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Spectral polarization of Gaussian Schell-model beams

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Abstract: We present a class of broadband electromagnetic Gaussian Schell-model sources whose state of polarization is both uniform and identical for all frequencies, but whose far-zone polarization properties strongly depend on wavelength. Also, these sources can produce beams whose polarized portion is always linearly polarized but with a polarization angle that evolves on propagation. Our results offer new insights into the behavior of broadband partially coherent sources.

1. Introduction

One of the key insights of coherence theory is that field properties such as directionality, spectrum and degree of polarization are dependent on the statistical properties of the source [1]. In many researches Gaussian Schell-model (GSM) sources, either scalar or electromagnetic (EGSM), are used to illustrate these findings. These are planar, secondary sources whose spectral density and spectral degree of coherence both have a Gaussian form [2]. GSM sources can be divided into two categories. The first one consists of sources with an rms width of the spectral density (denoted σ) and an rms width of the spectral degree of coherence (denoted δ) that are independent of the wavelength. Even in this restricted case a great variety of coherence-related phenomena like the Wolf shift [3] and a propagation-dependent state of polarization [4,5] can occur. The second category is formed by so-called *spectral* GSM sources for which σ and δ explicitely depend on frequency [6–8]. (Non-Gaussian spectral sources were discussed in [9–12]). It is interesting to note that the studies dealing with spectral EGSM sources consider their behavior at only a single frequency. An exception is [13] in which the propagation of an EGSM beam through $2f$ and $4f$ systems is described.

In this study we present a new class of broadband spectral EGSM sources whose state of polarization is both uniform and the same at every frequency that is present in the source spectrum. However, as will be shown, their far-zone Stokes parameters strongly depend on wavelength. In particular, we investigate beam-generating sources and derive expressions for their on-axis state of polarization. It is found that these sources can produce beams whose polarized part on the axis is always linearly polarized, but with an angle that evolves differently for different frequencies.

In principle, all EGSM parameters can be taken to be frequency-dependent, with each one having a different functional dependence on the source frequency. In order to provide physical insight, we have chosen the simplest system that provides interesting polarization effects. In

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such a system only the coherence radius δ_{xy}, defined below, needs to vary with wavelength. Furthermore, it will be seen that this parameter plays a central role in the so-called realizability conditions that the source must satisfy.

2. Electromagnetic Gausian Schell-model sources

A planar, secondary EGSM source (see Fig. 1) is described by a cross-spectral density matrix [2, Sec. 9.4.2]

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}_1; \omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}_2; \omega)}\mu_{ij}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1; \omega), \quad (i, j = x, y). \tag{1}$$

Here $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two-dimensional position vectors in the source plane $z = 0$, $S_i^{(0)}(\boldsymbol{\rho}; \omega)$ denotes the spectral density of the i^{th} component of the electric field vector $\mathbf{E}$ at frequency ω, and $\mu_{ij}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1; \omega)$ represents the degree of correlation between E_i at $\boldsymbol{\rho}_1$ and E_j at $\boldsymbol{\rho}_2$. The superscript (0) indicates quantities in the source plane. The spectral densities and the degrees of correlation are assumed to have a Gaussian form, i.e.,

$$S_i^{(0)}(\boldsymbol{\rho}; \omega) = A_i^2(\omega) e^{-\rho^2/2\sigma_i^2}, \tag{2}$$

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1; \omega) = B_{ij} e^{-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2/2\delta_{ij}^2(\omega)}. \tag{3}$$

