Optics Letters

Tunable, anomalous Mie scattering using spatial coherence

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Received 11 September 2015; accepted 24 September 2015; posted 28 September 2015 (Doc. ID 249814); published 14 October 2015

We demonstrate that a J_0-Bessel-correlated beam that is incident on a homogeneous sphere produces a highly unusual distribution of the scattered field, with the maximum no longer occurring in the forward direction. Such a beam can be easily generated using a spatially incoherent, annular source. Moreover, the direction of maximal scattering can be shifted by changing the spatial coherence length. In this process, the total power that is scattered remains constant. This new tool to control scattering directionality may be used to steer the scattered field away from the forward direction and selectively address detectors situated at different angles.

OCIS codes: (030.1640) Coherence; (290.2558) Forward scattering; (290.4020) Mie theory; (290.5825) Scattering theory.

http://dx.doi.org/10.1364/OL.40.004779

Mie scattering, which is the scattering of an optical field by a spherical object, has a long and venerable history [1–3]. In its classical form, it deals with the scattering of a fully coherent, monochromatic plane wave by a homogeneous, deterministic sphere. Its many applications in, for example, spectroscopy, optical trapping, astronomy, and atmospheric studies [4,5] have led to a large literature, a substantial part of which can be divided into two broad categories. The first one, inspired by the seminal work of Kerker *et al.* [6], consists of efforts to design objects with a prescribed scattering profile, such as a suppressed scattering in the forward or backward directions [7–13]. The second category consists of studies in which scattering theory is extended to incident fields that are not deterministic but, rather, partially coherent [14–21]. These researchers are motivated by the fact that light that travels through atmospheric turbulence suffers a loss of coherence.

In this Letter, we bridge both categories by reporting a novel coherence technique that allows one to steer most of the scattered intensity away from the forward direction. In contrast to previous works, this is done by manipulating the incident beam rather than the scatterer. We show that changing the spatial coherence of the beam allows one to dynamically control the scattering distribution. In this tuning of the scattering process, the total power that is scattered remains unchanged.

A special class of partially coherent beams is formed by those with a J_0-Bessel correlation. Such beams are easily produced with the help of uncorrelated, annular sources. Unlike, for example, Gaussian correlation functions, Bessel functions can take on negative values, which leads to qualitatively different physical effects. For example, when a Gaussian-correlated field is focused, the diffraction pattern gets washed out, with the maximum remaining at the focal point. In contrast, a Bessel-correlated field creates a minimum at focus [22–24]. Likewise, when a Gaussian-correlated beam is scattered, the scattering remains predominantly in the forward direction [18]. Here, we show that scalar Mie scattering with J_0-correlated fields leads to a radically different profile in which the maximum occurs in a cone centered around the forward direction. We examine the influence of the transverse coherence length of the incident field and find that, by reducing this length, the angle of maximum scattering can be gradually moved from 0° to 29°. We show that the extinguished power is independent of the coherence length. That means that changing this length results in a redistribution of the total scattered field.

In the space-frequency domain, the time-independent part of an incident wave field at position $\mathbf{r}$ and frequency ω can be represented in terms of an angular spectrum of plane waves propagating in directions $\mathbf{u} = (\mathbf{u}_\perp, u_z)$ into the half-space $z > 0$, namely, [25, Section 3.2]:

$$U^{(\mathrm{inc})}(\mathbf{r}, \omega) = \int_{|\mathbf{u}_\perp|^2 \leq 1} a(\mathbf{u}_\perp, \omega) \exp(ik\mathbf{u} \cdot \mathbf{r}) \mathrm{d}^2 u_\perp, \quad (1)$$

where k denotes the wavenumber associated with frequency ω. Restricting the domain of integration to $|\mathbf{u}_\perp|^2 \leq 1$ implies that evanescent fields are neglected. The correlation properties of the field are characterized by its angular correlation function [25, Section 5.6.3]:

$$\mathcal{A}(\mathbf{u}_{1\perp}, \mathbf{u}_{2\perp}, \omega) = \langle a^*(\mathbf{u}_{1\perp}, \omega) a(\mathbf{u}_{2\perp}, \omega) \rangle, \quad (2)$$

where the angular brackets indicate the average taken over an ensemble of field realizations.

