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Unpolarized light beams with different coherence properties

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We investigate the coherence properties of unpolarized beams. Such beams form a much richer class than has been previously realized. We illustrate our results by examples.

Keywords: polarization; coherence; beams

1. Introduction

The connection between the state of polarization and the state of coherence of optical wave fields has only recently been investigated [1–6]. Polarization phenomena have generally been described by means of 'one-point quantities' (i.e. quantities that are a function of a single point in space) such as the Stokes parameters, Wiener's coherency matrix and the degree of polarization [7]. Several years ago James [8] showed that the degree of polarization can change on propagation. To understand such changes it is necessary to generalize Wiener's coherency matrix and other correlation matrices used in polarization optics to become functions of two points rather than of a single point. Very recent researches have shown that such a generalization is necessary in order to elucidate, for example, the following well-known theorem due to Gabriel Stokes regarding the decomposition of an arbitrary beam into polarized and unpolarized components [9]: *…it is always possible to represent the given group by a stream of common light combined with a stream of elliptically polarized light independent of the former*. It has recently been shown that this assertion is incorrect [10]: the decomposition of a beam into a polarized part and an unpolarized part is local (i.e. the decomposition may be different at different points) rather than global [11–16]. To further clarify the situation, it is necessary to gain a better understanding of unpolarized beams. The behavior of such beams on propagation has not previously been studied. Moreover, it is not generally appreciated that unpolarized beams can differ in their coherence properties. It is with the latter subject that this paper is concerned.

Let us first discuss how an unpolarized beam can be generated. One way to do so is to superpose two independent beams that are linearly polarized in two mutually orthogonal directions. The spectral density (intensity at a fixed frequency ω) of the composite, unpolarized beam can be varied by the use of suitable gray filters. Its coherence properties can be tailored, for example, by passing the beam through a rotating diffuser [17], or by reflecting it off (or transmitting it through) a spatial light modulator [18].

We will illustrate our analysis by examining unpolarized beams produced by three different kinds of sources: a source with a Gaussian spectral density distribution, a source with a truncated, uniform spectral density that emits blackbody radiation, and a source that generates a Laguerre–Gauss beam.

2. Beams generated by partially coherent electromagnetic sources

In the space–frequency representation the state of coherence and the state of polarization of a planar stochastic source that generates a beam may be characterized by its electric cross-spectral density matrix ([4], Chapter 9), namely,

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\begin{pmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\end{pmatrix}, \tag{1}$$

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where

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle,\quad (i=x,y;\ j=x,y), \tag{2}$$

$$=\left[S_i^{(0)}(\boldsymbol{\rho}_1,\omega)S_j^{(0)}(\boldsymbol{\rho}_2,\omega)\right]^{1/2}\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega). \tag{3}$$

Here $E_i(\boldsymbol{\rho},\omega)$ are the Cartesian components of the electric vector at frequency ω at a point specified by a position vector $\boldsymbol{\rho}$ in the source plane. The angular brackets denote an ensemble average [4]. Furthermore, $S_i^{(0)}(\boldsymbol{\rho},\omega)\equiv W_{ii}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)$ is the spectral density associated with component E_i, and $\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$ represents the correlation between E_i at the point $\boldsymbol{\rho}_1$ and E_j at the point $\boldsymbol{\rho}_2$. The superscript (0) labels quantities in the source plane. From knowledge of the cross-spectral density matrix $\mathbf{W}^{(0)}$ various properties of the field in the source plane can be derived ([4], Chapter 9). In particular, the spectral density at a point $\boldsymbol{\rho}$ is given by the expression

$$S^{(0)}(\boldsymbol{\rho},\omega)=S_x^{(0)}(\boldsymbol{\rho},\omega)+S_y^{(0)}(\boldsymbol{\rho},\omega)=\operatorname{Tr}\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega), \tag{4}$$

the spectral degree of coherence at a pair of points is given by the formula

$$\eta^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{\operatorname{Tr}\mathbf{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{\left[S^{(0)}(\boldsymbol{\rho}_1,\omega)S^{(0)}(\boldsymbol{\rho}_2,\omega)\right]^{1/2}}, \tag{5}$$

and the degree of polarization at the point $\boldsymbol{\rho}$ is given by the expression

$$\mathcal{P}^{(0)}(\boldsymbol{\rho},\omega)=\left(1-\frac{4\operatorname{Det}\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)}{[\operatorname{Tr}\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)]^2}\right)^{1/2}. \tag{6}$$

In these formulas Tr and Det denote the trace and the determinant, respectively.

