

Strong suppression of forward or backward Mie scattering by using spatial coherence

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We derive analytic expressions relating Mie scattering with partially coherent fields to scattering with fully coherent fields. These equations are then used to demonstrate how the intensity of the forward- or backward-scattered field can be suppressed several orders of magnitude by tuning the spatial coherence properties of the incident field. This method allows the creation of cone-like scattered fields, with the angle of maximum intensity given by a simple formula. © 2016 Optical Society of America

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1. INTRODUCTION

In 1908 Gustav Mie obtained, on the basis of Maxwell's equations, a rigorous solution for the diffraction of a plane monochromatic wave by a homogeneous sphere [1]. His seminal work has since been applied in a wide range of fields such as astronomy, climate studies, atomic physics, optical trapping, etc. Extending and generalizing the theory of Mie scattering remains an important activity to this day [2,3].

In recent years, a large number of studies have been devoted to the question of how the angular distribution of scattered fields may be controlled. This line of research was initiated by Kerker *et al.* in 1983 [4]. They derived conditions under which the forward or backward scattering by magnetic spheres is strongly suppressed. Since then, both the influence of the particle's composition [5–11] and that of the coherence properties of the incident field on the scattering process have been examined [12–18].

We recently demonstrated that a J_0 Bessel-correlated beam, which is incident on a homogeneous sphere, produces a highly unusual distribution of the scattered field [19]. In the present study we derive expressions that relate the scattered field for this particular case to that of an incident field that is spatially fully coherent. These expressions allow us to tailor the transverse coherence length of the field to obtain strongly suppressed forward or backward scattering. We also derive an approximate formula for the angle at which the scattered intensity reaches its maximum value. This expression is found to work surprisingly well.

In Section 2, we briefly review scalar Mie theory. Besselcorrelated fields are discussed in Section 3. In Section 4 we derive an equation for the intensity of the forward-scattered field. We show by example how this expression can be used to reduce the forward-scattering signal by several orders of magnitude. An expression for the backward-scattered field is derived in Section 5. This is then applied to design a partially coherent incident field that causes strongly suppressed back-scattering. In the last section, we discuss possible ways to realize J_0 Bessel-correlated fields and offer some conclusions.

2. SCALAR MIE SCATTERING

Let us begin by considering a plane, monochromatic scalar wave of frequency ω and with unit amplitude, which is propagating in a direction specified by a real unit vector **u**. If this wave is incident on a deterministic, spherical scatterer with radius *a* and refractive index *n* (see Fig. 1), then the scattering amplitude in an observation direction **s** in the far zone can be expressed as (see [Eq. (4.66)] [20], with a trivial change in notation)

$$f(\mathbf{s} \cdot \mathbf{u}, \omega) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) \exp[i\delta_l(\omega)] \sin[\delta_l(\omega)] P_l(\mathbf{s} \cdot \mathbf{u}),$$
(1)

where k is the free-space wavenumber, P_l denotes a Legendre polynomial of order l, and the phase shifts $\delta_l(\omega)$ are given by the expressions [Secs. 4.3.2 and 4.4.1] [20]

$$\tan[\delta_{l}(\omega)] = \frac{k j_{l}(ka) j_{l}'(ka) - k j_{l}(ka) j_{l}'(ka)}{\bar{k} j_{l}'(\bar{k}a) n_{l}(ka) - k j_{l}(\bar{k}a) n_{l}'(ka)}.$$
 (2)



Fig. 1. Sphere with radius a is illuminated by a plane wave propagating in the direction **u**, which is taken to be the *z* axis. The scattering angle θ is the angle between the direction of the incident field and a far-zone observation point *r***s**.

Here, j_l and n_l are spherical Bessel functions and spherical Neumann functions, respectively, of order l. Furthermore,

$$\bar{k} = nk$$
 (3)

is the wavenumber associated with the reduced wavelength within the scatterer, and the primes denote differentiation. The intensity of the scattered field equals

$$S_{\rm fc}^{\rm (sca)}(\theta,\omega) = \frac{1}{r^2} |f(\cos\,\theta,\omega)|^2,\tag{4}$$

where r is the distance between the scattering sphere and the point of observation, and the subscript "fc" indicates an incident field that is fully coherent.