A general formalism for Mie scattering with partially coherent fields was presented in [19]. In particular, it was derived

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there that the radiant intensity $J^{(\mathrm{sca})}(\mathbf{s},\omega)$, the rate at which energy at frequency ω is scattered, per unit solid angle around the direction $\mathbf{s}$, is given by the expression

$$J^{(\mathrm{sca})}(\mathbf{s},\omega) = \int_{|\mathbf{u}_{1\perp}|^2\le 1}\int_{|\mathbf{u}_{2\perp}|^2\le 1} \mathcal{A}(\mathbf{u}_{1\perp},\mathbf{u}_{2\perp},\omega) \times f^*(\mathbf{s}\cdot\mathbf{u}_1)f(\mathbf{s}\cdot\mathbf{u}_2)\mathrm{d}^2u_{1\perp}\mathrm{d}^2u_{2\perp}. \tag{3}$$

[The symbol $J^{(\mathrm{sca})}$ is not to be confused with that for the Bessel function J_0.] Here, $f(\mathbf{s}\cdot\mathbf{u})$ denotes the amplitude scattered in direction $\mathbf{s}$ by a plane wave traveling along $\mathbf{u}$; see Fig. 1. For a homogeneous sphere of radius a and with refractive index n, it can be expressed as (see Eq. (4.66) in [26] with a trivial change in notation)

$$f(\mathbf{s}\cdot\mathbf{u}) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\exp[i\delta_l(\omega)]\sin[\delta_l(\omega)]P_l(\mathbf{s}\cdot\mathbf{u}), \tag{4}$$

where P_l denotes a Legendre polynomial of order l, and the phase shifts $\delta_l(\omega)$ are given by the expression [26, Sections 4.3.2 and 4.4.1]:

$$\tan[\delta_l(\omega)] = \frac{\bar{k}j_l(ka)j_l'(\bar{k}a) - kj_l(\bar{k}a)j_l'(ka)}{\bar{k}j_l'(\bar{k}a)n_l(ka) - kj_l(\bar{k}a)n_l'(ka)}. \tag{5}$$

Here, j_l and n_l are spherical Bessel functions and spherical Neumann functions, respectively, of order l. Furthermore,

$$\bar{k} = nk \tag{6}$$

is the wavenumber associated with the reduced wavelength within the scatterer, and the primes denote differentiation with respect to the spatial variable.

Let us now consider an incident field with a uniform spectral density $S^{(0)}(\omega)$, which is J_0 correlated. This means that its cross-spectral density function in the plane $z = 0$ (the plane that passes through the center of the sphere) is of the form

$$W^{(\mathrm{inc})}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = S^{(0)}(\omega)J_0(\beta|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|), \tag{7}$$

where J_0 denotes the Bessel function of the first kind and zeroth order, and $\boldsymbol{\rho}_1 = (x_1, y_1)$ and $\boldsymbol{\rho}_2 = (x_2, y_2)$ are 2D position vectors in the $z = 0$ plane. The parameter β is, roughly speaking, the inverse of the transverse coherence length of the incident field. The generation of such a beam requires an idealized, infinitely thin, δ-correlated ring source. However, a J_0-correlated beam may be approximated quite well by using an incoherent annular source, as was employed in [24].

In order to evaluate Eq. (3), we need to calculate the angular correlation function $\mathcal{A}(\mathbf{u}_{1\perp},\mathbf{u}_{2\perp},\omega)$. This function is related to the cross-spectral density through the expression [25, Section 5.6.3]

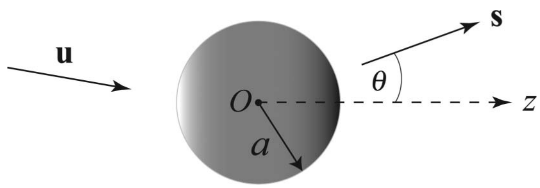

Fig. 1. A homogeneous sphere with radius a is illuminated by a plane wave propagating in the direction $\mathbf{u}$. The scattering angle θ is the angle between the positive z axis and an observation direction $\mathbf{s}$. Both $\mathbf{u}$ and $\mathbf{s}$ are unit vectors. The point O is the origin of the coordinate system.

$$\mathcal{A}(\mathbf{u}_{1\perp},\mathbf{u}_{2\perp},\omega) = \left(\frac{k}{2\pi}\right)^4 \iint_{-\infty} W^{(\mathrm{inc})}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \times\exp[-ik(\mathbf{u}_{2\perp}\cdot\boldsymbol{\rho}_2 - \mathbf{u}_{1\perp}\cdot\boldsymbol{\rho}_1)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \tag{8}$$