As our notation indicates, the spectral amplitudes A_i and the four coherence widths (or coherence radii) δ_{ij} are assumed to depend on the wavelength, but the rms widhts σ_i and the correlation coefficient B_{ij} do not. Furthermore, we take $\sigma_x = \sigma_y = \sigma$, and $A_x(\omega) = A_y(\omega) = A(\omega)$. The source parameters must satisfy certain constraints [2], namely

$$B_{xx} = B_{yy} = 1, \tag{4}$$

$$|B_{xy}| \leq 1, \tag{5}$$

$$B_{xy} = B_{yx}^*, \tag{6}$$

$$\delta_{xy}(\omega) = \delta_{yx}(\omega). \tag{7}$$

Additionally, the realizability conditions are [14],

$$\sqrt{\frac{\delta_{xx}^2(\omega) + \delta_{yy}^2(\omega)}{2}} \leq \delta_{xy}(\omega) \leq \sqrt{\frac{\delta_{xx}(\omega)\delta_{yy}(\omega)}{|B_{xy}|}}, \tag{8}$$

and

$$|B_{xy}| \leq \frac{2}{\delta_{xx}(\omega)/\delta_{yy}(\omega) + \delta_{yy}(\omega)/\delta_{xx}(\omega)}. \tag{9}$$

We note that expression (5) is implied by (9). An EGSM source will produce a beam-like field if the inequality [15]

$$\frac{1}{4\sigma^2} + \frac{1}{\delta_{ii}^2(\omega)} \ll \frac{\omega^2}{2c^2}, \quad (i = x, y), \tag{10}$$

with c the speed of light, holds true. Clearly, the above constraints must be satisfied for every frequency that is present in the source spectrum.

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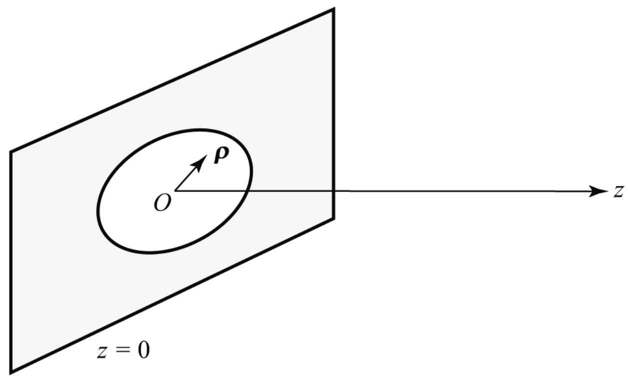

Fig. 1. A spectral electromagnetic, planar, secondary Gaussian Schell-model source occupies the plane $z = 0$ and radiates a beam that propagates parallel to the z axis. The vector $\boldsymbol{\rho}$ denotes a position in the xy plane.

As the beam propagates to a transverse plane z, the cross-spectral density matrix at two coincident points on the axis ($\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \mathbf{0}$) evolves into [2, p. 184]

$$W_{ij}(z;\omega) = \frac{A^2(\omega)B_{ij}}{\Delta_{ij}^2(z;\omega)}, \tag{11}$$

where

$$\Delta_{ij}^2(z;\omega) = 1 + \left[\frac{z}{k\sigma\Omega_{ij}(\omega)}\right]^2, \tag{12}$$

and

$$\frac{1}{\Omega_{ij}^2(\omega)} = \frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2(\omega)}, \tag{13}$$

with the wavenumber $k = \omega/c$. As explained by Wolf [2], the matrix elements expand at different rates. This results in a change of the state of polarization as the beam propagates. Furthermore, this change can become quite complex when the source parameters are allowed to vary with frequency.

The state of polarization (SOP) of an electromagnetic beam is characterized by the four spectral Stokes parameters, which can be expressed in terms of the cross-spectral density matrix as [2]

$$S_0(\boldsymbol{\rho};\omega) = W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega) + W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega), \tag{14}$$

$$S_1(\boldsymbol{\rho};\omega) = W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega) - W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega), \tag{15}$$

$$S_2(\boldsymbol{\rho};\omega) = W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega) + W_{yx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega), \tag{16}$$

$$S_3(\boldsymbol{\rho};\omega) = \mathrm{i}\left[W_{yx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega) - W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)\right]. \tag{17}$$