The propagation of the cross-spectral density matrix into the half-space $z>0$ is given by the formula ([19], Section 5.6.1)

$$\begin{aligned}\mathbf{W}&(\boldsymbol{\rho}_1,z_1,\boldsymbol{\rho}_2,z_2,\omega)\\ &=\iint_{(z=0)}\mathbf{W}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)G^*(\boldsymbol{\rho}_1-\boldsymbol{\rho}_1',z_1,\omega)\\ &\quad\times G(\boldsymbol{\rho}_2-\boldsymbol{\rho}_2',z_2,\omega)\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2',\end{aligned} \tag{7}$$

where

$$G(\boldsymbol{\rho}-\boldsymbol{\rho}',z,\omega)=-\frac{\mathrm{i}k}{2\pi z}\exp(\mathrm{i}kz)\exp\left[\mathrm{i}k\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|^2/2z\right], \tag{8}$$

is the Green's function pertaining to the paraxial Helmholtz equation, $k=\omega/c=2\pi/\lambda$ is the wavenumber associated with frequency ω, with c denoting the speed of light in vacuum, and λ is the wavelength. From knowledge of $\mathbf{W}(\boldsymbol{\rho}_1,z_1,\boldsymbol{\rho}_2,z_2,\omega)$ the spectral density of the beam, its degree of coherence and its degree of polarization can all be derived using expressions that are strictly similar to Equations (4)–(6).

For a Schell-model source ([19], Section 5.3.2) the field correlations depend only on the difference $\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$, i.e.

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\equiv\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega). \tag{9}$$

An important sub-class of Schell-model sources are the so-called quasi-homogeneous sources. For such sources the width of $|\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega)|$ is much smaller than that of $S_i^{(0)}(\boldsymbol{\rho},\omega)$. This behavior is illustrated in Figure 1.

For a quasi-homogeneous, uniformly polarized source (i.e. a quasi-homogeneous source for which the spectral degree of polarization and the spectral polarization ellipse associated with the polarized portion of the beam are the same at each source point) the

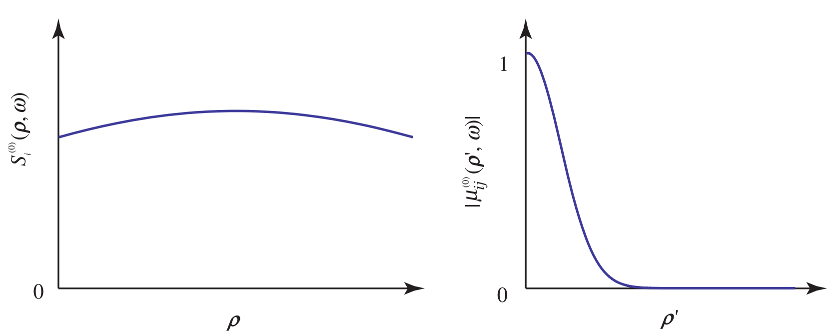

Figure 1. Illustrating the concept of a quasi-homogeneous source. (The color version of this figure is included in the online version of the journal.)

field in the source plane and the field in the far zone are related by the following two reciprocity relations [20]:

$$S^{(\infty)}(r\mathbf{s},\omega)=(2\pi k)^2\tilde{S}^{(0)}(0,\omega)\tilde{\eta}^{(0)}(k\mathbf{s}_{\perp},\omega)\cos^2\theta/r^2, \tag{10}$$

$$\eta^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2,\omega)=\tilde{S}^{(0)}[k(\mathbf{s}_{2\perp}-\mathbf{s}_{1\perp}),\omega]\exp[\mathrm{i}k(r_2-r_1)]/\tilde{S}^{(0)}(0,\omega), \tag{11}$$

with

$$\tilde{S}^{(0)}(\mathbf{f},\omega)=\frac{1}{(2\pi)^2}\int S^{(0)}(\boldsymbol{\rho},\omega)\exp(-\mathrm{i}\mathbf{f}\cdot\boldsymbol{\rho})\mathrm{d}^2\rho, \tag{12}$$

$$\tilde{\eta}^{(0)}(\mathbf{f},\omega)=\frac{1}{(2\pi)^2}\int \eta^{(0)}(\boldsymbol{\rho},\omega)\exp(-\mathrm{i}\mathbf{f}\cdot\boldsymbol{\rho})\mathrm{d}^2\rho. \tag{13}$$

In these expressions $\mathbf{s}_{\perp}$ is the projection (considered as a two-dimensional vector) of the unit directional vector $\mathbf{s}$ onto the xy-plane, and θ is the angle which the $\mathbf{s}$-direction makes with the z-axis (see Figure 2). The superscript (∞) indicates points in the far zone. Since we will study coherence properties of the far field, we will only need the second reciprocity relation, Equation (11). That equation implies that the spectral degree of coherence of the beam in the far zone of a planar, uniformly polarized, quasi-homogeneous source is, apart from a geometrical factor, proportional to the spatial Fourier transform of the spectral density of the source.

