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Polarization properties of stochastic electromagnetic beams

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Abstract

The behavior of the degree of polarization of a Gaussian Schell-model beam propagating in free space is investigated. Contour diagrams for the degree of polarization, and for the spectral density ('intensity') of the polarized and the unpolarized portions of the beam are presented.

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1. Introduction

Until relatively recently the degree of polarization of a stochastic electromagnetic beam (which in general is partially coherent and partially polarized) was regarded as one of its intrinsic properties. In 1994, it was shown by a simple example that the degree of polarization can in fact change, even on propagation in free space [1]. Since then many different examples of this kind of behavior have been found, (see for example [2–6]) and clarified from general considerations [7, Ch. 9].

In this paper, we present a detailed analysis of the behavior of the degree of polarization of a wide class of beams, the so-called electromagnetic Gaussian Schellmodel beams, on propagation in free space. Specifically, we obtain contour diagrams for:

- (1) the degree of polarization of a typical beam of this class;
- (2) the total intensity of the beam;
- (3) the intensity of the polarized part of the beam; and
- (4) the intensity of the unpolarized part of the beam.

Our analysis indicates the richness of polarization features of stochastic electromagnetic beams.

2. Electromagnetic Gaussian Schell-model beams

Consider a planar, secondary source, located in the plane z = 0, that generates a stochastic electromagnetic beam which propagates into the half-space z > 0, in a direction close to the positive z-axis. The source is assumed to be statistically stationary, at least in the wide sense. The *electric cross-spectral density matrix*, which may be used to represent both the state of coherence and the state of polarization of the beam in the source plane is defined as [8–10]

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) = \begin{pmatrix} W^{(0)}_{xx}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) & W^{(0)}_{xy}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) \\ W^{(0)}_{yx}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) & W^{(0)}_{yy}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) \end{pmatrix},$$
(1)

where

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \left\langle E_i^*(\boldsymbol{\rho}_1, \omega) E_j(\boldsymbol{\rho}_2, \omega) \right\rangle, \quad (i = x, y; j = x, y).$$
(2)

Here $E_i(\rho,\omega)$ is a Cartesian component, at frequency ω , of the (complex) electric vector in two mutually orthogonal *x*and *y* directions, perpendicular to the direction of propagation of the beam (the *z*-direction), at a point in the source plane specified by the two-dimensional transverse position vector ρ , of a typical realization of the statistical ensemble representing the field. The asterisk denotes the complex

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conjugate, and the angular brackets the ensemble average in the sense of coherence theory in the space-frequency domain [11, Section 4.3]. For an electromagnetic Gaussian Schell-model source, the elements of the cross-spectral density matrix are of the form

$$W_{ij}^{(0)}(\rho_{1},\rho_{2};\omega) = \sqrt{S_{i}^{(0)}(\rho_{1},\omega)} \sqrt{S_{j}^{(0)}(\rho_{2},\omega)} \mu_{ij}^{(0)} \times (\rho_{2}-\rho_{1};\omega).$$
(3)

Here $S_i^{(0)}(\rho, \omega)$ represents the spectral density of the component E_i of the electric vector at frequency ω in the source plane, and $\mu_{ij}^{(0)}(\rho_2 - \rho_1; \omega)$ is the coefficient characterizing the correlation between the *i*- and the *j* components of the electric field. Both these quantities are Gaussian functions, i.e.

$$S_i^{(0)}(\boldsymbol{\rho},\omega) = A_i^2 \exp(-\boldsymbol{\rho}^2/2\sigma_i^2), \qquad (4)$$

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1; \omega) = B_{ij} \exp[-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2 / 2\delta_{ij}^2].$$
(5)

