Center for Frontiers in Nuclear Science

# 3D structure of QCD and entanglement

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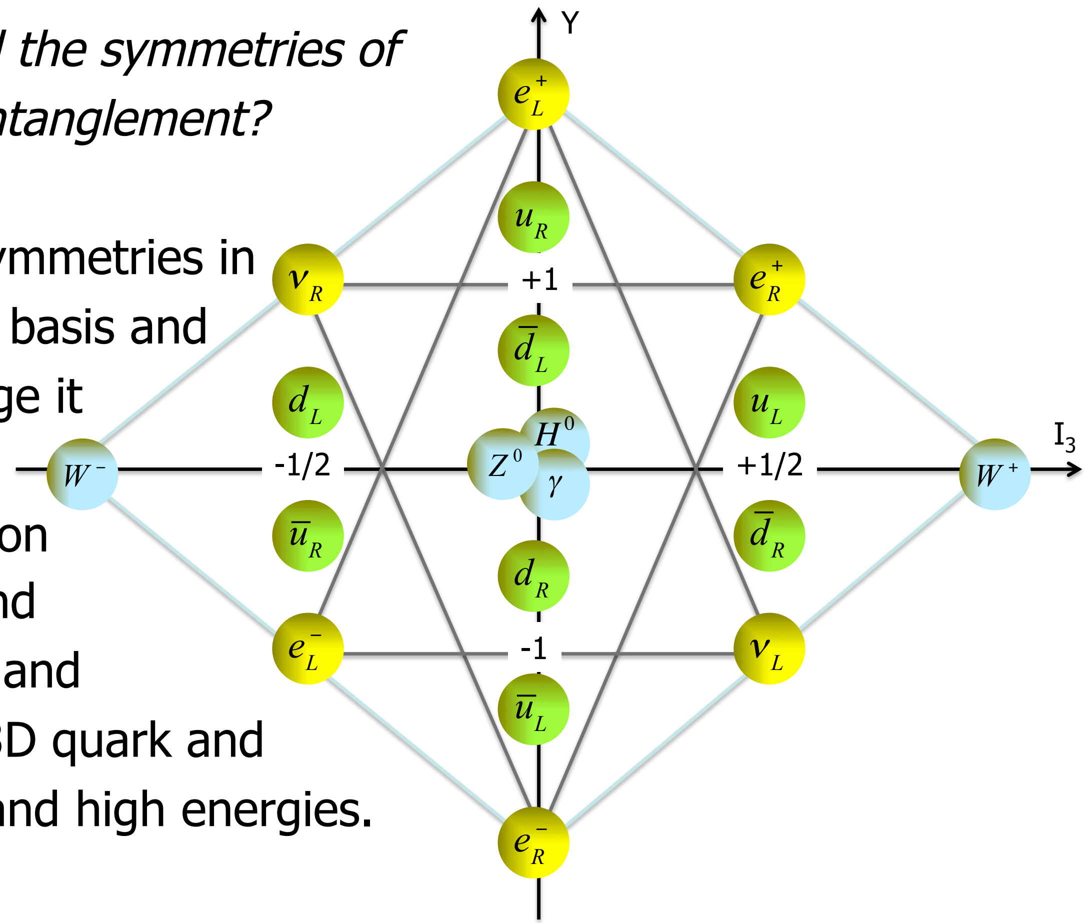
# abstract

*Could the 3D structure of QCD and the symmetries of the Standard Model be linked to entanglement?*

Look at a possible emergence of symmetries in the standard model from a simpler basis and less dimensions. Even if at this stage it may not make striking predictions, my hope is that it could shed light on the peculiarities of the spectrum and symmetries of the Standard Model and simplify our understanding of the 3D quark and gluon structure of hadrons at low and high energies.