An example of the angular distribution of the field for fully coherent Mie scattering is shown in Fig. 2. Many deep minima can be seen, with the first one occurring at $\theta = 0.69^{\circ}$, where $S_{\rm fc}^{\rm (sca)} = 3.15 \times 10^{-6}$. We will show that such a minimum can be "moved" to the forward direction ($\theta = 0^{\circ}$) by using an incident field that is not fully coherent but rather is J_0 -correlated. This then results in a strongly suppressed forward-scattered field.

Note that, just like the vast majority of previous studies that deal with scattering of partially coherent fields, we use a scalar theory rather than a vector approach. The partial wave expansion in Eq. (1) is quite similar to what is obtained in an electromagnetic theory. In the latter, the scattered field is written as the sum of two infinite series: one electric the other magnetic. However, when the field is either unpolarized or linearly polarized, it is to be expected that a scalar approach will give an accurate description.



Fig. 2. Angular distribution, on a logarithmic scale, of the normalized intensity of the scattered field $S_{fc}^{(sca)}(\theta, \omega)$ for a fully coherent incident field. In this example, the sphere radius $a = 50\lambda$, the refractive index n = 1.33, and the wavelength $\lambda = 632.8$ nm. The inset shows the first few scattering minima up to $\theta = 2.5^{\circ}$.

3. MIE SCATTERING WITH J₀ BESSEL-CORRELATED FIELDS

In the space-frequency domain the second-order coherence properties of a stochastic field $U(\mathbf{r}, \omega)$ are characterized by its cross-spectral density function at two positions \mathbf{r}_1 and \mathbf{r}_2 [Sec. 4.3.2] [21], namely,

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^*(\mathbf{r}_1, \omega) U(\mathbf{r}_2, \omega) \rangle,$$
 (5)

where the angular brackets denote an average taken over an ensemble of field realizations. The spectral density (the intensity at frequency ω) at a point **r** is defined as

$$S(\mathbf{r},\omega) = \langle U^*(\mathbf{r},\omega)U(\mathbf{r},\omega) \rangle = W(\mathbf{r},\mathbf{r},\omega).$$
(6)

We will consider an incident field with a uniform spectral density $S^{(0)}(\omega)$, that is J_0 -correlated. This means that its cross-spectral density function in the plane z = 0 (the plane that passes through the center of the sphere) is of the form

$$W^{(inc)}(\rho_1, \rho_2, \omega) = S^{(0)}(\omega) J_0(\beta | \rho_2 - \rho_1 |).$$
 (7)

Here, J_0 denotes the Bessel function of the first kind and zeroth order, and $\rho_1 = (x_1, y_1)$ and $\rho_2 = (x_2, y_2)$ are 2D position vectors in the z = 0 plane. The inverse of the parameter β is a rough measure of the effective transverse coherence length of the incident field. The generation of such a beam was reported in [22].

In a previous publication [19], we derived that in the case of a J_0 -correlated field, the angular distribution of the intensity of the scattered field is given by the expression

$$S_{\rm pc}^{\rm (sca)}(\theta,\omega) = \frac{S^{(0)}(\omega)}{2\pi r^2 k^2} \int_0^{2\pi} \sum_l \sum_m (2l+1)(2m+1) \\ \times \exp[\mathrm{i}(\delta_l - \delta_m)] \sin \delta_l \sin \delta_m \\ \times P_l \left[\beta k^{-1} \cos \alpha \sin \theta + \cos \theta \sqrt{1 - \beta^2/k^2}\right] \\ \times P_m \left[\beta k^{-1} \cos \alpha \sin \theta + \cos \theta \sqrt{1 - \beta^2/k^2}\right] \mathrm{d}\alpha,$$
(8)