On substituting from Eq. (7) into Eq. (8), we find after some algebra and making use of two Dirac δ-function representations [27], the expression

$$\mathcal{A}(\mathbf{u}_{1\perp},\mathbf{u}_{2\perp},\omega) = \frac{k^2}{2\pi\beta}S^{(0)}(\omega)\delta^2(\mathbf{u}_{2\perp} - \mathbf{u}_{1\perp}) \times\delta(\beta - k|\mathbf{u}_{1\perp} + \mathbf{u}_{2\perp}|/2). \tag{9}$$

Substituting this result into Eq. (3), we obtain for the radiant intensity, the formula

$$J^{(\mathrm{sca})}(\mathbf{s},\omega) = \frac{S^{(0)}(\omega)k^2}{2\pi\beta}\int_{|\mathbf{u}_{1\perp}|^2\le 1}\delta(\beta - k|\mathbf{u}_{1\perp}|) \times|f(\mathbf{s}\cdot\mathbf{u}_1)|^2\mathrm{d}^2u_{1\perp}. \tag{10}$$

Since $\mathbf{u}$ is a unit vector, it follows from the δ function in Eq. (10) that the incident Bessel-correlated beam must satisfy the condition

$$0 < \beta < k \tag{11}$$

to produce a nonzero scattered field. Because $1/\beta$ is a rough measure of the transverse coherence length, this implies that this length must exceed $1/k = \lambda/2\pi$. It is shown in Appendix A that under typical circumstances, $\beta < k/2$. We may thus, without loss of generality, set $|\mathbf{u}_{1\perp}| = \beta/k$. Next, let us write the vector $\mathbf{u}_1$, indicating a direction of incidence, and the vector $\mathbf{s}$, indicating the direction of scattering, as

$$\mathbf{u}_1 = \left(\beta k^{-1}\cos\alpha, \beta k^{-1}\sin\alpha, \sqrt{1 - \beta^2/k^2}\right), \tag{12}$$

$$\mathbf{s} = (\sin\theta\cos\gamma, \sin\theta\sin\gamma, \cos\theta). \tag{13}$$

Expressing the integral of Eq. (10) in polar coordinates gives

$$J^{(\mathrm{sca})}(\mathbf{s},\omega) = \frac{S^{(0)}(\omega)k^2}{2\pi\beta}\int_0^1\int_0^{2\pi}\delta(\beta - k|\mathbf{u}_{1\perp}|)|f(\mathbf{s}\cdot\mathbf{u}_1)|^2 \times|\mathbf{u}_{1\perp}|\mathrm{d}\alpha\mathrm{d}u_{1\perp}, \tag{14}$$

$$= \frac{S^{(0)}(\omega)}{2\pi k^2}\int_0^{2\pi}\mathrm{d}\alpha\sum_l\sum_m(2l+1)(2m+1) \times\exp[i(\delta_l - \delta_m)]\sin\delta_l\sin\delta_m \times P_l\left[\beta k^{-1}\cos\alpha\sin\theta + \cos\theta\sqrt{1 - \beta^2/k^2}\right] \times P_m\left[\beta k^{-1}\cos\alpha\sin\theta + \cos\theta\sqrt{1 - \beta^2/k^2}\right], \tag{15}$$

where we have used that $\delta(\beta - k|\mathbf{u}_\perp|) = k^{-1}\delta(\beta/k - |\mathbf{u}_\perp|)$ and the dependence on γ drops out on integration over α. Equation (15) shows how the angular distribution of the scattered field arises from an intricate interplay of the sphere radius a, its refractive index n, and the inverse coherence length β of the incident field. Formidable as Eq. (15) may look, it can easily be solved numerically.

The dependence of the scattered radiant intensity on the normalized coherence parameter β/k is shown in Fig. 2 for scattering angles up to 90°. The left-most curve (red) is for

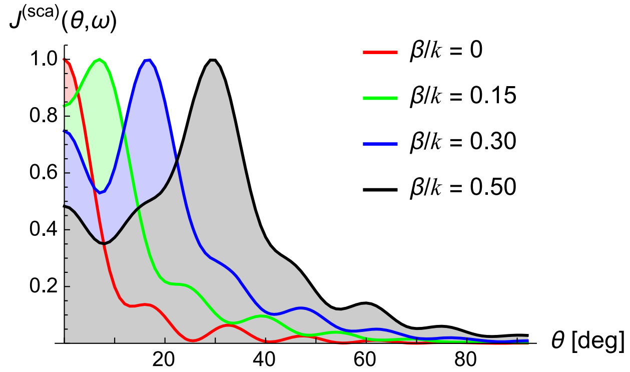

Fig. 2. Angular distribution of the normalized radiant intensity of the scattered field for selected values of the normalized coherence parameter β/k, namely: 0 (red), 0.15 (green), 0.30 (blue), and 0.50 (black). In this example, the wavelength $\lambda = 632.8$ nm, the refractive index $n = 1.33$, and the sphere radius $a = 2\lambda$.

