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# Application of the Hanbury Brown–Twiss effect to scattering from quasi-homogeneous media

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## 1. Introduction

Up to now the only application of the famous Hanbury Brown-Twiss effect [1–4] in the optical region of the electromagnetic spectrum has been the determination of angular diameters of stars. The effect is based on the surprising discovery that when a light beam illuminates two photo-detectors, the photo-electrons ejected from them are, in general, correlated and the correlation is proportional to the correlation between the intensity fluctuations of the light incident on them. In more recent years such intensityintensity correlation (or particle–particle correlation) experiments have been successfully used in fields such as nuclear physics [5], atomic physics [6], condensed matter physics [7], and in the characterization of rough surfaces [8].

A novel use of the Hanbury Brown–Twiss effect in optics was recently proposed by Xin et al. [9] who studied correlations between the intensity fluctuations in light scattered by so-called quasi-homogeneous random media. Shortly afterwards, Jacks and Korotkova [10] extended the analysis of [9] to a wider class of incident fields.

Quasi-homogeneous media are characterized by the property that the strength of their scattering potential  $S_F(\mathbf{r}, \omega)$  at a particular frequency  $\omega$  varies much more slowly with position than

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ABSTRACT

The scattering from a wide class of random scatterers, so-called quasi homogeneous scattering media, is studied by the use of the Hanbury Brown–Twiss effect. In particular the two-point correlation of intensity fluctuations and their variance in the far field are analyzed. A new reciprocity relation is derived, and expressions for the correlation of intensity fluctuations for several different types of scattering potentials are obtained. The results indicate the possibility of distinguishing, for example, hollow scatterers from solid ones.

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the correlation coefficient  $\mu_F(\mathbf{r}_1,\mathbf{r}_2,\omega) = \mu_F(\mathbf{r}_2-\mathbf{r}_1,\omega)$  varies with the position difference  $\mathbf{r}_2-\mathbf{r}_1$  [4, Section 6.3.3]. Examples of such media are the atmosphere, and confined plasmas.

Quasi-homogeneous scatterers have been the subject of several studies. It has been shown that, within the accuracy of the first-order Born approximation, the spectral density of the scattered field in the far zone is proportional to the Fourier transform of the correlation coefficient of the scatterer; and that the spectral degree of coherence of the scattered field in the far zone is proportional to the Fourier transform of the strength of the scattering potential [11]. These two reciprocity relations can be applied to study certain inverse problems [12–15]. Reciprocity relations pertaining to the scattering of light generated by quasihomogeneous sources by quasi-homogeneous media were presented in [16]. However, unlike Ref. [9] which deals with fourthorder (intensity) correlations, all these studies are concerned with second-order (field) correlations.

In the present paper we use the approach of Xin et al. to study the scattering of a monochromatic, plane wave by a quasihomogeneous medium. We analyze the correlation between the intensity fluctuations of the scattered field and their variance. We derive a new reciprocity relation, and we also investigate the correlation of intensity fluctuations produced by several types of scattering potentials.

We begin, in Section 2, with a brief discussion of the intensity– intensity correlations that occur when a monochromatic plane wave is scattered by a quasi-homogeneous medium. In the third

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section two fourth-order reciprocity relations are discussed. In the fourth section analytic expressions for the intensity correlations for different types of scattering strengths are derived. We find that for hollow scatterers such correlations differ significantly from those for solid scatterers.

#### 2. Scattering from quasi-homogeneous media

Consider a monochromatic, plane wave of frequency  $\omega$ , with space-dependent part

$$U^{(1)}(\mathbf{r},\omega) = a(\omega)e^{i\mathbf{k}\mathbf{s}_{0}\cdot\mathbf{r}}$$
(1)

(a time-dependent part  $\exp(-i\omega t)$  being understood) which is incident on a quasi-homogeneous scatterer. Here **r** denotes a position vector of a point in space,  $a(\omega)$  is a (generally complexvalued) amplitude, **s**<sub>0</sub> is a real unit vector in the direction of incidence, and  $k = |\mathbf{k}| = \omega/c$ , with *c* being the speed of light in vacuum, is the wavenumber associated with frequency  $\omega$ . On making use of the first-order Born approximation, the scattered field is given by the expression [17, Section 13.1]

$$U^{(s)}(\mathbf{r},\omega) = \int_{D} U^{(i)}(\mathbf{r}',\omega) F(\mathbf{r}',\omega) G(\mathbf{r},\mathbf{r}',\omega) \, \mathrm{d}^{3}r', \qquad (2)$$

where D denotes the domain occupied by the scatterer (see Fig. 1), and the scattering potential

$$F(\mathbf{r},\omega) = \frac{k^2}{4\pi} [n^2(\mathbf{r},\omega) - 1], \qquad (3)$$

