Hanbury Brown–Twiss effect with partially coherent electromagnetic beams

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Received February 20, 2014; revised March 18, 2014; accepted March 19, 2014; posted March 21, 2014 (Doc. ID 206322); published April 17, 2014

We derive expressions that allow us to examine the influence of different source parameters on the correlation of intensity fluctuations (the Hanbury Brown–Twiss effect) at two points in the same cross section of a random electromagnetic beam. It is found that these higher-order correlations behave quite differently than the lower-order amplitude-phase correlations that are described by the spectral degree of coherence.

OCIS codes: (030.1640) Coherence; (050.1960) Diffraction theory; (260.2110) Electromagnetic optics; (260.5430) Polarization.

http://dx.doi.org/10.1364/OL.39.002561

Ever since Hanbury Brown–Twiss (HBT) determined the angular diameter of radio stars by analyzing the correlation of intensity fluctuations of their radiation [1,2], the eponymous "HBT effect" has been applied to many branches of physics [3–8]. In many cases a scalar analysis as given in [9, Chap. 7] turns out to be sufficient. However, since the formulation of the unified theory of coherence and polarization [10–12], several studies have been devoted to the question of how the HBT effect in random electromagnetic beams can be analyzed [13–17]. It is well known that the fundamental properties of these beams, such as their spectrum, degree of polarization, state of polarization, and degree of coherence, can all change significantly on propagation, even when the propagation is through free space [18–24]. However, until now a detailed investigation of the evolution of the HBT effect in random electromagnetic beams has been lacking. In this Letter, we intend to fill this void by examining the correlation of intensity fluctuations occurring in a wide class of partially coherent beams, namely those of the Gaussian Schell-model (GSM) type [20]. We derive expressions that allow us to examine the influence of different source parameters on the HBT effect at two points in the same cross-sectional plane.

Let us consider a stochastic, wide-sense stationary, electromagnetic beam propagating close to the z direction into the half-space $z > 0$ (see Fig. 1). The source plane is defined as the plane $z = 0$. The vector $\boldsymbol{\rho} = (x, y)$ indicates a position in a transverse plane. $E_x(\boldsymbol{\rho}, z, \omega)$ and $E_y(\boldsymbol{\rho}, z, \omega)$ are the Cartesian components of the electric field at frequency ω along two mutually orthogonal x and y directions, perpendicular to the beam axis. The intensity of a single realization of the beam at a point $(\boldsymbol{\rho}, z)$ at frequency ω can be expressed as

$$I(\boldsymbol{\rho}, z, \omega) = |E_x(\boldsymbol{\rho}, z, \omega)|^2 + |E_y(\boldsymbol{\rho}, z, \omega)|^2. \quad (1)$$

From now on we will suppress the dependence on the frequency ω in our notation. The intensity $I(\boldsymbol{\rho}, z, \omega)$ is a random quantity and its variation from its mean value is

$$\Delta I(\boldsymbol{\rho}, z) = I(\boldsymbol{\rho}, z) - \langle I(\boldsymbol{\rho}, z) \rangle, \quad (2)$$

where the angular brackets denote the ensemble average. The statistical properties of the beam at a pair of points in cross-section z are described by the electric cross-spectral density matrix $\mathbf{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$, whose elements are defined as

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \langle E_i^*(\boldsymbol{\rho}_1, z) E_j(\boldsymbol{\rho}_2, z) \rangle, \qquad (i, j = x, y). \quad (3)$$

It follows from this definition that the ensemble-averaged intensity can be expressed as

$$\langle I(\boldsymbol{\rho}, z) \rangle = \mathrm{Tr}\, \mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z), \quad (4)$$

where Tr denotes the trace.

The correlation of the intensity fluctuations at two points, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, in the same cross-section z is defined as

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \langle \Delta I(\boldsymbol{\rho}_1, z) \Delta I(\boldsymbol{\rho}_2, z) \rangle. \quad (5)$$

We assume that the statistical properties of the beam are Gaussian. It then follows, by use of the Gaussian moment theorem for complex random processes, that the correlation of the intensity fluctuations at two positions may be expressed as [25, Chap. 8]

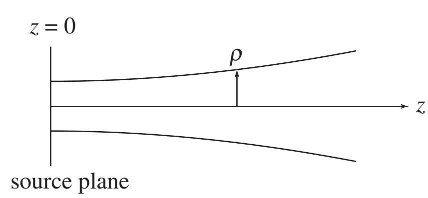

Fig. 1. Illustrating the notation.

