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Fourier processing with partially coherent fields

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We describe how Fourier signal processing techniques can be generalized to partially coherent fields. Using standard coherence theory, we first show that focusing of a partially coherent beam by a lens modifies its coherence properties. We then consider a $4f$ imaging system composed of two lenses and discuss how spatial filtering in the Fourier plane allows one to tune the coherence properties of the beam. This, in turn, provides control over the beam's directionality, spectrum, and degree of polarization.

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Fourier processing of signals and images with coherent light is a well-known technique discussed in several textbooks [1–3]. In essence, it involves the manipulation or filtering of the spatial Fourier components of a wave field. The field that results from synthesizing the altered spectrum leads, for example, to images that have less blur [3], or images in which phase differences are rendered visible [4]. A commonly used setup to achieve this is the $4f$ system, where the filtering takes place in the Fourier plane of a lens. The field then passes through a second lens in whose back focal plane the reconstituted field is imaged.

Here we describe an analogous process for fields that are not fully coherent, but rather spatially partially coherent. The correlation function that characterizes such fields in the space-frequency domain is the so-called cross-spectral density [5]. As we will show, focusing partially coherent fields produces the four-dimensional spatial Fourier transform of this function in the focal plane. Spatial filtering, followed by passage through a second lens, produces a synthesized field with altered coherence properties. Since the state of coherence of a beam governs its directionality, spectrum, and state of polarization, this form of Fourier processing has the potential to be a powerful tool to tailor the properties of partially coherent fields.

Changes in the cross-spectral density function on propagation through free space [6] or through a linear system [7] have been examined before. In addition, the use of ABCD systems [8] or apertures [9] has been considered. In a study that is more directly related to the present one, Indebetouw [10] analyzed the tuning of far-field spectra by applying spatial filtering techniques to quasi-homogeneous sources. More recently, the effects of focusing on specific types of cross-spectral density functions have been discussed [11,12].

In this Letter, we first show that focusing of a partially coherent beam by a lens modifies its coherence properties through changes in the cross-spectral density function. We then consider a $4f$ imaging system composed of two lenses and discuss how spatial filtering in the Fourier plane allows one to tune the coherence properties of the beam by tailoring the cross-spectral density. More specifically, we derive two general expressions for the effects of lenses on the statistical properties of wave fields and discuss their potential applications.

We begin by recalling that the optical field in the focal plane of a lens is proportional to the spatial Fourier transform of the field in the entrance plane (Section 5.2, [1]):

$$U^{(f)}(\boldsymbol{\rho},\omega)=\frac{e^{jk\rho^2/2f}}{j\lambda f}\int_{-\infty}^{\infty}U^{(\mathrm{in})}(\boldsymbol{\rho}',\omega)P(\boldsymbol{\rho}')\times\exp(-jk\boldsymbol{\rho}\cdot\boldsymbol{\rho}'/f)\mathrm{d}^2\rho'. \tag{1}$$

Here $U^{(f)}(\boldsymbol{\rho},\omega)$ denotes the field in the focal plane at frequency ω at a transverse position $\boldsymbol{\rho}=(x,y)$; $U^{(\mathrm{in})}(\boldsymbol{\rho}',\omega)$ is the field in the entrance plane; the wavenumber is $k=2\pi/\lambda=\omega/c$, with c being the speed of light. The lens, with focal length f, is taken to have its central axis along the z direction. $P(\boldsymbol{\rho}')$ is a pupil function, i.e., $P(\boldsymbol{\rho}')=1$ for points within the lens aperture, and 0 elsewhere.

The statistical properties of a partially coherent field are characterized by its cross-spectral density function [5]:

$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle U^*(\boldsymbol{\rho}_1,\omega)U(\boldsymbol{\rho}_2,\omega)\rangle, \tag{2}$$

where the angular brackets indicate an average taken over an ensemble of field realizations. Quite often, it is advantageous to use a normalized correlation function, the spectral degree of coherence, which is defined as

$$\mu(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{\sqrt{S(\boldsymbol{\rho}_1,\omega)S(\boldsymbol{\rho}_2,\omega)}}, \tag{3}$$

with the spectral density given by

$$S(\boldsymbol{\rho},\omega)=W(\boldsymbol{\rho},\boldsymbol{\rho},\omega). \tag{4}$$

We can now easily calculate the cross-spectral density in the focal plane of the lens. On substituting from Eq. (1) into Eq. (2) and interchanging the order of integration and ensemble averaging, we find that

$$W^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{e^{jk(\rho_2^2-\rho_1^2)/2f}}{\lambda^2 f^2}\iint_{-\infty}W^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)\times P(\boldsymbol{\rho}_1')P(\boldsymbol{\rho}_2')\times\exp[-jk(\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_1')/f]\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \quad (5)$$

Hence, the cross-spectral density of the field in the focal plane is proportional to the four-dimensional (4D) Fourier transform of $W^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)$, truncated by the two pupil functions that account for the finite aperture of the lens. This general result is quite powerful. It can readily be applied to obtain both the cross-spectral density and the spectral density of focused, partially coherent fields. We illustrate this with two examples.