For an unpolarized source the field components E_x and E_y *at each point* are uncorrelated and therefore, the cross-spectral density matrix at coincident points $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$ is diagonal. Furthermore, we have for an unpolarized source that ([4], Chapter 8)

$$W^{(0)}_{xx}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)=W^{(0)}_{yy}(\boldsymbol{\rho},\boldsymbol{\rho},\omega), \tag{14}$$

and hence

$$\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)=A(\boldsymbol{\rho},\omega)\begin{pmatrix}1&0\\0&1\end{pmatrix}, \tag{15}$$

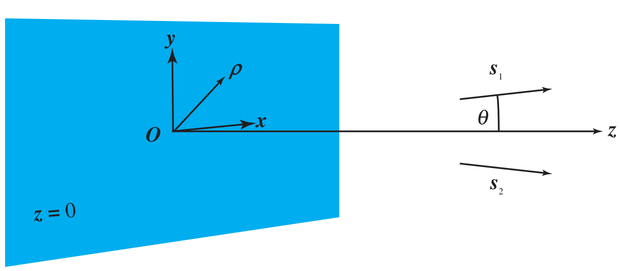

Figure 2. Notation relating to radiation generated by a planar, stochastic source. (The color version of this figure is included in the online version of the journal.)

with $A(\boldsymbol{\rho},\omega)$ being positive. We note that Equation (15) does *not* imply that when $\boldsymbol{\rho}_1\neq\boldsymbol{\rho}_2$, $\langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle$ is necessarily zero $(i,j=x,y)$. The fact that there are no additional constraints on the function $A(\boldsymbol{\rho},\omega)$ (which equals $S^{(0)}(\boldsymbol{\rho},\omega)/2$) suggests, in view of Equation (11), that unpolarized sources may differ in their coherence properties, and in the coherence properties of the beams that they generate. This will be illustrated by example in the next section.

3. Some examples

We will analyze three different kinds of unpolarized beams that are generated by planar, quasi-homogeneous electromagnetic sources.

First we consider an unpolarized Gaussian Schell-model source. We choose

$$S^{(0)}_x(\boldsymbol{\rho},\omega)=S^{(0)}_y(\boldsymbol{\rho},\omega) \tag{16}$$

$$=A^2\exp(-\boldsymbol{\rho}^2/2\sigma^2) \tag{17}$$

and

$$\mu^{(0)}_{ii}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega)=\exp[-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta^2], \tag{18}$$

$$\mu^{(0)}_{ij}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega)=0 \quad (\text{if } i\neq j). \tag{19}$$

If $\sigma\gg\delta$ the source will be quasi-homogeneous. The parameters A, σ and δ have to satisfy certain constraints due to the beam-like nature of the field [21], and also because $\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)$ is non-negative definite and Hermitian [22,23]. Furthermore, the fact that the effective coherence length δ is the same for both diagonal elements of the cross-spectral density matrix ensures that the beam generated by the source will remain unpolarized on propagation ([4], Section 9.4.3).

The reciprocity relation (11) now gives

$$\eta^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2,\omega)=\exp[-k^2\sigma^2(\mathbf{s}_{\perp2}-\mathbf{s}_{\perp1})^2/2]\times\exp[\mathrm{i}k(r_2-r_1)]. \tag{20}$$

In this example and in the ones that follow, we take the two observation points in the far zone to be symmetrically located with respect to the z-axis (see Figure 3), with θ denoting the angle which

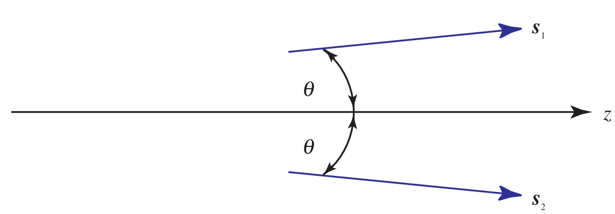

Figure 3. Two directions of observation that are symmetrically located with respect to the z-axis. (The color version of this figure is included in the online version of the journal.)