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with  $n(\mathbf{r},\omega)$  the index of refraction of the scattering medium. Furthermore,

$$G(\mathbf{r},\mathbf{r}',\omega) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$
(4)

is the outgoing free-space Green's function of the Helmholtz operator. Sufficiently far away from the scatterer:

$$G(r\mathbf{u},\mathbf{r}',\omega) \sim \frac{e^{ikr}}{r} e^{-ik\mathbf{u}\cdot\mathbf{r}'} \quad (kr \to \infty, \text{ with } \mathbf{u} \text{ kept fixed})$$
(5)

and  $|\mathbf{u}| = 1$ . From now on we no longer display the  $\omega$ -dependence of the various quantities.

The correlation function of the scattering potential is given by the expression:

$$C_F(\mathbf{r}_1, \mathbf{r}_2) = \langle F^*(\mathbf{r}_1) F(\mathbf{r}_2) \rangle, \tag{6}$$

where the angular brackets indicate the average, taken over an ensemble of realizations of the scatterer. The normalized correlation coefficient of the scattering potential is defined by the formula:

$$\mu_F(\mathbf{r}_1, \mathbf{r}_2) = \frac{C_F(\mathbf{r}_1, \mathbf{r}_2)}{\sqrt{S_F(\mathbf{r}_1)S_F(\mathbf{r}_2)}},\tag{7}$$

with the strength of the scattering potential

$$S_F(\mathbf{r}) = C_F(\mathbf{r}, \mathbf{r}). \tag{8}$$

The spectral density (intensity at frequency  $\omega$ ) of the scattered field at a point **r** = *r***u** in the far zone is given by the formula [18,



**Fig. 1.** A plane wave incident on a quasi-homogeneous scatterer occupying a volume *D*. The unit vectors  $\mathbf{s}_0$  and  $\mathbf{u}$  indicate the directions of incidence and of scattering, respectively.

Section 4.3.2]

$$S^{(s)}(r\mathbf{u}) = \langle U^{(s)*}(r\mathbf{u})U^{(s)}(r\mathbf{u}) \rangle.$$
(9)

The intensity fluctuation of the scattered field at a point specified by the position vector  $r\mathbf{u}$  is defined as

$$\Delta I^{(s)}(\mathbf{r}\mathbf{u}) = I^{(s)}(\mathbf{r}\mathbf{u}) - S^{(s)}(\mathbf{r}\mathbf{u}),\tag{10}$$

with  $I^{(s)}(\mathbf{ru})$  denoting the intensity of the scattered field in the far zone due to a single realization of the scatterer, i.e.,

$$I^{(s)}(r\mathbf{u}) = \frac{|a|^2}{r^2} \iint_D e^{-ik(\mathbf{s}_0 - \mathbf{u}) \cdot (\mathbf{r}' - \mathbf{r}'')} F^*(\mathbf{r}') F(\mathbf{r}'') \, \mathrm{d}^3 r' \, \mathrm{d}^3 r''.$$
(11)

Here we have made use of Eqs. (1), (2) and (5). The correlation of the intensity fluctuations at a pair of points  $r_1$ **u**<sub>1</sub> and  $r_2$ **u**<sub>2</sub> in the far zone is defined by the formula:

$$D(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \langle \Delta I^{(s)}(r_1\mathbf{u}_1)\Delta I^{(s)}(r_2\mathbf{u}_2) \rangle.$$
(12)

In the remainder we will examine the properties of this correlation function.

