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Coherence vortices in partially coherent beams

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Abstract

It is demonstrated that the spectral degree of coherence of a partially coherent beam may possess isolated pairs of points at which its phase is singular, and that in the neighborhood of these points the phase may possess a vortex structure. Partially coherent beams consisting of Hermite–Gaussian modes are considered as an example. The physical consequences of these so-called coherence vortices are discussed.

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Keywords: Singular optics; Optical vortices; Coherence

1. Introduction

The field of singular optics [1], which has primarily been studied with fully coherent, monochromatic, scalar wavefields, has recently been extended considerably by a variety of workers who have revealed many new effects. For instance, Gbur et al. [2,3] predicted that the spectrum of a fully coherent focused polychromatic field will undergo drastic changes in the vicinity of phase singularities of the central frequency. These predictions have since been verified experimentally [4] and such spectral changes have been shown by Berry to be a characteristic feature of polychromatic fields near phase singularities [5]. Kessler and Freund [6] have investigated the polarization singularities of two-color vector fields and characterized the so-called Lissajous singularities which arise. Phase singularities which arise at zeros of the intensity of partially coherent fields have also been investigated [7,8].

More recently, an examination of the phase singularities of *two-point* coherence functions has been undertaken by Schouten et al. [9]. In that paper, it was shown that the field produced by a Young's interference experiment may possess pairs of points in the region of superposition at which the *spectral degree of coherence* of the field is equal to zero. It was further shown that these pairs of points can be associated with pairs of surfaces for which the spectral degree of coherence vanishes if the observation points are confined to the complimentary surfaces.

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As yet, however, the existence of optical *vortices* of the spectral degree of coherence has not been demonstrated. In this paper, we examine a class of partially coherent beams comprised of Hermite–Gaussian modes, and show that in general there exist pairs of points in these beams for which the spectral degree of coherence vanishes. The phase of the spectral degree of coherence is shown to possess a vortex structure around these singular points and we use the term *coherence vortices* to refer to them. It is to be noted that the intensity of the field at such a pair of points is not required to vanish, and in general will not. The physical consequences of these coherence vortices are discussed.

2. Partially coherent beams

We consider a fluctuating, statistically stationary field $U(\mathbf{r},t)$ propagating from the plane $z=0$ into the half-space $z>0$ (see Fig. 1). Spatial and temporal correlations between pairs of points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ may be characterized by use of the mutual coherence function [10, Section 4.3.1],

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau) = \langle U^*(\mathbf{r}_1,t)U(\mathbf{r}_2,t+\tau)\rangle, \tag{1}$$

where the angular brackets denote time or ensemble averaging.

Because singular optics is typically investigated with monochromatic fields, it is advantageous to work instead with the Fourier transform of the mutual coherence function, the *cross-spectral density* [10, Section 4.3.2], defined as

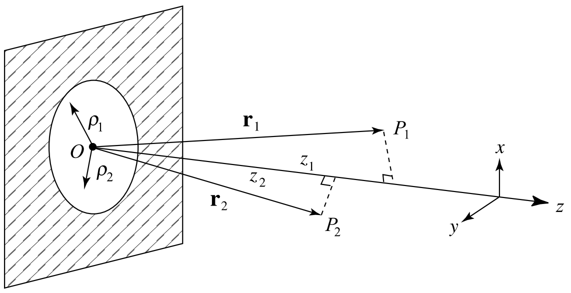

Fig. 1. Illustrating the notation relating to the propagation of a partially coherent beam. Here $\boldsymbol{\rho}_i = (x_i, y_i)$, with $i = 1, 2$.

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \frac{1}{2\pi}\int \Gamma(\mathbf{r}_1,\mathbf{r}_2,\omega)\,\mathrm{e}^{\mathrm{i}\omega\tau}\,\mathrm{d}\tau. \tag{2}$$

The cross-spectral density characterizes the spatial correlations of the field at a single frequency ω. It can be shown that it satisfies a pair of scalar Helmholtz equations with respect to its two spatial variables [10, Section 4.4.1], i.e.,

$$\nabla_i^2 W(\mathbf{r}_1,\mathbf{r}_2,\omega) + k^2 W(\mathbf{r}_1,\mathbf{r}_2,\omega) = 0, \tag{3}$$

where $k=\omega/c$, c being the speed of light in vacuum, and ∇_i represents the gradient with respect to the spatial variable $\mathbf{r}_i$, $i=1,2$. From this point on we will consider only a single frequency ω and suppress its depiction in the function arguments.

