WHEPP X
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Non-collinearity in high energy scattering processes

Piet Mulders

mulders@few.vu.nl

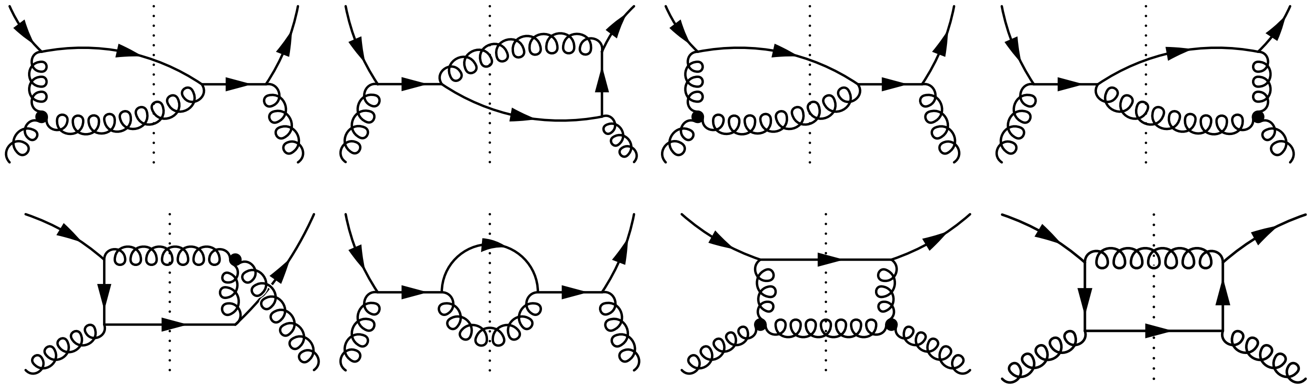
Outline

- Introduction: partons in high energy scattering processes
- (Non-)collinearity: collinear and non-collinear parton correlators
  - OPE, twist
  - Gauge invariance
  - Distribution functions (collinear, TMD)
- Observables
  - Azimuthal asymmetries
  - Time reversal odd phenomena/single spin asymmetries
- Gauge links
  - Resumming multi-gluon interactions: Initial/final states
  - Color flow dependence
- Applications
- Universality: an example gq→gq
- Conclusions

QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate cross sections, e.g. $\gamma^*q \to q$, $q\bar{q} \to \gamma^*$, $\gamma^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- E.g.

  $qg \to qg$



- Calculations work for plane waves

$$\left\langle 0 \left| \psi_i^{(s)}(\xi) \right| p, s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

Confinement in QCD

- Confinement limits us to hadrons as 'quark sources' or 'targets' (with $P_X = P - p$)

$$\left\langle X \left| \psi_i^{(s)}(\xi) \right| P \right\rangle e^{+ip.\xi}$$

$$\left\langle X \left| \psi_i^{(s)}(\xi) A^\mu(\eta) \right| P \right\rangle e^{+i(p-p_1).\xi + ip_1.\eta}$$

- These involve nucleon states
- At high energies interference terms between different hadrons disappear as $1/P_1.P_2$
- Thus, the theoretical description/calculation involves for hard processes, a forward matrix element of the form

$$\Phi_{ij}(p,P) = \int \frac{d^3 P_X}{(2\pi)^3\, 2E_X} < P \,|\, \overline{\psi}_j(0) \,|\, X >< X \,|\, \psi_i(0) \,|\, P > \delta(P - P_X - p)$$

quark momentum

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{i\,p.\xi} < P \,|\, \overline{\psi}_j(0)\, \psi_i(\xi) \,|\, P >$$

Correlators in high-energy processes

- Look at parton momentum p
- Parton belonging to a particular hadron P: p.P ~ M^2
- For all other momenta K: p.K ~ P.K ~ s ~ Q^2
- Introduce a generic vector n ~ satisfying P.n = 1, then we have n ~ 1/Q, e.g. n = K/(P.K)

$$p = x P^\mu + p_T^\mu + \sigma n^\mu$$

x ~ Q, p_T^μ ~ M, σ ~ M^2/Q

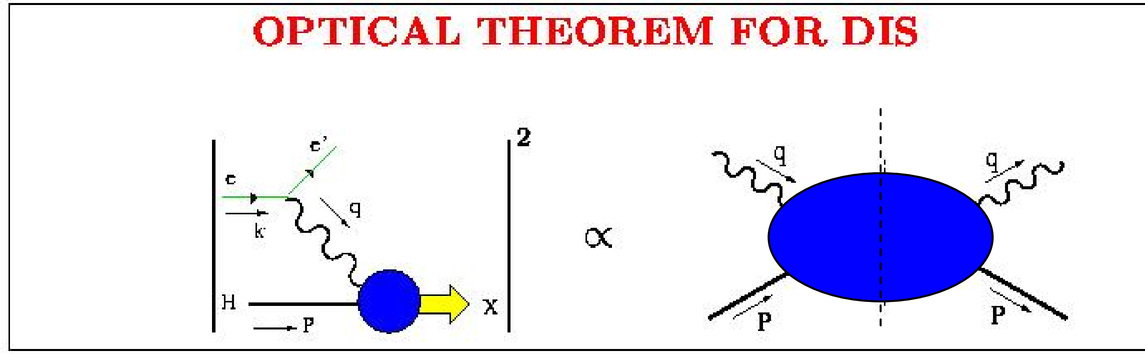
$$x = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

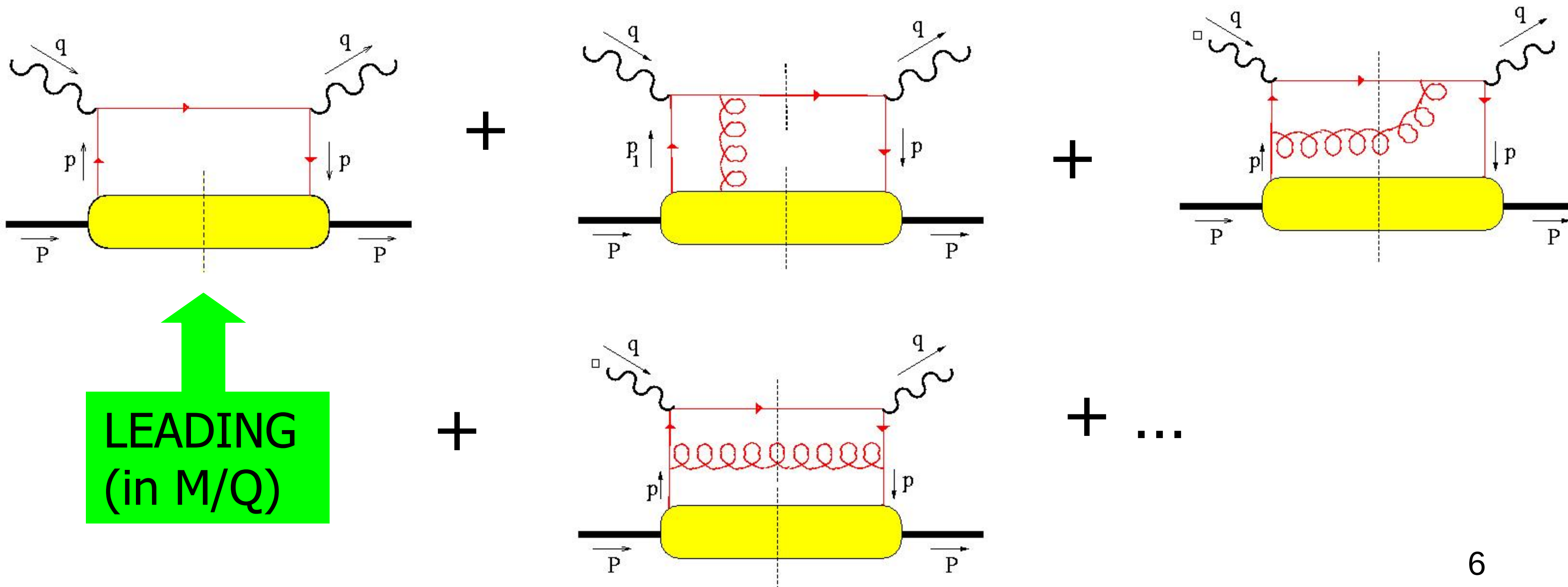
- Up to corrections of order M^2/Q^2 one can perform the σ-integration

$$\Phi_{ij}(x, p_T) = \int d(p.P)\,\Phi_{ij}(p,P) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \psi_j^\dagger(0)\,\psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