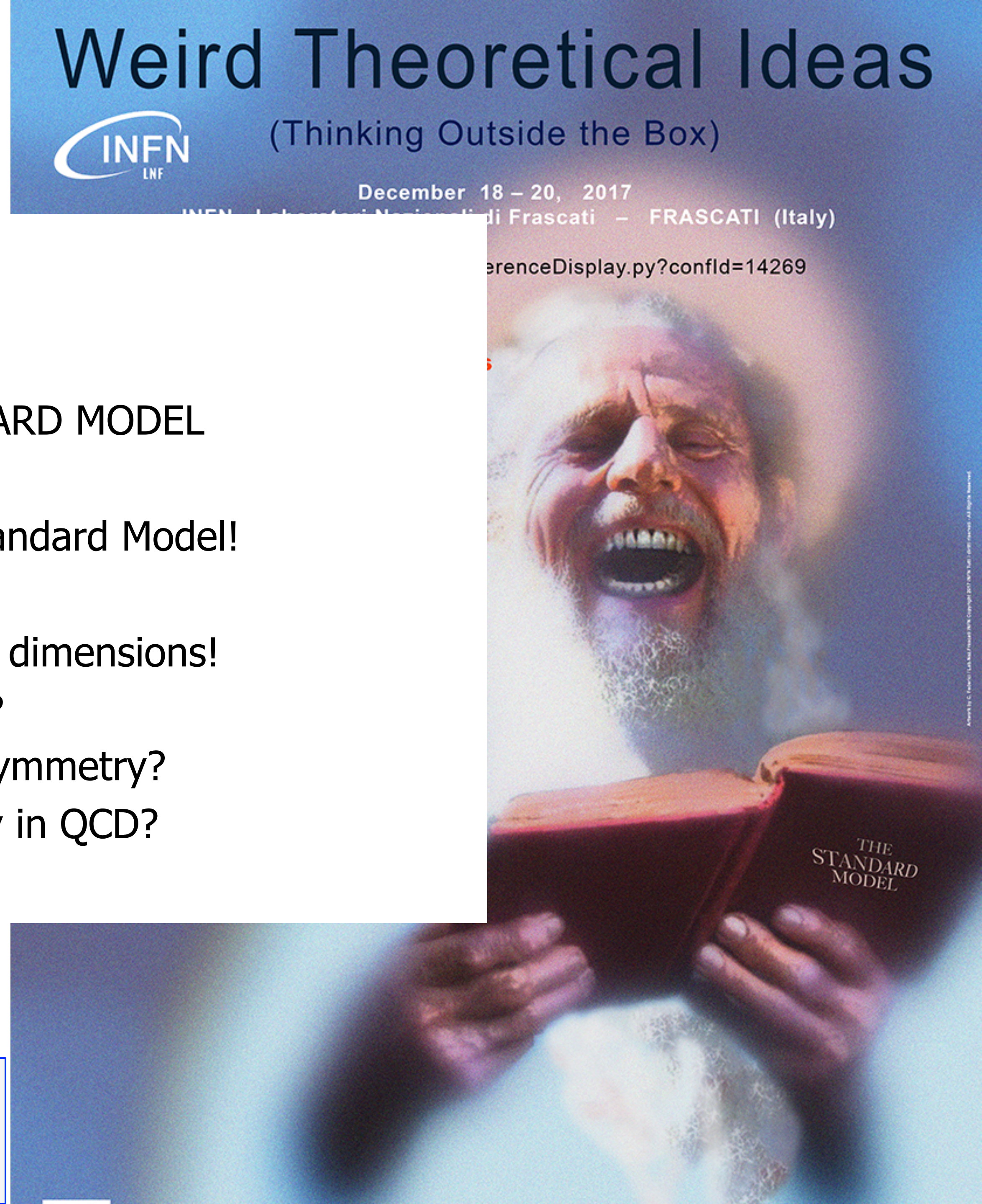


# A selection of basic questions about Standard Model

- Why are leptons color-blind while quarks have electroweak structure?
- How is a pure nucleon state entangled, leading to ensemble of partons?
  - Nucleons are composite and resolved at high energies!
- Partons are pure states that fragment into an ensemble of hadrons?
- What about the jet structure and substructure in QCD?
- Why is SCET so successful and are transverse modes less relevant?
  - Confinement and scale of QCD!
- Phenomena at low x (saturation, color glass condensate)?

EIC questions

- Where is basic supersymmetry (essential,  even showing up in hadron spectrum, nuclear spectra)?
- What about scalar sector, naturalness, ... (Higgs, XQCD, ...)


- Motivation
  (NOT) HAPPY WITH STANDARD MODEL

- In spite of the success of Standard Model!

- Three families, colors, space dimensions!
- Left-right (a)symmetry? B-L?
- Naturalness? Missing supersymmetry?
- Confinement and Collinearity in QCD?

PJM 1601.00300 (POETIC)
PJM 1801.03664 (Lightcone)
PJM 1806.09797 (Phys. Lett. B)

# QCD – entangled states and QIT

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states) [see for instance Kharzeev & Levin (1702.03489)]
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states

- Entangled (pure) states $|\Phi>$ in multipartite space, with a density matrix $\rho = |\Phi><\Phi|$, lead to ensembles (non-pure state) in the reduced spaces.
  - EPR bipartite pure state leads to a 50% - 50% ensemble in both subspaces.

- Both hadrons and partons might live in a tripartite $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$ space!

- Possibly combined with a principle of maximal entanglement (MaxEnt), such as hinted at in Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989): maximally entangled chiral left/right two-particle states are consistent with QED ($g_A$=0) & electroweak ($g_V$=0), at least if $\sin \Theta_W = ½$

# Conjecture

- My conjectures goes one step further:

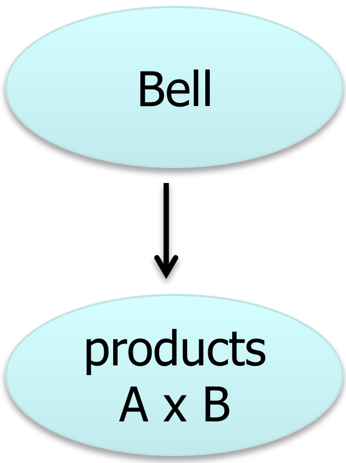
- Quarks and leptons are entangled states belonging to different classes of maximally entangled (MaxEnt) states in a multipartite space $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$
  - Criterion for these classes is equivalence under Stochastic Local Operations and Classical Communication (SLOCC), for our purposes Local Unitary (LU) equivalence
  - Nonequivalence of classes (not locally connected) corresponds to absence of leptoquarks in SM

- Furthermore spatial degrees of freedom and internal degrees of freedom are intrinsically connected (as is the concept LOCAL in QIT). This corresponds to local gauge invariance in SM

# Bipartite entangled states

- Bell states are maximally entangled (MaxEnt) states in product space $\mathcal{H}^A \otimes \mathcal{H}^B$ :
  $|RR\rangle + e^{i\varphi}|LL\rangle$ or $|RL\rangle + e^{i\varphi}|LR\rangle$

- They belong to the same class (SLOCC, for us local unitary, local = subspace)



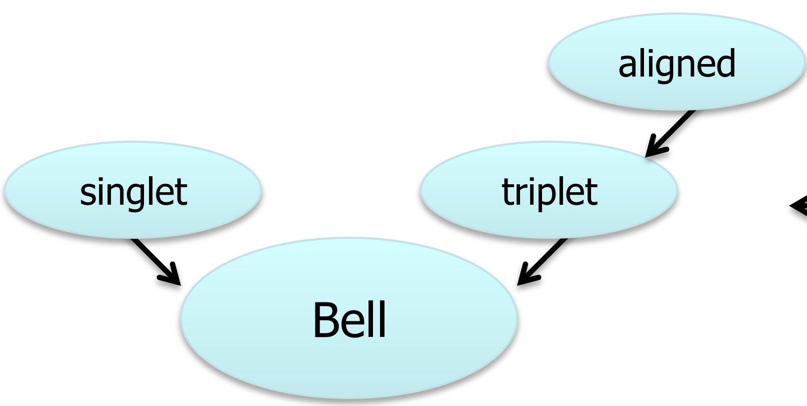

$$\rho = |\mathrm{Bell}\rangle\langle\mathrm{Bell}| \implies \rho_A = \tfrac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$

$$|\Phi\rangle = a|RR\rangle + b|RL\rangle + c|LR\rangle + d|LL\rangle$$
$$= \sqrt{p_1}|a_1 b_1\rangle + \sqrt{p_2}|a_2 b_2\rangle \quad \text{(Schmidt decomp.)}$$

entanglement measure:

$$0 \leq \Delta = \sqrt{2(1 - \mathrm{Tr}(\rho^2))} = 2|ad - bc| \to 2\sqrt{p_1 p_2} \leq 1$$