The parameters A_i , B_{ij} , σ_i and δ_{ij} are independent of position, but may depend on frequency. They have to satisfy certain constraints due to the beam-like nature of the field, viz. [12],

$$\frac{1}{4\sigma_i^2} + \frac{1}{\delta_{ii}^2} \ll \frac{2\pi^2}{\lambda^2} \quad (i = x, y),$$
(6)

where $\lambda = 2\pi c/\omega$ is the wavelength, *c* being the speed of light in vacuum. Also, because the cross-spectral density matrix is non-negative definite and Hermitian, it follows that (see [2], and also [13])

$$B_{ij} = 1 \quad \text{if } i = j, \tag{7}$$

$$|B_{ij}| \leqslant 1 \quad \text{if } i \neq j, \tag{8}$$

$$B_{ij} = B_{ji}^*, \tag{9}$$

$$\delta_{ij} = \delta_{ji},\tag{10}$$

$$\max\{\delta_{xx}, \delta_{yy}\} \leqslant \delta_{xy} \leqslant \min\left\{\frac{\delta_{xx}}{\sqrt{|B_{xy}|}}, \frac{\delta_{yy}}{\sqrt{|B_{xy}|}}\right\}.$$
 (11)

From Eqs. (3)–(5) it follows that

$$W_{xx}^{(0)}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) = A_{x}^{2} \exp[-(\boldsymbol{\rho}_{1}^{2} + \boldsymbol{\rho}_{2}^{2})/4\sigma_{x}^{2}] \\ \times \exp[-(\boldsymbol{\rho}_{2} - \boldsymbol{\rho}_{1})^{2}/2\delta_{xx}^{2}], \qquad (12)$$

$$W_{xy}^{(0)}(\rho_1, \rho_2; \omega) = A_x A_y B_{xy} \exp[-(\rho_1^2 / 4\sigma_x^2 + \rho_2^2 / 4\sigma_y^2)]$$

$$\times \exp[-(\rho_2 - \rho_1)^2 / 2\delta_{xy}^2],$$
(13)
$$W_{yx}^{(0)}(\rho_1, \rho_2; \omega) = A_x A_y B_{yx} \exp[-(\rho_1^2 / 4\sigma_y^2 + \rho_2^2 / 4\sigma_x^2)]$$

$$\times \exp[-(\rho_{2} - \rho_{1})^{2}/2\delta_{xy}^{2}], \qquad (14)$$
$$W_{w}^{(0)}(\rho_{1}, \rho_{2}; \omega) = A_{y}^{2} \exp[-(\rho_{1}^{2} + \rho_{2}^{2})/4\sigma_{y}^{2}]$$

$$\times \exp[-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2 / 2\delta_{yy}^2]. \tag{15}$$

We will assume that the variances of the two intensity distributions are equal, i.e.

$$\sigma_x = \sigma_y = \sigma. \tag{16}$$

The electric cross-spectral density matrix of the beam in any transverse plane z > 0 is given by the formula [9, Eq. (7)]

$$\mathbf{W}(\boldsymbol{\rho}_{1},\boldsymbol{\rho}_{2},z;\omega) = \int \int_{(z=0)} \mathbf{W}^{(0)}(\boldsymbol{\rho}_{1}',\boldsymbol{\rho}_{2}';\omega) \\ \times K(\boldsymbol{\rho}_{1}-\boldsymbol{\rho}_{1}',\boldsymbol{\rho}_{2}-\boldsymbol{\rho}_{2}',z;\omega) \mathrm{d}^{2}\boldsymbol{\rho}_{1}' \mathrm{d}^{2}\boldsymbol{\rho}_{2}', \quad (17)$$

where

$$K(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_1', \boldsymbol{\rho}_2 - \boldsymbol{\rho}_2', z; \omega) = G^*(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_1', z; \omega) \times G(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_2', z; \omega),$$
(18)

with G denoting the Green's function for paraxial propagation from the point $Q(\rho',0)$ in the source plane z = 0 to the field point $P(\rho,z)$ [11, Section 5.6.1], viz.,

$$G(\boldsymbol{\rho} - \boldsymbol{\rho}', z; \omega) = -\frac{ik}{2\pi z} \exp(ikz) \exp[ik|\boldsymbol{\rho} - \boldsymbol{\rho}'|^2/2z].$$
(19)

