The role of correlation functions in the theory of optical wave fields

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We discuss the fundamental role of correlation functions in optical wave fields. These functions determine important properties such as the spectrum, the state of polarization, and the state of coherence of light. These properties generally change on propagation, even when the field travels through free space. Two sources which have the same spectrum and the same state of polarization can produce fields whose spectrum and state of polarization are different. These effects can be understood by considering certain correlation functions and the laws which govern their propagation. © *2008 American Association of Physics Teachers.*
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I. INTRODUCTION

The monochromatic wave fields that are treated in many textbooks are idealizations. All fields encountered in practice have some randomness associated with them. This randomness can be a small effect, as in the output of a well-stabilized, single-mode laser, or it can be large, as in the field produced by a thermal source. In between these two extremes is the field generated by a multimode laser. The random fluctuations can be characterized by means of correlation functions. Coherence theory is concerned with these functions and their many applications in diffraction, propagation, and the scattering of light.

One of the surprising predictions of coherence theory is that the spectrum of an optical field can change on propagation, even when the field travels through free space.[1] Two sources with identical spectra can generate fields whose spectra are different. Likewise, the state of polarization of light can also change as the field travels through free space.[2] It was shown not long ago that two sources with the same state of polarization (that is, with identical sets of Stokes parameters) can generate fields whose polarization properties are different.[3] These effects, most of which have been confirmed experimentally, can be understood by analyzing certain correlation functions that characterize the source and field fluctuations.

In this paper we review the role of correlation functions in optical fields and explain how basic properties of the field such as its spectral density, its state of polarization, and its state of coherence are determined by these functions. As will be discussed, the correlation functions satisfy precise propagation laws. Their change on propagation implies that secondary field properties that are determined by them, such as the spectrum, can also change as the field propagates.

Although several monographs[4–10] and articles[11–13] are devoted to optical coherence, the importance of correlation functions in optics seems to be poorly appreciated by most scientists. Our hope is that a summary of the principal results of coherence theory will lead to its greater understanding. We discuss scalar fields as well as electromagnetic beams.

II. CORRELATION FUNCTIONS FOR SCALAR FIELDS

Consider a random, scalar wave field $V(\mathbf{r},t)$, where $\mathbf{r}$ is the position vector of a point in space, and t the time. In free space the field satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)V(\mathbf{r},t) = 0, \tag{1}$$

where c is the speed of light in vacuum. It follows from Eq. (1) that the the Fourier transform with respect to time

$$U(\mathbf{r},\omega) = \int_{-\infty}^{\infty} V(\mathbf{r},t)e^{i\omega t}\mathrm{d}t, \tag{2}$$

satisfies the Helmholtz equation,

$$(\nabla^2 + k^2)U(\mathbf{r},\omega) = 0, \tag{3}$$

where $k=\omega/c$ is the wavenumber associated with the angular frequency ω. The field is complex valued because we are using the analytic signal representation (Ref. 9, Sec. 3.1).

The lowest-order correlations of the field may be described by *the mutual coherence function*, which, for stationary fields is defined as (Ref. 9, Sec. 4.3.1)

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau) = \langle V^*(\mathbf{r}_1,t)V(\mathbf{r}_2,t+\tau)\rangle. \tag{4}$$

The angular brackets denote an ensemble average, and the asterisk denotes the complex conjugate. It is useful to normalize the mutual coherence function by introducing the *complex degree of coherence*

$$\gamma(\mathbf{r}_1,\mathbf{r}_2;\tau) = \frac{\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)}{\sqrt{I(\mathbf{r}_1)I(\mathbf{r}_2)}}, \tag{5}$$

where

$$I(\mathbf{r}) = \Gamma(\mathbf{r},\mathbf{r};0) = \langle V^*(\mathbf{r},t)V(\mathbf{r},t)\rangle \tag{6}$$

is the average intensity at position $\mathbf{r}$. The meaning of the function $\gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)$ can be seen from the following example.[14–16] Consider Young's experiment in which two pinholes located at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ in an opaque screen are illuminated with light of equal intensity. The light emanating from the pinholes interferes on an observation screen. It can be shown that the visibility of the fringes that are formed is equal to $|\gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)|$, where τ is the time difference between the light traveling from the two pinholes to a particular fringe.

