'Transverse spin physics'
RIKEN Spinfest, June 28 & 29, 2007

Transverse spin physics

Piet Mulders

mulders@few.vu.nl

Abstract

QCD is the theory underlying the strong interactions and the *structure of hadrons*. The properties of hadrons and their response in scattering processes provide in principle a large number of observables. For comparison with theory (lattice calculations or models), it is convenient if these observables can be identified with well-defined *correlators, hadronic matrix elements* that involve only one hadron and known local or nonlocal combinations of quark and gluon operators. Well-known examples are static properties, such as mass or charge, form factors and parton distribution and fragmentation functions. For the *partonic structure*, accessible in high-energy (hard) scattering processes, a lot of information can be obtained, in particular if one finds ways to probe the '*transverse structure*' (*momenta* and *spins*) of partons. Relevant scattering experiments to extract such correlations usually require polarized beams and targets and measurements of azimuthal asymmetries. Among these, *single spin asymmetries* are special because of their particular *time-reversal behavior*. The strength of single spin asymmetries depends on the *flow of color* in the hard scattering process, which affects the nonlocal structure of quark and gluon field operators in the correlators.

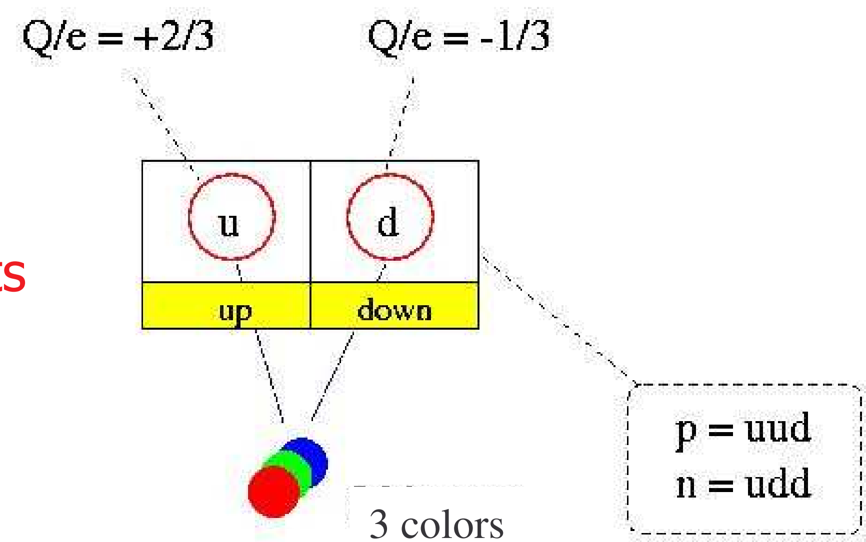
Content

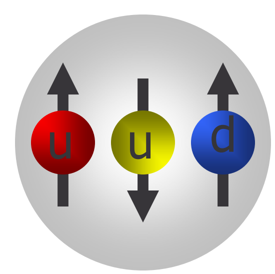
- Lecture 1:
  - Partonic structure of hadrons
  - correlators: distribution/fragmentation
- Lecture 2:
  - Correlators: parameterization, interpretation, sum rules
  - Orbital angular momentum?
- Lecture 3:
  - Including transverse momentum dependence
  - Single spin asymmetries
- Lecture 4:
  - Hadronic scattering processes
  - Theoretical issues on universality and factorization

Valence structure of hadrons: global properties of nucleons

mass	$M_p \approx M_n \approx 940$ MeV
charge	$Q_p = 1$, $Q_n = 0$
spin	$s = ½$
magnetic moment	$g_p \approx 5.59$, $g_n \approx -3.83$
isospin, strangeness	$I = ½$: (p,n) $S = 0$
baryon number	$B = 1$