The strength of coherence between a pair of points at locations $\mathbf{r}_1$ and $\mathbf{r}_2$ may be described by the so-called spectral degree of coherence, defined as

$$\mu(\mathbf{r}_1,\mathbf{r}_2) \equiv \frac{W(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{S(\mathbf{r}_1)}\sqrt{S(\mathbf{r}_2)}}, \tag{4}$$

where

$$S(\mathbf{r}_i) \equiv W(\mathbf{r}_i,\mathbf{r}_i) \tag{5}$$

is the spectral density (often referred to as intensity) of the field at point $\mathbf{r}_i$. It can be shown that the modulus of the spectral degree of coherence takes on values between 0 and 1, zero representing complete incoherence, unity representing complete coherence.

It is well known (see [10, Section 4.5.3] or [11]) that the cross-spectral density W^{coh} of a fully coherent field ($|\mu|=1$) may be represented in the factorized form

$$W^{\mathrm{coh}}(\mathbf{r}_1,\mathbf{r}_2) = \psi^*(\mathbf{r}_1)\psi(\mathbf{r}_2), \tag{6}$$

where $\psi(\mathbf{r})$ is a generally complex function which satisfies the Helmholtz equation. One can construct a partially coherent field by taking an incoherent superposition of a number of such coherent modes, so that

$$W(\mathbf{r}_1,\mathbf{r}_2) = \sum_n \lambda_n \psi_n^*(\mathbf{r}_1)\psi_n(\mathbf{r}_2). \tag{7}$$

In the above equation the mode weights λ_n are necessarily real and non-negative, and the subscript

n may represent more than one index of summation. The cross-spectral density of the field produced by a multimode laser or by the superposition of multiple independent coherent lasers would be of the form of Eq. (7).

If the modes are mutually orthonormal with respect to a particular chosen domain (such as a finite volume or, often in the case of beams, a plane of constant z), Eq. (7) represents a diagonal decomposition of the cross-spectral density within that domain, referred to as the coherent mode representation [12]. Such a decomposition is unique up to the selection of those modes with equal weights. In this paper, we will be considering incoherent superpositions of modes which are not necessarily orthogonal; we will refer to such a representation of the field as a mode representation but it is important to note that it is not a coherent mode representation.

We restrict ourselves exclusively to the so-called Hermite–Gaussian modes $u_{lm}(x, y, z; z_0)$, defined by [13, Section 4.7.4]

$$u_{lm}(x, y, z; z_0) = \frac{w_0}{w(z - z_0)} H_l\left(\frac{\sqrt{2}x}{w(z - z_0)}\right) \times H_m\left(\frac{\sqrt{2}y}{w(z - z_0)}\right) \times e^{-(x^2+y^2)/w^2(z-z_0)} \times e^{-ik(x^2+y^2)/2R(z-z_0)} e^{i(1+l+m)\phi(z-z_0)}, \tag{8}$$

where H_l represents the Hermite polynomial of order l, z_0 represents the position of the waist plane of the mode, w_0 represents the width of the mode at the waist, and

$$\phi(z) = \tan^{-1}\left(\frac{2kz}{(kw_0)^2}\right), \tag{9}$$

$$w(z) = w_0\sqrt{1 + \left(\frac{2kz}{(kw_0)^2}\right)^2}, \tag{10}$$

and

$$R(z) = z\left[1 + \left(\frac{(kw_0)^2}{2kz}\right)^2\right]. \tag{11}$$

In Eq. (8), $w(z - z_0)$ represents the width of the mode at distance z, $R(z - z_0)$ represents the radius of curvature of the equiphase surfaces, and $\phi(z - z_0)$ represents a longitudinal phase. These modes represent beams paraxially propagating into the half-space $z > 0$ along the z-axis. We will take $k = 9921\ \text{mm}^{-1}$ and $w_0 = 1$ mm in all following calculations; such parameters represent the typical output of a He–Ne laser. For purposes of clarity, we will investigate the cross-spectral density of beams consisting of Hermite–Gaussian modes with both points $\mathbf{r}_1$, $\mathbf{r}_2$ constrained to a single plane normal to the direction of propagation of the beam, typically the plane $z = 0$.