For many applications it is advantageous to work in the space-frequency domain by using the *cross-spectral density function*, the temporal Fourier transform of the mutual coherence function:

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)=\int_{-\infty}^{\infty}\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)e^{i\omega\tau}\mathrm{d}\tau. \tag{7}$$

We can show that $W(\mathbf{r}_1,\mathbf{r}_2;\omega)$, just like the mutual coherence function, is a correlation function (Ref. 9, Sec. 4.7.2), that is,

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)=\langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle, \tag{8}$$

where $U(\mathbf{r},\omega)$ is a member of an ensemble of monochromatic realizations of the field (which differ in phase and amplitude). Often it is useful to consider a normalized version of W, the *spectral degree of coherence*, which is given by

$$\mu(\mathbf{r}_1,\mathbf{r}_2;\omega)=\frac{W(\mathbf{r}_1,\mathbf{r}_2;\omega)}{\sqrt{S(\mathbf{r}_1,\omega)S(\mathbf{r}_2,\omega)}}, \tag{9}$$

where

$$S(\mathbf{r},\omega)=W(\mathbf{r},\mathbf{r};\omega) \tag{10}$$

is the *spectral density* (or the intensity at frequency ω) at position $\mathbf{r}$. It can be shown that the spectral degree of coherence is bounded (Ref. 9, Sec. 4.3.2):

$$0\le|\mu(\mathbf{r}_1,\mathbf{r}_2;\omega)|\le 1. \tag{11}$$

The lower bound occurs when the field at the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is completely uncorrelated (at frequency ω). The upper bound occurs when the field is fully correlated. For intermediate values the field is said to be partially coherent. Just like the complex degree of coherence, the spectral degree of coherence can be determined by Young's interference experiment but now with filters in front of the pinholes.[17]

Modern coherence theory began in 1954 when Wolf found that the mutual coherence function in free space satisfies the wave equations:[15,16]

$$\left(\nabla_1^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Gamma(\mathbf{r}_1,\mathbf{r}_2;\omega)=0, \tag{12a}$$

$$\left(\nabla_2^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Gamma(\mathbf{r}_1,\mathbf{r}_2;\omega)=0, \tag{12b}$$

where ∇_1^2 and ∇_2^2 represent the Laplace operator acting on $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Hence not only the field but also the mutual coherence function satisfies rigorous propagation laws. From Eq. (12) it follows that in free space the cross-spectral density satisfies the Helmholtz equations

$$(\nabla_1^2+k^2)W(\mathbf{r}_1,\mathbf{r}_2;\omega)=0, \tag{13a}$$

$$(\nabla_2^2+k^2)W(\mathbf{r}_1,\mathbf{r}_2;\omega)=0, \tag{13b}$$

implying that this correlation function also has a wave-like character. As will become evident, this observation has far-reaching consequences. As an example, consider a planar, secondary source located in the plane $z=0$ that radiates a stochastic field into the half-space $z>0$. The source is assumed to be stationary, at least in the wide sense (Ref. 9, Sec. 2.2). By using the first Rayleigh diffraction formula[18] we obtain for the cross-spectral density at a pair of observation points the relation (Ref. 9, Sec. 4.4.2)

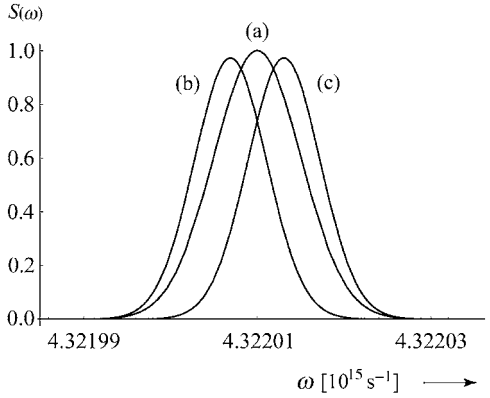

Fig. 1. The field spectrum generated by two identical, correlated point sources. (a) The spectral density of each point source. (b) The observed redshifted field spectrum. (c) The observed blueshifted field spectrum. For details see Ref. 23.