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \sum_{i,j} |W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)|^2. \tag{6}$$

We will study the correlation properties of a wide class of random beams, namely, those of the GSM type [20]. For these beams the elements of the cross-spectral density matrix in the source plane $z = 0$ read

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 0) = \sqrt{S_i(\boldsymbol{\rho}_1) S_j(\boldsymbol{\rho}_2)}\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1), \tag{7}$$

with the spectral densities $S_i(\boldsymbol{\rho}) = W_{ii}(\boldsymbol{\rho}, \boldsymbol{\rho})$ and the correlation coefficients $\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)$ both Gaussian functions; i.e.,

$$S_i(\boldsymbol{\rho}) = A_i^2 \exp(-\rho^2/2\sigma_i^2), \tag{8}$$

$$\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1) = B_{ij} \exp[-(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)^2/2\delta_{ij}^2]. \tag{9}$$

The parameters A_i, B_{ij}, σ_i and δ_{ij} are independent of position, but may depend on the frequency ω. They cannot be chosen arbitrarily. In particular, it follows from the definition of the cross-spectral density matrix that

$$B_{xx} = B_{yy} = 1, \tag{10}$$

$$B_{xy} = B_{yx}^*, \tag{11}$$

$$|B_{xy}|, \ |B_{yx}| \leq 1, \tag{12}$$

$$\delta_{xy} = \delta_{yx}. \tag{13}$$

In addition, the source parameters must satisfy certain constraints to ensure that, for the choice $\sigma_x = \sigma_y = \sigma$, the field is beam-like at wavelength λ [26], and that the cross-spectral density matrix is definitely positive, viz. [27]

$$\frac{1}{4\sigma^2} + \frac{1}{\delta_{ii}^2} \ll \frac{2\pi^2}{\lambda^2}, \tag{14}$$

$$\sqrt{\frac{\delta_{xx}^2 + \delta_{yy}^2}{2}} \leq \delta_{xy} \leq \sqrt{\frac{\delta_{xx}\delta_{yy}}{|B_{xy}|}}, \tag{15}$$

and

$$|B_{xy}| \leq \frac{2}{\delta_{yy}/\delta_{xx} + \delta_{xx}/\delta_{yy}}. \tag{16}$$

The matrix elements of the propagated beam in a plane z read (see [9], where the last minus sign of Eq. (10) on p. 184 should be a plus sign)

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{A_i A_j B_{ij}}{\Delta_{ij}^2(z)} \exp\left[-\frac{(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)^2}{8\sigma^2 \Delta_{ij}^2(z)}\right] \times \exp\left[-\frac{(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2}{2\Omega_{ij}^2 \Delta_{ij}^2(z)} + \frac{ik(\rho_2^2 - \rho_1^2)}{2R_{ij}(z)}\right], \tag{17}$$

where

$$\Delta_{ij}^2(z) = 1 + (z/\sigma k \Omega_{ij})^2, \tag{18}$$

$$\frac{1}{\Omega_{ij}^2} = \frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}, \tag{19}$$

$$R_{ij}(z) = [1 + (\sigma k \Omega_{ij}/z)^2] z, \tag{20}$$

and the wave number $k = 2\pi/\lambda$. In the following we take the reference point $\boldsymbol{\rho}_1$ to be on the z axis, i.e., $\boldsymbol{\rho}_1 = 0$. On substituting from Eq. (16) into Eq. (6), we obtain the expression

$$C(0, \boldsymbol{\rho}_2, z) = \sum_{i,j} \frac{A_i^2 A_j^2 |B_{ij}|^2}{\Delta_{ij}^4(z)} \times \exp\left[-\frac{\rho_2^2}{4\sigma^2 \Delta_{ij}^2(z)} - \frac{\rho_2^2}{\Omega_{ij}^2 \Delta_{ij}^2(z)}\right]. \tag{21}$$