where the subscript "pc" indicates partial coherence. We note that β/k cannot exceed 1. On comparing Eq. (8), which pertains to a partially coherent field, with Eq. (1), which is for a fully coherent field, we see that this result can be written in the form

$$S_{\rm pc}^{\rm (sca)}(\theta,\omega) = \frac{S^{(0)}(\omega)}{2\pi r^2} \times \int_0^{2\pi} \left| f(\beta k^{-1} \cos \alpha \sin \theta + \cos \theta \sqrt{1 - \beta^2 / k^2}) \right|^2 \mathrm{d}\alpha.$$
(9)

This expression, which relates the scattering of a J_0 -correlated field with that by a plane wave, will be used in the next sections. To simplify the notation, we will set the spectral density of the incident field equal to unity $(S^{(0)}(\omega) = 1)$, and, from now on, we no longer display the ω dependence.

4. SUPPRESSION OF FORWARD SCATTERING

The intensity of the scattered field, as given by Eq. (9), greatly simplifies when we consider the forward direction ($\theta = 0^{\circ}$). We then have that

$$S_{\rm pc}^{\rm (sca)}(\theta=0^{\circ}) = \frac{1}{r^2} \left| f(\sqrt{1-\beta^2/k^2}) \right|^2.$$
 (10)

This expression has a clear physical meaning. Because both **s** and **u** are unit vectors, the argument $\mathbf{s} \cdot \mathbf{u}$ of the scattering amplitude $f(\mathbf{s} \cdot \mathbf{u})$ in Eq. (1) can be interpreted as the cosine of an angle, ϕ say, such that $\cos \phi = \mathbf{s} \cdot \mathbf{u}$. It follows from Eq. (10) that, for the case of a J_0 -correlated field, this angle is such that

$$\cos\phi = \sqrt{1 - \beta^2/k^2},$$
 (11)

as is illustrated in Fig. 3. This result implies that, for an incident J_0 -correlated field with coherence parameter β/k , the forward-scattered intensity ($\theta = 0^\circ$) is equal to the intensity that is scattered in the fully coherent case in the direction ϕ , which is given by Eq. (11). This observation can be expressed as

$$S_{\rm pc}^{\rm (sca)}(\theta = 0^{\circ}) = S_{\rm fc}^{\rm (sca)}(\phi).$$
 (12)

The connection between fully coherent scattering and scattering with a J_0 -correlated fields that is expressed by Eq. (12) allows us to suppress the forward scattered intensity by "moving" a minimum of the scattering distribution to $\theta = 0^{\circ}$ by altering the coherence parameter β/k . To illustrate this, we return to the example of a sphere with radius $a = 50\lambda$ and refractive index n = 1.33 illuminated by a fully coherent field with wavelength $\lambda = 632.8$ nm, which was presented in Fig. 2. The first scattering minimum occurs at $\theta = 0.69^\circ$, where the normalized scattered intensity is 3.15×10^{-6} . Using Eq. (11) with $\cos \phi = \cos(0.69^\circ) = 0.999$ gives $\beta/k = 0.0121$. This implies that a J_0 Bessel-correlated field with this particular value of β/k will have a forward-scattered intensity that is almost 6 orders of magnitude less than its fully coherent counterpart. The intensity of the forward-scattered field as a function of the coherence parameter β/k is plotted in Fig. 4. We note that the value $\beta/k = 0$ corresponds to the fully coherent case. It is seen that, near $\beta/k = 0.0121$, the forward-scattered field is indeed strongly suppressed. In fact, the forward-scattered intensity is reduced by more than 5 orders of magnitude compared with the case of an incident field that is spatially fully coherent.