#### 3. Two reciprocity relations

In Appendix A it is shown that, under the assumption that the fluctuations of the scattering potential are governed by Gaussian statistics, the correlation of the intensity fluctuations of light scattered by a quasi-homogeneous medium can be expressed as

$$D(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{|a|^4}{r_1^2 r_2^2} |\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]\tilde{\mu}_F[k(\mathbf{s}_0 - (\mathbf{u}_1 + \mathbf{u}_2)/2)]|^2,$$
(13)

where  $\tilde{S}_F[\mathbf{K}]$  and  $\tilde{\mu}_F[\mathbf{K}]$  denote the three-dimensional spatial Fourier transforms of the strength and of the correlation coefficient of the scattering potential, respectively. We define the normalized correlation function of the intensity correlations of the scattered field in the far zone by the following expression:

$$\mathcal{I}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{D(r_1\mathbf{u}_1, r_2\mathbf{u}_2)}{S^{(s)}(r_1\mathbf{u}_1)S^{(s)}(r_2\mathbf{u}_2)}.$$
(14)

The spectral density of the scattered field and the normalized correlation function of the scattering potential are related by a second-order reciprocity relation [11,16], namely

$$S^{(s)}(r\mathbf{u}) = \frac{|a|^2 \tilde{S}_F(0)}{r^2} \tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u})].$$
(15)

On making use of Eq. (15) it follows that

$$\mathcal{I}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{\left|\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]\right|^2}{\tilde{S}_F^2(0)} \times \frac{\tilde{\mu}_F^2[k(\mathbf{s}_0 - (\mathbf{u}_1 + \mathbf{u}_2)/2)]}{\tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u}_1)]\tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u}_2)]}.$$
 (16)

In deriving Eq. (16) we made use of the fact that  $\tilde{\mu}_F(\mathbf{K})$  is realvalued (see Appendix A). For quasi-homogeneous scatterers  $\mu_F(\mathbf{r})$ is a "fast" function of its argument, and hence its Fourier transform  $\tilde{\mu}_F(\mathbf{K})$  is a "slow" function. We have, therefore, to a good approximation

$$\tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u}_1)] \approx \tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u}_2)],\tag{17}$$

$$\approx \tilde{\mu}_F[k(\mathbf{s}_0 - (\mathbf{u}_1 + \mathbf{u}_2)/2)]. \tag{18}$$

On making use of these approximations in Eq. (16) we find that the normalized correlation coefficient of the intensity fluctuations at the two points  $r_1$ **u**<sub>1</sub> and  $r_2$ **u**<sub>2</sub> is given by the expression:

$$\mathcal{I}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{\left|\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]\right|^2}{\tilde{S}_F^2(\mathbf{0})}.$$
(19)

Eq. (19) is a reciprocity relation, in agreement with a result derived by Xin et al. [9], which asserts that the normalized correlation coefficient of the intensity fluctuations of the scattered field in the far zone of a quasi-homogeneous scatterer with Gaussian statistics is proportional to the squared modulus of the Fourier transform of the strength of the scattering potential.

Formula (19), which is a fourth-order result, can be cast in a different form with the help of another of the aforementioned second-order reciprocity relations, namely [11,16]

$$\mu^{(s)}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]}{\tilde{S}_F(0)} \exp[ik(r_2 - r_1)].$$
(20)

This formula shows that the spectral degree of coherence of the scattered field in the far zone of a quasi-homogeneous scatterer with Gaussian statistics is proportional to the Fourier transform of the normalized strength of the scattering potential. On substituting from Eq. (20) into Eq. (19) one finds at once that

$$\mathcal{I}(r_1 \mathbf{u}_1, r_2 \mathbf{u}_2) = |\mu^{(s)}(r_1 \mathbf{u}_1, r_2 \mathbf{u}_2)|^2.$$
(21)

Eq. (21) shows that the normalized correlation coefficient of the intensity fluctuations of the scattered field in the far zone of a quasi-homogeneous scatterer with Gaussian statistics is proportional to the squared modulus of the spectral degree of coherence of the scattered field. Also, since the modulus of the spectral degree of coherence is bounded by zero and by unity [18, Section 4.3.2], it follows from Eq. (21) that

$$0 \le \mathcal{I}(r_1 \mathbf{u}_1, r_2 \mathbf{u}_2) \le 1. \tag{22}$$

The variance  $V(r\mathbf{u})$  of the intensity fluctuations at a point  $r\mathbf{u}$  in the far zone is given by the "diagonal elements" of the correlation of the intensity fluctuations, viz.

$$V(r\mathbf{u}) \equiv D(r\mathbf{u}, r\mathbf{u}),\tag{23}$$

$$=\frac{|a|^4}{r^4}\tilde{S}_F^2(0)\tilde{\mu}_F^2[k(\mathbf{s}_0-\mathbf{u})],$$
(24)

where we have used Eq. (13). Expression (24) brings into evidence a second reciprocity relation, which may be stated as follows: The variance of the intensity fluctuations of the scattered field in the far zone of a quasi-homogeneous scatterer with Gaussian statistics is proportional to the square of the Fourier transform of the correlation coefficient of the scattering potential.