the unit vectors s_1 and s_2 make with the axis. We then have

$$r_1 = r_2 = r, \quad s_{\perp 2} = -s_{\perp 1}, \tag{21}$$

and evidently

$$(s_{\perp 2} - s_{\perp 1})^2 = 4 \sin^2 \theta. \tag{22}$$

Consequently, Equation (20) then reduces to

$$\eta^{(\infty)}(rs_1, rs_2, \omega) = \exp[-2k^2\sigma^2 \sin^2 \theta], \quad (s_{\perp 2} = -s_{\perp 1}). \tag{23}$$

It is to be noted that the spectral degree of coherence $\eta^{(\infty)}(rs_1, rs_2, \omega)$ is now real. It is also positive and a monotonic function of the angle θ. This behavior is illustrated in Figure 4. It is seen that as the (normalized) effective source size $k\sigma$ increases, the distribution of the spectral degree of coherence of the far field becomes narrower.

As a second example we consider a beam generated by an unpolarized source with spectral density $S^{(0)}(\omega)$ given by Planck's law. We take the source to be an illuminated circular aperture with radius a, in an opaque screen (see Figure 5). Such a secondary source can be produced, for example, by placing the aperture in the far zone of a blackbody source. In this case

$$S_x^{(0)}(\boldsymbol{\rho}, \omega) = S_y^{(0)}(\boldsymbol{\rho}, \omega), \tag{24}$$

$$= S^{(0)}(\omega)\,\mathrm{circ}\left(\frac{\rho}{a}\right), \tag{25}$$

with

$$\mathrm{circ}(x) = \begin{cases} 1 & \text{if } x \leq 1, \\ 0 & \text{if } x > 1. \end{cases} \tag{26}$$

It has recently been shown (contrary to claims in the literature) that the radiation in the far zone of a blackbody source is unpolarized [24]. The aperture radius a is taken so that the angle it subtends at the blackbody cavity is much larger than the angular width of the Schell-model correlation function [25]. The source is then quasi-homogeneous and we may apply the reciprocity relation (11). Again we choose the two points of observation to be symmetrically located with respect to the z-axis (see Equation (21)). Then

$$\eta^{(\infty)}(rs_1, rs_2, \omega) = 2\frac{J_1(2ka \sin\theta)}{2ka \sin\theta}, \quad (s_{\perp 2} = -s_{\perp 1}), \tag{27}$$

where $J_1(x)$ is the Bessel function of the first kind and first order. It is seen that $\eta^{(\infty)}(rs_1, rs_2, \omega)$ is again real, but the degree of coherence is now 'jinc-like'.[1] In this case, when the angle θ increases the spectral degree of coherence decreases, becomes negative and then positive again. This behavior is illustrated in Figure 6. It is seen that as the radius a of the aperture is increased, the effective angular width of the spectral degree of coherence becomes smaller.

As a final example we consider a beam generated by a quasi-homogeneous, unpolarized

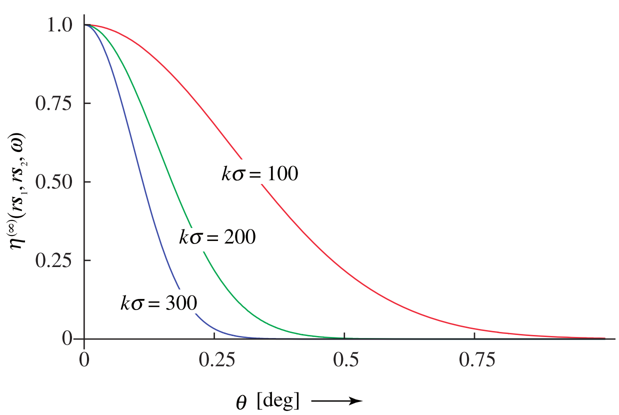

Figure 4. The spectral degree of coherence of the far field generated by a planar, unpolarized Gaussian Schell-model source. The three curves correspond to different values of the normalized effective source size $k\sigma$. (The color version of this figure is included in the online version of the journal.)

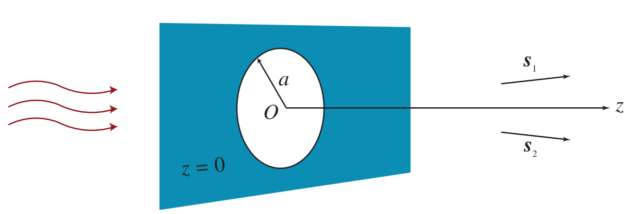

Figure 5. Notation relating to collimated blackbody radiation emerging from a circular aperture. (The color version of this figure is included in the online version of the journal.)