- Symmetry eigenstates are in general aligned and/or entangled




← Generators $T^{\otimes 2} = T \otimes T$

← generators $T \otimes \mathbb{I} + \mathbb{I} \otimes T$

# Symmetries and multipartite states

Relevant symmetry in tripartite space is Z(3), S(3), SO(3), and SU(3)

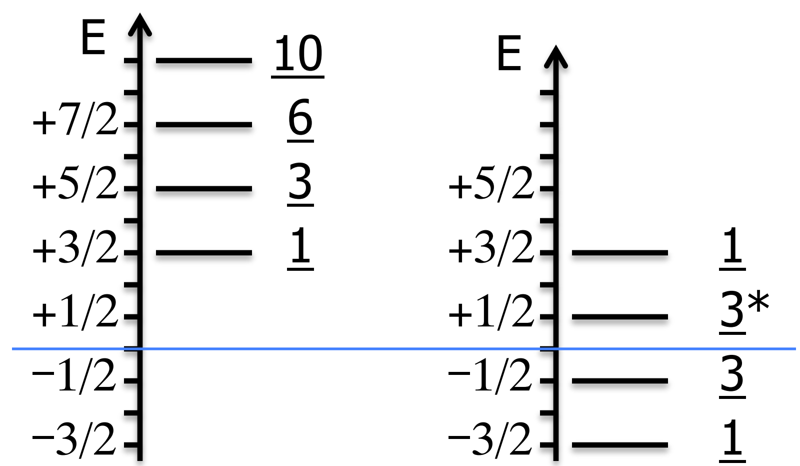
- Example: a 3D harmonic oscillator: states $|n_x n_y n_z>$ or $|n_r lm>$

| level | degeneracy | $(n_x, n_y, n_z)$ | SO(3) $(n_r \ell)$ | SU(3) $(\underline{n})$ |
|---|---|---|---|---|
| 0 | 1 | (0,0,0) | 0s | $\underline{1}$ |
| 1 | 3 | (1,0,0), ... | 0p | $\underline{3}$ |
| 2 | 6 | (2,0,0), (1,1,0), ... | 1s $\oplus$ 0d | $\underline{6}$ |
| 3 | 10 | (3,0,0), (2,1,0), (1,1,1), ... | 1p $\oplus$ 0f | $\underline{10}$ |
| 4 | 15 | ... | 2s $\oplus$ 1d $\oplus$ 0g | $\underline{15}_s$ |

- 3D HO is separable, has rotational [SO(3)] and more [SU(3)] symmetry. Symmetry eigenstates are in general entangled states (see H. Georgi, Group Theory, Ch. 13)

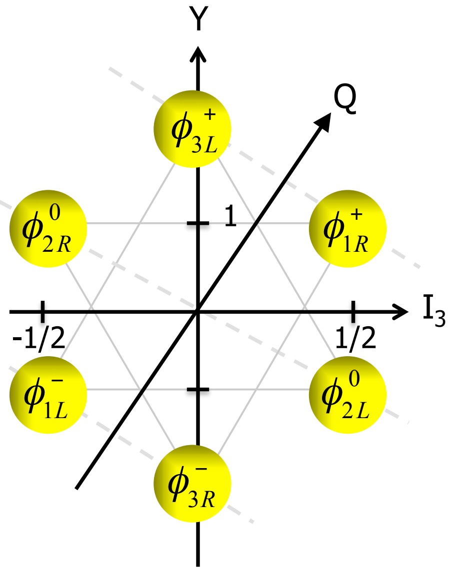
- Bosons & Fermions

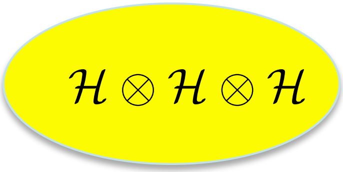
$$H = \frac{\omega}{2} \sum_k \{a_k^\dagger, a_k\} + \frac{\omega}{2} \sum_k [b_k^\dagger, b_k]$$

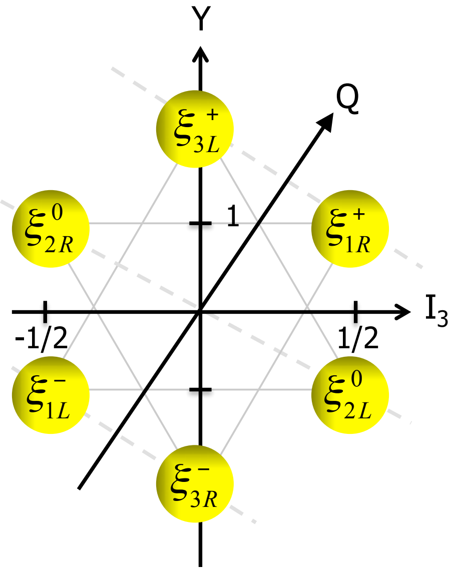


# Basic Hilbert space for Standard Model

- Basic Hilbert space needs right- and left-states (R and L): $\mathcal{H} = \mathcal{H}_R \oplus \mathcal{H}_L$
- Basic Hilbert space needs SU(3) symmetry eigenstates: (1, 2, 3) or (+, −, 0)