One particular result that can be immediately obtained from this reciprocity relation and from the expression for the spectral density equation (15) is that the value of the scintillation index [20] of the scattered field in the far zone is unity:

$$\gamma(\mathbf{r}\mathbf{u}) \equiv \sqrt{\frac{V(\mathbf{r}\mathbf{u})}{\left[S^{(s)}(\mathbf{r}\mathbf{u})\right]^2}} = 1.$$
(25)

More generally, it can be shown from Eqs. (A.1)–(A.6) that the assumption of quasi-homogeneity is not needed to obtain this result. All that is needed is that the scatterer obeys Gaussian statistics. Hence we find that the scintillation index of the far zone field generated by weak scattering of a monochromatic plane wave on a random medium with Gaussian statistics equals unity.

## 4. Examples of correlations of intensity fluctuations

We will now examine the implications of Eq. (19) for scattering from several different kinds of the strength  $S_F(\mathbf{r})$  of the scattering potential. Three examples will be discussed, namely (a) a scattering strength given by a Gaussian function, (b) one given by an exponential function, and (c) one described by a shifted Gaussian function (see Fig. 2). Whilst the first two examples represent solid media, the last one, the shifted Gaussian distribution, represents a hollow scatterer.



**Fig. 2.** Examples of different forms of the strength of the scattering potential  $S_F(\mathbf{r})$ , namely: a Gaussian function (a), an exponential function (b), and a shifted Gaussian function (c).



Fig. 3. Two directions of scattering,  $u_1$  and  $u_2$ , that are located symmetrically with respect to the direction of incidence  $s_0$ .



**Fig. 4.** The normalized correlation coefficient  $\mathcal{I}(\mathbf{ru}_1,\mathbf{ru}_2)$  of the intensity fluctuations for a scatterer with a Gaussian distribution of the scattering strength, as a function of the angle  $\theta$  between the two directions of scattering.

#### 4.1. Gaussian function

Let us first assume that the strength of the scattering potential  $S_F(\mathbf{r})$  is given by a Gaussian function, i.e.,

$$S_F(\mathbf{r}) = A_1 e^{-r^2/2\alpha^2},$$
 (26)

with  $A_1$  and  $\alpha$  being positive constants that may depend on the frequency. Its three-dimensional spatial Fourier transform is then given by the expression:

$$\tilde{S}_F(\mathbf{K}) = A_1 \int_0^\infty \int_0^{2\pi} \int_0^\pi e^{-r^2/\alpha^2} e^{iKr \cos\theta} r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \, \mathrm{d}r, \qquad (27)$$

$$=A_1\alpha^3(2\pi)^{3/2}e^{-K^2\alpha^2/2},$$
(28)

where  $K = |\mathbf{K}|$ . In this example, and in the following ones, we consider two symmetrically located points of observation whose position vectors,  $\mathbf{r}_1$  and  $\mathbf{r}_2$ , each make an angle  $\theta/2$  with the direction of incidence (see Fig. 3). In this case

$$|\mathbf{u}_2 - \mathbf{u}_1|^2 = 4\sin^2(\theta/2).$$
<sup>(29)</sup>

On substituting from Eqs. (28) and (29) into Eq. (19) we find that

$$\mathcal{I}(r_1 \mathbf{u}_1, r_2 \mathbf{u}_2) = e^{-4k^2 \alpha^2 \sin^2(\theta/2)}.$$
(30)

The behavior of the correlation coefficient of the intensity fluctuations for this case is shown in Fig. 4 for several values of the parameter  $k\alpha$ , the normalized effective range of the scattering strength. It is seen that the angular width of the correlation coefficient decreases when the width of the scattering strength

increases. This is in agreement with the reciprocity relation given by Eq. (19).