Let us first consider an incident field that is spatially partially correlated and has a homogeneous spectral density, i.e.,

$$W^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)=S^{(\mathrm{in})}(\omega)\mu^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2'), \quad (6)$$

with the spectral degree of coherence assumed to be constant across the support of $S^{(\mathrm{in})}(\omega)$. According to Eqs. (4) and (5), the spectral density $S^{(f)}(\boldsymbol{\rho},\omega)$ in the focal plane is then given by

$$S^{(f)}(\boldsymbol{\rho},\omega)=S^{(\mathrm{in})}(\omega)\frac{1}{\lambda^2 f^2}\iint_{-\infty}\mu^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')P(\boldsymbol{\rho}_1')P(\boldsymbol{\rho}_2')\times\exp[-jk\boldsymbol{\rho}\cdot(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')/f]\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \quad (7)$$

Equation (7) states that the spectral density in the focal plane is not equal to that of the incident field, but rather is modified by two factors. The first factor, $1/\lambda^2 f^2$, may be said to be diffraction-induced. The second factor, the integral, describes the effect of partial coherence. It is, in fact, the 4D Fourier transform of the (truncated) spectral degree of coherence, evaluated at spatial frequencies $-\boldsymbol{\rho}/\lambda f$ and $\boldsymbol{\rho}/\lambda f$.

As a second example, we assume that the incident field has an inhomogeneous spectral density and is δ-correlated, i.e.,

$$W^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)=S(\boldsymbol{\rho}_1',\omega)\delta^2(k\boldsymbol{\rho}_2'-k\boldsymbol{\rho}_1'). \quad (8)$$

Using this form in Eq. (5), we obtain

$$W^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{e^{jk(\rho_2^2-\rho_1^2)/2f}}{4\pi^2 f^2}\int_{-\infty}^{\infty}S(\boldsymbol{\rho}',\omega)P(\boldsymbol{\rho}')\times\exp[-jk(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)\cdot\boldsymbol{\rho}'/f]\mathrm{d}^2\rho'. \quad (9)$$

Equation (9) states that the cross-spectral density in the focal plane is proportional to the two-dimensional Fourier transform of the spectral density distribution in the entrance pupil. This is the analogue for focused fields of the far-zone form of the van Cittert–Zernike theorem (Section 4.4.4, [5]).

We illustrate the use of Eq. (9) by considering an incident beam with a spectral density of the form

$$S(\boldsymbol{\rho}',\omega)=s(\omega)\exp[-(\rho'^2/w_0^2)], \quad (10)$$

where $s(\omega)$ is the uniform spectral density of a Gaussian beam with spot size w_0. For simplicity, we assume that the beam is much narrower than the lens radius a, and we may therefore set $P(\boldsymbol{\rho}')=1$. The integration in Eq. (9) can be performed analytically, with the result

$$W^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=e^{jk(\rho_2^2-\rho_1^2)/2f}\frac{s(\omega)w_0^2}{4\pi f^2}\exp\left(-\frac{|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2}{\sigma_c^2}\right), \quad (11)$$

where $\sigma_c=\lambda f/(\pi w_0)$ is the coherence radius of the Gaussian beam in the focal plane of the lens. It is evident that the initially incoherent Gaussian beam becomes partially coherent and that its degree of coherence can be controlled through the focal length f of the lens.

Another application of Eq. (5) is in calculating the correlation function in the entrance plane of a lens from knowledge of the cross-spectral density in the focal plane. This is relevant for experiments in which the field is focused onto two point detectors from which then its correlation is determined from visibility measurements (Section 4.3.2, [5]).

The Fourier relation expressed by Eq. (1) is modified when the incident field is taken to be in the front focal plane of the lens. In that case (Section 5.2, [1]),

$$U^{(f)}(\boldsymbol{\rho},\omega)=\frac{1}{j\lambda f}\int_{-\infty}^{\infty}U^{(\mathrm{in})}(\boldsymbol{\rho}',\omega)P(\boldsymbol{\rho}')\exp(-jk\boldsymbol{\rho}\cdot\boldsymbol{\rho}'/f)\mathrm{d}^2\rho'. \quad (12)$$

In exactly the same way as before, we then find that the cross-spectral density in the back focal plane of the lens is given by

$$W^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{1}{\lambda^2 f^2}\iint_{-\infty}W^{(\mathrm{in})}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)P(\boldsymbol{\rho}_1')P(\boldsymbol{\rho}_2')\times\exp[-jk(\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_1')/f]\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \quad (13)$$

Equation (13) expresses a 4D Fourier-transform relationship between the cross-spectral densities of the field at the front and back focal planes of a lens.