3. Coherence vortices

To illustrate the differences between coherence vortices and their traditional counterparts, we first briefly review the latter topic from the point of view of the cross-spectral density $W(\mathbf{r}_1, \mathbf{r}_2)$.

The traditional singular optics of monochromatic coherent fields, with cross-spectral density given by Eq. (6), deals with the singularities of the phase that arise at points where the field amplitude is zero, i.e.,

$$|\psi(\mathbf{r})|^2 = S(\mathbf{r}) = 0, \tag{12}$$

or, equivalently, where

$$\text{Re}\{\psi(\mathbf{r})\} = 0, \tag{13}$$

$$\text{Im}\{\psi(\mathbf{r})\} = 0. \tag{14}$$

In three-dimensional space, this pair of equations is underdetermined and will typically have solutions in the form of lines. Around such lines, the phase ϕ_ψ of the mode $\psi(\mathbf{r})$ will typically have a helicoidal, or vortex, structure. In a plane of constant z, these singular lines will usually intersect the plane at isolated points.

Several conserved quantities may be associated with any vortex. One of the most important of these is the topological charge s of the singularity, defined as

$$s \equiv \frac{1}{2\pi} \oint_C \nabla\phi_\psi \cdot d\mathbf{r}, \tag{15}$$

where the path C is taken along a closed counterclockwise path of winding number 1 in a plane of constant z enclosing the (point) singularity.[1] It is well known that the topological charge of a singularity is a conserved quantity which may take on only integer values, and that such singularities may only be created and annihilated in ways such that the total topological charge is conserved. The vortex is referred to as positive or negative if the topological charge is positive or negative, respectively.

It should be pointed out that it is also possible for the singular points of the phase to take the form of surfaces in three-dimensional space. For instance, it can be seen that the amplitude of a beam consisting only of the Hermite–Gaussian mode u_{10} vanishes everywhere in the plane defined by $x = 0$. This behavior can be connected to the fact that, for this example, Eqs. (13) and (14) can be recombined in such a way that one equation is trivially satisfied. This is most obvious in the waist plane, where $\mathrm{Im}\{\psi(\mathbf{r})\} \equiv 0$ without any recombination. Such a case is not common, however, as most arbitrary wavefields will not satisfy such a requirement; it is said that surfaces of singular phase are not 'generic'.[2]

For partially coherent fields, with cross-spectral density given by Eq. (7), singularities of the spectral density again arise at points such that $S(\mathbf{r}) = 0$. For this to occur at a given point $\mathbf{r}$ the following set of equations must be satisfied:

$$|\psi_n(\mathbf{r})| = 0 \quad \text{for all } n. \tag{16}$$

One can ask how common or generic such zeros of intensity are in a partially coherent field. For a cross-spectral density with N modes, Eq. (16) represents $2N$ equations that must be solved simultaneously in a three-dimensional space, for the real and imaginary parts of each mode must equal zero. For a fully coherent field ($N = 1$), we have seen that such solutions are likely to occur in the form of lines; for $N > 1$, however, Eq. (16) represents an overspecified set of equations which do not, in general, have a solution.[3]

Zeros of the spectral density of a partially coherent field are therefore typically not present. For the spectral degree of coherence, however, which is a function of two spatial variables $\mathbf{r}_1$ and $\mathbf{r}_2$, zeros are quite common, as we now show. We first note that because zeros of the spectral density are uncommon in partially coherent fields, the spectral degree of coherence [given by Eq. (4)] is typically well-defined throughout space, and the phase of the spectral degree of coherence and the phase of the cross-spectral density of the field are the same; hence results which apply to the phase of one quantity also apply to the other. We may therefore work with the mathematically simpler cross-spectral density in the following calculations. For a fixed value of $\mathbf{r}_1$, the cross-spectral density is a generally complex function of position $\mathbf{r}_2$. Therefore two equations must be satisfied for $W(\mathbf{r}_1, \mathbf{r}_2)$ to vanish,

$$\mathrm{Re}\{W(\mathbf{r}_1, \mathbf{r}_2)\} = 0, \tag{17}$$

$$\mathrm{Im}\{W(\mathbf{r}_1, \mathbf{r}_2)\} = 0. \tag{18}$$

In a three-dimensional space, these equations will typically have simultaneous solutions in the form of lines; this is identical to the situation that arises for singularities of the spectral density of fully coherent fields. Again, in a plane of fixed z these singularities will take the form of points.