## 4.2. Exponential function

Let us next assume that the strength of the scattering potential  $S_F(\mathbf{r})$  is given by an exponential function, i.e.,

$$S_F(\mathbf{r}) = A_2 e^{-r/\beta},\tag{31}$$

with  $A_2$  and  $\beta$  being positive constants that may depend on the frequency. Then

$$\tilde{S}_{F}(\mathbf{K}) = A_{2} \iiint e^{-r/\beta} e^{iKr \cos \theta} r^{2} \sin \theta \, \mathrm{d}\theta \, \mathrm{d}\phi \, \mathrm{d}r, \tag{32}$$

$$=\frac{4\pi A_2}{K} \int_0^\infty e^{-r/\beta} r \sin(Kr) \, \mathrm{d}r,$$
 (33)

$$=\frac{8\pi A_2\beta^3}{(1+K^2\beta^2)^2}.$$
(34)

On substituting from Eqs. (34) and (29) into Eq. (19) we find that

$$\mathcal{I}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{1}{\left[1 + 4k^2\beta^2\sin^2(\theta/2)\right]^4}.$$
(35)

The behavior of the correlation coefficient of the intensity fluctuations for selected values of the parameter  $k\beta$ , the normalized width of the scattering strength, is shown in Fig. 5. A comparison with Fig. 4 shows the same general trends, but the angular half-width of the correlation coefficient is significantly smaller for the exponential distribution.

## 4.3. Shifted Gaussian function

So far we have examined solid scatterers. A hollow scatterer can be modeled by assuming that the scattering strength is given by a shifted Gaussian function, i.e.,

$$S_F(\mathbf{r}) = A_4 e^{-(r-\Delta)^2/2\sigma^2}$$
. (36)

with  $\Delta \ge \sigma$ . This represents a hollow-shell scatterer with radius  $\Delta$  and with thickness  $\sigma$ , see Fig. 2. We now have that

$$\tilde{S}_F(\mathbf{K}) = A_4 \iiint e^{-(r-\Delta)^2/2\sigma^2} e^{iKr \cos \theta} r^2 \sin \theta \, \mathrm{d}\theta \, \mathrm{d}\phi \, \mathrm{d}r, \tag{37}$$

$$=\frac{4\pi A_4}{K}\int_0^\infty e^{-(r-\Delta)^2/2\sigma^2}r\sin{(Kr)}\,\mathrm{d}r,$$
(38)

$$= \frac{4\pi A_4}{K} \sqrt{\frac{\pi}{2}} \sigma e^{-K^2 \sigma^2/2} \\ \times \operatorname{Im} \left\{ e^{iKA} (\varDelta + iK\sigma^2) \left[ \operatorname{erf} \left( \frac{\varDelta}{\sqrt{2}\sigma} + \frac{iK\sigma}{\sqrt{2}} \right) + 1 \right] \right\}.$$
(39)

A derivation of Eq. (39) is given in Appendix B. The normalized correlation coefficient  $\mathcal{I}(r\mathbf{u}_1, r\mathbf{u}_2)$  can be obtained by substituting Eqs. (39) and (29) into Eq. (19). The result is plotted in Fig. 6. It is



**Fig. 5.** The normalized correlation coefficient  $\mathcal{I}(\mathbf{ru}_1,\mathbf{ru}_2)$  of the intensity fluctuations for a scatterer with an exponential distribution of the scattering strength, as function of the angle  $\theta$  between the two directions of scattering.



**Fig. 6.** The normalized correlation coefficient  $\mathcal{I}(\mathbf{ru}_1, \mathbf{ru}_2)$  of the intensity fluctuations for a scatterer with a shifted Gaussian distribution of the scattering strength, as function of the angle  $\theta$  between the two directions of observation. In these examples  $k = 10^7 \text{ m}^{-1}$ ,  $\sigma = 10^{-6} \text{ m}$ , and  $\Delta = 15 \times 10^{-6} \text{ m}$  (a) and  $30 \times 10^{-6} \text{ m}$  (b).

seen that, in contrast to the solid scatterers considered in the previous examples, the hollow sphere shows a different behavior. Rather than being a monotonically decreasing function of the separation angle  $\theta$ , the correlation coefficient is now a damped oscillating function.

#### 5. Summary and conclusions

In previous studies concerning the scattering of a monochromatic plane wave by a quasi-homogeneous random scatterer two second-order reciprocity relations have been derived, namely

$$\mu^{(s)}(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]}{\tilde{S}_F(0)} \exp[ik(r_2 - r_1)],$$
(40)

$$S^{(s)}(r\mathbf{u}) = \frac{|a|^2 \tilde{S}_F(0)}{r^2} \tilde{\mu}_F[k(\mathbf{s}_0 - \mathbf{u})].$$
(41)

The strength of the scattering potential  $S_F(\mathbf{r})$  can be reconstructed with the help of Eq. (40), by measuring the spectral degree of coherence  $\mu^{(s)}(r_1\mathbf{u}_1, r_2\mathbf{u}_2)$  in the far zone.