On substituting from Eq. (19) into Eq. (17) we obtain for the matrix elements evaluated at 'coincident points' $\rho_1 = \rho_2 \equiv \rho$ the expressions

$$W_{ij}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \left(\frac{k\sigma\gamma_{ij}}{z}\right)^2 A_i A_j B_{ij} \exp(-k^2 \rho^2 \gamma_{ij}^2 / 2z^2), \quad (20)$$

where

$$\frac{1}{\gamma_{ij}^2} = \frac{1}{\delta_{ij}^2} + \frac{1}{4\sigma^2} + \frac{k^2\sigma^2}{z^2}, \quad (i = x, y).$$
(21)

This result can be expressed more concisely as

$$W_{ij}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \frac{A_i A_j B_{ij}}{\Delta_{ij}^2(z, \omega)} \exp[-\rho^2 / 2\sigma^2 \Delta_{ij}^2(z, \omega)], \qquad (22)$$

where

$$\Delta_{ij}^{2}(z,\omega) = 1 + \frac{z^{2}}{k^{2}\sigma^{2}} \left(\frac{1}{4\sigma^{2}} + \frac{1}{\delta_{ij}^{2}}\right), \quad (i = x, y).$$
(23)

The spectral density (the 'intensity at frequency ω ') of the beam at a point (ρ ,z) is, apart from a proportionality factor which depends on the choice of units, given by the trace of the cross-spectral density matrix at that point, i.e.

$$S(\boldsymbol{\rho}, z; \omega) = \operatorname{tr} \mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega), \qquad (24)$$

$$= W_{xx}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) + W_{yy}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega), \qquad (25)$$

$$= S_x(\boldsymbol{\rho}, z; \omega) + S_y(\boldsymbol{\rho}, z; \omega).$$
(26)

At each point the cross-spectral density matrix can be uniquely decomposed into a sum of two matrices, one of which, $\mathbf{W}^{(p)}$ say, represents a fully polarized field, whereas the other, $\mathbf{W}^{(u)}$ say, represents a completely unpolarized field [15,16], i.e.

$$\mathbf{W}(\boldsymbol{\rho},\boldsymbol{\rho},z;\omega) = \mathbf{W}^{(\mathrm{p})}(\boldsymbol{\rho},z;\omega) + \mathbf{W}^{(\mathrm{u})}(\boldsymbol{\rho},z;\omega), \qquad (27)$$

with

$$\mathbf{W}^{(p)}(\boldsymbol{\rho}, z; \omega) = \begin{pmatrix} B & D \\ D^* & C \end{pmatrix}, \tag{28}$$

and

$$\mathbf{W}^{(\mathrm{u})}(\boldsymbol{\rho}, \boldsymbol{z}; \boldsymbol{\omega}) = \begin{pmatrix} A & 0\\ 0 & A \end{pmatrix},\tag{29}$$

with A, B, $C \ge 0$, and D in general a complex number. The dependence of the matrix elements A, B, C and D on position and frequency is not displayed. These quantities can be expressed in terms of the cross-spectral density matrix $\mathbf{W}(\boldsymbol{\rho},\boldsymbol{\rho},z;\omega)$ as

$$A = \frac{1}{2} \left\{ W_{xx} + W_{yy} - \left[(W_{xx} - W_{yy})^2 + 4 |W_{xy}|^2 \right]^{1/2} \right\}, \quad (30)$$

$$B = \frac{1}{2} \left\{ W_{xx} - W_{yy} + \left[(W_{xx} - W_{yy})^2 + 4|W_{xy}|^2 \right]^{1/2} \right\}, \quad (31)$$

$$C = \frac{1}{2} \left\{ W_{yy} - W_{xx} + \left[(W_{xx} - W_{yy})^2 + 4 |W_{xy}|^2 \right]^{1/2} \right\}, \quad (32)$$

$$D = W_{xy}.$$
 (33)