Notice that Eq. (21) implies that $C(0, \boldsymbol{\rho}_2, z)$ is rotationally symmetric about the z axis, i.e., it only depends on $\rho_2 = |\boldsymbol{\rho}_2|$. We define the normalized correlation function as

$$C_N(0, \boldsymbol{\rho}_2, z) = \frac{C(0, \boldsymbol{\rho}_2, z)}{\langle I(0, z)\rangle \langle I(\boldsymbol{\rho}_2, z)\rangle}, \tag{22}$$

where

$$\langle I(0, z)\rangle = \frac{A_x^2}{\Delta_{xx}^2(z)} + \frac{A_y^2}{\Delta_{yy}^2(z)}, \tag{23}$$

and

$$\langle I(\rho_2, z)\rangle = \frac{A_x^2}{\Delta_{xx}^2(z)} \exp\left[-\frac{\rho_2^2}{2\sigma^2 \Delta_{xx}^2(z)}\right] + \frac{A_y^2}{\Delta_{yy}^2(z)} \exp\left[-\frac{\rho_2^2}{2\sigma^2 \Delta_{yy}^2(z)}\right]. \tag{24}$$

It can be shown that $C_N(0, \boldsymbol{\rho}_2, z)$ is bounded by zero and unity [16]. It is easily derived that

$$\lim_{z\to\infty} C_N(0, \boldsymbol{\rho}_2, z) = \frac{\sum_{i,j} A_i^2 A_j^2 |B_{ij}|^2 \Omega_{ij}^4}{(A_x^2 \Omega_{xx}^2 + A_y^2 \Omega_{yy}^2)^2}. \tag{25}$$

Notice that this asymptotic value is independent of the choice of the point $\boldsymbol{\rho}_2$. Equation (25) is generally valid, in contrast to the much more restricted analysis presented in [17]. We will compare the fourth-order correlation function $C_N(0, \boldsymbol{\rho}_2, z)$ with the second-order spectral degree of coherence. The latter is defined as ([9], Section 9.2)

$$\eta(0,\boldsymbol{\rho}_2,z) = \frac{\mathrm{Tr}\,\mathbf{W}(0,\boldsymbol{\rho}_2,z)}{\sqrt{\langle I(0,z)\rangle}\sqrt{\langle I(\boldsymbol{\rho}_2,z)\rangle}}, \tag{26}$$

and is a direct measure of the visibility of the fringe pattern produced in Young's experiment. Note that, in contrast to Eq. (25),

$$\lim_{z\to\infty}\eta(0,\boldsymbol{\rho}_2,z) = 1. \tag{27}$$

We now employ the above theoretical development to study the evolution of the second- and fourth-order correlations of a GSM beam when propagated in free space. In the examples we set $\lambda = 0.6328$ μm, $\sigma = 4$ mm, $A_x = 1$, $A_y = 3$, $|B_{xy}| = 0.2$, $\delta_{xx} = 3$ mm, $\delta_{xy} = 2.7$ mm, and $\delta_{yy} = 1$ mm, unless specified otherwise. For these values, the conditions (14)–(16) are all satisfied. A comparison of the contours of $C_N(0,\boldsymbol{\rho}_2,z)$ and those of $|\eta(0,\rho_2,z)|$ in the $z\rho_2$-plane (Figs. 2 and 3) indicates that the evolution of the correlation of intensity fluctuations is more complicated than that of the spectral degree of coherence. This is further illustrated by Fig. 4, which shows that $|\eta(0,\rho_2,z)|$ increases monotonically to the value 1, whereas $C_N(0,\boldsymbol{\rho}_2,z)$ quickly rises to its maximum value, then decreases, after which it slowly rises to its asymptotic limit.

An essential difference between the spectral degree of coherence and the correlation of intensity fluctuations is that $\eta(0,\rho_2,z)$ only depends on the diagonal elements of the cross-spectral density matrix, whereas the definition of $C_N(0,\boldsymbol{\rho}_2,z)$ contains all four matrix elements. A direct consequence is that the spectral degree of coherence is unaffected by changes in the coherence length δ_{xy}. (Notice, however, that this not the case for an alternative definition as proposed in Ref. [28]). The correlation of intensity fluctuations, on the other hand, is quite sensitive to changes in this parameter, as Fig. 5 shows. The influence of the coherence length δ_{xx} at a fixed point in the beam is shown in Fig. 6. It is seen that, $|\eta(0,\rho_2,z)|$ is less sensitive than $C_N(0,\boldsymbol{\rho}_2,z)$. A similar result is obtained when the amplitude A_y is varied. This is illustrated in Fig. 7.