In this paper two fourth-order reciprocity relations for the scattered field in the far zone of a quasi-homogeneous random scatterer with Gaussian statistics were discussed, namely

$$\mathcal{I}(r_1 \mathbf{u}_1, r_2 \mathbf{u}_2) = \frac{\left|\tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)]\right|^2}{\tilde{S}_F^2(0)},$$
(42)

$$V(r\mathbf{u}) = \frac{|a|^4}{r^4} \tilde{S}_F^2(0) \tilde{\mu}_F^2[k(\mathbf{s}_0 - \mathbf{u})].$$
(43)

We note that the fourth-order relations appear to be the "squared versions" of the second-order expressions. Just like Eq. (40), Eq. (42) can also be used to reconstruct the scattering potential  $S_F(\mathbf{r})$ . The latter case, however, requires relatively simple intensity correlation experiments. Both approaches yield the Fourier transform of  $S_F(\mathbf{r})$ . Therefore, in either approach there is the usual loss of information because of the well-known restrictions imposed by the Ewald sphere [17, Section 13.1].

We have used Eq. (42) to investigate the intensity fluctuations correlations for different forms of the scattering strength. It was found that a hollow sphere can be distinguished from solid spherical scatterers by examining the angular dependence of the correlation coefficient. We also showed that spherical scatterers with an exponential form of the scattering strength display a behavior that differs from scatterers with a scattering strength that is Gaussian.

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## Appendix A. Derivation of Eq. (13)

On substituting from Eqs. (1), (2), (5) and (6) into Eq. (9) we find the following expression for the spectral density of the scattered field in the far zone:

$$S^{(s)}(r\mathbf{u}) = \frac{|a|^2}{r^2} \iint_D e^{-ik(\mathbf{s}_0 - \mathbf{u}) \cdot (\mathbf{r}' - \mathbf{r}'')} C_F(\mathbf{r}', \mathbf{r}'') \, \mathrm{d}^3 r' \, \mathrm{d}^3 r''.$$
(A.1)

On making use of Eqs. (A.1), (10) and (11) in the definition of the correlation of the intensity fluctuations [Eq. (12)], it follows that

$$D(r_{1}\mathbf{u}_{1}, r_{2}\mathbf{u}_{2}) = \frac{|a|^{4}}{r_{1}^{2}r_{2}^{2}} \iiint \int_{D} \langle [F^{*}(\mathbf{r}')F(\mathbf{r}'') - C_{F}(\mathbf{r}', \mathbf{r}'')] \\ \times [F^{*}(\mathbf{p}')F(\mathbf{p}'') - C_{F}(\mathbf{p}', \mathbf{p}'')] \rangle \\ \times e^{-ik(\mathbf{s}_{0}-\mathbf{u}_{1})\cdot(\mathbf{r}'-\mathbf{r}'')}e^{-ik(\mathbf{s}_{0}-\mathbf{u}_{2})\cdot(\mathbf{p}'-\mathbf{p}'')} d^{3}r' d^{3}r'' d^{3}p' d^{3}p''.$$
(A.2)

Under the assumption that the fluctuations of the scattering potential are governed by Gaussian statistics, the fourth-order correlations occurring in Eq. (A.2) may be expressed in terms of second-order correlations with the help of the Gaussian moments theorem [19]. The expression for the correlation of the intensity fluctuations then factorizes, and one finds that

$$D(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = A(r_1\mathbf{u}_1, r_2\mathbf{u}_2)B(r_1\mathbf{u}_1, r_2\mathbf{u}_2),$$
(A.3)

where

$$A(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{|a|^2}{r_1r_2} \iint_D C_F(\mathbf{r}', \mathbf{p}'') e^{-ik(\mathbf{s}_0 - \mathbf{u}_1) \cdot \mathbf{r}'} e^{ik(\mathbf{s}_0 - \mathbf{u}_2) \cdot \mathbf{p}''} d^3r' d^3p''$$
(A.4)

and

$$B(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{|a|^2}{r_1r_2} \iint_D C_F(\mathbf{p}', \mathbf{r}'') e^{ik(s_0 - \mathbf{u}_1) \cdot \mathbf{r}''} e^{-ik(s_0 - \mathbf{u}_2) \cdot \mathbf{p}'} d^3 r'' d^3 p'.$$
(A.5)

Since  $C_F(\mathbf{p}', \mathbf{r}'') = [C_F(\mathbf{r}'', \mathbf{p}')]^*$  it is seen that  $B(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = [A(r_1\mathbf{u}_1, r_2\mathbf{u}_2)]^*$ .