The correspondence between traditional vortices of coherent fields and coherence vortices of partially coherent fields is perhaps not surprising for it can be seen from Eq. (3) with $\mathbf{r}_1$ fixed, $W(\mathbf{r}_1, \mathbf{r}_2)$ is a solution of the Helmholtz equation with respect to $\mathbf{r}_2$, and it is expected to have essentially the same behaviors as a fully coherent field. The interpretation of the zeros, however, is somewhat different, as we will discuss in Section 4.

[1] The path is counterclockwise with respect to an observer facing the oncoming beam.

[2] Genericity is described in further detail in Chapter 1 of [14].

[3] It is to be noted that such partially coherent fields can be found, as described for instance in [7,8]. The fields described in these papers have been carefully constructed to possess optical vortices, however, and are not typical of the behavior of partially coherent fields.

As examples, let us consider partially coherent beams consisting of only two modes, i.e., with a cross-spectral density given by the expression

$$W(\mathbf{r}_1,\mathbf{r}_2) = \lambda_0 u_{00}^*(\mathbf{r}_1;0)u_{00}(\mathbf{r}_2;0) + \lambda_1 u_{lm}^*(\mathbf{r}_1;z_0)u_{lm}(\mathbf{r}_2;z_0), \quad (19)$$

where the u_{lm} are defined in Eq. (8). Eq. (19) represents an incoherent superposition of a Gaussian beam with waist plane at $z = 0$ and a Hermite–Gaussian beam of order lm with waist plane at $z = z_0$. The waist planes are taken to not coincide, because coincident waists would result in a real-valued cross-spectral density in the waist plane, and the singular points of $W(\mathbf{r}_1,\mathbf{r}_2)$ would then take the form of surfaces. It is to be noted that because the waist planes do not coincide, the modes are not necessarily orthogonal. The spectral density of a partially coherent beam of the form (19) is nonzero throughout space because the mode $u_{00}(\mathbf{r},0)$ has no zeros.

The zeros of the real and imaginary parts of the cross-spectral density were computed numerically for fixed $\mathbf{r}_1$ and a variety of values of $\mathbf{r}_2$, the computation being done in a single z-plane, i.e., $z_1 = z_2$. A typical example of such a calculation is shown in Fig. 2. Those points for which the phase is singular are located at the intersection of the curves. In this example, the points labeled A and B represent a positive (charge $s = +1$) and negative ($s = -1$) phase vortex, respectively. This can be seen explicitly in the phase contours shown in Fig. 3. The vortices presented here are highly anisotropic;[4] that is, the phase changes extremely fast in the neighborhood of a set of cophasal lines, in this case the $\pi/2$ and $3\pi/2$ phase lines.

Even in this simple example, there exist a large number of parameters that can be smoothly varied to alter the position and behavior of the singular points. Creation and annihilation of singular points can be observed by allowing the beam to propagate to a different z-plane, by changing the relative position of the waist planes of the modes, by changing the relative weights of the modes, or even by changing the position $\mathbf{r}_1$. An example of this latter variation is shown in Fig. 4. As the point $\mathbf{r}_1$ is continuously varied, two pairs of new coherence vortices are created in the region under consideration.

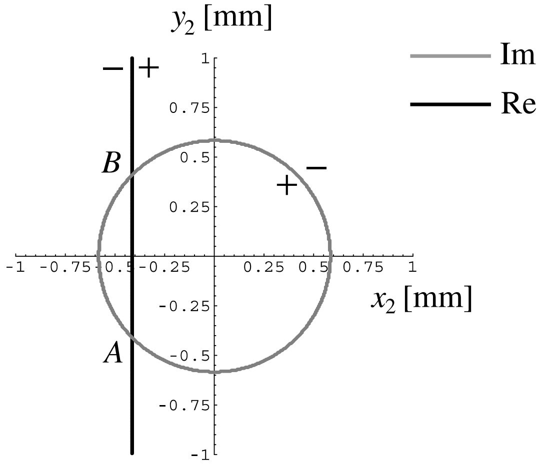

Fig. 2. Illustration of the zeros of the real (Re) and imaginary (Im) part of the cross-spectral density $W(\mathbf{r}_1,\mathbf{r}_2)$, with $\mathbf{r}_1$ kept fixed. The + and − indicate the side of the zero lines on which the function is positive or negative. Here $x_1 = 0.3$ mm, $y_1 = 0.5$ mm, $\lambda_0 = \lambda_1 = 1$, $z_0 = -50$ mm, $z_1 = z_2 = 0$, and $l = 1$, $m = 0$. The points A and B represent a positive and negative phase vortex, respectively.

