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Collapse and revival of spatial coherence on free-space propagation

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ABSTRACT

We demonstrate analytically and numerically that the transverse spatial coherence (the *spectral degree of coherence*) of the field radiated by two partially correlated electromagnetic sources oscillates as a function of propagation distance in free space. In particular, it is possible for its magnitude to completely collapse to zero on propagation, and then return again to unity at a further distance. Our results are a direct demonstration of the wave character of the optical correlation functions. A similar oscillatory effect occurs for the degree of polarization.

One of the key results of optical coherence theory is the insight that two-point correlation functions satisfy a pair of wave equations or, in the space–frequency domain, two Helmholtz equations. These relations, named after their discoverer Emil Wolf [1,2], allow one to study how the state of coherence of an optical field evolves on propagation, while using the tools of diffraction theory. This state of coherence determines the field's evolution in space and time [3] and its interaction with material structures [4]. The many predictions of coherence theory, based on the Wolf equations, have been tested successfully under a wide variety of circumstances [5–10]. However, these are all indirect verifications of the wave nature of correlation functions. To the best of our knowledge, a direct demonstration of the Wolf equations has not been provided yet. Here we report that two partially correlated electromagnetic point sources give rise to a field whose spectral degree of coherence, denoted $\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)$, oscillates on propagation, and indeed undergoes the same interference process as the field itself. In particular, it is possible for $|\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ to become zero, corresponding to complete incoherence, and then increase again to unity, meaning that the field has become spatially fully coherent. This repeating collapse and revival occurs in complete unison with the spectral density at one of the observation points, thus demonstrating directly the wave character of correlations.

The collapse and revival of entirely different forms of coherence have been reported previously, in quantum optics [11] and Optical Coherence Tomography [12]. It is to be noted that those studies are concerned with temporal coherence, whereas as the present analysis pertains to spatial coherence.

The configuration that we study is Young's setup. An opaque screen $\mathcal{A}$ in the plane $z = 0$ contains two small identical apertures located at $Q_1 = (d, 0, 0)$ and $Q_2 = (-d, 0, 0)$ (see Fig. 1). From the pinholes

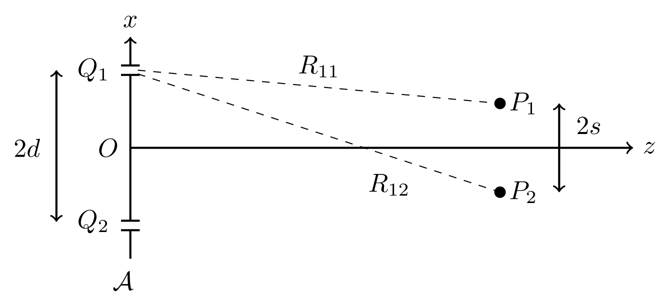

Fig. 1. The superposition of the fields radiated by two identical pinholes at Q_1 and Q_2 in a screen $\mathcal{A}$ is observed at two symmetrically located positions P_1 and P_2. The origin O is located midway between the apertures. $R_{11} = |Q_1 P_1|$, and $R_{12} = |Q_1 P_2|$; R_{22} and R_{21} are not shown. The separation between Q_1 and Q_2 is $2d$, that between P_1 and P_2 is $2s$.

a stochastic, statistically stationary electromagnetic field emerges. The superposition of the two radiated fields is observed at two symmetrically located points $P_1 = (s, 0, z)$ and $P_2 = (-s, 0, z)$. When P_1 and P_2 are in the far zone and close to the z axis, the z component of the field there may be neglected. The electric field vector $\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$ at P_m at frequency ω is then given by the expression

$$\mathbf{E}(P_m, \omega) = K_{1m}\mathbf{E}(Q_1, \omega) + K_{2m}\mathbf{E}(Q_2, \omega), \tag{1}$$

with the propagators

$$K_{nm} = -\frac{\mathrm{i}D}{\lambda}\frac{e^{\mathrm{i}kR_{nm}}}{R_{nm}}, \quad (n, m = 1, 2), \tag{2}$$

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where D denotes the area of the apertures, λ and k are the free-space wavelength and wavenumber corresponding to ω, and the distance $R_{nm} = |Q_n P_m|$ [13, Sec. 8.2]. In the following we keep the separation distances $2d$ and $2s$ fixed, while varying the observation plane z.