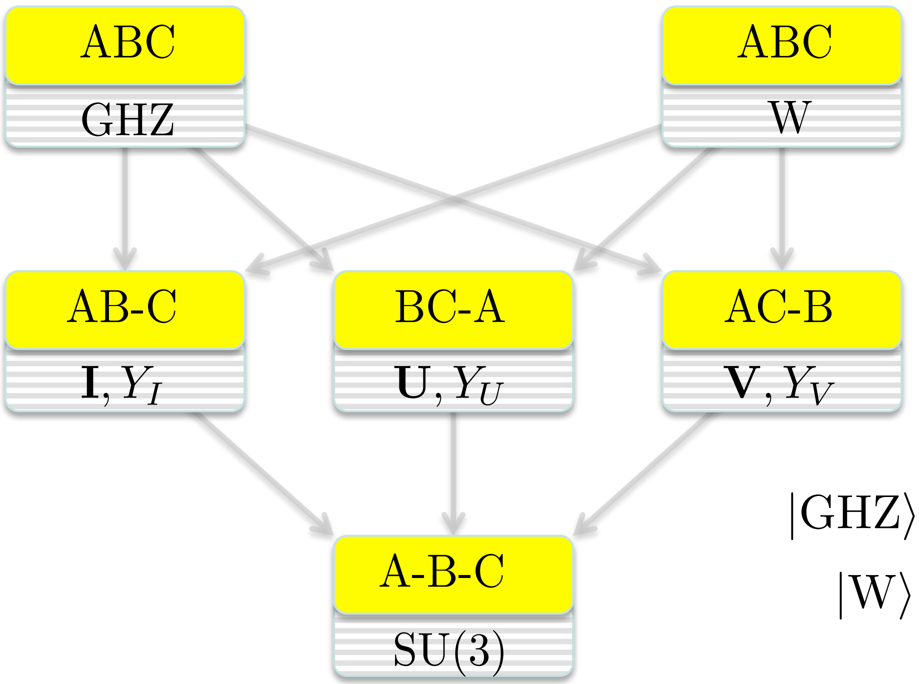




- Spectrum in SM (quarks, leptons, gauge bosons, Higgs) is CP symmetric, thus also starting basis
- Supersymmetry is natural (bosons $\phi$ and fermions $\xi$) in basic space(s), but will be hidden in tripartite space

# Tripartite entangled chiral states

- Two classes of maximally entangled ABC states: (Dur, Vidal, Cirac 2000)

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$




$$|\mathrm{GHZ}\rangle \implies \rho_{AB} = \frac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$
$$|\mathrm{W}\rangle \implies \rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$

GHZ: fragile
W: robust

- Beyond tripartites there is an infinite number of classes!

# EMERGENCE OF SPACE-TIME DEPENDENCE

# All Possible Symmetries of the $S$ Matrix*

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(Received 16 March 1967)

We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the $S$ matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the $S$ matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the $S$ matrix. Let $G$ be a connected symmetry group of the $S$ matrix, and let the following five conditions hold: (1) $G$ contains a subgroup locally isomorphic to the Poincaré group. (2) For any $M>0$, there are only a finite number of one-particle states with mass less than $M$. (3) Elastic scattering amplitudes are analytic functions of $s$ and $t$, in some neighborhood of the physical region. (4) The $S$ matrix is nontrivial in the sense that any two one-particle momentum eigenstates scatter (into something), except perhaps at isolated values of $s$. (5) The generators of $G$, written as integral operators in momentum space, have distributions for their kernels. Then, we show that $G$ is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

## I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on $SU(6)$[1]; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finally settled by the discovery of

symmetry group of the $S$ matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the $S$ matrix be nontrivial and let elastic scattering amplitudes be analytic functions of $s$ and $t$ in some neighborhood of the physical region. Finally, let the generators of $G$ be representable as integral operators in momentum space, with kernels that are distributions. Then $G$ is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)

# Basic symmetries including SUSY

- Hilbert space

$\{(a^\dagger)^n|0\rangle, b^\dagger|0\rangle\}$ $\qquad [a,a^\dagger]=1,\ \{b,b^\dagger\}=1$

- Supercharges

$Q_{ik}^\dagger = b_i\, a_k^\dagger$ and $Q_{ik} = b_i^\dagger a_k$ $\qquad \{Q_{ik}^\dagger, Q_{jl}\} = \frac{1}{2}\,\delta_{ij}\{a_l^\dagger, a_k\} + \frac{1}{2}\,\delta_{kl}[b_i^\dagger, b_j]$

$a_k^\dagger \xrightarrow{Q_{ik}} b_i^\dagger \qquad a_k^\dagger \xleftarrow{Q_{ik}^\dagger} b_i^\dagger$

hamiltonian/number operators (i=j, k=l)
& unitary rotations

- For boson and fermion fields

$$\varphi = \frac{1}{\sqrt{2\omega}}(a+a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(b+b^\dagger)$$

$$Q = \sqrt{\omega}(a^\dagger b - b^\dagger a)$$

$$[Q,\varphi] = \xi \qquad \{Q,\xi\} = \{Q,[Q,\varphi]\} = F = iD\varphi$$
$$[Q,F] = [Q,\{Q,\xi\}] = iD\xi$$

Single (free) field

$$F = [\varphi, H]$$
$$= iD\varphi = i\dot{\varphi}$$

$$iD = i\partial + gA$$

↑ unitary rotations

- Implement symmetries via constraints F

... and a nontrivial vacuum (not everything is for free!)

$$\phi(t) = \mathcal{T}\exp\left(-i\int_0^t ds{\cdot}D\right)\phi$$

# Emerging symmetries and space-time

| Fields | Generators: Space-time & Internal |
|---|---|
| Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$ | H |
| $\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino) | $P^+$, $P^-$    K, <u>SU(3)</u> |

$U(1)_R$ x $U(1)_L$ x SU(3)

# Emerging symmetries and space-time

| Fields | Generators: Space-time | & Internal | |
|---|---|---|---|
| ■ Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$ | ■ H | | |
| ■ $\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino → gauge theory) | ■ P⁺, P⁻ | K, SU(3) | Z(2) |
| ■ 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$ | ■ H, P, K | SU(3) | |

$$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \psi^j$$

P(1,1) x SU(3)

# Emerging symmetries and space-time

## Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \psi^j$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$iD_\mu\psi^i = i\partial_\mu\psi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \psi^j$$

## Generators

Space-time & Internal

- H
- $P^+$, $P^-$ — K, SU(3)

Z(2)

- H, P, K — SU(3) = [SO(3),SU(2)xU(1)]

Z(3)

- H, **P**, **K**, **J** — SU(2)xU(1)

P(1,3) x SU(2) x U(1)

# Emerging symmetries and space-time

| Fields | Generators Space-time & Internal |
|---|---|
| Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$ | H |
| $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino → gauge theory) | P+, P–   K, SU(3)   Z(2) |
| 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$ | H, P, K   SU(3) = [SO(3),SU(2)xU(1)] |
| $$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\dots 8} A_\sigma^a (T_a)^i_j \psi^j$$ | Z(3) |
| 3D: $\phi_S \quad A_k^a \quad \psi$ | H, **P**, **K**, **J**   SU(2)xU(1) |
| $$iD_\mu\psi^i = i\partial_\mu\psi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \psi^j$$ | **P(1,3) x SU(2) x U(1)** |

- and ....