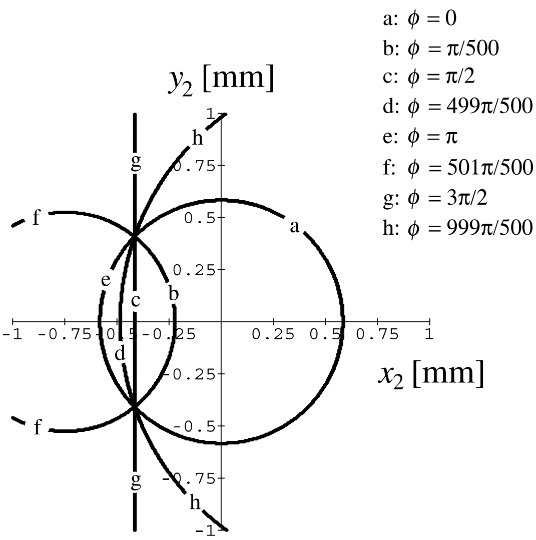

Fig. 3. Phase contour lines of the cross-spectral density as a function of $\mathbf{r}_2$, with $\mathbf{r}_1$ kept fixed, for the example illustrated in Fig. 2. It can be seen from the choice of contours that the singularities are extremely anisotropic.

[4] For a discussion of anisotropic phase singularities, see [15].

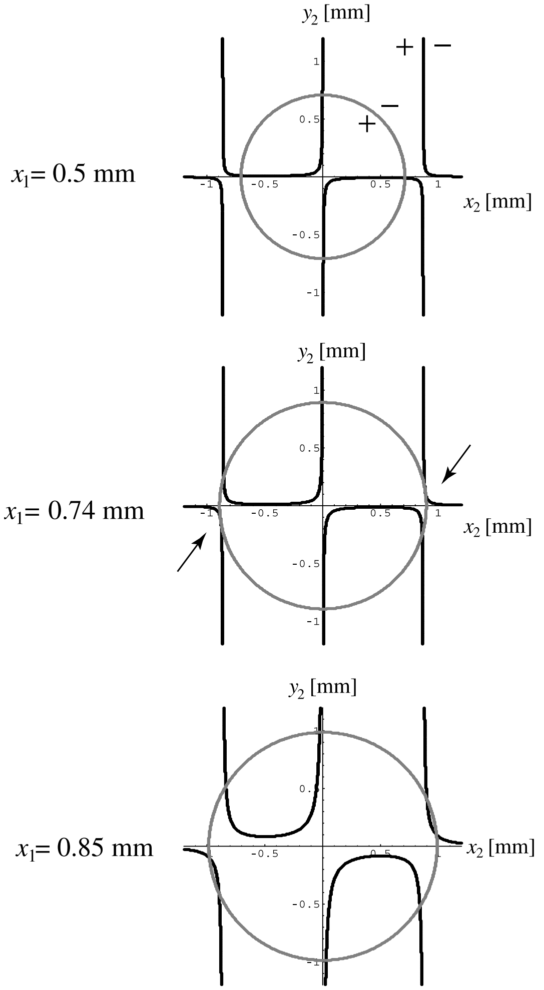

Fig. 4. Illustration of the creation (annihilation) of coherence vortices, for $y_1 = 0.5$ mm, $\lambda_0 = \lambda_1 = 1$, $z_0 = -50$ mm, $l = 3$, $m = 1$, as the parameter x_1 is varied. Two pairs of singular points are clearly created when $x_1 = 0.74$ mm. In the center plot, arrows indicate the locations where the singular points are created.