In the frequency domain the state of coherence of the field is described by the cross-spectral density (CSD) matrix $\mathbf{W}$ with elements [2, Ch. 9]

$$W_{ij}(P_n, P_m) = \langle E_i^*(P_n) E_j(P_m)\rangle, \quad (i, j = x, y), \tag{3}$$

where the angled brackets indicate an ensemble average, and with the frequency dependence suppressed for brevity. It is readily found that

$$\begin{aligned} W_{ij}(P_n, P_m) = {} & K_{1n}^* K_{1m} W_{ij}^{(0)}(Q_1, Q_1) + K_{1n}^* K_{2m} W_{ij}^{(0)}(Q_1, Q_2) \\ & + K_{2n}^* K_{1m} W_{ij}^{(0)}(Q_2, Q_1) + K_{2n}^* K_{2m} W_{ij}^{(0)}(Q_2, Q_2), \end{aligned} \tag{4}$$

and hence

$$\begin{aligned} W_{ij}(P_n, P_n) = {} & |K_{1n}|^2 W_{ij}^{(0)}(Q_1, Q_1) + K_{1n}^* K_{2n} W_{ij}^{(0)}(Q_1, Q_2) \\ & + K_{2n}^* K_{1n} W_{ij}^{(0)}(Q_2, Q_1) + |K_{2n}|^2 W_{ij}^{(0)}(Q_2, Q_2), \end{aligned} \tag{5}$$

where

$$W_{ij}^{(0)}(Q_n, Q_m) = \langle E_i^*(Q_n) E_j(Q_m)\rangle, \tag{6}$$

is the CSD matrix of the incident field in the plane $z = 0$. The spectral density at each observation point is defined as the trace of the CSD matrix, i.e., $S(P_n) = \mathrm{Tr}\,\mathbf{W}(P_n, P_n)$.

The spectral degree of coherence, whose magnitude is directly related to the fringe visibility in Young's interference experiment [2], equals

$$\eta(P_1, P_2) = \frac{\mathrm{Tr}\,\mathbf{W}(P_1, P_2)}{\sqrt{S(P_1) S(P_2)}}. \tag{7}$$

The magnitude $|\eta(P_1, P_2)|$ is bounded by zero and unity. The lower bound corresponds to the complete absence of coherence, whereas the upper bound indicates full coherence. It should be noted that other definitions of the degree of coherence have been suggested [14].

Another quantity of interest is the spectral degree of polarization, which is defined as

$$\mathcal{P}(\mathbf{r}) = \sqrt{1 - \frac{4\,\mathrm{Det}\,\mathbf{W}(\mathbf{r}, \mathbf{r})}{[\mathrm{Tr}\,\mathbf{W}(\mathbf{r}, \mathbf{r})]^2}}. \tag{8}$$

This real-valued quantity is also bounded by zero and one.

As an illustration of the behavior of the spectral degree of coherence we consider a Gaussian Schell-model source [2, Sec. 9.4.2], located in the plane $z = 0$ and covered by the screen $\mathcal{A}$ that contains the two pinholes. In that case the CSD elements are

$$W_{ij}^{(0)}(\rho_1, \rho_2) = \sqrt{S_i^{(0)}(\rho_1) S_j^{(0)}(\rho_2)}\, \mu_{ij}^{(0)}(\rho_2 - \rho_1). \tag{9}$$

Here $\rho = (x, y)$ is a two-dimensional position vector, $S_i^{(0)}(\rho)$ denotes the spectral density of the Cartesian component E_i, and $\mu_{ij}^{(0)}(\rho_2 - \rho_1)$ is the correlation between E_i and E_j. Furthermore

$$S_i^{(0)}(\rho) = A_i^2 \exp[-\rho^2/(2\sigma_i^2)], \tag{10}$$

$$\mu_{ij}^{(0)}(\rho_2 - \rho_1) = B_{ij} \exp[-(\rho_2 - \rho_1)^2/(2\delta_{ij}^2)]. \tag{11}$$