It was shown in [19] that the total scattered power remains constant when the coherence parameter β/k is varied. This means that the scattered intensity is merely redistributed. The precise form of the scattered field for the case $\beta/k =$ 0.0121 is shown in Fig. 5 (blue curve). The intensity of the forward-scattered field $S_{\rm pc}^{\rm (sca)}(\theta = 0^{\circ})$ is about 5×10^{-5} times smaller than the maximum that occurs at $\theta \approx 0.65^{\circ}$ (see inset). Note that the deep minima of Fig. 2 are no longer present.



Fig. 3. Illustrating the connection between the scattering angle ϕ and the coherence parameter β/k .



Fig. 4. Logarithmic plot of the forward-scattered intensity as function of the coherence parameter β/k . In this example, the wavelength $\lambda = 632.8$ nm, the sphere radius $a = 50\lambda$ and the refractive index n = 1.33.

If we plot the scattered intensity for another value of the refractive index, namely, n = 1.50, we see that the angular distribution becomes quite different (red curve), but the maximum occurs at precisely the same position; in fact, the two curves in the region shown in the inset are indistinguishable. Apparently, the angle θ_{max} at which the scattered intensity reaches its highest value is quite insensitive to the precise value of the refractive index. It is possible to explain this behavior by analyzing the relation between the coherence parameter β/k and θ_{max} in a simpler but related situation, namely, the scattering of J_0 -correlated light by a Gaussian random scatterer while using the first-order Born approximation. As explained in Appendix A, one then finds that

$$\sin \theta_{\rm max} \approx \beta/k.$$
 (13)

This equation implies that, to the first order, the angle θ_{max} at which the scattered intensity reaches its peak does not depend on the refractive index *n* or on the sphere radius *a* but only on the coherence parameter. In Fig. 6, the value of θ_{max} is plotted as a function of β/k for several values of the sphere radius *a*. It is seen that the agreement between the result of the Born approximation given by Eq. (13) (red curve) and a numerical evaluation of Eq. (8) for $a = 50\lambda$ (blue curve) is surprisingly good. It is only for small values of β/k that a



Fig. 5. Logarithmic plot of the angular distribution of the scattered intensity for a J_0 Bessel-correlated field with coherence parameter $\beta/k = 0.0121$, for two values of the refractive index: n = 1.33 (blue curve) and n = 1.50 (red curve). The inset shows the same, but for scattering angles up to 2° (non-logarithmic, with both curves normalized by their respective values for the fully coherent case). The other parameters in this example are the same as for Fig. 2.



Fig. 6. Angle θ_{max} , the angle at which the scattered field has its maximum intensity, as a function of the coherence parameter β/k for selected values of the sphere radius *a*. The result of the first-order Born approximation, Eq. (13), is given by the straight red curve. The other curves are obtained by numerical evaluation of Eq. (8). In all cases, the refractive index n = 1.33.

discrepancy is seen. This is can be understood as follows. Because the first zero of $J_0(x)$ is at x = 2.4, Eq. (7) implies that, as long as $\beta 2a < 2.4$, the function J_0 will be positive and the cross-spectral density function between all possible pairs of points within the scatterer will be qualitatively similar to a Gaussian. It is known from earlier studies [16] that, for such a correlation function, the scattering remains predominantly in the forward direction, i.e., $\theta_{max} = 0^{\circ}$. Only when $\beta 2a > 2.4$ will there be pairs of points that are negatively correlated, which gives rise to a qualitatively different scattering profile. For a sphere radius of 50 λ , this means that β/k must exceed 0.004 in order for any significant suppression of the forward-scattered intensity to occur. It is indeed seen that only for values somewhat larger than this threshold, the angle θ_{\max} is very well approximated by Eq. (13). For the three smaller spheres (corresponding to the orange, green, and purple curves) a similar result holds.