A(4)

$$n_\pm^\sigma \longrightarrow n_\alpha^\mu \qquad \gamma^\sigma = \begin{bmatrix} 0 & n_-^\sigma \\ n_+^\sigma & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

in order to match space-time and field symmetries and respect Coleman-Mandula

# DYNAMICS

# Dynamics

- Right-Left symmetry
- Supersymmetry (Wess-Zumino structure)
  - Bosons: $$\phi\sqrt{2} = e^{i\pi/4}\phi_R + e^{-i\pi/4}\phi_L$$ $$= \phi_S + i\phi_P = \chi\, e^{i\theta}$$
  - Fermions: $$\xi\sqrt{2} = \begin{bmatrix} \xi_R \\ -i\,\xi_L \end{bmatrix}$$
- Wess-Zumino in 1+1 dim including potential and vev determining symmetry

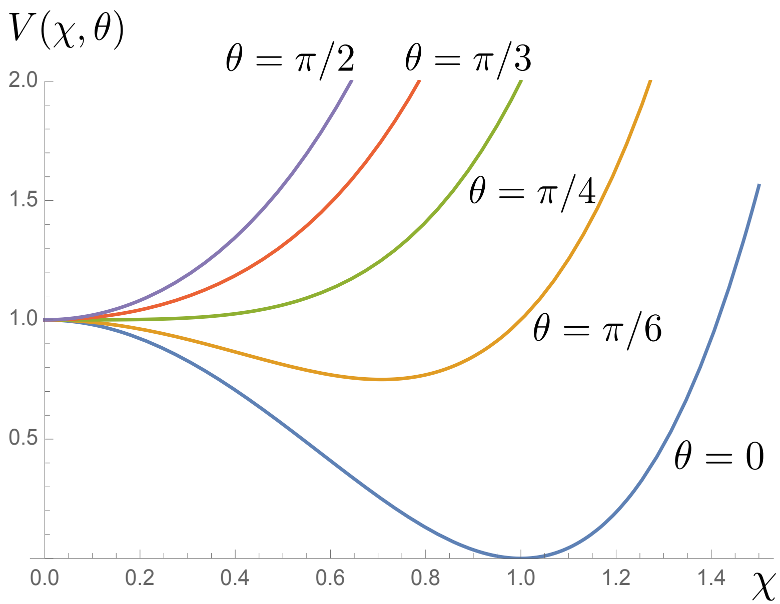
$$V(\phi) = \tfrac{1}{8}M^2\left(4\phi_S^2\phi_P^2 + (1-\phi_S^2+\phi_P^2)^2\right)$$
$$= \tfrac{1}{8}M^2\left(\chi^4\sin^2(2\theta) + (\chi^2\cos(2\theta)-1)^2\right)$$

- Pseudoscalar fields (θ) → gauge fields

$$\phi^\dagger\partial_\sigma\phi = \tfrac{1}{2}\,\chi^T D_\sigma\chi$$

- ... + masses through symmetry breaking
- Link to gravity (?) via constraint

$$\lambda\left(\chi^2\cos^2(2\theta) - 1\right)$$

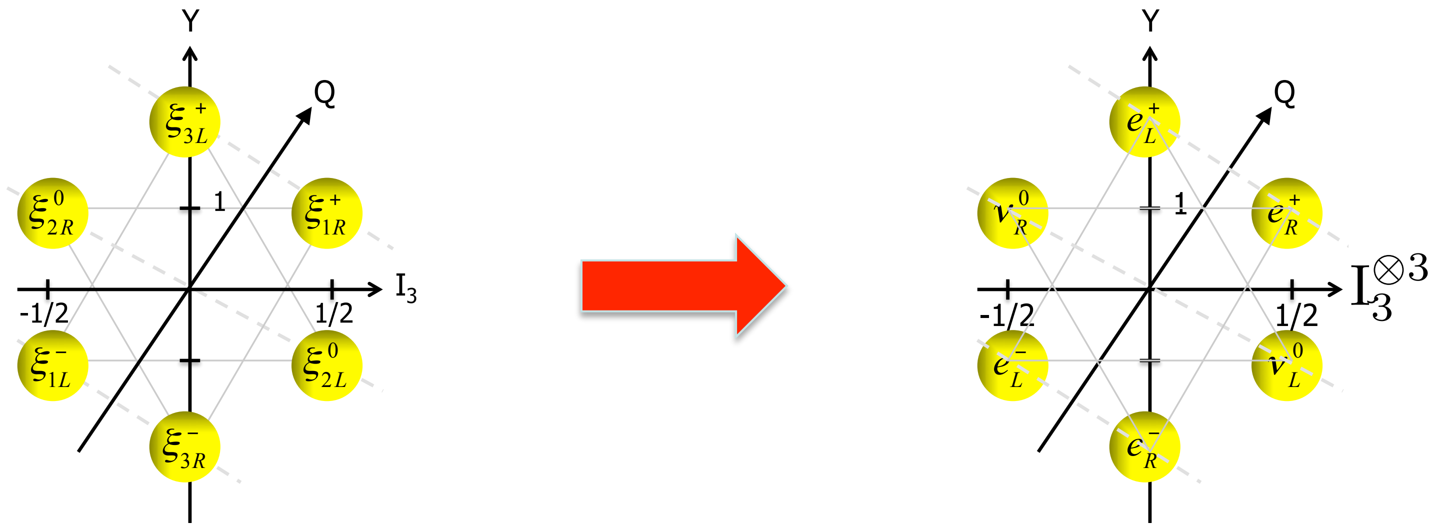



SO(3) symmetry via vev

# FERMIONS AND BOSONS AS TRIPARTITE STATES

# Leptons

- GHZ class, $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$ has same symmetry as basis (including chirality)