(calculation of) cross section in DIS

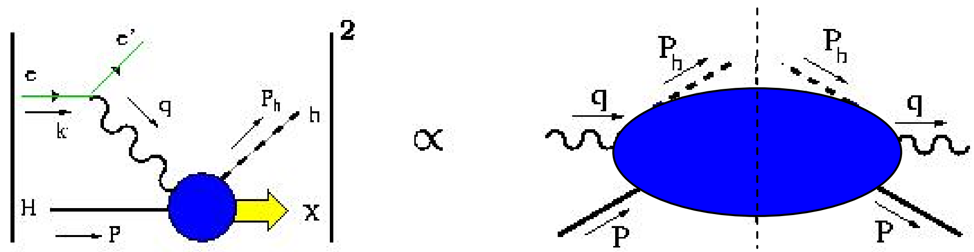


Full calculation

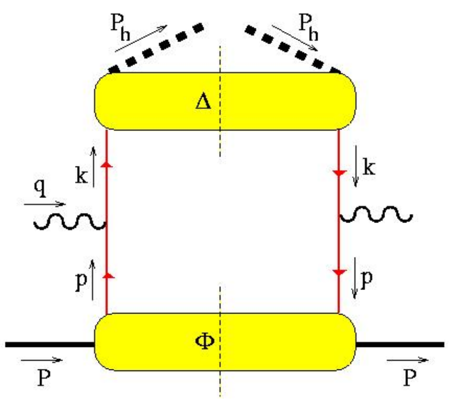


(calculation of) cross section in SIDIS

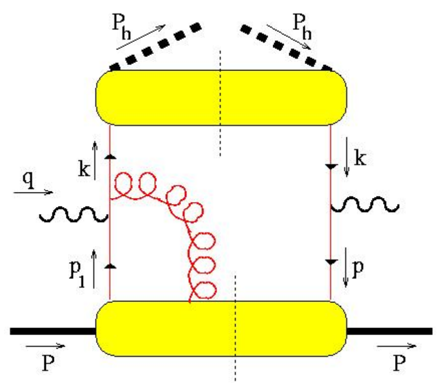
OPTICAL THEOREM FOR SIDIS



Full calculation



+



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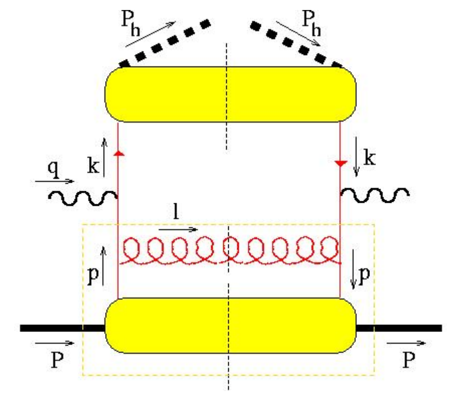
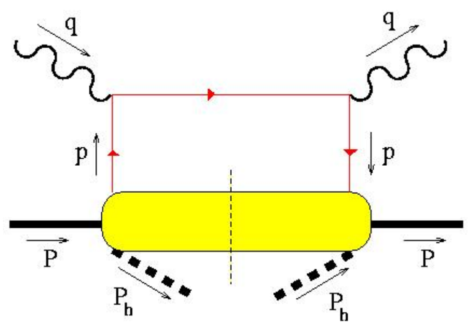


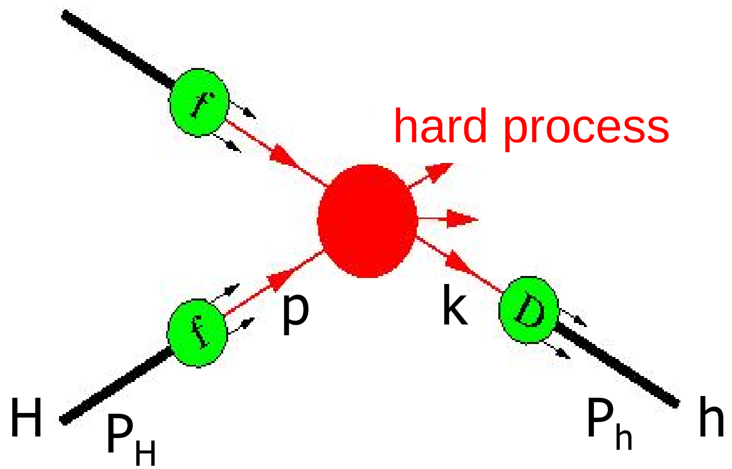
LEADING
(in M/Q)

+



+ …

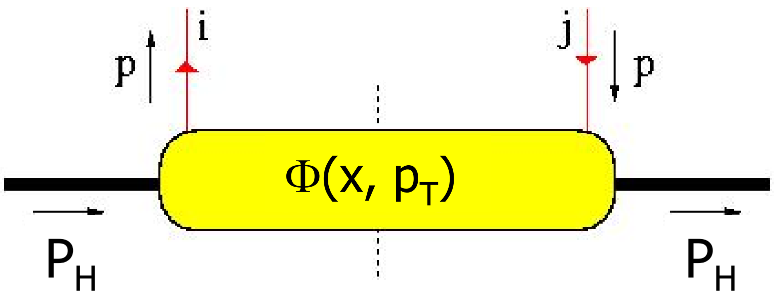
Leading partonic structure of hadrons



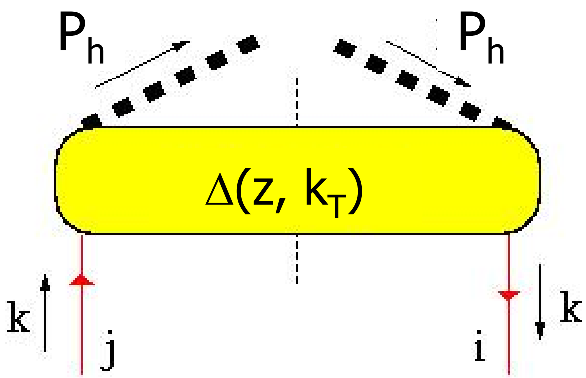
Need $P_H.P_h \sim s$ (large) to get separation of soft and hard parts

Allows $\int d\sigma \ldots = \int d(p.P) \ldots$

distribution correlator



fragmentation correlator



Partonic correlators

The cross section can be expressed in hard squared QCD-amplitudes and distribution and fragmentation functions entering in forward matrix elements of nonlocal combinations of quark and gluon field operators ($\phi \rightarrow \psi$ or G). These are the (hopefully universal) objects we are after, useful in parametrizations and modelling.

Distribution functions

$$p^\mu = x P^\mu + p_T^\mu + \frac{p.P - x M^2}{P.n} n^\mu$$

$$\Phi(x, p_T) = \int \frac{d(\xi.P) d^2\xi_T}{(2\pi)^3} e^{i\, p.\xi} \langle P | \phi^\dagger(0) \phi(\xi) | P \rangle_{\xi.n=0}$$

Fragmentation functions

$$k^\mu = \frac{1}{z} K^\mu + k_T^\mu + \frac{k.K - z^{-1} M_h^2}{K.n} n^\mu$$

$$\Delta(z, k_T) = \int \frac{d(\xi.K) d^2\xi_T}{(2\pi)^3} e^{-ik.\xi} \langle 0 | \phi(0) | K, X \rangle \langle K, X | \phi^\dagger(\xi) | 0 \rangle_{\xi.n=0}$$

(non-)collinearity of parton correlators

The cross section can be expressed in hard squared QCD-amplitudes and distribution and fragmentation functions entering in forward matrix elements of nonlocal combinations of quark and gluon field operators ($\phi \rightarrow \psi$ or G). These are the (hopefully universal) objects we are after, useful in parametrizations and modelling.

Distribution functions

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TMD

lightfront: $\xi^+ = 0$

$$\Phi(x) = \int d^2p_T\, \Phi(x, p_T) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \langle P | \phi^\dagger(0)\phi(\xi) | P \rangle_{\xi.n=\xi_T=0}$$

collinear

lightcone

$$\Phi = \int dx\, \Phi(x) = \langle P | \phi^\dagger(0)\phi(0) | P \rangle$$

local

Spin and twist expansion

Spin n:
~ ($P^{\mu 1} \ldots P^{\mu n}$ – traces)
Twist t:
dimension – spin

- Local matrix elements in Φ
  Operators can be classified via their canonical dimensions and spin (OPE)
- Nonlocal matrix elements in $\Phi(x)$
  Parametrized in terms of (collinear) distribution functions f…(x) that involve operators of different spin but with one specific twist t that determines the power of $(M/Q)^{t-2}$ in observables (cross sections and asymmetries).
  Moments give local operators.

$$M^{(n)} = \int dx\, x^{n-1} f(x)$$

- Nonlocal matrix elements in $\Phi(x,p_T)$
  Parametrized in terms of TMD distribution functions f…(x,p_T^2) that involve operators of different spin and different twist. The lowest twist determines the operational twist t of the TMD functions and determines the power of $(M/Q)^{t-2}$ in observables.
  Transverse moments give collinear functions.