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)=\frac{1}{(2\pi)^2}\int\int_{(z_1',z_2'=0)}W^{(0)}(\mathbf{r}_1',\mathbf{r}_2';\omega)\times\left[\frac{\partial}{\partial z_1}\left(\frac{e^{-ikR_{11}}}{R_{11}}\right)\right]\times\left[\frac{\partial}{\partial z_2}\left(\frac{e^{ikR_{22}}}{R_{22}}\right)\right]\mathrm{d}^2r_1'\mathrm{d}^2r_2', \tag{14}$$

where $R_{11}=|\mathbf{r}_1-\mathbf{r}_1'|$, $R_{22}=|\mathbf{r}_2-\mathbf{r}_2'|$, and

$$W^{(0)}(\mathbf{r}_1',\mathbf{r}_2';\omega)=\langle U^*(x_1',y_1',0;\omega)U(x_2',y_2',0;\omega)\rangle \tag{15}$$

is the cross-spectral density across the source plane. Equation (14) shows that knowledge of $W^{(0)}(\mathbf{r}_1',\mathbf{r}_2';\omega)$, the cross-spectral density in the source plane which can be deduced from Young's experiment, allows in principle the calculation of the cross-spectral density function everywhere in the half-space $z>0$.

According to Eqs. (10) and (14) the spectral density of the field in the half-space $z>0$ is given by

$$S(\mathbf{r},\omega)=\frac{1}{(2\pi)^2}\int\int_{(z_1',z_2'=0)}W^{(0)}(\mathbf{r}_1',\mathbf{r}_2';\omega)\left[\frac{\partial}{\partial z_1}\left(\frac{e^{-ikR_1}}{R_1}\right)\right]\times\left[\frac{\partial}{\partial z_2}\left(\frac{e^{ikR_2}}{R_2}\right)\right]\mathrm{d}^2r_1'\mathrm{d}^2r_2', \tag{16}$$

where $R_1=|\mathbf{r}-\mathbf{r}_1'|$ and $R_2=|\mathbf{r}-\mathbf{r}_2'|$.

We can now draw several important conclusions: (1) According to Eq. (14), the cross-spectral density function generally changes on propagation, even through free space; (2) As the cross-spectral density changes, the spectral density as given by Eq. (16) will generally also change; (3) Two sources with different cross-spectral densities can have identical spectra. However, the spectra of the fields that they produce may be quite different.

An illustration of the second conclusion is shown in Fig. 1. There the spectral density of two identical, correlated point sources is plotted [curve (a)]. This curve is also the spectral density of the field that is observed when the two sources are completely uncorrelated. If the two sources are partially coherent, then, depending on how the spectral degree of coherence varies with frequency, the spectral density of the field can, for example, be redshifted [curve (b)], or

blueshifted [curve (c)]. This prediction was verified in several experiments.[19,20] A review of such coherence-induced spectral changes was given Ref. 21.

An illustration of the third conclusion is provided by planar Gaussian Schell-model sources (Ref. 9, Sec. 5.4.2). Let us represent the location of a point in the source plane $z=0$ by a two-dimensional position vector $\boldsymbol{\rho}$. For Gaussian Schell-model sources the spectral degree of coherence depends on the positions $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ only through the difference $\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$:

$$W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)=\sqrt{S^{(0)}(\boldsymbol{\rho}_1,\omega)S^{(0)}(\boldsymbol{\rho}_2,\omega)}\,\mu^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega). \tag{17}$$

Also, the spectral density and the spectral degree of coherence at each frequency both have a Gaussian dependence on position:

$$S^{(0)}(\boldsymbol{\rho},\omega)=C^2e^{-\boldsymbol{\rho}^2/2\sigma^2}, \tag{18}$$

$$\mu^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega)=e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta^2}. \tag{19}$$