Let us now consider a paraxial $4f$ setup consisting of two lenses as sketched in Fig. 1. An incident beam that propagates in the positive z direction has, in the front focal plane of lens L1, a cross-spectral density $W^{(\mathrm{in})}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},\omega)$. The Fourier transform of this function is spatially filtered in the back focal plane of L1. This plane coincides with the front focal plane of lens L2. The cross-spectral density $W^{(f_2)}(\boldsymbol{\rho}_{21},\boldsymbol{\rho}_{22},\omega)$ in its back focal plane is controlled by the filtering process.

As an example, let us choose an incident field with a uniform spectral density, which is δ-correlated, i.e.,

$$W^{(\mathrm{in})}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},\omega)=S(\omega)\delta^2(k\boldsymbol{\rho}_{01}-k\boldsymbol{\rho}_{02}). \quad (14)$$

The first lens is taken to have a focal length f_1 and a radius a_1. According to Eq. (13), the cross-spectral density in the

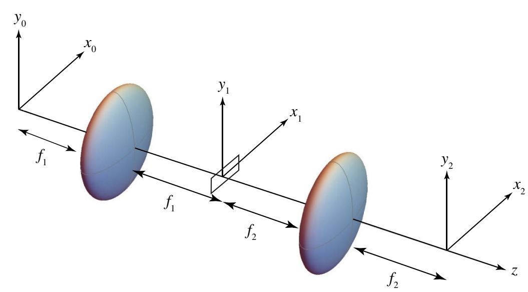

Fig. 1. 4f setup for Fourier processing consisting of two lenses of focal lengths f_1 and f_2. A narrow slit is used for spatial filtering in the back focal plane of the first lens. The object and image planes are also shown.

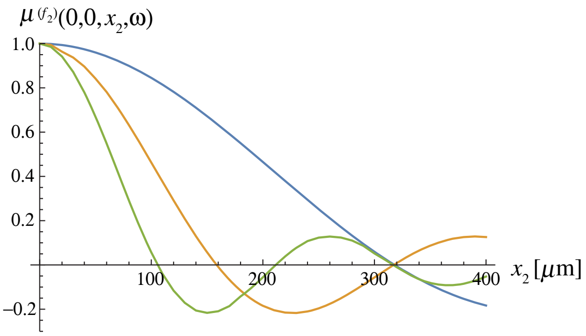

Fig. 2. Spectral degree of coherence $\mu^{(f_2)}(0, 0, x_2, \omega)$ in the back focal plane of lens L2 for three different values of the slit width. In these examples, the length $2b = 1$ mm (blue curve), 2 mm (orange curve), and 3 mm (green curve). The other parameters are $\lambda = 632.8$ nm, $a_1 = 1$ cm, and $f_1 = f_2 = 50$ cm.

back focal plane, $W^{(f_1)}(\boldsymbol{\rho}_{11}, \boldsymbol{\rho}_{12}, \omega)$, then becomes (see Section 4.4.4, [5])

$$W^{(f_1)}(\boldsymbol{\rho}_{11},\boldsymbol{\rho}_{12},\omega) = \frac{S(\omega)}{4\pi^2 f_1^2}\int_{-\infty}^{\infty} P(\boldsymbol{\rho}_{01}) \times \exp[-jk(\boldsymbol{\rho}_{12}-\boldsymbol{\rho}_{11})\cdot\boldsymbol{\rho}_{01}/f_1]\mathrm{d}^2\rho_{01} \tag{15}$$

$$= \frac{2\pi a_1 S(\omega)}{4\pi^2 k f_1 |\boldsymbol{\rho}_{12}-\boldsymbol{\rho}_{11}|} J_1\left(\frac{k a_1 |\boldsymbol{\rho}_{12}-\boldsymbol{\rho}_{11}|}{f_1}\right). \tag{16}$$

Here J_1 denotes a Bessel function of the first kind of order 1.