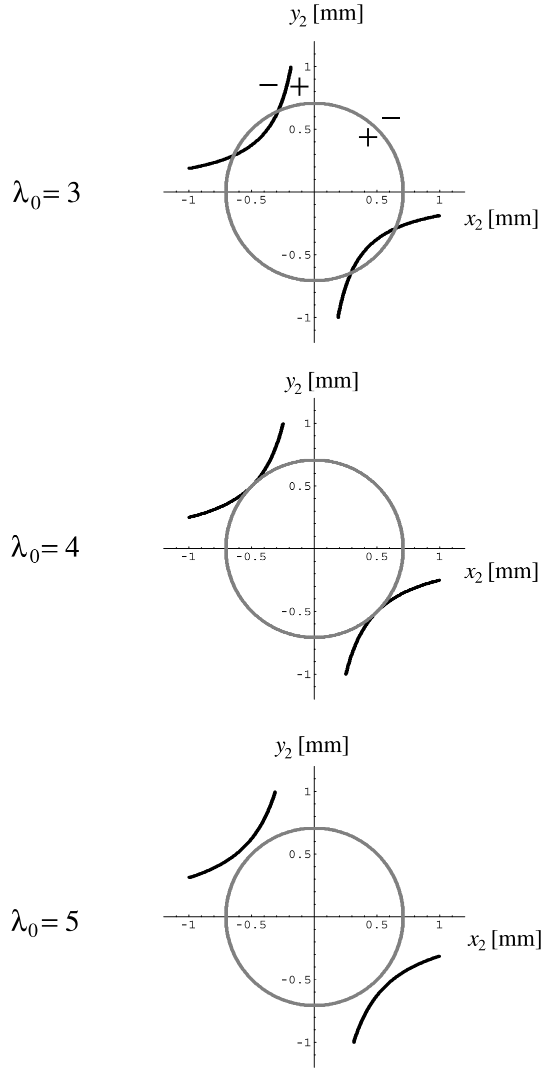

Fig. 5. Illustration of the annihilation (creation) of coherence vortices, for $x_1 = y_1 = 0.5$ mm, $\lambda_1 = 1$, $z_0 = -50$ mm, $l = 1$, $m = 1$, as the parameter λ_0 is varied. Two pairs of singular points are clearly annihilated when $\lambda_0 = 4$.

Fig. 5 shows an annihilation event produced by varying the relative weight of the modes.

In all the previous discussions, the point $\mathbf{r}_1$ was treated as a parameter of the system and the singularities of $W(\mathbf{r}_1, \mathbf{r}_2)$ were determined with respect to the variable $\mathbf{r}_2$. One may reverse this prescription and hold $\mathbf{r}_2$ fixed and determine the singularities with respect to the variable $\mathbf{r}_1$. Because $W(\mathbf{r}_1, \mathbf{r}_2)$ is also a solution of the Hemholtz equation with respect to the variable $\mathbf{r}_1$ (as displayed in Eq. (3)), it follows that the nature and behavior of the singularities will be comparable to those found by varying $\mathbf{r}_2$. In fact, given a pair of points $\mathbf{r}_1$ and $\mathbf{r}_2$ for which the cross-spectral density is zero, it follows that there exist "complementary" vortices with respect to this pair of points: one which may be observed by