The decomposition (27) makes it possible to express the spectral density at frequency ω at each point as the sum of two contributions, namely that of a polarized portion of the beam and that of an unpolarized portion, i.e.

$$S(\boldsymbol{\rho}, z; \omega) = S^{(p)}(\boldsymbol{\rho}, z; \omega) + S^{(u)}(\boldsymbol{\rho}, z; \omega),$$
(34)

with

$$S^{(p)}(\boldsymbol{\rho}, z; \omega) = \operatorname{tr} \mathbf{W}^{(p)}(\boldsymbol{\rho}, z; \omega) = B + C,$$
(35)

$$S^{(u)}(\boldsymbol{\rho}, z; \omega) = \operatorname{tr} \mathbf{W}^{(u)}(\boldsymbol{\rho}, z; \omega) = 2A.$$
(36)

The spectral degree of polarization of the beam at a point in the half-space $z \ge 0$, which is defined as the ratio of the intensity of the polarized part of the beam and the total beam intensity at that point, is given by the expression (see [11, Section 6.3.3] and Ref. [14])

$$P(\boldsymbol{\rho}, z; \omega) = \frac{S^{(\mathbf{p})}(\boldsymbol{\rho}, z; \omega)}{S(\boldsymbol{\rho}, z; \omega)},$$
(37)

$$= \sqrt{1 - \frac{4 \det \mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega)}{\left[\operatorname{tr} \mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) \right]^2}},$$
(38)

where det denotes the determinant and tr the trace. It can be readily shown that $0 \le P(\rho, z; \omega) \le 1$. The upper bound represents complete polarization, the lower bound represents a complete absence of polarization. For all intermediate values $0 \le P(\rho, z; \omega) \le 1$ the field is partially polarized.

3. Sources with a diagonal cross-spectral density matrix

The behavior of the beam on propagation can be studied by numerical evaluation of Eqs. (22), (24), (35), (36) and (38). We first consider beams generated by sources whose cross-spectral density matrix has a diagonal form, i.e. for which

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) = \begin{pmatrix} W^{(0)}_{xx}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) & 0\\ 0 & W^{(0)}_{yy}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \omega) \end{pmatrix}.$$
(39)

For sources of this kind, the x- and y components of the electric vector are uncorrelated. In this case, some of the preceeding expressions become simpler. In particular, we

readily find on substituting from Eq. (1) into Eq. (38) while using Eq. (16), that the spectral degree of polarization in the source plane is now given by the formula

$$P^{(0)}(\boldsymbol{\rho},\omega) = \frac{|A_x^2 - A_y^2|}{A_x^2 + A_y^2}.$$
(40)

It is seen from Eq. (40) that, because of the simplifying assumption (16), the spectral degree of polarization is constant across the entire source plane. Also, the spectral degree of polarization of the beam in a plane z > 0 is now given by the simple formula

$$P(\boldsymbol{\rho}, z; \omega) = \frac{|S_x(\boldsymbol{\rho}, z; \omega) - S_y(\boldsymbol{\rho}, z; \omega)|}{S_x(\boldsymbol{\rho}, z; \omega) + S_y(\boldsymbol{\rho}, z; \omega)}.$$
(41)

The formulas (30)-(33) now become

$$A = \min\{W_{xx}, W_{yy}\},$$
 (42)

$$B = \begin{cases} W_{xx} - W_{yy} & \text{if } W_{xx} > W_{yy}, \\ 0 & \text{otherwise,} \end{cases}$$
(43)

$$C = \begin{cases} W_{yy} - W_{xx} & \text{if } W_{yy} > W_{xx}, \\ 0 & \text{otherwise,} \end{cases}$$
(44)

$$D = 0. (45)$$

An example of the behavior of beam generated by an electromagnetic Gaussian Schell-model source, whose electric cross-spectral density matrix has a diagonal form, is given in Fig. 1a. We note that the on-axis value of the spectral degree of polarization $P(0,z;\omega)$ decreases first to zero and then increases gradually to its asymptotic value (dashed line). Panel (b) shows the axial behavior of the total spectral density $S(0,z;\omega)$, the spectral density $S^{(u)}(0,z;\omega)$ of the unpolarized part, and the spectral density $S^{(p)}(0,z;\omega)$ of the polarized part.