Next, the field is spatially filtered by a narrow horizontal slit of width h and length $2b$. The slit is assumed to be narrow enough to allow us to ignore the variation of $W^{(f_1)}(\boldsymbol{\rho}_{11}, \boldsymbol{\rho}_{12}, \omega)$ along the y_1 direction. We choose the first reference point in the back focal plane of lens L2 to be on the z axis, i.e., $\boldsymbol{\rho}_{21} = (0, 0)$. Applying Eq. (13) while using Eq. (16) gives us

$$W^{(f_2)}(0, 0, x_2, \omega) = \frac{a_1 S(\omega) h^2}{4\pi^2 f_1 f_2^2 \lambda}\int_{-b}^{b}\int_{-b}^{b}\frac{1}{|x_{12}-x_{11}|} \times J_1\left(\frac{k a_1 |x_{12}-x_{11}|}{f_1}\right) \times \cos(k x_2 x_{12}/f_2)\mathrm{d}x_{11}\mathrm{d}x_{12}, \tag{17}$$

where f_2 denotes the focal length of lens L2. Clearly, the cross-spectral density function is real-valued in this example. We note that, there being no y_2-dependence, the argument of $W^{(f_2)}$ is displayed as x_2 rather than $\boldsymbol{\rho}_{22}$. Similarly, we obtain for $S^{(f_2)}(x_2, \omega)$, the spectral density in the second back focal plane, the expression

$$S^{(f_2)}(x_2, \omega) = \frac{a_1 S(\omega) h^2}{4\pi^2 f_1 f_2^2 \lambda}\int_{-b}^{b}\int_{-b}^{b}\frac{1}{|x_{12}-x_{11}|} \times J_1\left(\frac{k a_1 |x_{12}-x_{11}|}{f_1}\right) \times \cos[k x_2 (x_{12}-x_{11})/f_2]\mathrm{d}x_{11}\mathrm{d}x_{12}. \tag{18}$$

A numerical evaluation of Eqs. (17) and (18) yields the spectral degree of coherence, $\mu^{(f_2)}(0, 0, x_2, \omega)$, as defined by Eq. (3). An example is shown in Fig. 2, in which the spectral degree of coherence is plotted for three selected values of the slit length $2b$. A measure of the transverse coherence length is the distance at which the spectral degree of coherence drops to 0.5. This distance is seen to be 65, 98, and 192 μm, respectively, demonstrating that the transverse coherence length of the field can be tuned by spatial filtering. Furthermore, at certain positions, $\mu^{(f_2)}(0, 0, x_2, \omega) = 0$. This indicates the presence of so-called correlation singularities, i.e., pairs of points at which the fields are completely uncorrelated [13,14].

In this Letter, we have discussed how the traditional Fourier image processing techniques can be generalized to partially coherent fields. Using standard coherence theory, we derived a general expression that provides the cross-spectral density in the focal plane of a lens in terms of a 4D Fourier transform of the cross-spectral density at the entrance plane. We used this general result to show that focusing of a partially coherent beam by a lens modifies both its spectral density and its coherence properties.

We then considered a $4f$ imaging system composed of two lenses and discussed how spatial filtering of the cross-spectral density in the Fourier plane allows one to tune the coherence properties of the beam. As an example, we used a narrow slit in the Fourier plane for spatial filtering and showed that an incoherent optical beam can be turned into a partially coherent beam whose degree of spatial coherence is controllable by changing the slit length. This, in turn, provides control over the beam's directionality and its optical spectrum. The theory presented in this Letter made use of scalar fields, justified by the assumption of paraxiality. However, our approach can be easily extended to vector fields using the concept of a coherency matrix. This should allow control over a beam's state of polarization. *Note*: After submission of this article we learned that some related results have been reported in [15].

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REFERENCES

1. J. W. Goodman, *Introduction to Fourier Optics*, 2nd ed. (McGraw Hill, 1996).
2. E. G. Steward, *Fourier Optics: An Introduction*, 2nd ed. (Dover, 2011).
3. K. Khare, *Fourier Optics and Computational Imaging* (Wiley, 2015).
4. F. Zernike, Physics **16**, 454 (1935).
5. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University, 1995).
6. F. Bloisi and L. Vicari, J. Opt. Soc. Am. A **8**, 1106 (1991).
7. E. Wolf and J. R. Fienup, Opt. Commun. **82**, 209 (1991).
8. F. Bloisi and L. Vicari, J. Phys. III **1**, 1569 (1991).
9. J. T. Foley, Opt. Commun. **75**, 347 (1990).
10. G. Indebetouw, J. Mod. Opt. **36**, 251 (1989).
11. D. G. Fischer and T. D. Visser, J. Opt. Soc. Am. A **21**, 2097 (2004).
12. S. B. Raghunathan, T. van Dijk, E. J. G. Peterman, and T. D. Visser, Opt. Lett. **35**, 4166 (2010).
13. H. F. Schouten, G. Gbur, T. D. Visser, and E. Wolf, Opt. Lett. **28**, 968 (2003).
14. G. J. Gbur, *Singular Optics* (CRC Press, 2016).
15. D. Mendlovic, G. Shabtay, and A. W. Lohmann,Opt. Lett. **24**, 361 (1999).