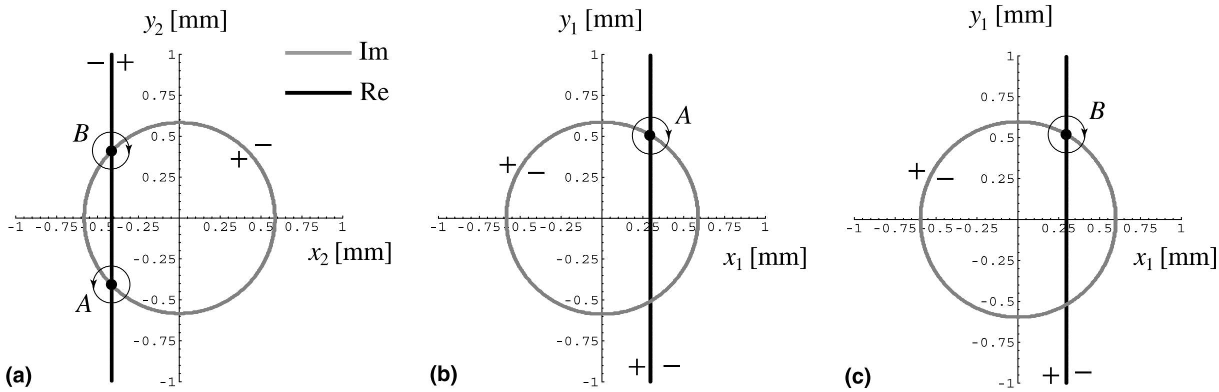

Fig. 6. An examination of the relation between the vortices produced by varying $\mathbf{r}_1$ and $\mathbf{r}_2$ in the neighborhood of a pair of points for which the cross-spectral density vanishes. In (a), $\mathbf{r}_1$ is kept fixed with $x_1 = 0.3$ mm, $y_1 = 0.5$ mm, $z_1 = z_2 = 0$, $\lambda_0 = \lambda_1 = 1$, $z_0 = -50$ mm, $l = 1$, $m = 0$, and the point $\mathbf{r}_2$ is varied, showing a pair of vortices A and B. In (b), the point $\mathbf{r}_2$ is kept fixed at the location of vortex A: $x_2 = -0.425$ mm, $y_2 = -0.405$ mm, and the point $\mathbf{r}_1$ is varied, showing the complementary vortex at $x_1 = 0.3$ mm, $y_1 = 0.5$ mm. Likewise, in (c), the point $\mathbf{r}_2$ is kept fixed at the location of vortex B: $x_2 = -0.425$ mm, $y_2 = 0.420$ mm, and the point $\mathbf{r}_1$ is varied, showing the complementary vortex at $x_1 = 0.3$ mm, $y_1 = 0.5$ mm. It can be seen that for A, the complementary vortices circulate in opposite directions, whereas for B, the complementary vortices circulate in the same direction.

varying $\mathbf{r}_1$ with $\mathbf{r}_2$ kept fixed, and one which may be observed by varying $\mathbf{r}_2$ with $\mathbf{r}_1$ kept fixed.

Given such a pair of points $\mathbf{r}_1$, $\mathbf{r}_2$ for which the cross-spectral density vanishes, one might wonder if there is a simple relation between the direction of the complementary vortices. In other words, if there is a positive vortex about the point $\mathbf{r}_2$, can we state with certainty that there is a positive (negative) vortex about the point $\mathbf{r}_1$? It can be shown by example that this is, in fact, not the case. In Fig. 6 such an example is shown. The behavior of two vortices with respect to $\mathbf{r}_2$, denoted A and B, are shown in (a) with an arrow indicating the direction of increasing phase. In (b), the vortex complementary to A is shown, and in (c), the vortex complementary to B is shown. It can be seen that the complementary vortices of A are in opposite directions, while the complementary vortices of B are in the same direction. Evidently there is no direct relation between the complementary vortices.

4. Discussion

Although the mathematical behaviors of the coherence vortices are essentially identical to the traditional vortices of intensity, their observable effects are quite different, as we now show. Let us consider as an observable quantity the interference pattern produced by a Young's interference experiment (Fig. 7) when the light incident upon the pinholes $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ is (a) fully coherent and possesses traditional intensity vortices and (b) partially coherent and possesses coherence vortices. The spectral interference pattern $S(\mathbf{r})$ observed at a point $P(\mathbf{r})$ on the screen $\mathscr{B}$ depends on the light at the pinholes through the *spectral interference law* [10, Section 4.3.2],

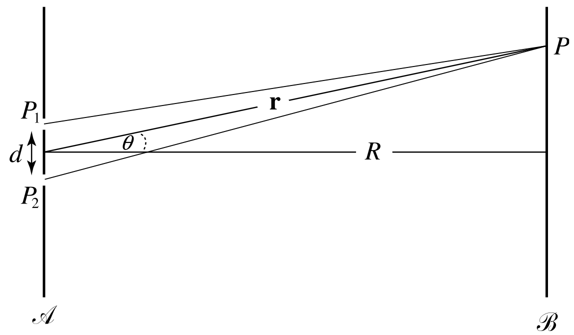

Fig. 7. Illustration of the notation relating to Young's interference experiment and the spectral interference law. The planes $\mathscr{A}$ and $\mathscr{B}$ are assumed to be parallel.

$$S(\mathbf{r}) = S^{(1)}(\mathbf{r}) + S^{(2)}(\mathbf{r}) + 2[S^{(1)}(\mathbf{r})S^{(2)}(\mathbf{r})]^{1/2} \times |\mu(\mathbf{r}_1,\mathbf{r}_2)| \cos[\phi_\mu(\mathbf{r}_1,\mathbf{r}_2) - \omega d \sin\theta/c], \tag{20}$$

where $S^{(i)}(\mathbf{r})$ is the spectrum of the light at position $\mathbf{r}$ if the ith pinhole is opened alone, ϕ_μ is the phase of the spectral degree of coherence of the field at the two pinholes, θ is the angle between the direction of observation and the normal to the observation plane, and d is the separation of the pinholes. Eq. (20) describes how the spectrum of the field at the observation point $P(\mathbf{r})$ depends upon the spectral density of the field at the two pinholes P_1 and P_2 and the degree of coherence between them.