Quarks as constituents

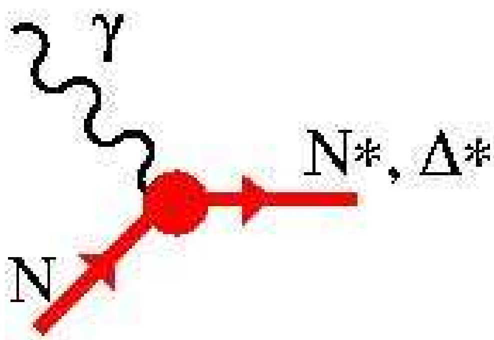




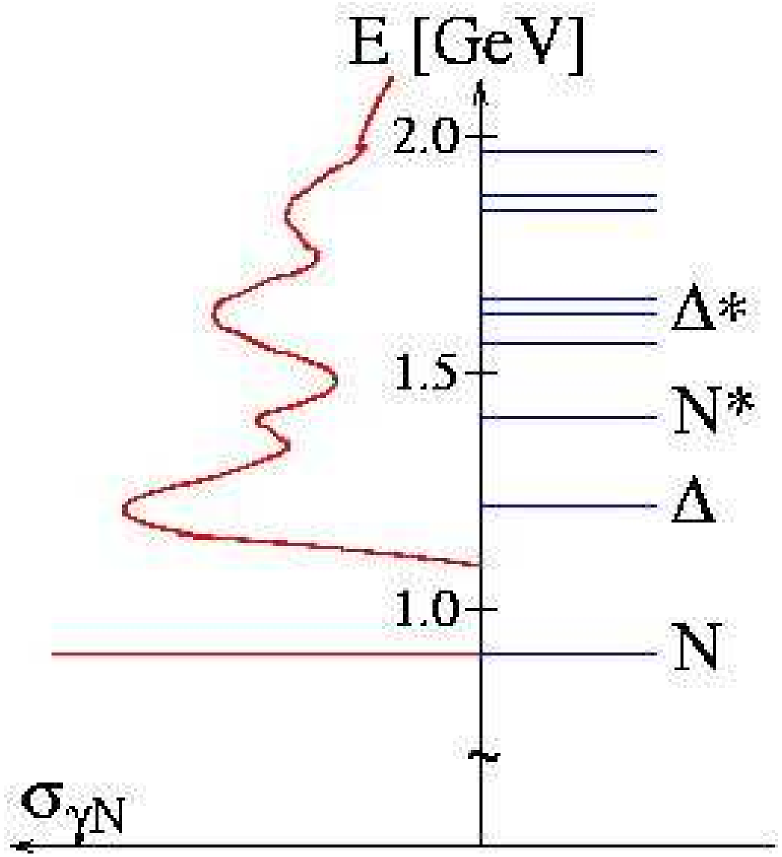
proton

A real look at the proton

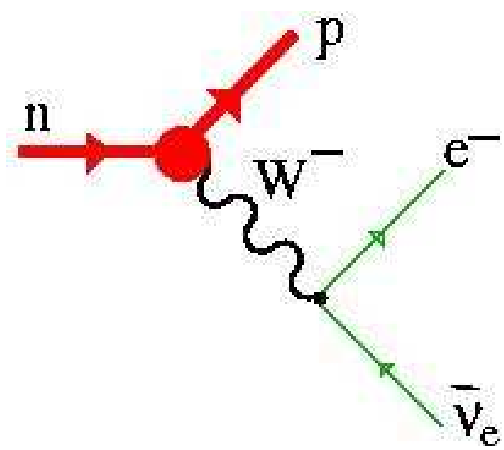
γ + N à ....