The integral A in Eq. 
$$(A.4)$$
 can be calculated by invoking the quasi-

homogeneous character of the scatterer, which implies that

$$C_F(\mathbf{r}',\mathbf{p}'') \approx S_F[(\mathbf{r}'+\mathbf{p}'')/2]\mu_F(\mathbf{p}''-\mathbf{r}').$$
 (A.7)

On changing to sum and difference variables  $\mathbf{R}^+ = (\mathbf{r}' + \mathbf{p}'')/2$  and  $\mathbf{R}^- = \mathbf{r}' - \mathbf{p}''$ , the integral *A* factorizes into two integrals. Extending the limits of integration in each variable formally to minus and plus infinity one finds that

$$A(r_{1}\mathbf{u}_{1}, r_{2}\mathbf{u}_{2}) = \frac{|a|^{2}}{r_{1}r_{2}} \int_{D} S_{F}(\mathbf{R}^{+}) e^{ik(\mathbf{u}_{1}-\mathbf{u}_{2})\cdot\mathbf{R}^{+}} d^{3}R^{+} \\ \times \int_{D} \mu_{F}(-\mathbf{R}^{-}) e^{-ik[\mathbf{s}_{0}-(\mathbf{u}_{1}+\mathbf{u}_{2})/2]\cdot\mathbf{R}^{-}} d^{3}R^{-},$$
(A.8)

$$= \frac{|a|^2}{r_1 r_2} \tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)] \tilde{\mu}_F[k(\mathbf{s}_0 - (\mathbf{u}_1 + \mathbf{u}_2)/2)], \tag{A.9}$$

with  $\tilde{S}_{F}[\mathbf{K}]$  and  $\tilde{\mu}_{F}[\mathbf{K}]$  being the three-dimensional spatial Fourier transforms of the strength of the scattering potential and of the correlation coefficient of the scatterer, respectively. In the last step we used the fact that  $\mu_{F}(-\mathbf{K}) = [\mu_{F}(\mathbf{K})]^{*}$  and consequently

 $\mu_F[\mathbf{K}]$  is real-valued. On making use of Eq. (A.9), together with Eqs. (A.3) and (A.6), we obtain for the correlation of the intensity fluctuations the expression:

$$D(r_1\mathbf{u}_1, r_2\mathbf{u}_2) = \frac{|a|^4}{r_1^2 r_2^2} \Big| \tilde{S}_F[k(\mathbf{u}_1 - \mathbf{u}_2)] \tilde{\mu}_F[k(\mathbf{s}_0 - (\mathbf{u}_1 + \mathbf{u}_2)/2)] \Big|^2, \quad (A.10)$$

which is Eq. (13).

#### Appendix B. Derivation of Eq. (39)

Eq. (38) can be re-written in the form:

$$\tilde{S}_F(\mathbf{K}) = B \operatorname{Im}\left\{\int_0^\infty e^{-(r-\varDelta)^2/2\sigma^2} r e^{i\mathbf{K}r} \, \mathrm{d}r\right\},\tag{B.1}$$

where  $B = 4\pi A_4/K$ . On making the change of variables  $x = r - \Delta$ , one finds that

$$\tilde{S}_{F}(\mathbf{K}) = B \operatorname{Im} \left\{ e^{iK\Delta} \int_{-\Delta}^{\infty} e^{-x^{2}/2\sigma^{2}} x \, e^{iKx} \, \mathrm{d}x + e^{iK\Delta} \Delta \int_{-\Delta}^{\infty} e^{-x^{2}/2\sigma^{2}} e^{iKx} \, \mathrm{d}x \right\}.$$
(B.2)

The first term of Eq. (B.2), Q say, can be transformed by partial integration into

$$Q = B \operatorname{Im} \left\{ e^{iKA} \int_{-\Delta}^{\infty} e^{-x^2/2\sigma^2} x \, e^{iKx} \, \mathrm{d}x \right\},\tag{B.3}$$

$$= B \operatorname{Im} \left\{ e^{iK\Delta} iK\sigma^2 \int_{-\Delta}^{\infty} e^{-x^2/2\sigma^2} e^{iKx} \, \mathrm{d}x \right\}.$$
(B.4)