The parameters A_i, B_{ij}, σ_i and δ_{ij} are independent of position, but may depend on frequency. They cannot be chosen arbitrarily, but have to satisfy certain constraints relating to physical realizability. If $\sigma_x = \sigma_y = \sigma$ these are given by Eqs. (7a)–(9) of [2, Sec. 9.4.2], and Eqs. (25) and (29) of [15].

Let us first analyze the special case that a) $B_{xy} \in \mathbb{R}$, b) the two component-wise spectral densities are equal at both pinholes, and c) the coherence radii $\delta_{xx} = \delta_{yy} \leq \delta_{xy}$. This inequality is in agreement with the realizability constraints outlined in [15]. We introduce the abbreviations

$$A \equiv \sqrt{S_i^{(0)}(Q_n)} = A_i e^{-d^2/(4\sigma^2)}, \quad (i = x, y), \tag{12}$$

Table 1
Evolution of the spectral density, degree of coherence, and degree of polarization on free-space propagation. In the last column it is assumed that $\delta_{xy} > \delta_{xx}$.

$\cos(k\Delta)$	$S(P_n)$	$\eta(P_1, P_2)$	$\mathcal{P}(P_n)$
1	max	max	max
0	(max+min)/2	μ	$\lvert B_{xy}\rvert$
−1	min	min	min

$$\mu \equiv \mu_{xx}^{(0)}(Q_1, Q_2) = \mu_{yy}^{(0)}(Q_1, Q_2) = e^{-2d^2/\delta_{xx}^2}, \tag{13}$$

$$\mu_{xy} \equiv e^{-2d^2/\delta_{xy}^2}, \tag{14}$$

and hence

$$\mu_{xy}^{(0)}(Q_1, Q_2) = \mu_{yx}^{(0)}(Q_1, Q_2) = B_{xy}\mu_{xy}. \tag{15}$$

It then follows that

$$W_{ij}^{(0)}(Q_n, Q_n) = B_{ij} A^2, \quad (n = 1, 2), \tag{16}$$

$$W_{xx}^{(0)}(Q_1, Q_2) = W_{yy}^{(0)}(Q_1, Q_2) = A^2 \mu, \tag{17}$$

$$W_{xy}^{(0)}(Q_1, Q_2) = W_{yx}^{(0)}(Q_1, Q_2) = A^2 B_{xy}\mu_{xy}, \tag{18}$$

where we have used that $B_{ii} = 1$, $B_{xy} = B_{yx}$, and $\delta_{xy} = \delta_{yx}$. Finally,

$$W_{ij}^{(0)}(Q_2, Q_1) = W_{ij}^{(0)}(Q_1, Q_2). \tag{19}$$

The assumptions of paraxiality and the observation points being in the far zone imply that $1/R_{ij} \approx 1/z$, and $R_{12} - R_{11} \approx 2sd/z \equiv \Delta$. From Eq. (5) it is then straightforward to derive that

$$S(P_n) = 4\left(\frac{DA}{\lambda z}\right)^2 [1 + \mu\cos(k\Delta)], \quad (n = 1, 2), \tag{20}$$

and thus, from Eq. (4), it follows that

$$\eta(P_1, P_2) = \frac{\mu + \cos(k\Delta)}{1 + \mu\cos(k\Delta)}, \tag{21}$$

$$\mathcal{P}(P_n) = |B_{xy}| \frac{1 + \mu_{xy}\cos(k\Delta)}{1 + \mu\cos(k\Delta)}, \quad (n = 1, 2). \tag{22}$$