Fig. 2 depicts the total spectral density $S(\rho,z;\omega)$, the spectral density $S^{(u)}(\rho,z;\omega)$ of the unpolarized part, and the spectral density $S^{(p)}(\rho,z;\omega)$ of the polarized part, in a cross-section of the beam. It is seen that whereas $S(\rho,z;\omega)$ decreases monotonically with increasing distance from the beam-axis, the spectral density of the polarized part first decreases to zero and then rises again, eventually surpassing the spectral density of the unpolarized part. Also shown in the figure is the degree of polarization $P(\rho,z;\omega)$ (dotted curve).

The behavior of the spectral degree of polarization of the same beam is shown in Fig. 3a, where the vertical scale is in centimeters, and the horizontal scale is in meters. The spectral degree of polarization $P^{(0)}(\rho, z = 0; \omega)$ in the source plane has the constant value 0.18. In the far zone $P(\rho, z; \omega)$ first decreases to zero with increasing ρ , and then increases to unity. This behavior is quite different from that of the normalized spectral density $S(\rho, z; \omega)/S(0, z; \omega)$ which, as is seen from Fig. 3b, decreases monotonically with increasing distance from the beam-axis. A comparison of Fig. 3a and b also shows that the contour representing the case $P(\rho, z; \omega) = 0$ lies closer to the propagation direction than the contour $S(\rho, z; \omega)/S(0, z; \omega) = 0.4$. In the far zone, the



Fig. 1. (a) The on-axis spectral degree of polarization $P(\rho = 0, z; \omega)$ of a stochastic electromagnetic Gaussian Schell-model beam as a function of the propagation distance z (solid curve). The asymptotic value (0.32) is indicated by the horizontal dashed line. The beam is generated by a source whose cross-spectral density matrix is of the form (39). In this example $A_x = 1.2$, $A_y = 1$, $\sigma = 1$ cm, $\delta_{xx} = 0.15$ mm, $\delta_{yy} = 0.25$ mm, $\lambda = 0.6328 \,\mu\text{m}$. For these parameter values the spectral degree of polarization across the source plane $P^{(0)}(\rho, \omega) = 0.18$. Panel (b) The on-axis total spectral density $S(0,z;\omega)$ of the beam, the spectral density of its unpolarized portion, $S^{(u)}(0,z;\omega)$ (dashed curve), and its polarized portion, $S^{(p)}(0,z;\omega)$, with all three curves divided by $S(0,0;\omega)$.



Fig. 2. The total spectral density $S(\rho,z;\omega)$ of a beam, the spectral density $S^{(u)}(\rho,z;\omega)$ of its unpolarized portion (dashed curve), the spectral density $S^{(p)}(\rho,z;\omega)$ of its polarized portion, and the degree of polarization $P(\rho,z;\omega)$ (dotted curve), in the transverse plane z = 100 m. All parameters are the same as in Fig. 1, with all three spectral density curves divided by $S(0,z;\omega)$.

latter contour makes an angle of 0.04 degrees with the beam-axis. This indicates that the spectral degree of polarization has zero values occuring at points well within the paraxial regime.



Fig. 3. (a) Contours of the spectral degree of polarization $P(\rho,z;\omega)$ of a stochastic electromagnetic Gaussian Schell-model beam. The successive contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled. The parameters are the same as in Fig. 1. Panel (b) Contours of the normalized spectral density $S(\rho,z;\omega)/S(0,z;\omega)$ of the beam. All contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled.