Their normalized versions are defined as $s_n(\boldsymbol{\rho};\omega) = S_n(\boldsymbol{\rho};\omega)/S_0(\boldsymbol{\rho};\omega)$ for $n = 1, 2, 3$. On making use of Eqs. (1)–(3), together with the previously mentioned assumptions $\sigma_x = \sigma_y = \sigma$, and $A_x(\omega) = A_y(\omega) = A(\omega)$, its readily found that in the source plane these parameters take on the form

$$s_1^{(0)}(\boldsymbol{\rho};\omega) = 0, \tag{18}$$

$$s_2^{(0)}(\boldsymbol{\rho};\omega) = \mathrm{Re}[B_{xy}], \tag{19}$$

$$s_3^{(0)}(\boldsymbol{\rho};\omega) = \mathrm{Im}[B_{xy}]. \tag{20}$$

We note that, since B_{xy} is assumed to be constant, this implies that the SOP of the field in the plane $z = 0$ is both uniform and identical at all frequencies at which the source radiates. The

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same holds true for the degree of polarization (DOP), which is defined as

$$P^{(0)}(\boldsymbol{\rho};\omega) \equiv \sqrt{\left[s_1^{(0)}(\boldsymbol{\rho};\omega)\right]^2 + \left[s_2^{(0)}(\boldsymbol{\rho};\omega)\right]^2 + \left[s_3^{(0)}(\boldsymbol{\rho};\omega)\right]^2} \tag{21}$$

$$= \left|B_{xy}\right|. \tag{22}$$

In the far-zone, indicated by the superscript (∞), the first term on the right-hand side of Eq. (12) may be neglected, and the normalized Stokes parameters on the beam axis ($\boldsymbol{\rho} = \mathbf{0}$) are seen to be

$$s_1^{(\infty)}(\omega) = \frac{\Omega_{xx}^2(\omega) - \Omega_{yy}^2(\omega)}{\Omega_{xx}^2(\omega) + \Omega_{yy}^2(\omega)}, \tag{23}$$

$$s_2^{(\infty)}(\omega) = 2\mathrm{Re}[B_{xy}]\frac{\Omega_{xy}^2(\omega)}{\Omega_{xx}^2(\omega) + \Omega_{yy}^2(\omega)}, \tag{24}$$

$$s_3^{(\infty)}(\omega) = 2\mathrm{Im}[B_{xy}]\frac{\Omega_{xy}^2(\omega)}{\Omega_{xx}^2(\omega) + \Omega_{yy}^2(\omega)}. \tag{25}$$

Hence the far-zone, on-axis DOP equals

$$P^{(\infty)}(\omega) = \frac{\sqrt{[\Omega_{xx}^2(\omega) - \Omega_{yy}^2(\omega)]^2 + 4\Omega_{xy}^4(\omega)|B_{xy}|^2}}{\Omega_{xx}^2(\omega) + \Omega_{yy}^2(\omega)}. \tag{26}$$

Let us first consider the case $\delta_{xx}(\omega) = \delta_{yy}(\omega) = \delta(\omega)$. It follows from the realizability condition (8) that then $\delta_{xy}(\omega) \geq \delta(\omega)$, and hence

$$2\Omega_{xy}^2(\omega) \geq \Omega_{xx}^2(\omega) + \Omega_{yy}^2(\omega). \tag{27}$$

On making use of (27) in Eqs. (23)–(26) we find that

$$s_1^{(\infty)}(\omega) = s_1^{(0)}(\boldsymbol{\rho};\omega) = 0, \tag{28}$$

$$s_2^{(\infty)}(\omega) \geq s_2^{(0)}(\boldsymbol{\rho};\omega), \tag{29}$$

$$s_3^{(\infty)}(\omega) \geq s_3^{(0)}(\boldsymbol{\rho};\omega), \tag{30}$$

$$P^{(\infty)}(\omega) \geq P^{(0)}(\boldsymbol{\rho};\omega). \tag{31}$$

In words, the SOP of the field has changed on propagation to the far-zone: for every frequency that is present in the source spectrum, the far-zone DOP is greater or equal than the DOP in the source plane.