Here C, σ, and δ are positive constants that may depend on frequency. Consider two such sources, A and B, for which

$$W_A^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)=C^2e^{-(\boldsymbol{\rho}_1^2+\boldsymbol{\rho}_2^2)/4\sigma^2}e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta_A^2}, \tag{20a}$$

$$W_B^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)=C^2e^{-(\boldsymbol{\rho}_1^2+\boldsymbol{\rho}_2^2)/4\sigma^2}e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta_B^2}. \tag{20b}$$

We assume that $\delta_A\neq\delta_B$. It is immediately seen from Eqs. (10) and (20) that the spatial variations of the spectra across the two sources are identical, that is, $S_A^{(0)}(\boldsymbol{\rho},\omega)=S_B^{(0)}(\boldsymbol{\rho},\omega)=C^2e^{-\boldsymbol{\rho}^2/2\sigma^2}$, even though their state of coherence is different. It is evident from Eq. (16) that, therefore, these two sources may produce fields whose spectra are quite different. Stated differently, knowledge of the source spectrum does not suffice to determine the spectrum of the field in the half space into which the source radiates.

III. CORRELATION FUNCTIONS FOR ELECTROMAGNETIC BEAMS

Thus far we have considered scalar fields, but the concept of correlation functions can be generalized to electromagnetic beams. For such fields both the electric and the magnetic vector are, to a good approximation, perpendicular to the direction of propagation.

Consider a planar, secondary source that generates a stochastic, electromagnetic beam which propagates into the half-space $z>0$ in a direction close to the positive z axis. We assume that the source is statistically stationary, at least in the wide sense. The *electric cross-spectral density matrix*, which may be used to characterize the state of coherence and the state of polarization of the beam in the source plane $z=0$ is defined as

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)=\begin{pmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)\\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)\end{pmatrix}, \tag{21}$$

where

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)=\langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle,\quad (i,j=x,y). \tag{22}$$

Here $E_i(\boldsymbol{\rho},\omega)$ is a Cartesian component of the (complex) electric field in the x or y direction of a typical member of the ensemble that represents the field. The cross-spectral density matrix elements of the field at an arbitrary pair of observation points in the half-space $z>0$ are given by a similar expression:

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle,\quad (i,j=x,y). \tag{23}$$

The propagation of the electric field components is governed by (Ref. 9, Sec. 5.6.1)

$$E_i(\mathbf{r},\omega)=\int_{(z'=0)}E_i(\boldsymbol{\rho}',0;\omega)G(\boldsymbol{\rho}-\boldsymbol{\rho}',z;\omega)\mathrm{d}^2\rho', \tag{24}$$

with G denoting the Green's function for paraxial propagation from the point $Q(\boldsymbol{\rho}',0)$ in the source plane $z=0$ to the field point $P(\boldsymbol{\rho},z)$:

$$G(\boldsymbol{\rho}-\boldsymbol{\rho}',z;\omega)=-\frac{ik}{2\pi z}\exp(ikz)\exp[ik|\boldsymbol{\rho}-\boldsymbol{\rho}'|^2/2z]. \tag{25}$$

We substitute from Eq. (24) into Eq. (23) and find that the electric cross-spectral density matrix of the beam at a pair of observation points is given by

$$\mathbf{W}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\int\int_{(z_1'z_2'=0)}\mathbf{W}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)\times K(\boldsymbol{\rho}_1-\boldsymbol{\rho}_1',z_1,\boldsymbol{\rho}_2-\boldsymbol{\rho}_2',z_2;\omega)\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2', \tag{26}$$

where

$$K(\boldsymbol{\rho}_1-\boldsymbol{\rho}_1',z_1,\boldsymbol{\rho}_2-\boldsymbol{\rho}_2',z_2;\omega)=G^*(\boldsymbol{\rho}_1-\boldsymbol{\rho}_1',z_1;\omega)G(\boldsymbol{\rho}_2-\boldsymbol{\rho}_2',z_2;\omega). \tag{27}$$

From knowledge of the cross-spectral density matrix across the plane $z=0$, as can be obtained from straightforward interference experiments,[22] the cross-spectral density matrix in the half-space $z>0$ can be calculated by use of Eq. (26). Just as in the scalar case, the fundamental properties of the field are all determined by the (propagated) electric cross-spectral density matrix.