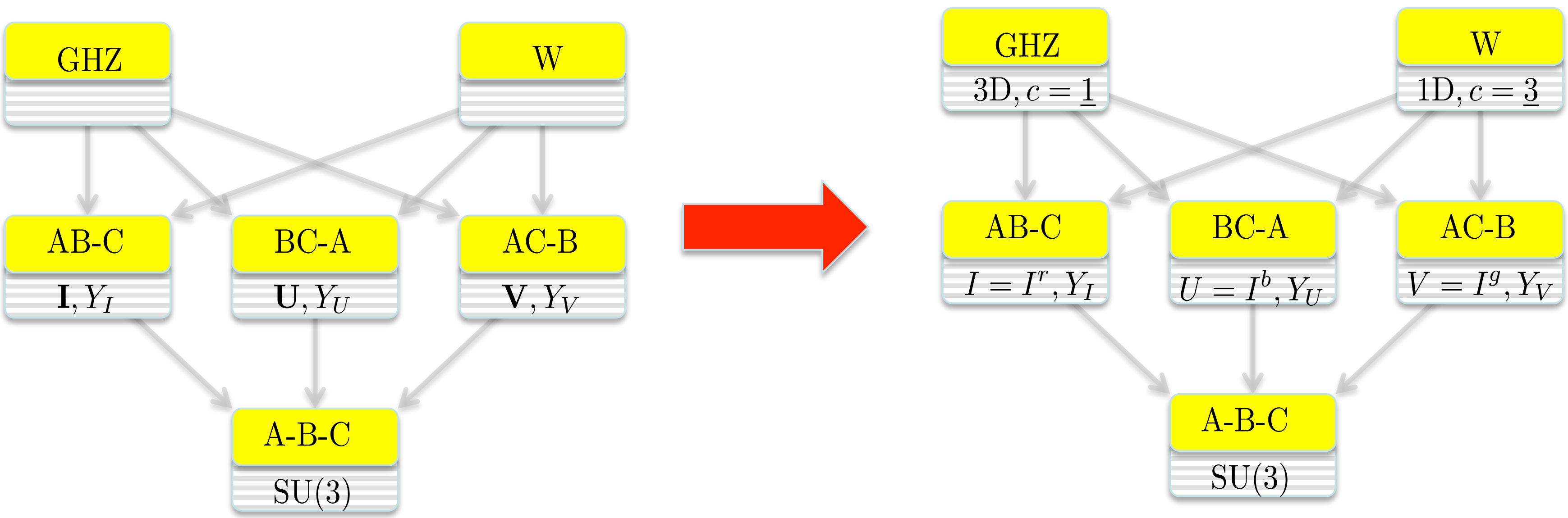

- Using $t_I \equiv (\mathbf{I}, Y_I)$ note that $t_I \otimes t_U \otimes t_V$ is LU equivalent to $t_I \otimes t_I \otimes t_I$ and the aligned GHZ states can be SO(3) multiplets (living in 3D) identified with leptons
- Embedding symmetry A(4) has three singlet representations: families
- This gives tri-bimaximal family – electroweak mixing [slightly different from the way obtained by Fritsch & Xing, or Harrison, Perkins & Scott]

$$U_{\mathrm{TB}} = W U_Q = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \omega^2 & 1 & \omega \\ \omega & 1 & \omega^2 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 1 & 0 & i \end{bmatrix} = \begin{bmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{bmatrix}$$

# Quarks

- For W-class chirality is more complex
- again employ SU(3) and SU(2) x U(1) subgroups (I, U, V) in bipartite classes
- $t_I \otimes t_U \otimes t_V \rightarrow t_I^r \otimes t_I^g \otimes t_I^b$

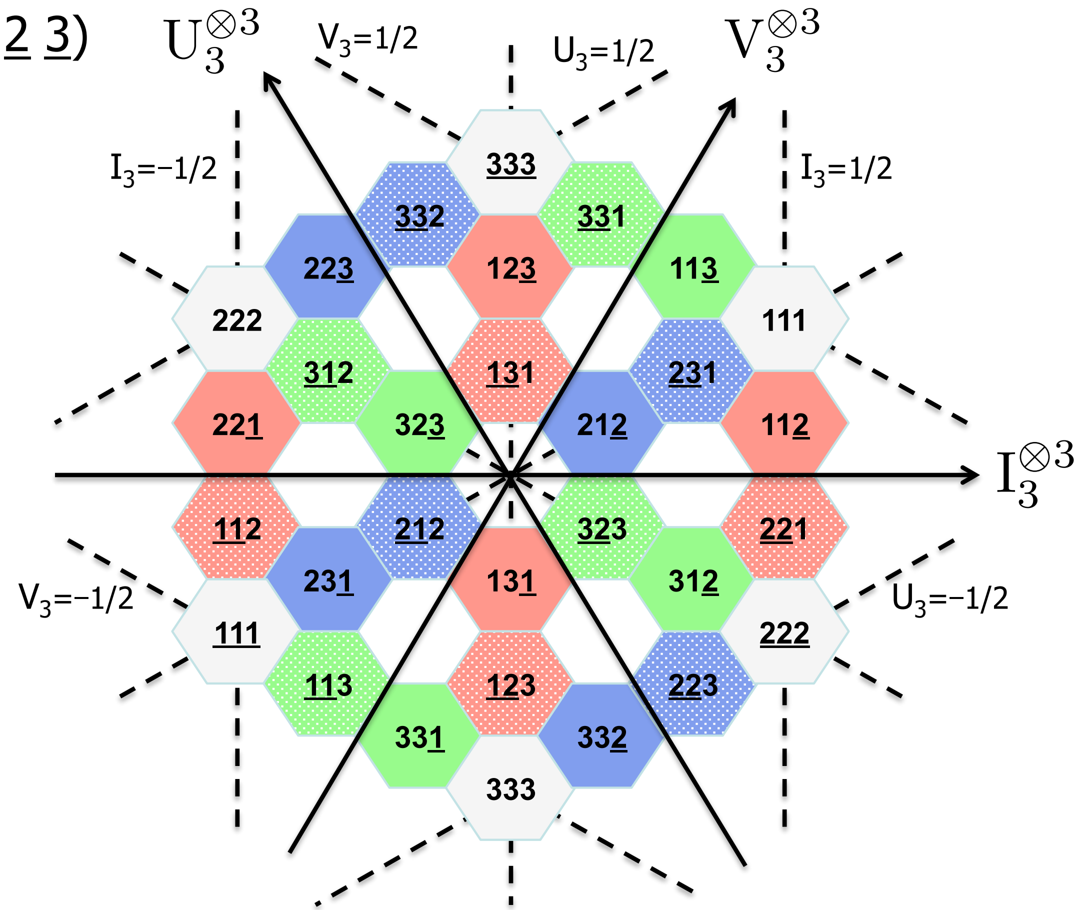
$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$