An example is shown in Fig. 2 in which the parameters $s_2^{(\infty)}(\omega)$, $s_3^{(\infty)}(\omega)$ and the far zone DOP are plotted as a function of δ_{xy}. It is seen that if the coherence radius δ_{xy} increases then so do the two normalized Stokes parameters, and hence the degree of polarization.

When $\delta_{xx}(\omega) \neq \delta_{yy}(\omega)$, the inequalities (29)–(31) are still valid. The proof of this is straightforward but somewhat tedious and is not presented here. It is readily seen from Eq. (23) that in this case $s_1^{(\infty)}(\omega) \neq 0$. However, this quantity does not depend on the coherence radius δ_{xy}. An example is presented in Fig. 3. It is again seen that the DOP increases with increasing δ_{xy}. This tendency can be understood by considering the quasi-homogeneous limit of $\sigma^2 \gg \delta_{ij}^2$ for all $i,j = x,y$ [2]. In that case $\Omega_{ij}^2 \approx \delta_{ij}^2$, and both $s_2^{(\infty)}(\omega)$ and $s_3^{(\infty)}(\omega)$ are seen to increase with increasing δ_{xy}. Furthermore, it then follows that if δ_{xy} nears its maximum value as given by Eq. (8), then $P^{(\infty)}(\omega)$ tends to 1. Although the sources in Figs. 2 and 3 are stricly speaking not quasi-homogeneous, their far-zone DOPs do display this behavior. Note that at optical frequencies ($\sim 10^{15}$ Hz) these sources satisfy the beam condition (10).

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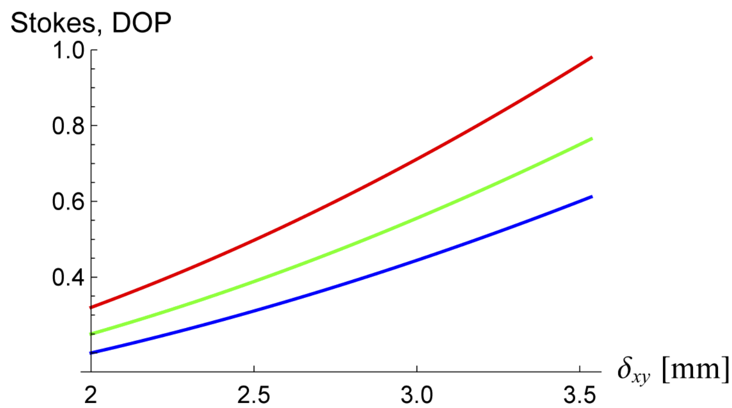

Fig. 2. The two non-zero normalized Stokes parameters in the far zone, $s_2^{(\infty)}$ (blue curve) and $s_3^{(\infty)}$ (green), together with the far-zone DOP $P^{(\infty)}$ (red) as a function of the coherence radius δ_{xy}. When we set $\delta_{xx} = \delta_{yy} = 2$ mm, $\sigma = 1$ cm, and $B_{xy} = 0.2 + i0.25$, the parameter δ_{xy} is, according to Eq. (8), restricted to the interval $[2.0, 3.5]$ mm.