We noted before that the asymptotic value of $C_N(0,\boldsymbol{\rho}_2,z)$ is independent of the choice of the point ρ_2. In Fig. 8(a) the variation of the correlation of intensity fluctuations is plotted for several values of ρ_2. Although these curves are quite distinct as $z < 100$ m, they eventually all approach the limiting value indicated by

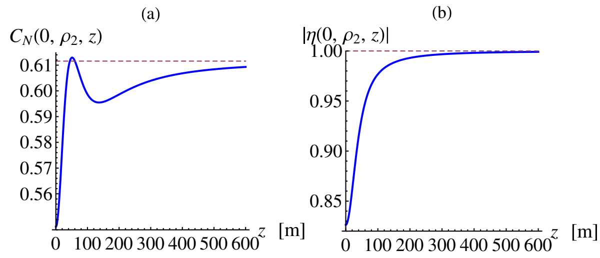

Fig. 4. Evolution of (a) the normalized correlation of intensity fluctuations and (b) the modulus of the spectral degree of coherence when $\rho_2 = 0.65$ mm. The dashed lines are the asymptotic values given by Eqs. (25) and (27), respectively.

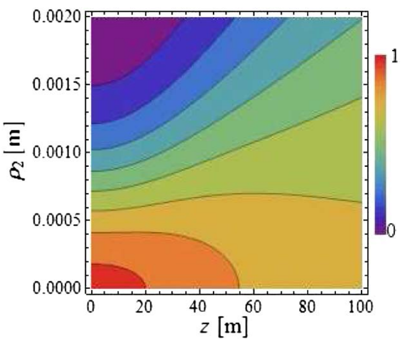

Fig. 2. Contours of the normalized correlation of intensity fluctuations $C_N(0,\boldsymbol{\rho}_2,z)$ in the z ρ_2 plane.

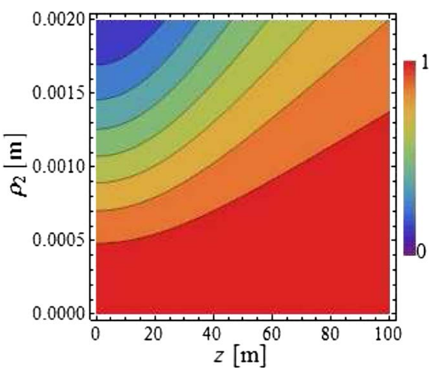

Fig. 3. Contours of the modulus of the spectral degree of coherence $|\eta(0,\rho_2,z)|$ in the z ρ_2 plane.

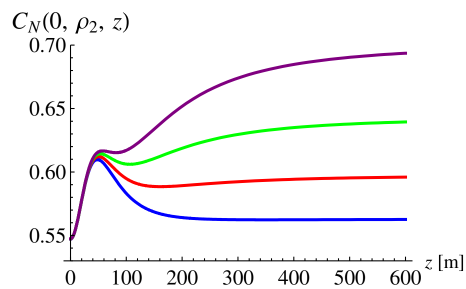

Fig. 5. Evolution of the normalized correlation of intensity fluctuations as a function of z for different values of the parameter δ_{xy}. From bottom to top: $\delta_{xy} = 2.3$ mm (blue), 2.6 mm (red), 2.9 mm (green), and 3.2 mm (purple). As in Fig. 4, $\rho_2 = 2$ mm.

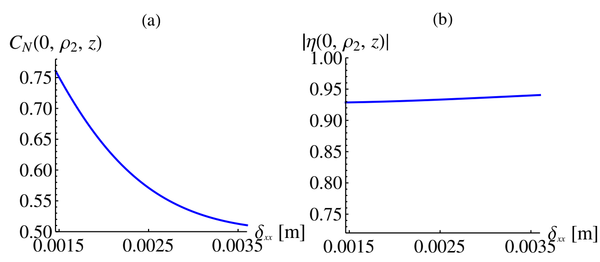

Fig. 6. Variation of (a) the normalized correlation of intensity fluctuations and (b) the modulus of the spectral degree of coherence as a function of δ_{xx} at the point $\rho_2 = 2$ mm, $z = 200$ m.