5. SUPPRESSION OF BACKWARD SCATTERING

Just as for the forward scattered-field, we find that the expression for the back-scattered intensity ($\theta = 180^{\circ}$), as given by Eq. (9), takes on a simpler form, namely

$$S_{\rm pc}^{\rm (sca)}(\theta = 180^{\circ}) = \frac{1}{r^2} \left| f(-\sqrt{1 - \beta^2/k^2}) \right|^2.$$
 (14)

This formula implies that, for an incident J_0 -correlated field with coherence parameter β/k , the backward-scattered intensity is equal to the intensity that is scattered in the fully coherent case in a direction $180^\circ - \phi$, with the angle ϕ defined by Eq. (11). Thus, we find that

$$S_{\rm pc}^{\rm (sca)}(\theta = 180^{\circ}) = S_{\rm fc}^{\rm (sca)}(180^{\circ} - \phi).$$
 (15)

Let us again return to the example of a sphere with radius $a = 50\lambda$ and refractive index n = 1.33, as shown in Fig. 2. From this plot, we find that the scattered field for the fully coherent case has an intensity minimum near $\theta = 179.62^{\circ}$ with a normalized value of 9.4×10^{-7} , whereas the backward-scattered intensity equals 1.7×10^{-5} . According to Eqs. (15) and (11), this minimum can be "moved" 0.38° to $\theta = 180^{\circ}$ by making



Fig. 7. Backward-scattered intensity as a function of the normalized coherence parameter β/k . Note that $\beta/k = 0$ corresponds to an incident field that is fully coherent. In this example, the wavelength $\lambda = 632.8$ nm, the sphere radius $a = 50\lambda$ and the refractive index n = 1.33.

the field partially coherent with $\beta/k = 6.5 \times 10^{-3}$. A plot of the backscattered intensity as a function of β/k is given in Fig. 7. It is indeed seen that $S_{\rm pc}^{\rm (sca)}(\theta = 180^{\circ})$ is strongly suppressed when β/k reaches this prescribed value. The backscattered intensity is now reduced to a mere 5.5% of that of the fully coherent case. This result is in exact agreement with the two intensities that were mentioned above.

6. DISCUSSION AND CONCLUSIONS

The practical generation of a J_0 Bessel-correlated beam has been reported in [22]. In that study, an optical diffuser was used to first obtain a spatially incoherent field that was passed through a thin annular aperture. Imaging this aperture with a lens then produces (according to the van Cittert–Zernike theorem [21]) a field with the desired correlation function. An alternative approach would be to use a spatial light modulator (SLM) to dynamically impart a J_0 Bessel correlation on the field.

In [22], the focusing of a J_0 -correlated beam was found to produce, instead of a maximum, an intensity minimum at the geometrical focus. Because of the similarity between focusing and scattering by a dielectric sphere, one can say with hindsight that suppression of the forward-scattered field is to be expected.

It is to be noted that a change in the scattering pattern implies a change in the force that is exerted on the sphere [5]. That means that control of the scattering direction can be used to dynamically vary the properties of an optical trap as reported in [22].

We also remark that the near-zero scattering in the forward direction that we obtain does not violate the optical theorem. This issue was addressed in [19].

In summary, we have investigated the scattering of a J_0 Bessel-correlated field by a dielectric sphere. Equations were derived that connect this situation with the scattering of a fully coherent field. These formulas were applied to design fields for which the forward- or the backward-scattered intensity is significantly reduced. Examples were presented that show a forwardscattering suppression of 5 orders of magnitude and a suppression of the back-scattered intensity by almost 2 orders of magnitude. An approximate expression for the angle at which the scattered field reaches its highest intensity was derived.

In contrast to earlier researches that aim at modifying Mie scattering, our approach is not based on changing the properties of the scattering object but rather those of the illuminating beam. Our results show that the use of spatial coherence offers a new tool to actively steer the scattered field.