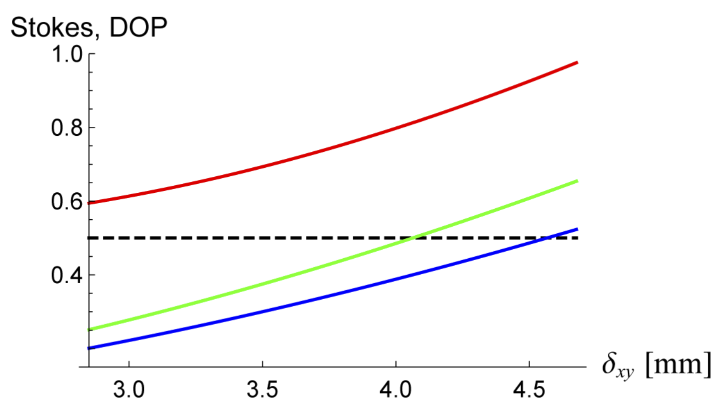

Fig. 3. The three normalized Stokes parameters in the far zone: $s_1^{(\infty)}$ (dashed black curve), $s_2^{(\infty)}$ (blue) and $s_3^{(\infty)}$ (green), together with the far-zone DOP $P^{(\infty)}$ (red) as a function of the coherence radius δ_{xy}. For $\delta_{xx} = 3.5$ mm, $\delta_{yy} = 2$ mm, $\sigma = 1$ cm, and $B_{xy} = 0.2 + i0.25$, the parameter δ_{xy} is, according to Eq. (8), restricted to the interval $[2.8, 4.6]$ mm.

3. Spectral behavior of the degree of polarization

It follows from Eqs. (23)–(26) that in general the state of polarization of the far-zone field is wavelength dependent, even though this is not the case for the source field [as evidenced by Eqs. (18)–(22)]. We illustrate this spectral dependence of the SOP with several examples.

We begin with the case $\delta_{xx} = \delta_{yy} = C$, with C a constant that does not depend on frequency. Furthermore, it is assumed that δ_{xy} has a frequency dependence given by

$$\delta_{xy}(\omega) = C + D\exp\left[-\frac{(\omega-\omega_0)^2}{F^2}\right], \quad (D>0), \tag{32}$$

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for all frequencies present in the source spectrum. This choice for δ_{xy} satisfies the realizability conditions (8) provided that

$$D \leq C\left(\frac{1}{\sqrt{|B_{xy}|}} - 1\right). \tag{33}$$

An example of the frequency dependence of the far-zone DOP is shown in Fig. 4. It is seen that, eventhough the degree of polarization of the source $P^{(0)}$ does *not* depend on frequency, $P^{(\infty)}$ strongly varies with wavelength. Near $\omega = \omega_0$ the beam is highly polarized, at other frequencies it is weakly polarized with $P^{(\infty)}<0.4$. However, at all frequencies $P^{(\infty)} \geq P^{(0)}$.

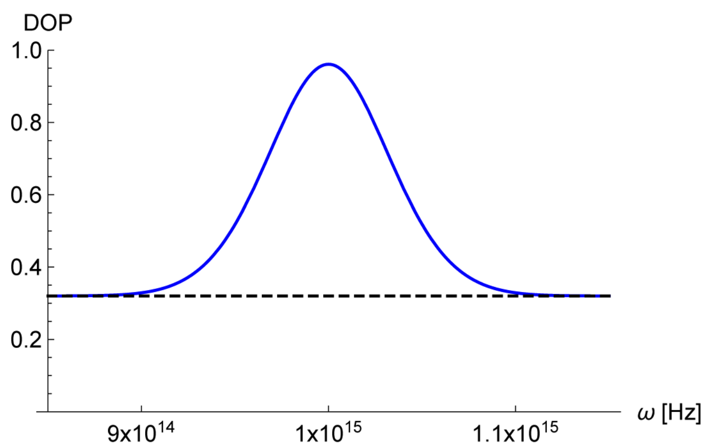

Fig. 4. The on-axis, far-zone DOP $P^{(\infty)}(\omega)$ as a function of frequency. In this example $\delta_{xx} = \delta_{yy} = 2\,\mathrm{mm}$, $\sigma = 1\,\mathrm{cm}$, $B_{xy} = 0.2 + \mathrm{i}0.25$. The frequency-dependent parameter δ_{xy} is given by Eq. (32), with $D = 1.5\,\mathrm{mm}$, $\omega_0 = 1.0 \times 10^{15}\mathrm{s}^{-1}$, and $F = 0.05 \times 10^{15}\mathrm{s}^{-1}$. The dashed line indicates $P^{(0)}$, the degree of polarization of the source.