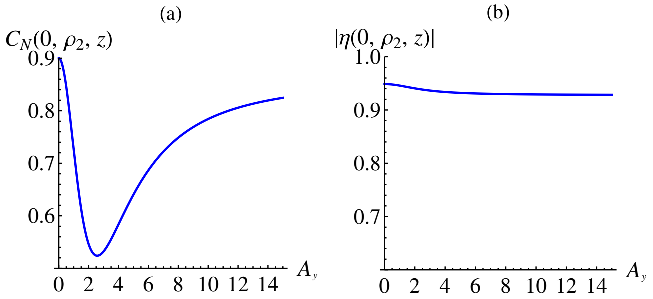

Fig. 7. Variation of (a) the normalized correlation of intensity fluctuations and (b) the modulus of the spectral degree of coherence as a function of A_y at the point $\rho_2 = 2$ mm, $z = 200$ m.

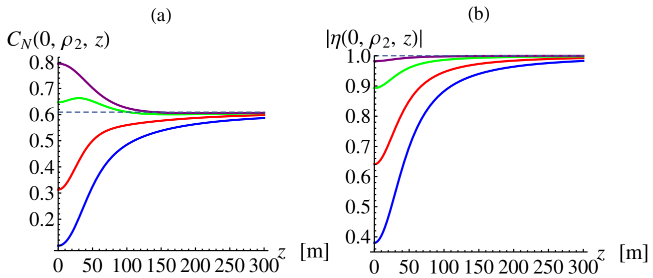

Fig. 8. Evolution of (a) the normalized correlation of intensity fluctuations and (b) the modulus of the spectral degree of coherence for different choices of ρ_2. From bottom to top, $\rho_2 = 1.5$ mm (blue), 1 mm (red), 0.5 mm (green), and 0.2 mm (purple); the dashed lines are the asymptotic values given by Eqs. (24) and (26), respectively.

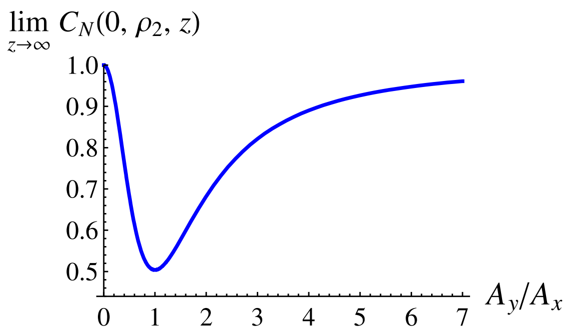

Fig. 9. Variation of the far-zone value of $C_N(0, \boldsymbol{\rho}_2, z)$ as a function of the ratio A_y/A_x.

the dashed line. For comparison's sake the evolution of $|\eta(0, \rho_2, z)|$ is shown in Fig. 8(b).

It is interesting to note that expression Eq. (25) offers several options to tailor the correlation of the intensity fluctuations in the far-field. One possibility is to change the ratio of the two spectral densities A_x and A_y. It immediately follows from Eq. (25) that $\lim_{z\to\infty} C_N(0, \boldsymbol{\rho}_2, z) = 1$ if one of the spectral densities is zero, i.e., if the beam is linearly polarized. As Fig. 9 shows, the asymptotic value of $C_N(0, \boldsymbol{\rho}_2, z)$ can be varied from its maximum value of 1 down to a value of 0.5. In this example $\sigma = 1$ mm, $|B_{xy}| = 0.1$, $\delta_{xx} = 3$ mm, $\delta_{xy} = 2.5$ mm and $\delta_{yy} = 3$ mm.

In conclusion, we have studied the evolution of the HBT effect on propagation of a electromagnetic GSM beam. The influence of the different source parameters was explored numerically. We found that the correlation of intensity fluctuations in the far-field can be tuned by adjusting, for example, the ratio of the amplitudes of the two components of the electric field.

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