- A(4) symmetry: three singlets and three triplets
- Construct SU(3) root diagram to see all GHZ- and W-states

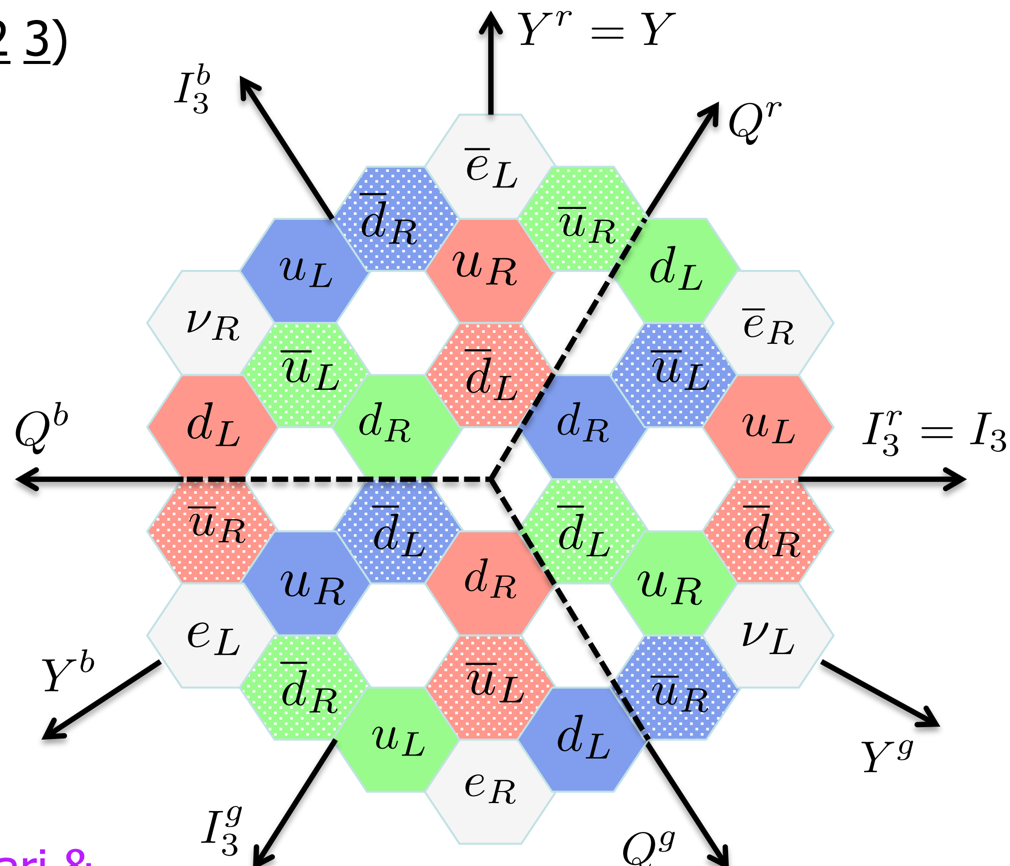
# Fermionic excitations: tripartite entanglement

- Tripartite states (R: 1 2 3 & L: 1 2 3)
- Aligned (RRR, LLL): GHZ states
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): W-states
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)



# Fermionic excitations: electroweak identification

- Tripartite states (R: 1 2 3 & L: 1 2 3)
- Aligned (RRR, LLL): LEPTONS
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): QUARKS
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)
- Resembles the rishon model (Harari & Seiberg 1982, Shupe 1979)



# Bosons

- Boson fields appear as Higgs field and in covariant derivatives:

$$\phi\sqrt{2} = \chi\, e^{i\theta} \qquad\qquad \phi^\dagger \partial_\sigma \phi = \frac{1}{2}\chi^T D_\sigma \chi$$

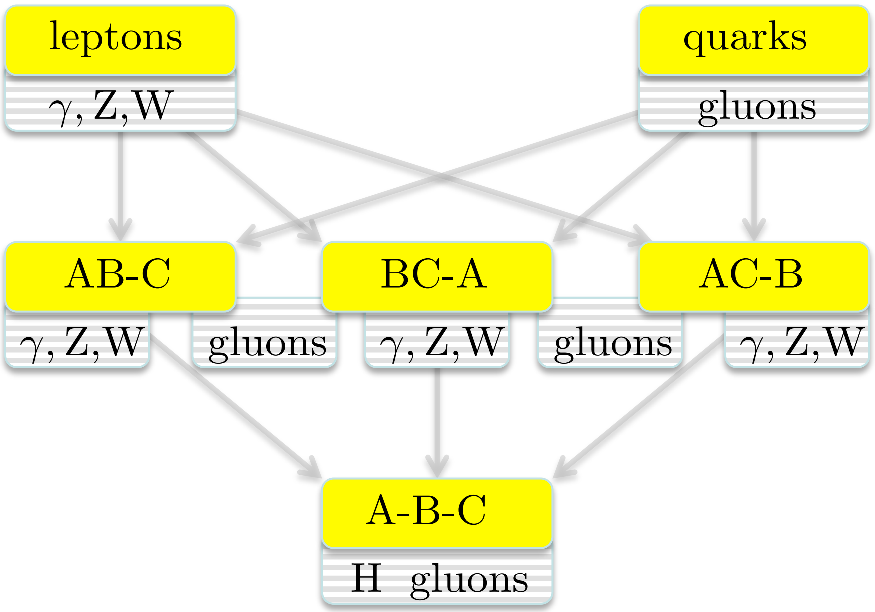
- Depending on implementation:

1D $$iD_\sigma \psi^i = i\partial_\sigma \psi^i + g_0 \sum_{a=1,\ldots 8} A^a_\sigma (T_a)^i_j \psi^j$$

3D $$iD_\mu \psi^i = i\partial_\mu \psi^i + g \sum_{a=1,2,3,8} A^a_\mu (T_a)^i_j \psi^j$$

- Gauge fields linked to symmetry generators

- More or less like SM starting with
  - $\sin\theta_W = 1/2$
  - $M_Z\sqrt{2} = M_H = M_{\rm top}/\sqrt{2}$
  - Need for radiative corrections



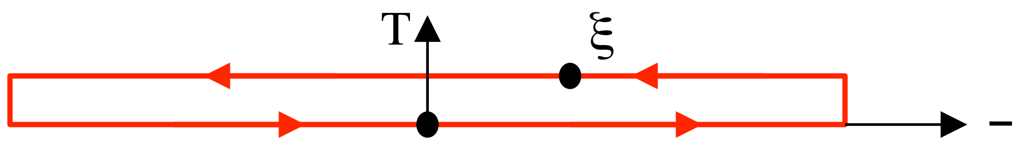
# Composites & local versus global symmetries

- Strong Interactions: resembles $XQCD_{1+1}$ (analogous to Kaplan 1306.5818), while dynamics governed via Wilson loop (including freezing of color at small x/high energies)

$$W[C] = \exp\left(-ig\oint_C ds^\mu A_\mu(s)\right)$$

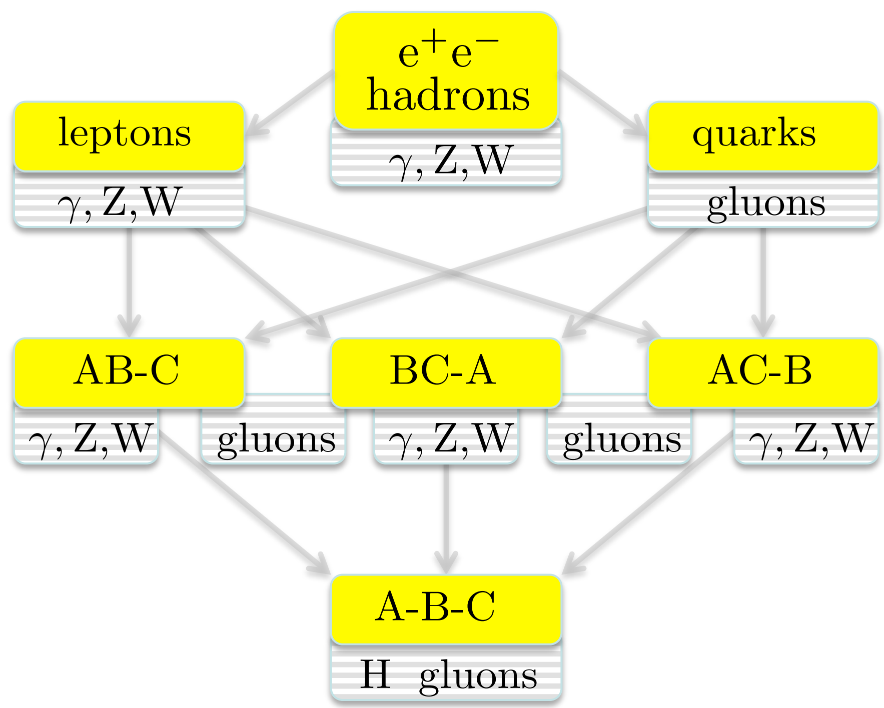
$$gF_{\tau\sigma} = \delta W[C]/\delta\sigma^{\tau\sigma}$$

lightcone (PDF) → light-front (TMD)




- Elementary constituents only involve entanglement in discrete R/L and 1,2,3 degrees of freedom; composites entangled in the continuous space-time degrees of freedom

- Quark-entangled states form hadrons that are global SU(3) color singlets. Color local in 1D and global in 3D valence – current quarks (ontological basis choice, G 't Hooft)
- More to incorporate in QCD:
  - collinearity and TMD
  - light-front dominance, OPE
  - jets, SCET
  - AdS/CFT
  - color-kinematic dualities



# SUMMARY AND OUTLOOK

# Emergence in Standard Model

- Choice of basic 'internal' degrees of freedom and embedding them in $R^{1,1}$ or $R^{1,3}$ via tripartite entanglement:
  - Advantageous for convergence: $d[\phi] = (d-2)/2 \rightarrow 0$, $d[\xi] = (d-1)/2 \rightarrow ½$, naturalness, … [see Stojkovic – 1406.2696]
  - What are consequences in higher order corrections, e.g. g-2, EW+strong, …
  - Supersymmetry included, but invisible in 3D !
  - Gravity also emerges in 1D → 3D.
- Does it provide a consistent framework to look at family structure and symmetries of standard model as emergent phenomena?
  - e.g. tri-bimaximal mixing for leptons, zeroth order parameters in SM
- Tripartite space for quarks naturally has color dual to space/electroweak:
  - explains why color decoupled from electroweak interactions (no lepto-quarks)
  - color invisible in 3D: local gauge invariance! No asymptotic quarks or gluons!
  - global color remains visible in 3D via color factors such as N vs 1/N, f x D (distribution x fragmentation)

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Let’s continue to look for new ways out