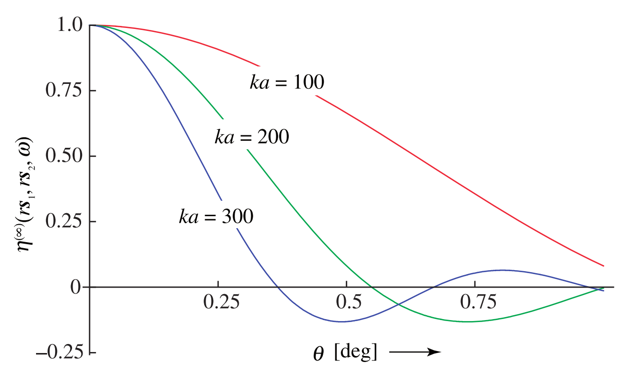

Figure 6. The spectral degree of coherence in the far zone of an unpolarized beam emerging a circular aperture. The three curves correspond to sources with different values of the normalized aperture size ka. (The color version of this figure is included in the online version of the journal.)

Laguerre–Gauss source ([26], Section 16.4) and [27,28]. In this case

$$S_x^{(0)}(\boldsymbol{\rho}, \omega) = S_y^{(0)}(\boldsymbol{\rho}, \omega) \tag{28}$$

$$= A^2 \rho^2 \exp[-\rho^2/2\beta^2], \tag{29}$$

and

$$\mu_{ii}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega) = \exp[-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2/2\Delta^2], \tag{30}$$

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega) = 0 \quad (\text{if } i \neq j). \tag{31}$$

If the effective coherence length, Δ, is much shorter than β the source will be quasi-homogeneous. Just as in the first example, the beam generated by the source remains unpolarized on propagation since the effective coherence length Δ is the same for both diagonal elements of the cross-spectral density matrix. Application of the reciprocity relation (11) (again assuming two symmetrically located points of observation) and using the well-known expressions for the Fourier transform of derivatives gives

$$\eta^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) = \left[1 - 2k^2\beta^2 \sin^2\theta\right] \exp(-2k^2\beta^2 \sin^2\theta), \quad (\boldsymbol{s}_{\perp 2} = -\boldsymbol{s}_{\perp 1}). \tag{32}$$

The degree of coherence $\eta^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega)$ of the far field is again seen to be real, but its behavior is different from that in the two cases discussed previously (see Figure 7). With increasing values of the angle of observation θ the spectral degree of coherence decreases, then becomes negative, and then tends asymptotically to zero. When the parameter $k\beta$ increases, the effective angular width of the distribution becomes smaller.

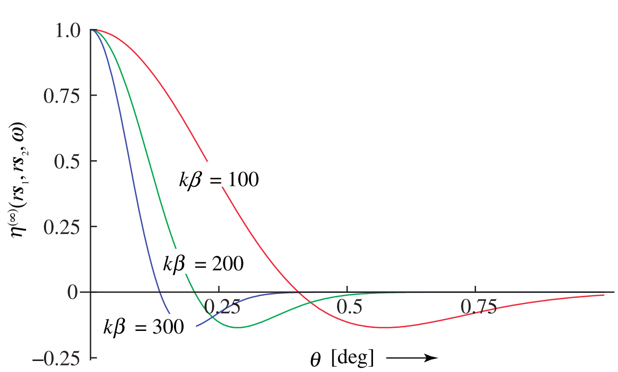

Figure 7. The spectral degree of coherence of the far field generated by a planar, unpolarized Laguerre–Gauss source. The three curves correspond to sources with different values of the parameter $k\beta$. (The color version of this figure is included in the online version of the journal.)

4. Conclusions

We examined unpolarized beams generated by secondary, planar, stochastic electromagnetic sources of different kinds. The quasi-homogeneous nature of the sources which we considered make it possible to employ reciprocity relations that yield analytic expressions for the spectral degree of coherence of the far field. We presented examples of beams produced by unpolarized sources with (a) a Gaussian spectral density distribution, (b) a uniform blackbody spectral distribution, and (c) a Laguerre–Gauss spectral distribution. The coherence properties of the three types of beams in the far zone were found to be quite different. The results indicate that unpolarized radiation can be generated not only by blackbodies, but by many sources with different coherence properties. The beams that such sources generate form a much broader class than was previously realized.

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Note

1. Michelson and Pease used a similar expression as Equation (27) in their famous work on determining stellar diameters. For a further discussion of this point see ([19], Section 7.2).

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