Completing the square, the right-hand side of Eq. (B.4) can be expressed in the form

$$Q = B \operatorname{Im} \left\{ e^{iK\Delta} iK\sigma^3 \sqrt{2} e^{-K^2 \sigma^2/2} \int_{-\Delta/(\sqrt{2}\sigma) - iK\sigma/\sqrt{2}}^{\infty - iK\sigma/\sqrt{2}} e^{-y^2} \, \mathrm{d}y \right\},$$
(B.5)

$$= B \operatorname{Im} \left\{ e^{iK\Delta} iK\sigma^3 e^{-K^2\sigma^2/2} \sqrt{\frac{\pi}{2}} \left[ \operatorname{erf} \left( \frac{\Delta}{\sqrt{2}\sigma} + \frac{iK\sigma}{\sqrt{2}} \right) + 1 \right] \right\}, \quad (B.6)$$

with erf(*z*) denoting the error function:

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$
 (B.7)

and where the fact that  $erf(\infty) = 1$  has been used. Similarly, the second term of Eq. (B.2), *T* say, can be re-written as

$$T = B \operatorname{Im} \left\{ e^{iK\Delta} \Delta \int_{-\Delta}^{\infty} e^{-x^2/2\sigma^2} e^{iKx} \, \mathrm{d}x \right\},$$
(B.8)

$$= B \operatorname{Im} \left\{ e^{iK\Delta} \Delta \sigma e^{-K^2 \sigma^2/2} \sqrt{\frac{\pi}{2}} \left[ \operatorname{erf} \left( \frac{\Delta}{\sqrt{2}\sigma} + \frac{iK\sigma}{\sqrt{2}} \right) + 1 \right] \right\}.$$
(B.9)

Adding Eqs. (B.6) and (B.9) yields Eq. (39).

#### References

(A.6)

- [1] R. Hanbury Brown, R.Q. Twiss, Nature 177 (1956) 27.
- [2] R. Hanbury Brown, R.Q. Twiss, Nature 178 (1956) 1046.
- [3] D. Kleppner, Physics Today, August (2008) 8.
- [4] E. Wolf, Introduction to the Theory of Coherence and Polarization of Light, Cambridge University Press, Cambridge, 2007.
- [5] G. Baym, Acta Physica Polonica B 29 (1998) 1839.
- [6] T. Jeltes, J.M. McNamara, W. Hogervorst, W. Vassen, V. Krachmalnicoff, M. Schellekens, A. Perrin, H. Chang, D. Boiron, A. Aspect, C.I. Westbrook, Nature 445 (2007) 402.
- [7] A.M. Rey, I.I. Satija, C.W. Clark, New Journal of Physics 8 (2006) 155.
- [8] G. Zhang, Z. Wu, Optics Express 20 (2012) 1491.
- [9] Y. Xin, Y. He, Y. Chen, J. Li, Optics Letters 35 (2010) 4000.
- [10] H.C. Jacks, O. Korotkova, Journal of Optical Society of America A 28 (2011) 1139.
- [11] W.H. Carter, E. Wolf, Optics Communications 67 (1988) 85.
- [12] D.G. Fischer, E. Wolf, Journal of Optical Society of America A 11 (1994) 1128.
- [13] D.G. Fischer, E. Wolf, Optics Communications 133 (1997) 17.
- [14] D.G. Fischer, Pure and Applied Optics 7 (1998) 1151.

- [15] D. Zhao, O. Korotkova, E. Wolf, Optics Letters 32 (2007) 3483.
- [16] T.D. Visser, D.G. Fischer, E. Wolf, Journal of Optical Society of America A 23 (2006) 1631.
- University Press, Cambridge, 1995. C.L. Mehta, Theory of photoelectron counting, in: E. Wolf (Ed.), Progress in Optics, vol. VIII, Elsevier, Amsterdam, 1970, p. 373.
- [17] M. Born, E. Wolf, Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light, seventh (expanded) ed., Cambridge University Press, Cambridge, 1999.
- [20] L.C. Andrews, R.L. Phillips, Laser Beam Propagation through Random Media, 2nd ed., SPIE, Bellingham, 2005. (see Chapter 8, Sections 1 and 2).

[18] L. Mandel, E. Wolf, Optical Coherence and Quantum Optics, Cambridge