$$f^{(n)}(x) = \int d^2 p_T \left(\frac{-p_T^2}{2M^2} \right)^n f(x, p_T^2)$$

Gauge invariance for quark correlators

- Presence of gauge link needed for color gauge invariance

$$U_{[0,\xi]}^{[C]} = \mathscr{P} \exp\left(-ig \int_0^\xi ds^\mu A_\mu \right)$$

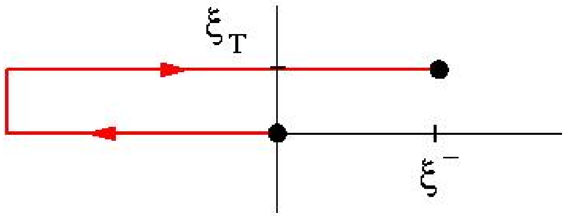
$$A^\mu = A^+ P^\mu + A_T^\mu + A^- n^\mu$$

- The gauge link arises from all 'leading' m.e.'s as $\psi\ A^+ \ldots A^+\ \psi$

$$\Phi_{ij}^q(x;n) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}_j(0) U_{[0,\xi]}^{[n]} \psi_i(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

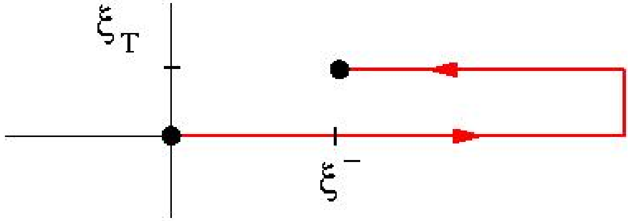
$$\Phi_{ij}^q(x, p_T; n, C) = \int \frac{d(\xi.P) d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}_j(0) U_{[0,\xi]}^{[C]} \psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- Transverse pieces arise from $A_T^\alpha \rightarrow G^{+\alpha} = \partial^+ A^\alpha + \ldots$
- Basic gauge links:

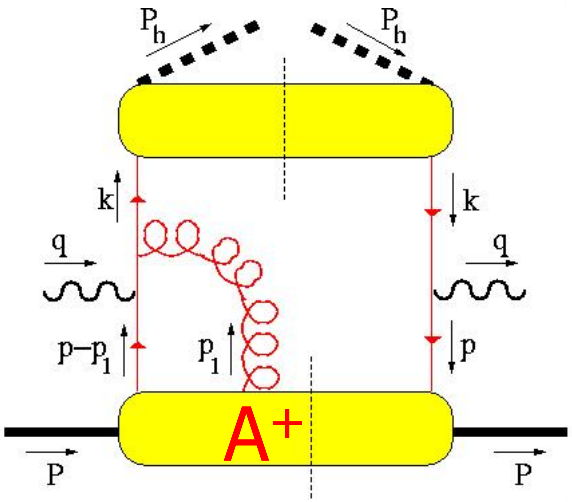


$\Phi^{[U^-]} = \Phi^{[-]}$

TIME RERVERSAL

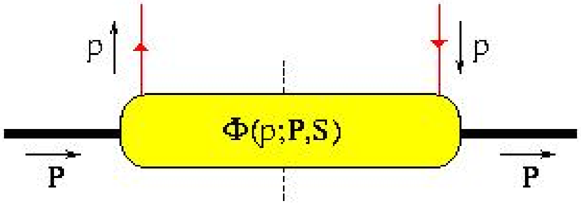


$\Phi^{[U^+]} = \Phi^{[+]}$



Distribution

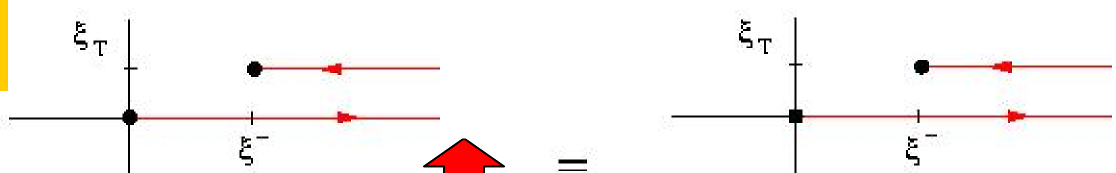
$x = p^+/P^+$



$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi} \langle P, S|\overline{\psi}_j(0)\psi_i(\xi)|P, S\rangle\Big|_{\xi^+=0}$$

including the gauge link (in SIDIS)

$$\Phi^{[+]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{ip\cdot\xi} \langle P, S|\overline{\psi}(0)\, U^-_{[0,\infty]} U^T_{[0_T,\infty_T]}\, U^T_{[\infty_T,\xi_T]} U^-_{[\infty,\xi]}\, \psi(\xi)|P, S\rangle\Big|_{\xi^+=0}$$



One needs also A_T

$G^{+\alpha} = \partial^+ A_T^\alpha$

$A_T^\alpha(\xi) = A_T^\alpha(\infty) + \int d\eta\; G^{+\alpha}$

Belitsky, Ji, Yuan 2002
Boer, M, Pijlman 2003

From $\langle\psi(0)A_T(\infty)\psi(\xi)\rangle$ m.e.

Gauge invariance for gluon correlators

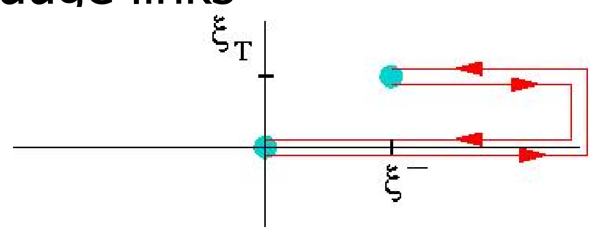
$$\Phi_g^{\alpha\beta}(x,p_T;C,C') = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| U^{[C]}_{[\xi,0]} F^{n\alpha}(0) U^{[C']}_{[0,\xi]} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- Using 3x3 matrix representation for U, one finds in TMD gluon correlator appearance of two links, possibly with different paths.
- Note that standard field displacement involves C = C′

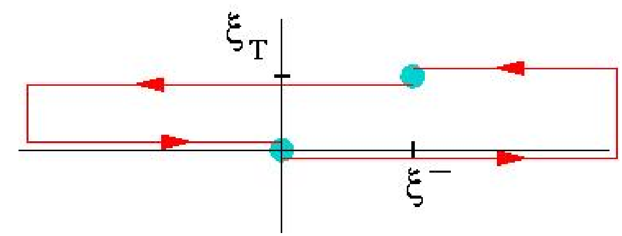
$$F^{\alpha\beta}(\xi) \to U^{[C]}_{[\eta,\xi]} F^{\alpha\beta}(\xi) U^{[C]}_{[\xi,\eta]}$$

- Basic gauge links

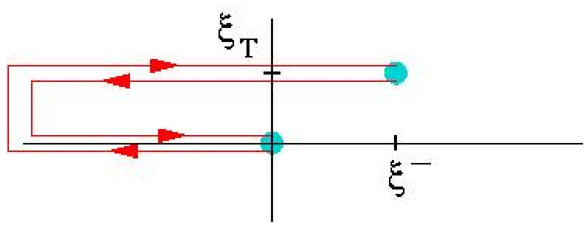
$\Phi_g^{[+,+]}$



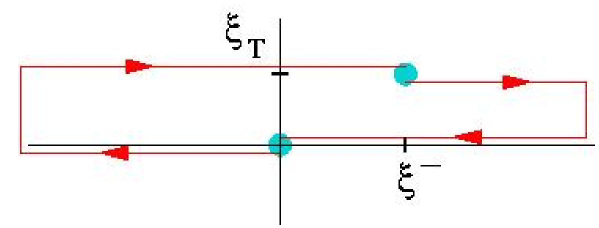
$\Phi_g^{[+,-]}$



$\Phi_g^{[-,-]}$



$\Phi_g^{[-,+]}$



Collinear parametrizations

- Gauge invariant correlators → distribution functions
- Collinear quark correlators (leading part, no n-dependence)

$$\Phi^q(x) = \left(f_1^q(x) + S_L\, g_1^q(x)\,\gamma_5 + h_1^q(x)\,\gamma_5 \not{S}_T \right) \frac{\not{P}}{2}$$

$$S \approx S_L \frac{P^\mu}{M} + S_T^\mu$$

- i.e. massless fermions with momentum distribution $f_1^q(x)$ = q(x), chiral distribution $g_1^q(x)$ = Δq(x) and transverse spin polarization $h_1^q(x)$ = δq(x) in a spin ½ hadron
- Collinear gluon correlators (leading part)

$$\Phi_g^{\mu\nu}(x) = \frac{1}{2x}\left(-g_T^{\mu\nu}\, f_1^g(x) + i S_L\, \varepsilon_T^{\mu\nu}\, g_1^g(x) \right)$$

- i.e. massless gauge bosons with momentum distribution $f_1^g(x)$ = g(x) and polarized distribution $g_1^g(x)$ = Δg(x)

TMD parametrizations

- Gauge invariant correlators → distribution functions
- TMD quark correlators (leading part, unpolarized)

$$\Phi^q(x,p_T) = \left(f_1^q(x,p_T^2) + i\,h_1^{\perp q}(x,p_T^2)\frac{\not{p}_T}{M}\right)\frac{\not{P}}{2}$$

- as massless fermions with momentum distribution $f_1^q(x,p_T)$ and transverse spin polarization $h_1^{\perp q}(x,p_T)$ in an unpolarized hadron
- The function $h_1^{\perp q}(x,p_T)$ is T-odd!