Fig. 4. (a) Contours of the normalized spectral density $S^{(u)}(\rho,z;\omega)/S(0,z;\omega)$ of the unpolarized part of the beam. All contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled. Panel (b) Contours of the normalized spectral density $S^{(p)}(\rho,z;\omega)/S(0,z;\omega)$ of the polarized part of the beam.

In Fig. 4, the spectral density $S^{(u)}(\rho, z; \omega)$ of the unpolarized part is shown in panel (a), and the spectral density $S^{(p)}(\rho, z; \omega)$ of the polarized part is shown in panel (b).

It is emphasized that the decomposition of the crossspectral density matrix into a polarized and an unpolarized part is a *local* decomposition, i.e. it pertains to the beam behavior at a *single point* in space. Our results do, therefore, *not* imply that the two parts represent beams (i.e. satisfy the paraxial time-independent wave equation), contrary to the impression given in the literature.

4. Sources with a non-diagonal cross-spectral density matrix

Let us now consider a broader class of sources, namely Gaussian Schell-model sources whose electric cross-spectral density matrix is of the general form given by Eq. (1), i.e. with non-vanishing off-diagonal elements. This implies that the two components of the electric vector are correlated. An example of the beam behavior in a crosssection of such a beam is shown in Fig. 5. In contrast to



Fig. 5. The total spectral density $S(\rho,z;\omega)$ of a beam, the spectral density $S^{(u)}(\rho,z;\omega)$ of its unpolarized portion (dashed curve), the spectral density $S^{(p)}(\rho,z;\omega)$ of its polarized portion, and the degree of polarization $P(\rho,z;\omega)$ (dotted curve), in the transverse plane z = 125 m. The beam is generated by a source whose cross-spectral density matrix is of the form (1). In this example $A_x = 1.0$, $A_y = 1.5$, $B_{xy} = 0.09$, $\sigma = 1$ cm, $\delta_{xx} = 0.15$ mm, $\delta_{yy} = 0.25$ mm, $\delta_{xy} = 0.40$ mm, $\lambda = 0.6328$ µm, with all three spectral density curves divided by $S(0,z;\omega)$.



Fig. 6. (a) Contours of the spectral degree of polarization $P(\rho,z;\omega)$ of a stochastic electromagnetic Gaussian Schell-model beam. The successive contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled. The parameters are the same as in Fig. 5. Panel (b) Contours of the normalized spectral density $S(\rho,z;\omega)/S(0,z;\omega)$ of the beam. All contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled.



Fig. 7. (a) Contours of the normalized spectral density $S^{(u)}(\rho,z;\omega)/S(0,z;\omega)$ of the unpolarized part of the beam. All contour levels are chosen with increments of 0.1, with only the levels that are multiples of 0.2 being labeled. The parameters are the same as in Fig. 5. Panel (b) Contours of the normalized spectral density $S^{(p)}(\rho,z;\omega)/S(0,z;\omega)$ of the polarized part of the beam.

the case presented in Fig. 2, the intensity $S^{(p)}(\rho,z;\omega)$ of the polarized part of the field does not reach zero value before increasing again, i.e. the degree of polarization (dotted curve) does now not become zero.

In Fig. 6, contours of the spectral degree of polarization $P(\rho,z;\omega)$ of the same beam as in Fig. 5 are shown, together with contours of the normalized spectral density $S(\rho,z;\omega)/S(0,z;\omega)$.

In Fig. 7, the spectral density $S^{(u)}(\rho,z;\omega)$ of the unpolarized part is shown in panel (a), and the spectral density $S^{(p)}(\rho,z;\omega)$ of the polarized part is shown in panel (b). Just as in the previous case, illustrated in Fig. 4, $S^{(u)}(\rho,z;\omega)$, and $S^{(p)}(\rho,z;\omega)$ are seen to have a more complicated structure than the normalized total spectral density $S(\rho,z;\omega)/S(0,z;\omega)$.

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