APPENDIX A: DERIVATION OF EQ. (13)

In [18] the scattering of a J_0 Bessel-correlated field by a random sphere was examined within the accuracy of the first-order Born approximation. The correlation of the scattering potential is taken to be Gaussian, i.e.,

$$C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = C_0 \exp[-(\mathbf{r}'_2 - \mathbf{r}'_1)^2/2\sigma_F^2],$$
 (A1)

where C_0 is a positive constant, and σ_F denotes the coherence length of the scattering potential. It was then derived that

$$S^{(\text{sca})}(r\mathbf{s},\omega) = \frac{C_0}{r^2} \iint_V W^{(\text{inc})}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)$$
$$\times \exp[-(\mathbf{r}'_2 - \mathbf{r}'_1)^2/2\sigma_F^2]$$
$$\times \exp[-ik\mathbf{s}\cdot(\mathbf{r}'_2 - \mathbf{r}'_1)]d^3\mathbf{r}'_1d^3\mathbf{r}'_2, \qquad (A2)$$

where V is the volume of the scatterer, and $\mathbf{r}_{i'} = (\boldsymbol{\rho}'_i, z_{i'})$ with i = 1, 2. If we assume the field to be longitudinally coherent [Sec. 5.2.1 21], then

$$W^{(\text{inc})}(\mathbf{r}'_{1},\mathbf{r}'_{2},\omega) = e^{ik(z'_{2}-z'_{1})} J_{0}(\beta|\boldsymbol{\rho}'_{2}-\boldsymbol{\rho}'_{1}|), \quad (A3)$$

where the transverse part of the cross-spectral density of the incident field is taken from Eq. (7) with $S^{(0)}(\omega) = 1$. We next change to the sum and difference variables

$$\rho_+ = (\rho'_1 + \rho'_2)/2,$$
 (A4)

$$\rho_{-} = \rho_{2}' - \rho_{1}',$$
 (A5)

$$z_{+} = (z'_{1} + z'_{2})/2,$$
 (A6)

$$z_{-} = z_{2}' - z_{1}'$$
. (A7)

The Jacobian of this transformation is unity, and we find that

$$S^{(\text{sca})}(r\mathbf{s},\omega) = \frac{C_0}{r^2} \int dz_+ \iint d^2 \rho_+ \int e^{ikz_-(1-s_z)} e^{-z_-^2/2\sigma_F^2} dz_- \\ \times \iint J_0(\beta \rho_-) e^{-\rho_-^2/2\sigma_F^2} e^{ik\mathbf{s}_\perp \cdot \ast \ast \rho_-} d^2 \rho_-.$$
(A8)

The product of the first two integrals yields the scattering volume V. If we assume that σ_F is small compared with the size of the scatterer, then the integration over z_- can be extended to the entire real axis, and we obtain the result:

$$\int_{-\infty}^{\infty} e^{ikz_{-}(1-s_{z})} e^{-z_{-}^{2}/2\sigma_{F}^{2}} dz_{-} = \sqrt{2\pi}\sigma_{F} e^{-k^{2}\sigma_{F}^{2}(1-s_{z})^{2}/2}.$$
 (A9)

The remaining integral over ρ_{-} is seen to be a Fourier-Bessel transform, i.e.,

$$\iint J_0(\beta \rho_-) e^{-\rho_-^2/2\sigma_F^2} e^{\mathbf{i} k \mathbf{s}_\perp \cdot \rho_-} \mathrm{d}^2 \rho_-$$
 (A10)

$$= 2\pi \int_0^\infty J_0(\beta \rho_-) J_0(k | \mathbf{s}_\perp | \rho_-) e^{-\rho_-^2/2\sigma_F^2} \rho_- \mathrm{d}\rho_-.$$
 (A11)

The right-hand side of Eq. (A11) contains the product of two oscillating Bessel functions that, in general, will tend to cancel each other on integration. Therefore, we expect the integral and, hence, the total scattered field to reach its maximum value when the arguments of the two Bessel functions are identical, i.e., when

$$\beta = k|\mathbf{s}_{\perp}|, \qquad (A12)$$

and such a cancellation does not occur. On using that $|\mathbf{s}_{\perp}| = \sin \theta$, we thus find for θ_{\max} , the angle at which the scattered intensity is maximal, that

$$\sin \theta_{\rm max} = \beta/k, \tag{A13}$$

which is Eq. (13).

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