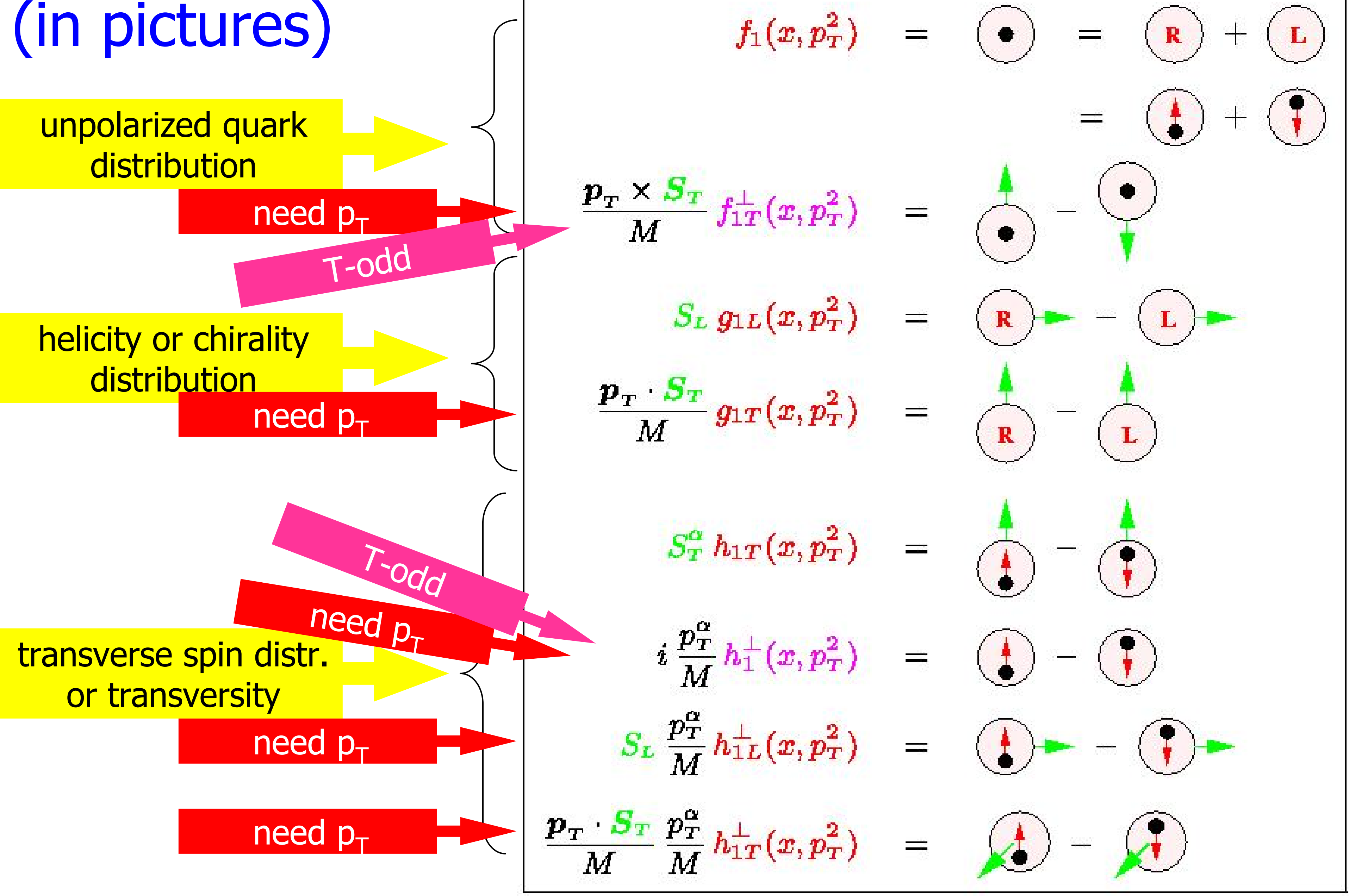
- TMD gluon correlators (leading part, unpolarized)

$$\Phi_g^{\mu\nu}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu + \frac{1}{2} g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g}(x,p_T^2)\right)$$

- as massless gauge bosons with momentum distribution $f_1^g(x,p_T)$ and linear polarization $h_1^{\perp g}(x,p_T)$ in an unpolarized hadron

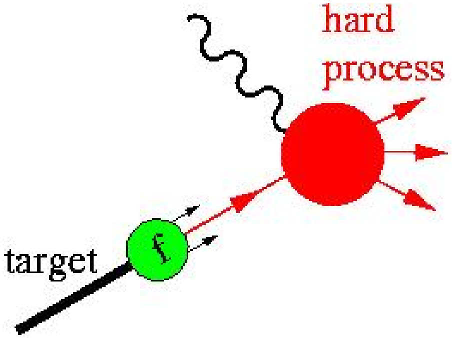
TMD correlators $\Phi_q^{[U]}$ and $\Phi_g^{[U,U']}$ do depend on gauge links!

The quark distributions (in pictures)



Results for deep inelastic processes

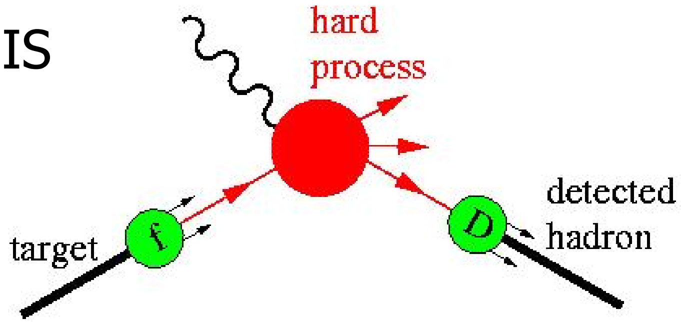
DIS



$$\sigma_{\gamma^* N \to X} = f_1^{N \to q}(x) \otimes \hat{\sigma}_{\gamma^* q \to q}$$

$$\Delta\sigma_{\gamma^* \vec{N} \to X} = g_1^{N \to q}(x) \otimes \Delta\hat{\sigma}_{\gamma^* \vec{q} \to \vec{q}}$$

SIDIS



$$\sigma_{\gamma^* N \to hX} = f_1^{N \to q}(x) \otimes \hat{\sigma}_{\gamma^* q \to q} \otimes D_1^{q \to N}(z)$$

Probing intrinsic transverse momenta

$$p \approx xP + p_T$$
$$k \approx z^{-1}P + k_T$$

- In a hard process one probes quarks and gluons
- Momenta fixed by kinematics (external momenta)

  DIS $\quad x = x_B = Q^2 / 2P.q$

  SIDIS $\quad z = z_h = P.K_h / P.q$

- Also possible for transverse momenta

  SIDIS $\quad q_T = q + x_B P - z_h^{-1} K_h = k_T - p_T$

  2-particle inclusive hadron-hadron scattering

  $$q_T = z_1^{-1}K_1 + z_2^{-1}K_2 - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T} - k_{1T} - k_{2T}$$

- Sensitivity for transverse momenta requires ≥ 3 momenta

  SIDIS: $\gamma^* + H \rightarrow h + X$

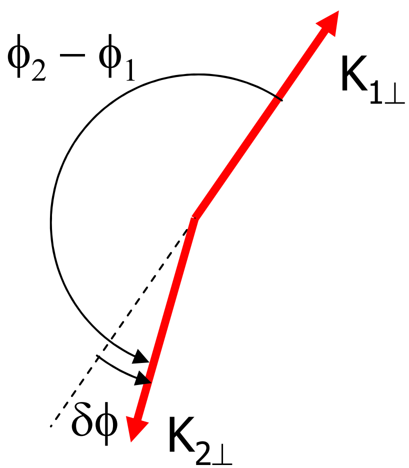
  DY: $H_1 + H_2 \rightarrow \gamma^* + X$

  e+e-: $\gamma^* \rightarrow h_1 + h_2 + X$

  hadronproduction: $H_1 + H_2 \rightarrow h_1 + h_2 + X$

  $\rightarrow h + X$ (?)

