch points.

Suppose that layer i with index $n_{0,i}$ is perturbed by an imaginary index $\delta n_i''$. The modified effective index, due to the perturbation, is given by the Taylor expansion

$$\begin{aligned} n_{\mathrm{eff}}(n_i) &= n_{\mathrm{eff}}(n_{0,i} + \delta n_i'') = n_{\mathrm{eff}}(n_{0,i}) \\ &\quad + \delta n_i'' \frac{\partial n_{\mathrm{eff}}}{\partial n_i}\bigg|_{n_i = n_{0,i}} + \frac{1}{2}\delta n_i''^2 \frac{\partial^2 n_{\mathrm{eff}}}{\partial n_i^2}\bigg|_{n_i = n_{0,i}} + O\left(\delta n_i^3\right) \end{aligned} \tag{16}$$

The dispersion relation being an analytic function, the Cauchy–Riemann conditions [13]

$$\frac{\partial\ \mathrm{Re}(n_{\mathrm{eff}})}{\partial\ \mathrm{Re}(n)} = \frac{\partial\ \mathrm{Im}(n_{\mathrm{eff}})}{\partial\ \mathrm{Im}(n)}, \quad \frac{\partial\ \mathrm{Re}(n_{\mathrm{eff}})}{\partial\ \mathrm{Im}(n)} = -\frac{\partial\ \mathrm{Im}(n_{\mathrm{eff}})}{\partial\ \mathrm{Re}(n)} \tag{17}$$

are fulfilled, expressing that the derivative in the complex plane of the dispersion relation in one refractive index point is independent of the direction. Practically, this means that the influence of a small imaginary perturbation can be calculated by adding a real perturbation to the index profile, as is shown by the first equality in (17). It follows also from (16) that the first-order correction on the effective index is purely imaginary (*cf.* (3)). This means that the trajectory of the effective index in the complex n_i-plane will move away from the real axis in a direction parallel to the imaginary axis. The difference between TE and TM polarization is accounted for by the dependence of the dispersion equation on the refractive index profile.

The third term in Equation 16 suggests that the theory can be extended to higher-order corrections. To evaluate this term it would be preferable to use again the analytic property (17). It is indeed true that the function $\partial n_{\mathrm{eff}}/\partial n$ is analytic. Knowledge of $\partial n_{\mathrm{eff}}/\partial n$ in the direct vicinity of the nominal refractive index point $n_{0,i}$ makes it possible to evaluate the second-order correction. Practically, the effective index in three real index points around $n_{0,i}$ defines a parabola. The coefficient of the quadratic term determines the second-order correction, which is found to be real, see Equation 16.

In practical calculations, the derivative in (17) will be numerically evaluated, which may lead to a loss of accuracy. It is therefore not obvious at first sight if this method predicts the modal gain correctly. From numerical simulations we found that if the normalized propagation constant $B = (n_{\text{eff}}^2 - n_{\text{cl},2}^2)/(n_{\text{cl},1}^2 - n_{\text{cl},2}^2)$, with $n_{\text{cl},1} > n_{\text{cl},2}$, is known up to three significant digits, the imaginary part of the effective index is calculated within an accuracy of a few per cent, compared with the result of a complex mode solver.

Finally, we remark that the theory has been presented for the case where the refractive index varies in only one layer. Where different layers are perturbed one could apply the aforementioned theory separately to each layer and add up all the effective index corrections, or one could alternatively parametrize the refractive index profile as

$$n_i = n_{0,i} + \rho\delta n_i'' \tag{18}$$

where $i = 1, 2, \ldots, N$ and take the Taylor expansion with respect to the parameter ρ. Evaluating the first-order term gives the modified effective index in only one calculation.

5. Numerical examples

5.1. Modal gain calculation

We consider three different waveguide structures and calculate both TE and TM modal gain using the different perturbation approaches outlined in Section 3. The numerical results are compared with the gain values predicted by a complex mode solver and by making use of the analyticity. The waveguide structures are chosen as representative for a broad class of potential structures and are depicted schematically in Fig. 1. The numerical results are summarized in Table I for TE and Table II for TM polarization.

Table I shows that the modal gain values for waveguide A calculated by the complex mode solver and the TE perturbation formula coincide. The error of the analytic calculation is very small. Turning to TM polarization, Table II reveals that only the perturbation result using Γ^{TM} reproduces the mode solver result. It is also seen that the perturbation approaches using $\Gamma^{\text{TM},\text{e}_{\text{tot}}}$ and Γ^{TM,e_x} approximate very well the correct solution. Modal gain values predicted by Γ^{TM,S_z} and Γ^{TM,h_y} are wrong. The error of the analytic approach is again marginal. The same conclusion applies for the strongly asymmetric waveguide B. We also notice that TE and TM modal gain values differ by a factor of two for waveguide A and even a factor of four for structure B.