The fact that the spectral density $S(P_n)$ varies sinusoidally as the plane of observation z is changed, comes as no surprise. It is a direct effect of the interference between the two wave contributions. In contrast, Eq. (21) has several striking consequences. These follow from the fact that the correlation coefficient μ, as defined by (14), is bounded by zero and unity. First of all, no matter how strongly the two aperture fields are correlated (with the exception of the limiting case of complete spatial coherence $\mu = 1$), at observation planes z such that $\cos(k\Delta) = -\mu$ the spectral degree of coherence is zero, indicating a complete absence of spatial coherence. Then, as z is increased, the magnitude of η grows to unity, then decreases, to eventually become zero again. The evolution of all three quantities (spectral density, degree of coherence, and degree of polarization) is governed by the same $\cos(k\Delta)$ term. This shown is in Table 1. If the cosine equals one (minus one), then $S(P_n)$, $\eta(P_1, P_2)$, and $\mathcal{P}(P_n)$ all attain their maximum (minimum) value. It follows that the spectral density, the degree of coherence, and the degree of polarization oscillate in complete unison.

This behavior is illustrated in Fig. 2. The cycles of collapse and revival of spatial coherence are repeated over and over again as the field propagates. The spatial extent of these cycles gradually becomes larger as $\eta(P_1, P_2)$ tends to its asymptotic value 1 as $z \to \infty$. The fact that the spectral degree of coherence oscillates just like the spectral density directly demonstrates the wave nature of the correlation functions and shows that the observed spatial coherence is, just like the spectral density, the result of interference. The degree of polarization shows a similar oscillatory behavior.

An example of a more general case is presented in Fig. 3. Now the two observation points are no longer symmetric with respect to the x axis: $P_1 = (s, 0, z)$, $P_2 = (-2s, 0, z)$ with $s = 5$ mm. Also, $\delta_{xx} = 1$ cm, $\delta_{yy} = 1.2$ cm, and $\delta_{xy} = 1.41$ cm. Unlike the highly symmetric case of Fig. 2, no

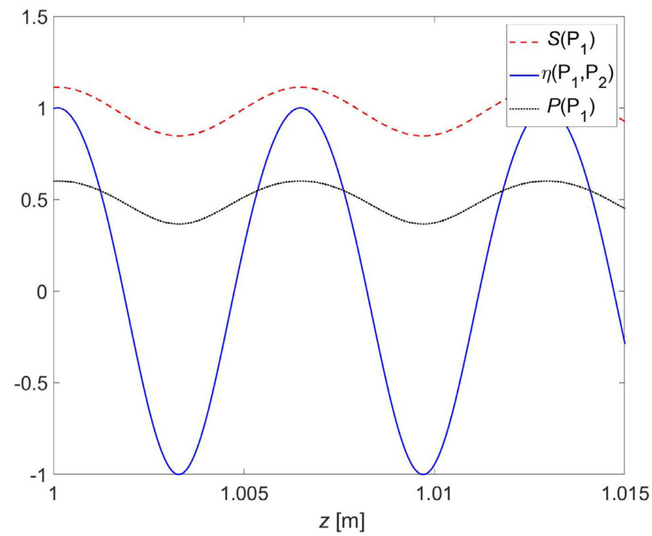

Fig. 2. The spectral degree of coherence $\eta(P_1, P_2)$ (solid blue curve), together with the scaled spectral density (red dashed) and the degree of polarization (black dotted) at the observation points as a function of the distance z. In this example $\delta_{xx} = 1$ cm, $\delta_{xy} = 1.41$ cm, $B_{xy} = 0.5$, $\lambda = 632.8$ nm, $d = 1$ cm, and $s = 5$ mm. Hence $\mu = 0.135$ and $\mu_{xy} = 0.365$.