...and knowledge of hard process(es)!



pp-scattering

Time reversal as discriminator

$$W_{\mu\nu}(q;P,S,P_h,S_h) = -W_{\mu\nu}(-q;P,S,P_h,S_h)$$ symmetry structure

$$W^*_{\mu\nu}(q;P,S,P_h,S_h) = W_{\nu\mu}(q;P,S,P_h,S_h)$$ hermiticity

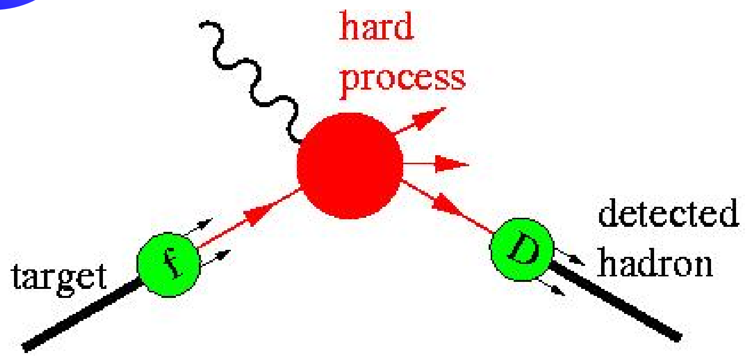
$$W_{\mu\nu}(q;P,S,P_h,S_h) = \overline{W}_{\mu\nu}(\overline{q};\overline{P},-\overline{S},\overline{P}_h,-\overline{S}_h)$$ parity

$$W^*_{\mu\nu}(q;P,S,P_h,S_h) = \overline{W}_{\mu\nu}(\overline{q};\overline{P},\overline{S},\overline{P}_h,\overline{S}_h)$$ time reversal

$$\boxed{W_{\mu\nu}(q;P,S,P_h,S_h) = W_{\nu\mu}(q;P,-S,P_h,-S_h)}$$ combined

- If time reversal can be used to restrict observable one has only even spin asymmetries
- If time reversal symmetry cannot be used as a constraint (SIDIS, DY, pp, …) one can nevertheless connect T-even and T-odd phenomena (since T holds at level of QCD).
- In hard part T is valid up to order α_s^2

Results for deep inelastic processes



$$q_T = q + x_B P - z_h^{-1} K_h = k_T - p_T$$

Sivers asymmetry

$$\left\langle \frac{|q_T|}{M} \sin(\phi_h^\ell + \phi_S^\ell)\, \sigma_{\gamma^* N^\uparrow \to \pi X} \right\rangle = h_1^q(x) \otimes \hat{\sigma}_{\gamma^* q^\uparrow \to q^\uparrow} \otimes H_1^{\perp(1)q}(z)$$

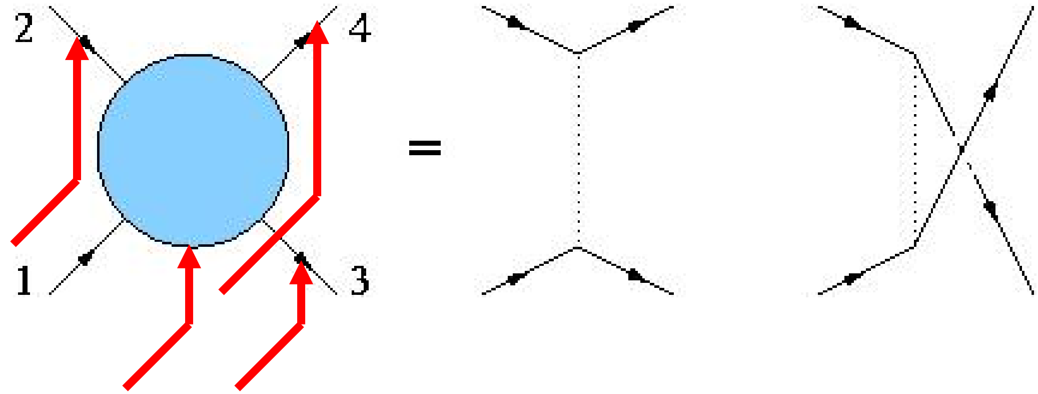
Collins asymmetry

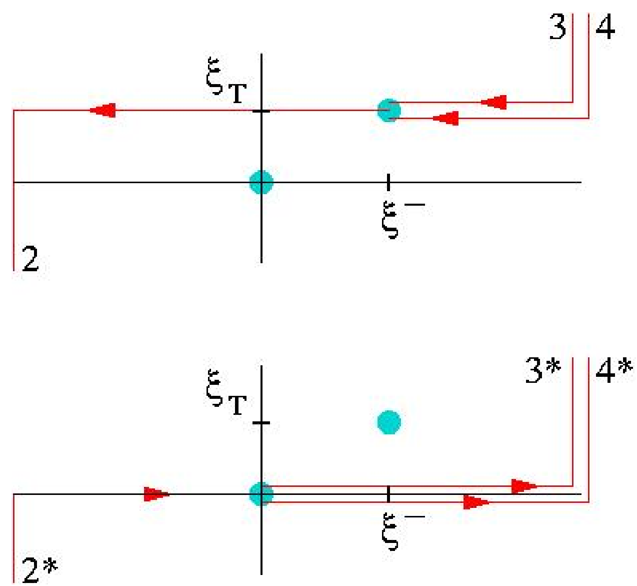
$$\left\langle \frac{|q_T|}{M} \sin(\phi_h^\ell - \phi_S^\ell)\, \sigma_{\gamma^* N^\uparrow \to \pi X} \right\rangle = f_{1T}^{\perp(1)q}(x) \otimes \hat{\sigma}_{\gamma^* q \to q} \otimes D_1^q(z)$$

Function as appearing in parametrization of $\Phi^{[+]}$

Generic hard processes

- Matrix elements involving parton 1 and additional gluon(s) $A^+ = A.n$ appear at same (leading) order in 'twist' expansion and produce link $\Phi^{[U]}(1)$
- insertions of gluons collinear with parton 1 are possible at many places
- this leads for correlator $\Phi(1)$ to gauge links running to lightcone ± infinity
- SIDIS → $\Phi^{[+]}(1)$
- DY → $\Phi^{[-]}(1)$





Link structure for fields in correlator 1

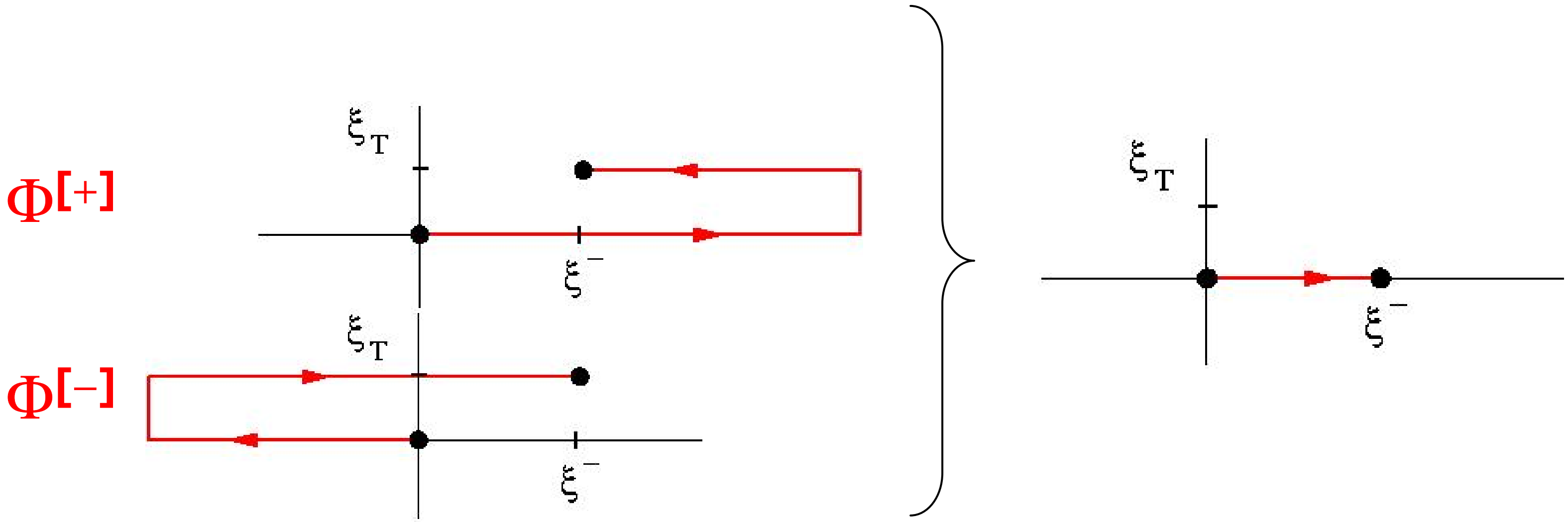
C. Bomhof, P.J. Mulders and F. Pijlman, PLB 596 (2004) 277 [hep-ph/0406099]; EPJ C 47 (2006) 147 [hep-ph/0601171]