$\beta/k = 0$, which corresponds to the case of a fully coherent incident field. We indeed retrieve the classical Mie result with strong forward scattering. However, when the coherence length is decreased to $\beta/k = 0.15$ (green curve), the forward scattering is somewhat suppressed and the maximum scattering occurs at an angle of $\theta = 7°$ deg. For the cases $\beta/k = 0.30$ (blue) and $\beta/k = 0.50$ (black), the angle of maximum scattering moves to 17° and 29°, respectively. Also, the minimums are raised from their near-zero value. We note that as long as $\beta/k < 0.12$, the maximum occurs in the forward direction. This maximum only shifts to larger angles when the correlation function, Eq. (7), takes on negative values for pairs of points in the sphere, i.e., when $J_0(\beta 2a) < 0$.

The unnormalized scattered field for all scattering angles is shown on a logarithmic scale in Fig. 3. For the fully coherent case, we obtain the well-known Mie resonances (red curve). These become much less pronounced for the partially coherent cases (green, blue, and black curves). In these last three cases, the backscattering is strongly decreased compared with the fully coherent case.

The question of how the coherence parameter β may be varied is addressed in Appendix A. There, it is shown that one possible way is to change the radius of the annular source. A practical implementation is also described in [24].

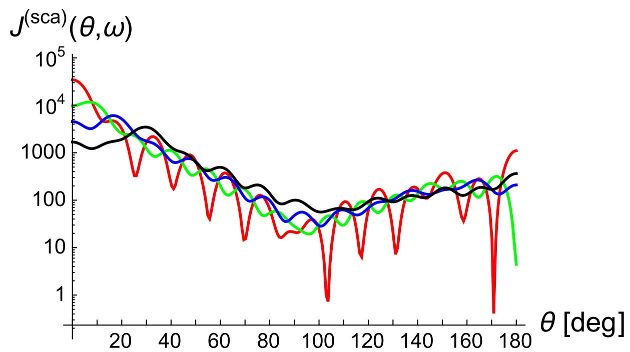

Fig. 3. Angular distribution of the logarithmic radiant intensity of the scattered field for selected values of the normalized coherence parameter β/k:0 (red), 0.15 (green), 0.30 (blue), and 0.50 (black). The parameters are the same as in Fig. 2.

The optical theorem in its classical form [2,3] relates the total extinguished power (due to scattering and absorption) to the scattering amplitude in the forward direction. Because this theorem assumes the incident field to be a monochromatic plane wave, rather than a partially coherent field, it does not apply to the present case. However, the theorem has been generalized to deal with stochastic fields by Carney *et al.* [16]. They derived that the ensemble-averaged extinguished power $\langle P_e(\omega)\rangle$ is then given by the expression

$$\langle P_e(\omega)\rangle = \frac{4\pi}{k}\,\mathrm{Im}\left[\int_{|\mathbf{u}_{1\perp}|\le 1}\int_{|\mathbf{u}_{2\perp}|\le 1} A(\mathbf{u}_1,\mathbf{u}_2,\omega) \times f(\mathbf{u}_1\cdot\mathbf{u}_2)\mathrm{d}^2u_{1\perp}\mathrm{d}^2u_{2\perp}\right]. \tag{16}$$

Because we are assuming a nonabsorbing scatterer, the extinction is entirely due to scattering, i.e., the extinguished power is equal to the radiant intensity of the scattered field integrated over a 4π solid angle:

$$\langle P_e(\omega)\rangle = \int_0^{2\pi}\int_0^{\pi} J^{(\mathrm{sca})}(\mathbf{s},\omega)\sin\theta\mathrm{d}\theta\mathrm{d}\phi. \tag{17}$$