Let us first assume that the light incident on the pinholes is fully coherent, and that the point P_2 is in the immediate neighborhood of a singular point of the spectral density. Because the light is fully coherent, we may write $\mu(\mathbf{r}_1,\mathbf{r}_2) = \exp[\mathrm{i}(\phi_\psi(\mathbf{r}_2) - \phi_\psi(\mathbf{r}_1))]$, where $\phi_\psi(\mathbf{r}_i)$ is the phase of the mode $\psi(\mathbf{r})$ at the pinhole i; the spectral interference law then reduces to the form

$$S(\mathbf{r}) = S^{(1)}(\mathbf{r}) + S^{(2)}(\mathbf{r}) + 2[S^{(1)}(\mathbf{r})S^{(2)}(\mathbf{r})]^{1/2} \times \cos[\phi_\psi(\mathbf{r}_2) - \phi_\psi(\mathbf{r}_1) - \omega d \sin\theta/c]. \tag{21}$$

If the point $\mathbf{r}_2$ is moved around the singularity in a counterclockwise manner, the phase $\phi_\psi(\mathbf{r}_2)$ will increase or decrease according as the vortex is positive or negative; this will cause the entire interference pattern to move to the left or right. After a complete circuit around the singularity, the interference pattern will have reproduced itself.

Because the pinhole P_2 is in the immediate neighborhood of a zero of the spectral density, it follows that typically $S^{(2)}$ will be much smaller than $S^{(1)}$. The latter two terms of Eq. (21) may be considered a perturbation of the uniform spectral density $S^{(1)}$ due to a *single* pinhole.

Next let us suppose that the light incident upon the pinholes is partially coherent, and that the point P_1 is fixed and the point P_2 is in the neighborhood of a coherence vortex. The pattern observed on the screen is given by the full formula (20), but now $S^{(1)}$ and $S^{(2)}$ are generally of comparable magnitude. Again, as the point $\mathbf{r}_2$ is moved around the singularity, the interference pattern will move to the left or right and will reproduce itself after a complete circuit. However, since P_2 is in the neighborhood of a coherence vortex, $|\mu(\mathbf{r}_1,\mathbf{r}_2)| \ll 1$ and only the last term of Eq. (20) may be considered negligible. The interference pattern is therefore a perturbation of the summed intensity due to *both* pinholes. The difference between the two cases is illustrated schematically in Fig. 8.

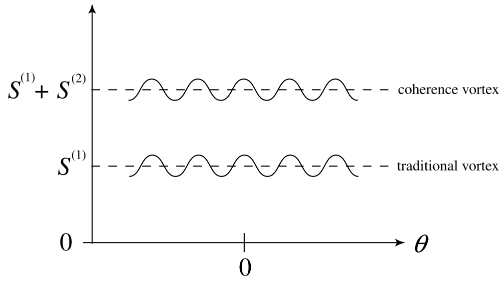

Fig. 8. A schematic comparison of the observed fringes in the neighborhood of a traditional vortex in a coherent beam and a coherence vortex in a partially coherent beam.

There is another important difference between traditional optical vortices and the coherence vortices discussed here. It is to be emphasized that a coherence vortex cannot be associated with any single point of a wavefield, but only pairs of points; it might be said that it is a 'virtual' feature of the wavefield. This can be seen by considering again the Young's interference experiments with coherent and partially coherent light and examining the effect of changing the point P_1 on the behavior of the vortex in the neighborhood of P_2. For a traditional optical vortex in a coherent beam, a change in the position of P_1 changes the phase $\phi_\psi(\mathbf{r}_1)$ and the spectrum $S^{(1)}$, but the location of the vortex near P_2 is unchanged. For a coherence vortex, however, a change in P_1 changes the location of the vortex near P_2 (recall Fig. 4). As P_1 is moved further away from its starting position, the coherence vortex will move further away from point P_2 and eventually no singular behavior will be observed there.

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