It is seen from Eq. (23) that the trace of the cross-spectral density matrix, $\mathrm{Tr}\,\mathbf{W}(\mathbf{r},\mathbf{r};\omega)$, is proportional to the electric energy density at the point $P(\mathbf{r})$. The spectral density of the field can therefore, apart from a prefactor, be identified as

$$S(\mathbf{r},\omega)=\mathrm{Tr}\,\mathbf{W}(\mathbf{r},\mathbf{r};\omega). \tag{28}$$

The *spectral degree of polarization* of the field at any point is defined as the ratio of the intensity of the polarized part of the beam to the total beam intensity and is given by (Ref. 9, Sec. 6.3.3)

$$\mathcal{P}(\mathbf{r},\omega)=\sqrt{1-\frac{4\det\mathbf{W}(\mathbf{r},\mathbf{r};\omega)}{[\mathrm{Tr}\,\mathbf{W}(\mathbf{r},\mathbf{r};\omega)]^2}}, \tag{29}$$

where det denotes the determinant. It can be shown that $0\leq\mathcal{P}(\mathbf{r},\omega)\leq1$. The upper bound represents a completely polarized field, and the lower bound represents a completely unpolarized field. For intermediate values the field is said to be partially polarized.

The polarized part of the beam can also be characterized by the *spectral Stokes parameters* (Ref. 10, Sec. 9.5). These parameters can be expressed as linear combinations of the elements of the electric cross-spectral density matrix, that is,

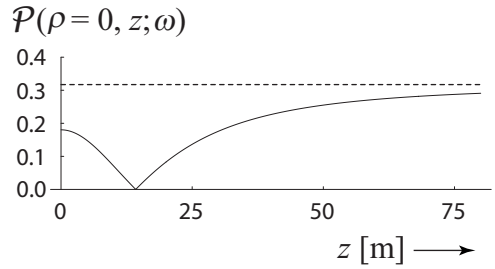

Fig. 2. The on-axis spectral degree of polarization $\mathcal{P}(\boldsymbol{\rho}=0,z;\omega)$ of a stochastic electromagnetic beam as a function of the propagation distance z (solid curve). The asymptotic value is indicated by the horizontal dashed line. For details see Ref. 24.

$$s_0(\mathbf{r},\omega) = W_{xx}(\mathbf{r},\mathbf{r};\omega) + W_{yy}(\mathbf{r},\mathbf{r};\omega), \tag{30a}$$

$$s_1(\mathbf{r},\omega) = W_{xx}(\mathbf{r},\mathbf{r};\omega) - W_{yy}(\mathbf{r},\mathbf{r};\omega), \tag{30b}$$

$$s_2(\mathbf{r},\omega) = W_{xy}(\mathbf{r},\mathbf{r};\omega) + W_{yx}(\mathbf{r},\mathbf{r};\omega), \tag{30c}$$

$$s_3(\mathbf{r},\omega) = i[W_{yx}(\mathbf{r},\mathbf{r};\omega) - W_{xy}(\mathbf{r},\mathbf{r};\omega)]. \tag{30d}$$

From these considerations we conclude: (4) according to Eq. (26) the cross-spectral density matrix generally changes on propagation, even if the beam propagates through free space. (5) As the cross-spectral density matrix changes, we may expect the quantities that are determined by it, namely the spectral density, the state of coherence, the degree of polarization, and the Stokes parameters of the beam [as given by Eqs. (28)–(30)] to change as well as the beam propagates. (6) Two sources with different cross-spectral density matrices can have spectra, degrees of polarization, and sets of Stokes parameters that are identical. However, the properties of the fields that they generate may be different.

An illustration of the fifth conclusion is provided by Fig. 2. It shows the changes in the state of polarization as the beam propagates. In this example the degree of polarization of the field on the axis of a stochastic beam first decreases to zero, and then gradually increases to an asymptotic value that is higher than the value at the source itself.