Integrating $\Phi^{[\pm]}(x,p_T) \to \Phi^{[\pm]}(x)$

$$\Phi^{[\pm]}(x,p_T) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \langle P|\psi^\dagger(0) U^n_{[0,\pm\infty]} U^T_{[0_T,\xi_T]} U^n_{[\pm\infty,\xi]} \psi(\xi)|P\rangle_{\xi.n=0}$$

collinear correlator

$$\Phi^{[\pm]}(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \langle P|\psi^\dagger(0) U^n_{[0,\xi]} \psi(\xi)|P\rangle_{\xi.n=\xi_T=0}$$



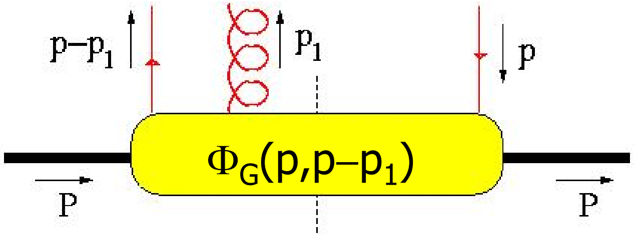
Integrating $\Phi^{[\pm]}(x,p_T) \rightarrow \Phi_\partial^{\alpha[\pm]}(x)$

transverse moments

$$\Phi_\partial^{\alpha[\pm]}(x) = \int d^2p_T\; p_T^\alpha\, \Phi^{[\pm]}(x,p_T)$$

$$\Phi_\partial^{\alpha[\pm]}(x) = \int d^2p_T \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \langle P|\psi^\dagger(0)U^n_{[0,\pm\infty]}\, i\partial_T^\alpha U^T_{[0_T,\xi_T]} U^n_{[\pm\infty,\xi]}\psi(\xi)|P\rangle_{\xi.n=0}$$

$$= \Phi_D^\alpha(x) + \int dx_1 \frac{i}{x_1 \mp i\varepsilon} \Phi_G^\alpha(x, x-x_1)$$



$$= \underbrace{\Phi_D^\alpha(x) + \int dx_1\, P\frac{i}{x_1} \Phi_G^\alpha(x, x-x_1)}_{\Phi_\partial^\alpha(x)} \pm \pi \Phi_G^\alpha(x,x)$$

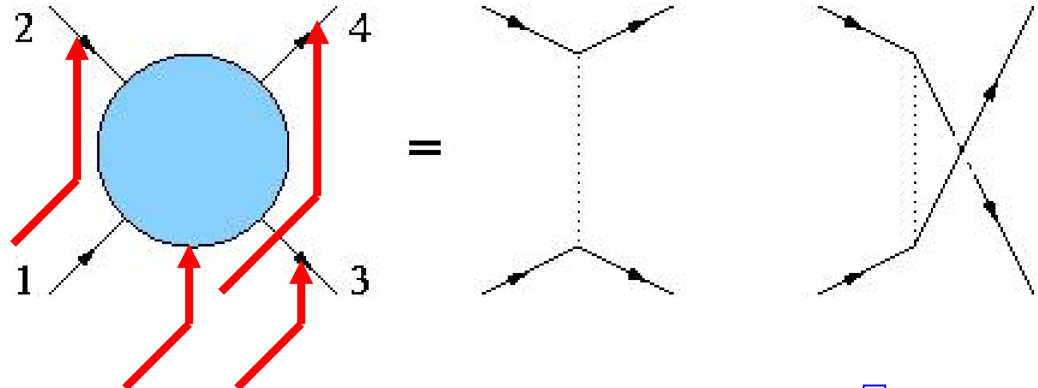
Gluonic pole m.e.

T-even

T-odd

A 2 → 2 hard processes: qq → qq

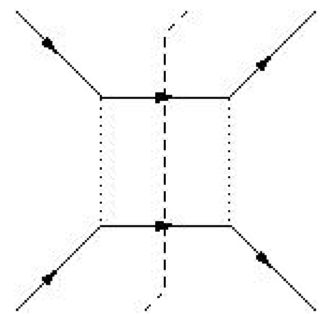
- E.g. qq-scattering as hard subprocess
- The correlator $\Phi(x,p_T)$ enters for each contributing term in squared amplitude with specific link

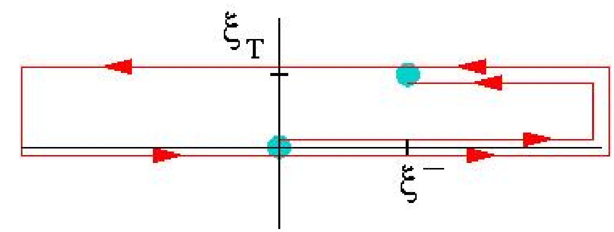


$U^\square = U^+U^{-\dagger}$

$\Phi^{[\mathrm{Tr}(U^\square)U^+]} = \Phi^{[(\square)+]}$

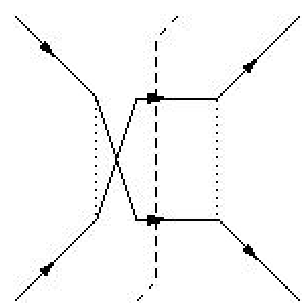
Traced loop

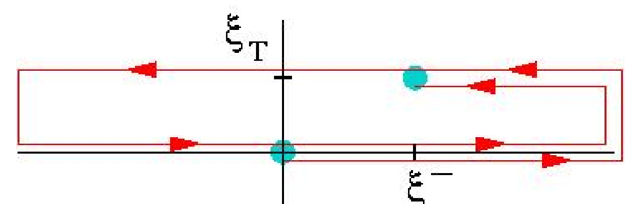




$\Phi^{[U^\square U^+]} = \Phi^{[\square+]}$

loop





Gluonic poles

- Thus: $\Phi^{[U]}(x) = \Phi(x)$

  $\Phi_{\partial}^{[U]\alpha}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\,\pi\Phi_G^{\alpha}(x,x)$

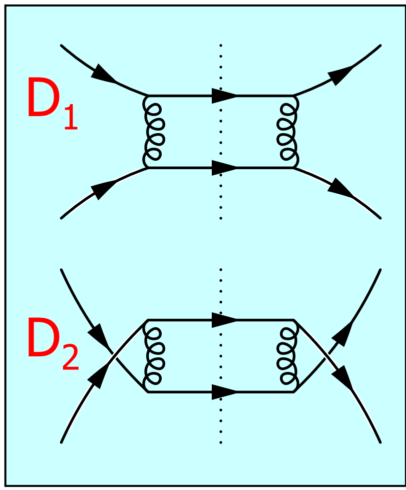
- Universal gluonic pole m.e. (T-odd for distributions)
- $\pi\Phi_G(x)$ contains the weighted T-odd functions $h_1^{\perp(1)}(x)$ [Boer-Mulders] and (for transversely polarized hadrons) the function $f_{1T}^{\perp(1)}(x)$ [Sivers]
- $\tilde{\Phi}_{\partial}(x)$ contains the T-even functions $h_{1L}^{\perp(1)}(x)$ and $g_{1T}^{\perp(1)}(x)$
- For SIDIS/DY links: $C_G^{[\pm]} = \pm 1$
- In other hard processes one encounters different factors:

  $C_G^{[\Box+]} = 3$, $C_G^{[(\Box)+]} = N_c$

Efremov and Teryaev 1982; Qiu and Sterman 1991
Boer, Mulders, Pijlman, NPB 667 (2003) 201
C. Bomhof, P.J. Mulders and F. Pijlman, EPJ C 47 (2006) 147

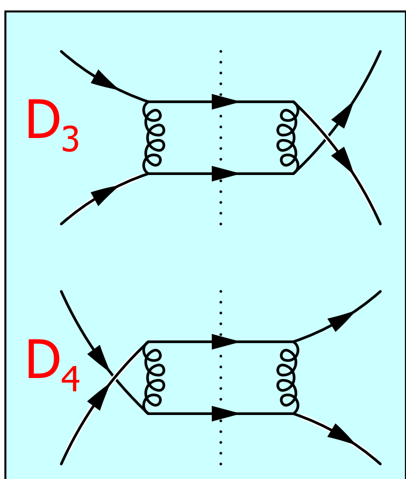
examples: qq→qq in pp

$\frac{Tr(U^{\square})}{N_c} U^{+}$ | $U^{\square} U^{+}$



$$\Phi_q = \frac{N_c^2 + 1}{N_c^2 - 1} \Phi^{[(\square)+]} - \frac{2}{N_c^2 - 1} \Phi^{[\square +]} \rightarrow \tilde{\Phi}_{\partial} + \frac{N_c^2 - 5}{N_c^2 - 1} \pi \Phi_G$$