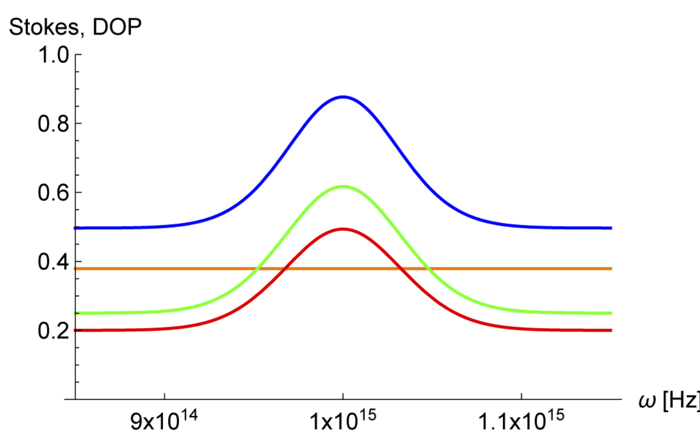

Fig. 5. The far-zone DOP $P^{(\infty)}(\omega)$ (blue curve), and the Stokes parameters $s_1^{(\infty)}(\omega)$ (orange), $s_2^{(\infty)}(\omega)$ (red) and $s_3^{(\infty)}(\omega)$ (green) as a function of frequency. In this example $\delta_{xx} = 3\,\mathrm{mm}$, $\delta_{yy} = 2\,\mathrm{mm}$, $\sigma = 1\,\mathrm{cm}$, $B_{xy} = 0.2 + \mathrm{i}0.25$. The parameter δ_{xy} is given by Eq. (32), with $D = 1.5\,\mathrm{mm}$, $\omega_0 = 1.0 \times 10^{15}\,\mathrm{s}^{-1}$, and $F = 0.05 \times 10^{15}\,\mathrm{s}^{-1}$.

We next consider the case that δ_{xx} and δ_{yy} are both constants, but not necessarily equal. Also, δ_{xy} is still given by (32), but with C replaced by the left-most term of (8). An example is shown

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in Fig. 5. Although $s_1^{(\infty)}$ is independent of frequency, it is, unlike its counterpart in the source plane, non-zero. The two other Stokes parameters and the far-zone DOP reach their maximum value at $\omega = \omega_0$.

An interesting polarization effect occurs when $\mathrm{Im}[B_{xy}] = 0$. In that case $s_3^{(0)}(\omega) = 0$, i.e., the polarized portion of the field in the source plane is linearly polarized. Since $s_1^{(0)} = 0$, and $s_2^{(0)} = \mathrm{Re}[B_{xy}]$ [see Eqs. (18) and (19)], this polarization is under an angle of 45° with the x axis. As the beam propagates $s_3(\omega)$ remains zero. However, this is not so for $s_1(\omega)$ which will, in general, become non-zero. This means that, although the polarized part of the field remains linearly polarized, the angle of vibration of the electric field $\phi(\omega)$, defined by [2, p. 170],

$$\tan[2\phi(\omega)] = \frac{s_2(\omega)}{s_1(\omega)}, \tag{34}$$

rotates on propagation to the far zone, in a way that depends on frequency. Moreover, we find that this behavior is non-trivial. By using Eq. (11) without applying the far-zone approximation we can track the polarization angle $\phi(\omega)$ as the beam propagates. This is shown in Fig. 6. Although the polarized part of the beam remains lineraly polarized, the direction of vibration of the electric field is seen to rotate on propagation, with an angle that strongly depends on frequency. Furthermore, the angle of polarization does not always change monotonically, e.g., it can first decrease and then increase again (green curve). Also, $\phi(\omega)$ at infinity can either be larger or smaller than in the source plane. We note that $\phi(\omega)$ is also the orientation angle for which a linear polarizer inserted in the beam yields a maximum transmission. Finally, also the evolution of the degree of polarization strongly depends on wavelength. This is illustrated in Fig. 7. The red curve, corresponding to frequency ω_0, is seen to rise considerably faster than the blue curve, which represents $0.8\omega_0$.