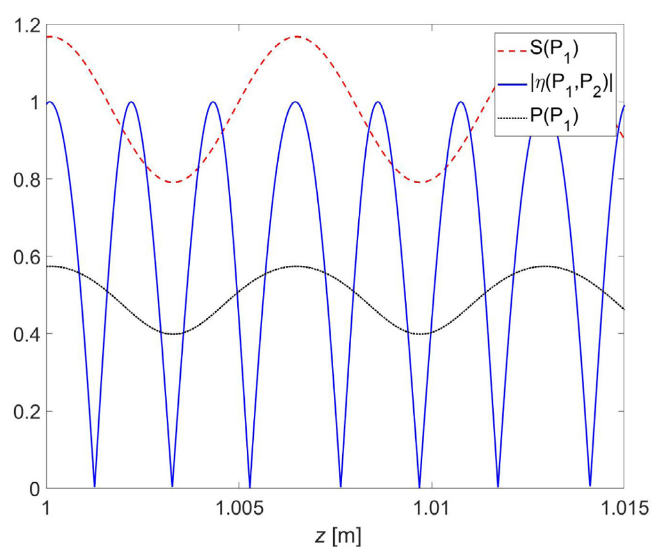

Fig. 3. The modulus of the spectral degree of coherence $\eta(P_1, P_2)$ (solid blue curve), together with the scaled spectral density (red dashed) and the degree of polarization (black dotted) for two asymmetric observation points $P_1 = (s, 0, z)$ and $P_2 = (-2s, 0, z)$, as a function of the propagation distance z. In this example $\delta_{xx} = 1$ cm, $\delta_{yy} = 1.2$ cm, $\delta_{xy} = 1.41$ cm, $B_{xy} = 0.5$, $\lambda = 632.8$ nm, $d = 1$ cm, and $s = 5$ mm.

simple analytic expressions are available. Also, $\eta(P_1, P_2)$ is now complex valued. Nevertheless, the numerical results show that in this case too the modulus of the spectral degree of coherence varies between zero and unity. Furthermore, the degree of polarization is again seen to have an oscillatory nature. We note that the (approximate) zeros of $|\eta|$ occur a distances z where the spectral density is appreciable, and not necessarily a minimum. In Fig. 4 the real and complex parts of $\eta(P_1, P_2)$ are plotted. It is seen that both parts exhibit a complicated oscillatory behavior.

In summary, the Wolf equations state that optical correlation functions satisfy a pair of wave equations or, in the frequency domain, a pair of Helmholtz equations. We have directly demonstrated that the spatial coherence of a partially coherent wavefield (i.e. the correlation of the wavefield at two different points in space) has indeed a wave-like

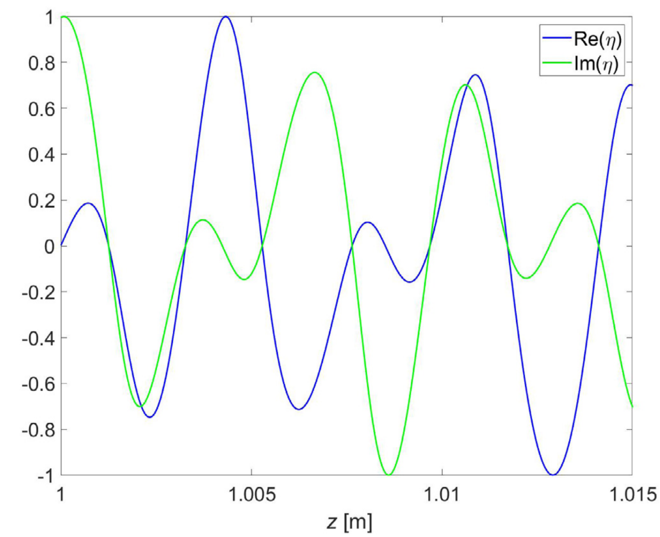

Fig. 4. The real (blue) and imaginary (green) parts of $\eta(P_1, P_2)$ as a function of the observation distance z. All parameters are as in Fig. 3.

character. For the field generated by two partially-correlated electromagnetic point sources this manifests as a continuing cycle of collapse and revival of coherence on propagation through free space. More specifically, it was shown that the spatial coherence that is observed is the direct result of interference, just as is the case for the spectral density and the degree of polarization. We note that the oscillatory behavior that we have described here does not occur for extended Gaussian Schell-model sources [16,17]. The mutual interference between all the pairs of points that make up such sources, washes out the oscillations.

We dedicate this study to the memory of our mentor and friend Emil Wolf.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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