$C_G^{[D_1]} = C_G^{[D_2]}$

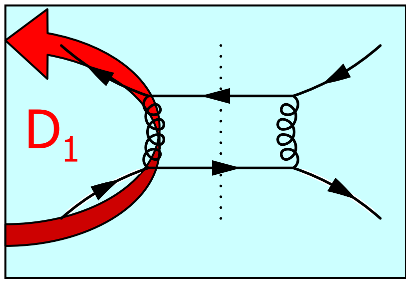


$$\Phi_q = \frac{2N_c^2}{N_c^2 - 1} \Phi^{[(\square)+]} - \frac{N_c^2 + 1}{N_c^2 - 1} \Phi^{[\square +]} \rightarrow \tilde{\Phi}_{\partial} - \frac{N_c^2 + 3}{N_c^2 - 1} \pi \Phi_G$$

$C_G^{[D_3]} = C_G^{[D_4]}$

Bacchetta, Bomhof, Pijlman, M, PRD 72 (2005) 034030; hep-ph/0505268

examples: $q\bar{q} \to q\bar{q}$ in pp

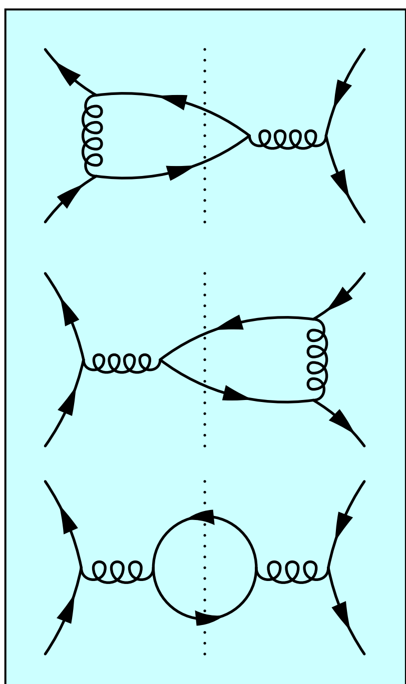


$$\Phi_q = \frac{1}{N_c^2 - 1}\Phi^{[(\Box^\dagger)+]} + \frac{N_c^2 - 2}{N_c^2 - 1}\Phi^{[-]} \quad \to \quad \tilde{\Phi}_\partial - \frac{N_c^2 - 3}{N_c^2 - 1}\pi\Phi_G$$

For $N_c \to \infty$:

$C_G^{[D_1]} \longrightarrow -1$

(color flow as DY)



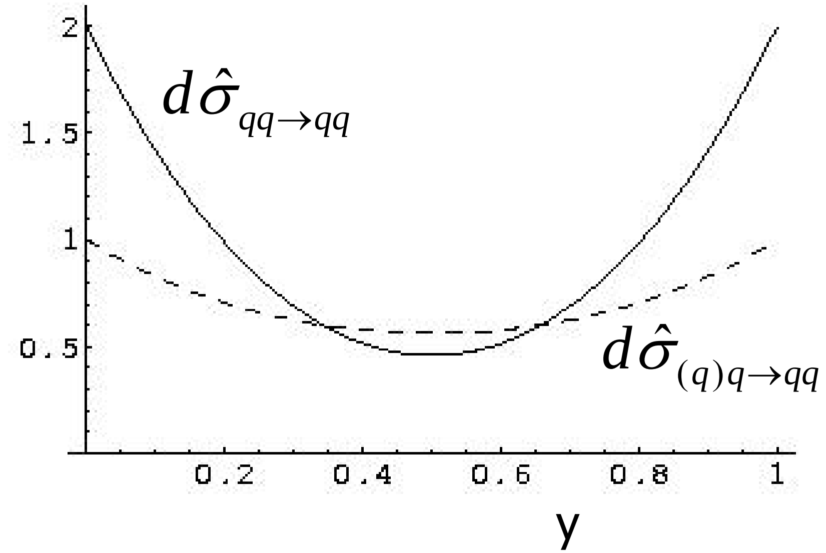
$$\Phi_q = \frac{N_c^2}{N_c^2 - 1}\Phi^{[(\Box^\dagger)+]} - \frac{1}{N_c^2 - 1}\Phi^{[-]} \quad \to \quad \tilde{\Phi}_\partial + \frac{N_c^2 + 1}{N_c^2 - 1}\pi\Phi_G$$

Bacchetta, Bomhof, D'Alesio,Bomhof, M, Murgia, PRL2007, hep-ph/0703153

Gluonic pole cross sections

- In order to absorb the factors $C_G^{[U]}$, one can define specific hard cross sections for gluonic poles (which will appear with the functions in transverse moments)
- for pp: $\hat{\sigma}_{qq\to qq} = \sum_{[D]} \hat{\sigma}^{[D]}$ $\qquad \hat{\sigma}_{[q]q\to qq} = \sum_{[D]} C_G^{[U(D)]}\hat{\sigma}^{[D]}$

  etc. (gluonic pole cross section)

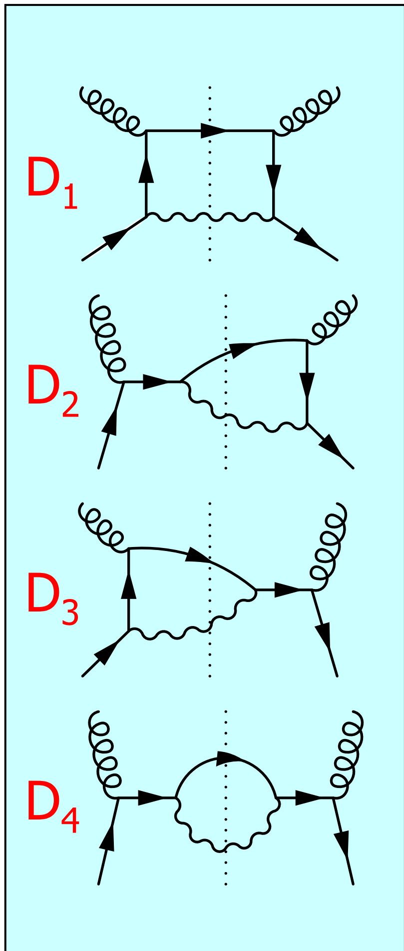


- for SIDIS: $\hat{\sigma}_{\ell[q]\to\ell q} = +\hat{\sigma}_{\ell q\to\ell q}$

  for DY: $\hat{\sigma}_{[q]\bar{q}\to\ell\bar{\ell}} = -\hat{\sigma}_{q\bar{q}\to\ell\bar{\ell}}$

Bomhof, Mulders, JHEP 0702 (2007) 029 [hep-ph/0609206]

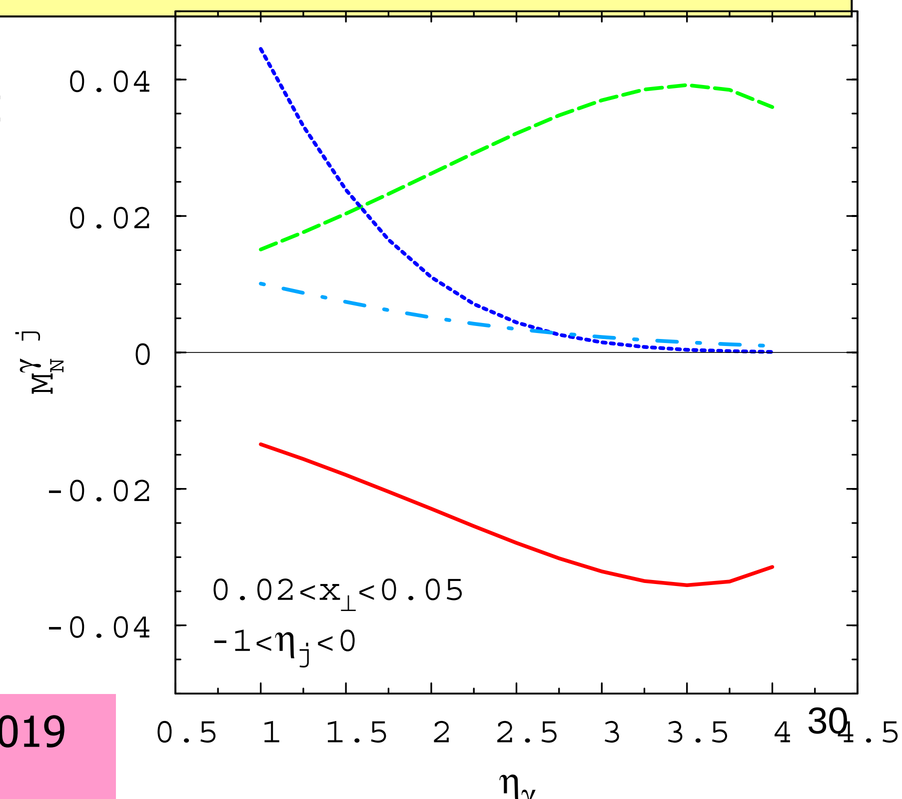
examples: qg→qγ in pp



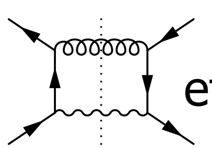
Transverse momentum dependent — weigted

$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \rightarrow \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