On substituting from Eq. (9) for the angular correlation function into Eq. (16), we obtain the formula

$$\langle P_e(\omega)\rangle = \frac{2kS^{(0)}(\omega)}{\beta} \times \mathrm{Im}\left[\int_{|\mathbf{u}_{1\perp}|\le 1}\delta(\beta - k|\mathbf{u}_{1\perp}|)f(\mathbf{u}_1\cdot\mathbf{u}_1)\mathrm{d}^2u_{1\perp}\right]. \tag{18}$$

Evaluating this in polar coordinates gives

$$\langle P_e(\omega)\rangle = \frac{4\pi S^{(0)}(\omega)}{k}\,\mathrm{Im}[f(\mathbf{u}\cdot\mathbf{u})]. \tag{19}$$

In Eq. (19), the two arguments of the scattering amplitude f are equal, i.e., $f(\mathbf{u}\cdot\mathbf{u})$ represents the forward scattering amplitude. It is seen from this expression that the total scattered power does *not* depend on the coherence parameter β. This implies that varying β, as was illustrated in Figs. 2 and 3, results in a redistribution of the scattered intensity with the total scattered power being unaffected. Also, it was verified numerically that Eqs. (19) and (17) yield the same result.

Some related results were reported in [28], where it was suggested that Bessel-correlated fields can give rise to strongly suppressed scattering in the forward direction. In contrast to the present study, this result was obtained for a random spherical scatterer while making use of the first-order Born approximation. However, it is well known that this approximation is incompatible with the optical theorem [29], i.e., it violates energy conservation. In contrast, using Mie theory allows us to make use of the optical theorem. We thus find that the extinguished power is independent of the coherence length. That means that changing the coherence length of the incident field results in a redistribution of the total scattered field. Also, our analysis pertains to the important class of scatterers that are deterministic, rather than random. Furthermore, the angular shifts of the direction of maximum scattering that we find while using Mie theory are significantly larger than those obtained using the Born approximation.

In conclusion, we have demonstrated that the angular distribution of a field that is scattered by a homogeneous sphere can be controlled. In contrast to previous works, this is done by manipulating the incident beam rather than the scatterer. In particular, an incident beam with a J_0-Bessel correlation gives rise to an unusual scattering profile. This profile can be changed by varying the spatial coherence length. The total power of the scattered field remains constant when the transverse coherence length is varied. This provides a new tool to steer the scattered field dynamically without losing energy. This method may be used to selectively address detectors that are not (or cannot be) located along the line of sight connecting the source and the scatterer. Such detectors have the advantage that they are not saturated by the illuminating beam.

APPENDIX A

For the idealized case of a δ-correlated, infinitely thin ring source of radius c, the cross-spectral density function of the source field reads

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S^{(0)}(\omega)\delta(\rho_1 - c)\delta^2(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1). \quad \text{(A1)}$$

Away from the source, this function is given by [25, Eqs. (4.4)–(18)]

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left(\frac{k}{2\pi}\right)^2 \iint_{z=0} W^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) \frac{\exp[ik(R_2 - R_1)]}{R_1 R_2} \times \cos\theta_1 \cos\theta_2 \mathrm{d}^2\rho_1 \mathrm{d}^2\rho_2, \quad \text{(A2)}$$

where $R_i = |\mathbf{r}_i - (\boldsymbol{\rho}_i, 0)|$ and $i = 1, 2$. On substituting from Eq. (A1) into Eq. (A2) and making the usual far-zone approximations described in [25, Section 4.4.4], we then find that

$$W^{(\infty)}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \omega) = S^{(0)}(\omega) J_0\left(\frac{kc|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|}{\Delta z}\right), \quad \text{(A3)}$$

where c denotes the ring radius, and Δz is the approximate distance from the center of the ring to the two observation points $(\boldsymbol{\rho}_1, z)$ and $(\boldsymbol{\rho}_2, z)$. For the far-zone approximation to be valid, the distance Δz should be at least, let us say, twice as large as the radius c. Comparing the condition $\Delta z \geq 2c$ with Eq. (7) gives $\beta \leq 0.5k$. This upper limit for the parameter β means that Eq. (11) is always satisfied. Also, it translates into a transverse coherence length $1/\beta \geq \lambda/\pi$.

A way to obtain a variable coherence parameter β is suggested by comparing Eqs. (7) and (A3). It is seen that β corresponds to $kc/\Delta z$. This means that the coherence length of the incident field can be varied by changing the radius of the annular source.

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