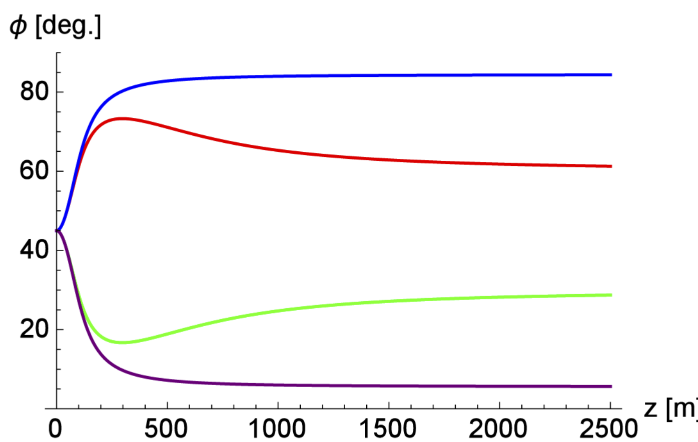

Fig. 6. Evolution of the angle of polarization $\phi(\omega)$ of the (linearly) polarized part of the field as the beam propagates. In this example $\sigma = 1$ cm, $B_{xy} = 0.1$. $\delta_{xx} = 2.0$ mm, $\delta_{yy} = 3.5$ mm, (red and blue curves); and $\delta_{xx} = 3.5$ mm, $\delta_{yy} = 2.0$ mm, (green and purple curves). The parameter δ_{xy} is given by Eq. (32), with $C = 2.85$ mm, $D = 5.5$ mm, $\omega_0 = 1.0 \times 10^{15}$ s^{-1}, and $F = 0.05 \times 10^{15}$ s^{-1}. The coherence radius δ_{xy} is evaluated at ω_0 (red and green curves), and at $0.8\omega_0$ (blue and purple curves).

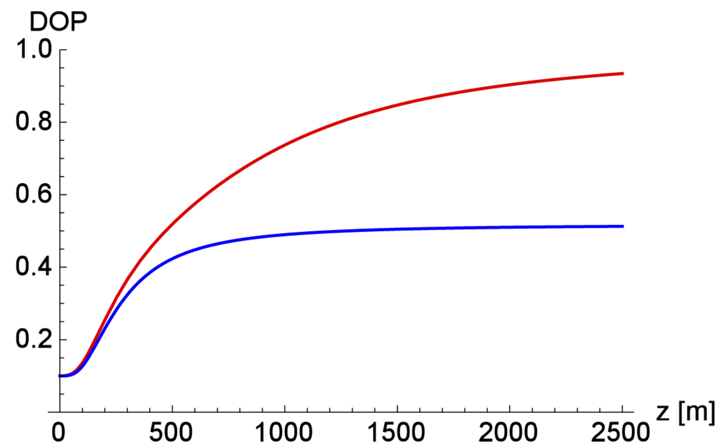

Fig. 7. Evolution of the degree of polarization on propagation for two different frequencies. The red curve is for ω_0, the blue curve is for $0.8\omega_0$. All other parameters are as in Fig. 6.

4. Conclusions

Spectral electromagnetic Gaussian Schell-model sources are characterized by parameters that are frequency dependent. Remarkably, to the best of our knowledge, the wavelength dependence of their state of polarization has up till now not been examined. We have presented a new class of broadband spectral EGSM sources whose state of polarization is the same at every source point, and the same at every wavelength. However, the Stokes parameters of the far-zone beams that they produce are shown to strongly vary with frequency. In particular the on-axis degree of polarization can be quite low ($P^{(\infty)}$<0.4) at certain frequencies and very high ($P^{(\infty)}$>0.9) at others. Examples were presented of beams whose polarized part is always linearly polarized, but with an angle of polarization that rotates non-trivially on propagation. Again, the precise nature of this behavior was seen to be frequency dependent. We have shown that the behavior of broadband partially coherent sources is more complicated than previously known. These results are relevant for the design of such sources with prescribed spectral polarization properties.

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Disclosures

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