Only one factor, but more DY-like than SIDIS



Note: also

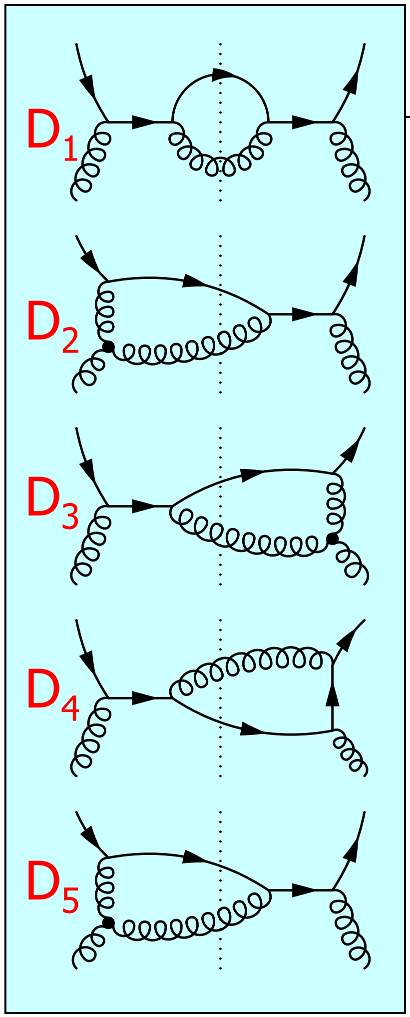


etc

Bomhof, M, Vogelsang, Yuan, PRD 75 (2007) 074019
Boer, M, Pisano, hep-ph/0712.0777

Universality (examples qg→qg)

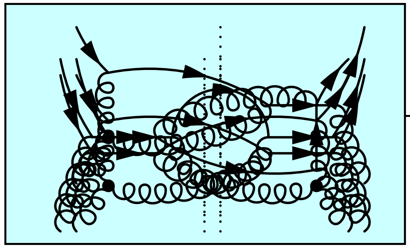
Transverse momentum dependent weighted



$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad \rightarrow \quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

Universality (examples qg→qg)

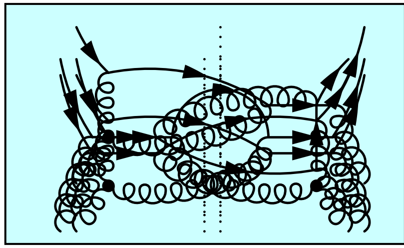
Transverse momentum dependent | weighted



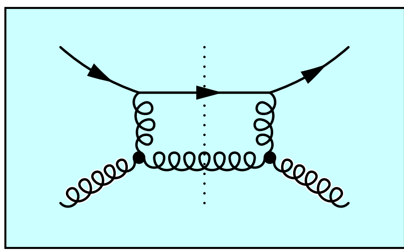
$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad \rightarrow \quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

Universality (examples qg→qg)

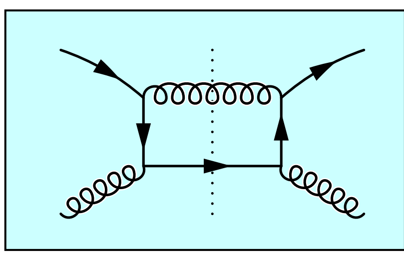
Transverse momentum dependent — weighted



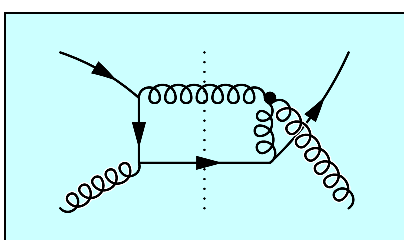
$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \rightarrow \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$



$$\Phi_q = \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)(\Box^\dagger)+]} + \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \rightarrow \tilde{\Phi}_\partial - \frac{1}{N_c^2-1}\pi\Phi_G$$



$$\Phi_q = \frac{N_c^4}{(N_c^2-1)^2}\Phi^{[(\Box)(\Box^\dagger)+]} - \frac{N_c^2}{(N_c^2-1)^2}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \rightarrow \tilde{\Phi}_\partial + \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$



$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)(\Box^\dagger)+]} - \frac{1}{N_c^2-1}\Phi^{[+]} \rightarrow \tilde{\Phi}_\partial + \pi\Phi_G$$

Universality (examples qg→qg)

Transverse momentum dependent

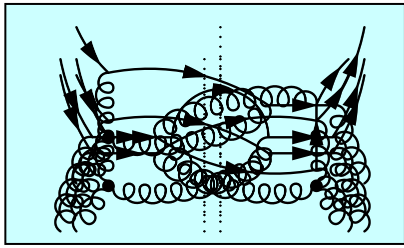
It is also possible to group the TMD functions in a smart way into two!
(nontrivial for nine diagrams/four color-flow possibilities)

$$\tilde{\Phi}_{\partial} = \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)(\Box^\dagger)+]} + \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]}$$

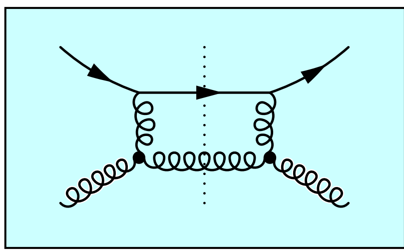
$$\pi\Phi_G = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)(\Box^\dagger)+]} - \frac{1}{N_c^2-1}\Phi^{[+]}$$

But still no factorization!

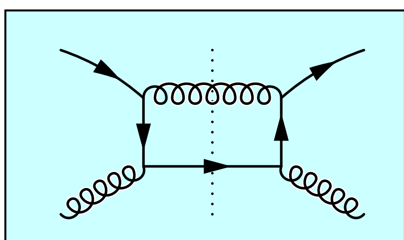
weighted



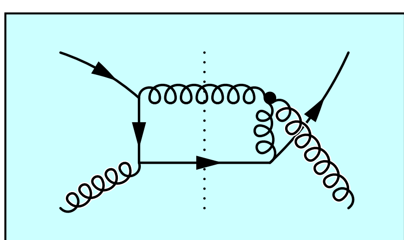
$$\rightarrow \quad \tilde{\Phi}_{\partial} - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$



$$\rightarrow \quad \tilde{\Phi}_{\partial} - \frac{1}{N_c^2-1}\pi\Phi_G$$



$$\rightarrow \quad \tilde{\Phi}_{\partial} + \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$



$$\rightarrow \quad \tilde{\Phi}_{\partial} + \pi\Phi_G$$

‘Residual’ TMDs

- We find that we can work with basic TMD functions $\Phi^{[\pm]}(x,p_T)$ + ‘junk’
- The ‘junk’ constitutes process-dependent residual TMDs

$$\Phi^{[(\Box)(\Box^\dagger)+]}(x,p_T) = \Phi^{[+]} + \underbrace{\left[\Phi^{[(\Box)(\Box^\dagger)+]}(x,p_T) - \Phi^{[+]}(x,p_T)\right]}_{\delta\Phi^{[(\Box)(\Box^\dagger)+]}(x,p_T)}$$

$$\Phi^{[\Box+]}(x,p_T) = 2\Phi^{[+]}(x,p_T) - \Phi^{[-]}(x,p_T) + \delta\Phi^{[\Box+]}(x,p_T)$$

no definite T-behavior

definite T-behavior

- The residuals satisfies $\delta\Phi_\partial(x) = 0$ and $\pi\delta\Phi_G(x,x) = 0$, i.e. cancelling k_T contributions; moreover they most likely disappear for large k_T

Bomhof, Mulders, Vogelsang, Yuan, NPB, hep-ph/0709.1390

Conclusions

- Beyond collinearity many interesting phenomena appear
- For integrated and weighted functions factorization is possible (collinear quark, gluon and gluonic pole m.e.)
- Accounted for by using gluonic pole cross sections (new gauge-invariant combinations of squared hard amplitudes)
- For TMD distribution functions the breaking of universality can be made explicit and be attributed to specific matrix elements
- Many applications in hard processes. Including fragmentation (e.g. polarized Lambda's within jets) even at LHC

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