The last example is the symmetric trimodal waveguide C. According to Table I, the TE perturbation theory is correct for all three modes. For the TM case only the perturbation result Γ^{TM} coincides with the mode solver value. The true confinement factor $\Gamma^{\text{TM},\text{e}_{\text{tot}}}$ is able to predict the modal gain for all three guided modes with good accuracy and the

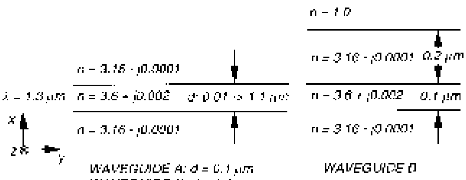

Figure 1 Waveguide geometries. The left waveguide is symmetric, the right strongly asymmetric.

TABLE I Comparison between the modal gain calculated by a complex slab solver, TE perturbation theory and the analytic approach. The numbers in parentheses indicate the relative deviation from the exact mode solver result

Waveguide	TE modal gain, cm^{-1}		
	Mode solver	Γ^{TE,e_y}	Analytic approach
A	48.68	48.68 (0%)	48.68 (0%)
B	52.76	52.76 (0%)	52.72 (–0.08%)
C – mode 0	191.6	191.6 (0%)	191.6 (0%)
C – mode 1	184.4	184.4 (0%)	184.3 (–0.05%)
C – mode 2	156.5	156.5 (0%)	156.4 (–0.06%)

expression based on Γ^{TM,e_x} underestimates the modal gain of the second-order mode significantly. This is expected, for the $e_z(x)$-field component increases for higher-order modes. We further remark that for all modes TE and TM modal gain values are comparable and that Γ^{TM,S_z} and Γ^{TM,h_y} lead to wrong results. The analytic approach proves again to be solid.

In every example the TE modal gain is larger than the TM modal gain, which leads to the suggestion that bulk semiconductor laser diodes with bulk active layers lase preferentially in the TE regime, not only because of the different mirror characteristics for TE and TM, but also because of the difference in modal gain.

5.2. Variation of TM modal gain as a function of the core thickness

We now calculate the evolution of the modal gain of the symmetric waveguide structure of Fig. 1 as a function of the core thickness d, and focus on the comparison of the different

TABLE II Comparison between the modal gain calculated by a complex slab solver, TM perturbation theory, the different TM confinement factor expressions and the analytic approach. The numbers in parentheses indicate the relative deviation from the exact mode solver result

Waveguide	TM modal gain, cm^{-1}						
	Mode solver	Γ^{TM}	$\Gamma^{TM,\mathbf{e}_{tot}}$	Γ^{TM,e_x}	Γ^{TM,S_z}	Γ^{TM,h_y}	Analytic approach
A	23.22	23.22 (0%)	22.98 (–1.1%)	23.34 (0.5%)	31.36 (+35%)	40.84 (+76%)	23.21 (–0.04%)
B	12.93	12.93 (0%)	12.74 (–1.4%)	12.70 (–1.8%)	18.60 (+44%)	25.72 (+98%)	12.88 (–0.39%)
C – mode 0	190.7	190.7 (0%)	190.3 (–0.2%)	191.1 (0.2%)	192.1 (+0.8%)	192.9 (+1.2%)	190.7 (0%)
C – mode 1	179.5	179.5 (0%)	178.3 (–0.7%)	179.9 (0.2%)	184.6 (+2.8%)	188.5 (+5%)	179.5 (0%)
C – mode 2	140.3	140.3 (0%)	139.1 (–0.8%)	133.8 (–4.6%)	146.6 (+4.5%)	158.2 (+13%)	140.2 (–0.07%)

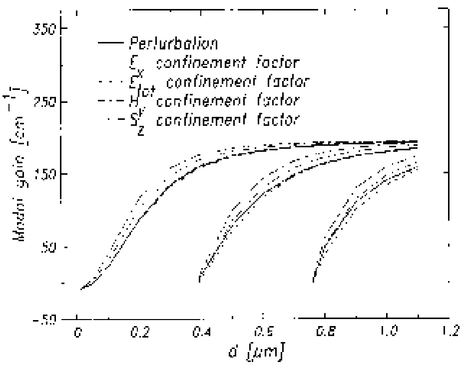

Figure 2 Variation of TM modal gain as a function of the core thickness of the waveguide. On the scale of the figure, the modal gain calculated by a complex slab solver, the correct perturbation expression and the analytic approach are indistinguishable. The three curves correspond (from left to right) to the fundamental, the first- and the second-order mode. The performance of the different confinement factor expressions are also compared.

TM perturbation approaches. The results are summarized in Fig. 2. One may argue that it would be preferable to plot the normalized propagation constant versus the normalized frequency. However, both quantities are in this case complex numbers and in the normalized propagation constant the real and imaginary parts of the effective index are not separable.

From Fig. 2 it is concluded that the curves calculated by $\Gamma^{\mathrm{TM}}$and $\Gamma^{\mathrm{TM,e_{tot}}}$ are hardly distinguishable on the scale of the plot. The accuracy of the modal gain values based on Γ^{TM,e_x} decreases progressively for higher-order modes. For the fundamental mode, the difference with the $\Gamma^{\mathrm{TM,e_{tot}}}$ result is, as expected, negligible. The confinement factor approaches based on Γ^{TM,S_z} and Γ^{TM,h_y} overestimate the modal gain significantly. Only under the conditions of cutoff or extreme guidance is the correct value approximated.