The sixth conclusion is exemplified in a manner that is similar to the scalar case. For electromagnetic Gaussian Schell-model sources each element of the cross-spectral density matrix can be expressed in the form

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}_1,\omega)S_j^{(0)}(\boldsymbol{\rho}_2,\omega)}\,\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega), \quad (i,j=x,y), \tag{31}$$

where

$$S_i^{(0)}(\boldsymbol{\rho},\omega) = C_i^2 e^{-\boldsymbol{\rho}^2/2\sigma^2}, \tag{32}$$

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega) = D_{ij}e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta_{ij}^2}. \tag{33}$$

Here C_i, D_{ij}, σ, and δ_{ij} are positive constants that may depend on frequency, and satisfy the constraints

$$D_{ij} = 1 \quad \text{if } i=j, \tag{34a}$$

$$|D_{ij}| \le 1 \quad \text{if } i \ne j. \tag{34b}$$

Additional constraints are mentioned in Ref. 10, Sec. 9.4.2.

Consider two such sources, A and B, for which

$$[W_{ij}^{(0)}]_A(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = C_iC_je^{-(\boldsymbol{\rho}_1^2+\boldsymbol{\rho}_2^2)/4\sigma^2}D_{ij}e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta_{ij,A}^2}, \tag{35a}$$

$$[W_{ij}^{(0)}]_B(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = C_iC_je^{-(\boldsymbol{\rho}_1^2+\boldsymbol{\rho}_2^2)/4\sigma^2}D_{ij}e^{-(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2\delta_{ij,B}^2}. \tag{35b}$$

Let us assume that $\delta_{ij,A} \ne \delta_{ij,B}$. It is seen from Eqs. (28)–(30) that the spectrum of these two sources and their state of polarization are identical, even though their state of coherence is different. However, these sources may produce beams whose properties are different. An example of this kind of behavior is presented in Ref. 3. Thus it follows that knowledge of the spectrum and the state of polarization of a source is not sufficient to determine these properties of the field in the half space into which the source radiates.

IV. SUMMARY AND CONCLUSIONS

We have discussed the central role played by correlation functions in optical wave fields. These functions can be measured by means of interference experiments. An indication of their importance is that they cannot be derived from the spectrum or the state of polarization of the field. Correlation functions are two-point functions, that is, they characterize the statistical similarity[10] of the field at two points in space. Because the spectrum, the state of polarization, and the degree of polarization are all one-point functions, the correlation functions can in general not be derived from them. In other words, two sources with different coherence properties may have identical spectra and identical polarization properties.

The correlation functions determine basic field properties such as the spectrum, the state of coherence, and the state of polarization. Because the correlation functions obey precise propagation laws, the field properties that are derived from them, generally all change on propagation, even when the field travels through free space. Several examples of such changes were presented. We noted that knowledge of the source spectrum and of its state of polarization is not sufficient to calculate these properties in the region into which the source radiates. Only by calculating the correlation function of the propagated field in the region of observation can the spectrum and the state of polarization there be determined.

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YOU JUST LOOK AT THE THING!

What are the most central and fundamental problems of biology today? They are questions like: What is the sequence of bases in the DNA? What happens when you have a mutation? How is the base order in the DNA connected to the order of amino acids in the protein? What is the structure of the RNA; is it single-chain or double-chain, and how is it related in its order of bases to the DNA? What is the organization of the microsomes? How are proteins synthesized? Where does the RNA go? How does it sit? Where do the proteins sit? Where do the amino acids go in? In photosynthesis, where is the chlorophyll; how is it arranged; where are the carotenoids involved in this thing? What is the system of the conversion of light into chemical energy?

It is very easy to answer many of these fundamental biological questions; you just look at the thing! You will see the order of bases in the chain; you will see the structure of the microsome. Unfortunately, the present microscope sees at a scale which is just a bit too crude. Make the microscope one hundred times more powerful, and many problems of biology would be made very much easier. I exaggerate, of course, but the biologists would surely be very thankful to you—and they would prefer that to the criticism that they should use more mathematics.

Richard P. Feynman, "There's Plenty of Room at the Bottom: An Invitation to Enter a New Field of Physics," Engineering & Science, February 1960. Presented at the annual meeting of the American Physical Society, 29 December 1959. Full transcript available at <www.zyvex.com/nanotech/feynman.html>.