5.3. Variation of the effective index as a function of the material gain

We investigate now the accuracy of the analytic approach. Therefore we consider a monomode waveguide, typical for semiconductor optical amplifiers at 1.3 μm and vary the material gain of the active layer in a very broad interval going from 10 cm^{-1} to 10^4 cm^{-1}. This extremely high (and physically unrealistic) upper boundary will enable us to explore the limits of the algorithm. The modal gain will be calculated using the analytic approach, perturbation theory and a complex mode solver. The influence of the gain on the real part of the effective index will be calculated using (16) and a complex mode solver. The variation of the effective index as a function of the core index n is approximated, in the vicinity of the nominal core index 3.60, for both polarizations by the parabolas

$$\begin{aligned} n_{\mathrm{eff}}^{\mathrm{TM}} &= 0.287n^2 - 1.778n + 5.908 \\ n_{\mathrm{eff}}^{\mathrm{TM}} &= 0.0915n^2 - 0.496n + 3.804 \end{aligned} \tag{19}$$

defined by the core index points 3.59, 3.60 and 3.61. Substitution of (19) in the Taylor expansion (16) leads to the curves labelled ‘analytic’ in Fig. 3. The modal gain values obtained by the complex mode solver, the analytic approach and the perturbation theory coincide perfectly, both for TE and TM polarization. The modal gain varies linearly. Considering the real part of the effective index it is seen that the expansion (16) predicts accurate results. Additional simulations have shown that the numerical results are insensitive to the precise choice of the core indices for the calculation of the parabolas (19).

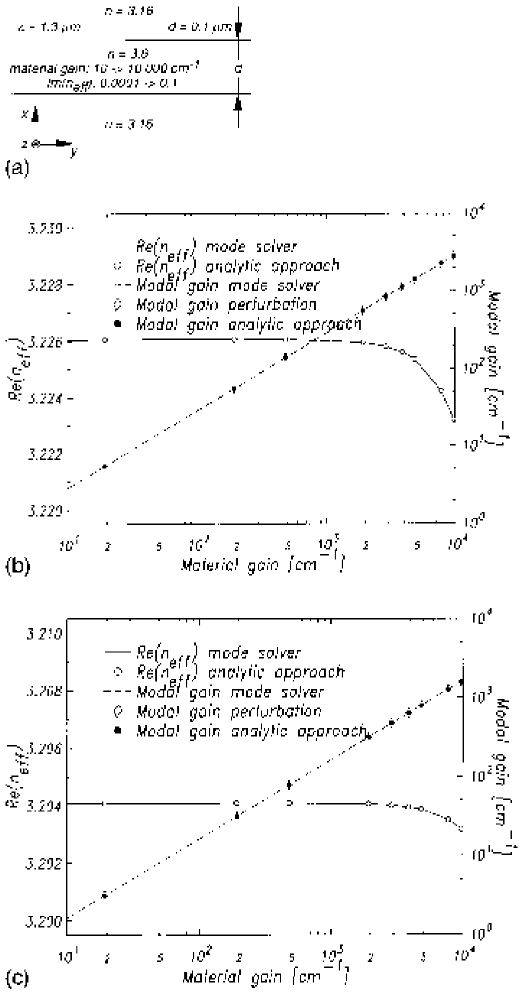

Figure 3 Variation of real and imaginary (expressed through the modal gain) parts of the effective index of the fundamental mode as a function of the material gain of the core. The waveguide is depicted in (a). On the scale of the figure, the differences between the results obtained using a complex mode solver, the correct perturbation formula and the analytic approach disappear. TE results can be found in (b) and TM results in (c).

6. Conclusion

It has been shown from a simple variational calculation (see [9]) that TE and TM modal gain behave fundamentally differently. Therefore, one has to be very cautious when generalizing TE formulae to the TM case, as is frequently done in the literature for the case of the confinement factor formulation, which relates the material gain of the waveguide layers to the modal gain of the waveguide mode. Starting from a general perturbation formula, an accurate expression, correct up to first order, for both TE and TM modal gain were derived. Unfortunately, the TM modal gain is not given by a true filling factor formulation. By approximating the TM relation we have shown that both TE and TM modal gain are given by the same expression

$$g_{\mathrm{mod}} = \frac{1}{n_{\mathrm{eff},0}} \sum_{i=1}^{N} n_{0,i} \Gamma_t^{\mathbf{e}_{\mathrm{tot}}} g_{\mathrm{mat},i} \tag{20}$$

which is exact for TE modes and approximate for TM modes.

An alternative way of calculating the modal gain to first order has also been investigated. This method relies on the fact that the dispersion equation for the effective index is an analytic function of the refractive index profile. The distinction between TE and TM polarization is accounted for in the form of the dispersion relation. Using this approach the effect of an imaginary index perturbation on the real part of the effective index can be calculated very accurately.

The advantage of both methods is that in order to obtain the complex effective index only a real index modal calculation has to be done and that the effect of different material gain or loss levels can be directly estimated without any additional simulation. The validity of the TM confinement factor approximation, as well as that of the analytic approach, has been numerically verified.

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