Nucleon excitation spectrum
E ~ 1/R ~ 200 MeV
R ~ 1 fm



A (weak) look at the nucleon



n à p + e^- + ν

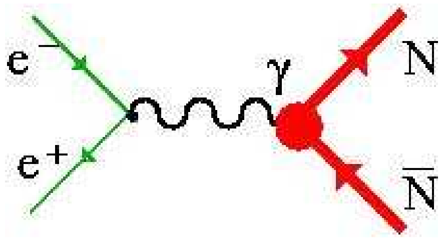
τ = 900 s

à Axial charge

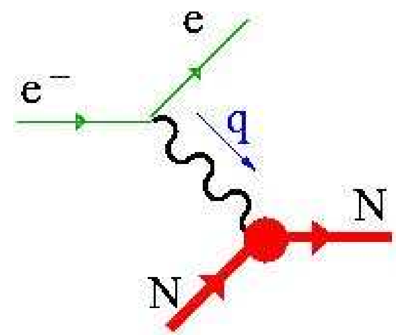
$G_A(0) = 1.26$

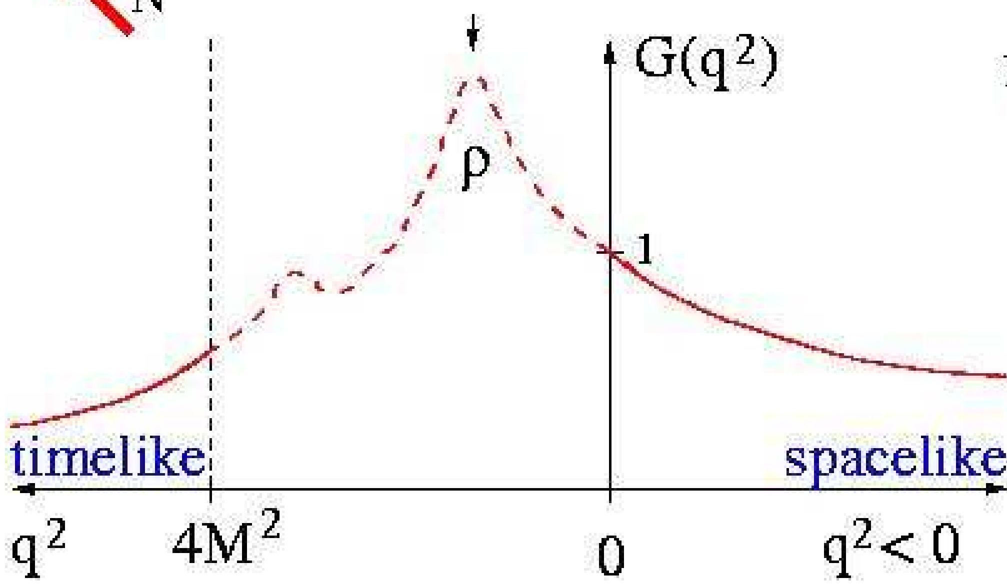
A virtual look at the proton

γ^* à N $\bar{\mathrm{N}}$



γ^* + N à N

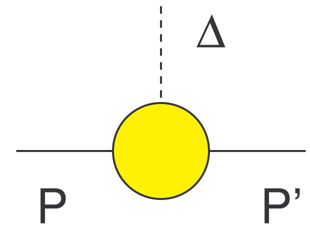




Local – forward and off-forward m.e.

Local operators (coordinate space densities):

$$< P'|O(x)|P> = e^{i\Delta . x}\left[G_1(t) - i\Delta_\mu G_2^\mu(t)\right]$$



$t = \Delta^2$

Form factors

Static properties:

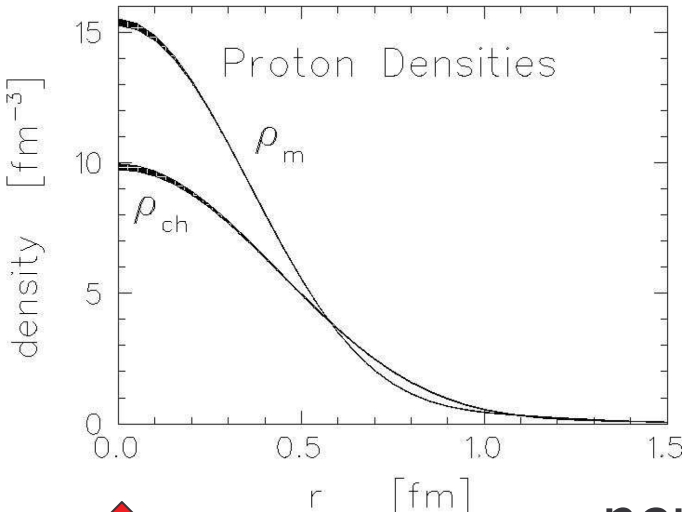
$$G_1(0) = < P|O(x)|P >$$

$$G_2^\mu(0) = < P|x^\mu O(x)|P >$$

<u>Examples</u>:
(axial) charge
mass
spin
magnetic moment
angular momentum

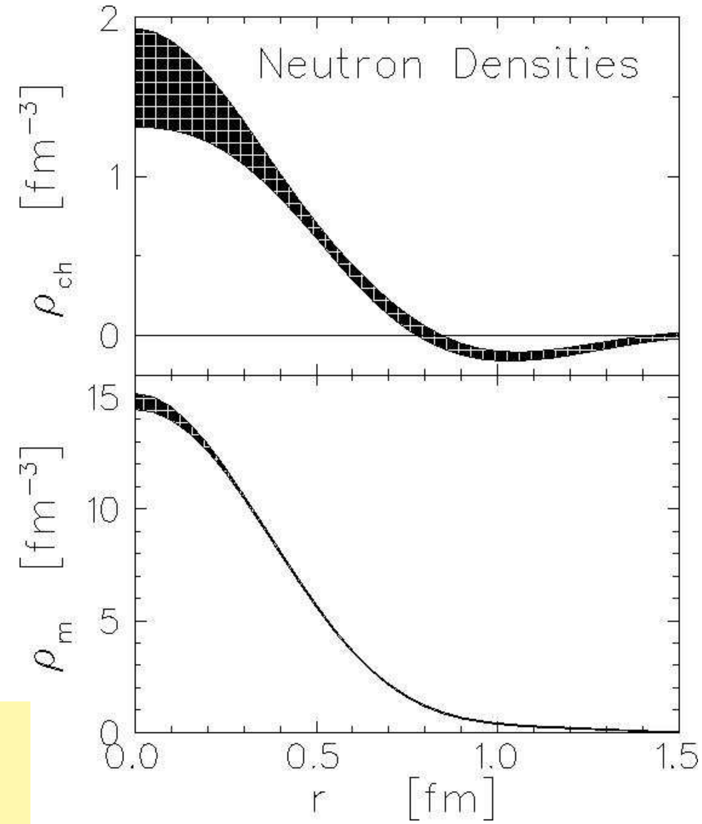
Nucleon densities from virtual look

$$G_i(t) \rightarrow \rho_i(x)$$



proton

neutron



- charge density $\neq 0$
- u more central than d?
- role of antiquarks?
- $n = n_0 + p\pi^- + \ldots$?

Quark and gluon operators

Given the QCD framework, the operators are known quarkic or gluonic currents such as

(axial) vector currents

$$V^q_\mu(x) = \overline{q}(x)\gamma_\mu q(x)$$

$$A^{q'q}_\mu(x) = \overline{q}(x)\gamma_\mu\gamma_5 q'(x)$$

probed in specific combinations by photons, Z- or W-bosons

$$J^{(\gamma)}_\mu = \tfrac{2}{3}V^u_\mu - \tfrac{1}{3}V^d_\mu - \tfrac{1}{3}V^s_\mu + \ldots$$

$$J^{(Z)}_\mu = \tfrac{1}{2}\left(V^u_\mu - A^u_\mu\right) - \tfrac{4}{3}\sin^2\theta_W V^u_\mu + \ldots$$

$$J^{(W)}_\mu = V^{ud'}_\mu - A^{ud'}_\mu + \ldots$$

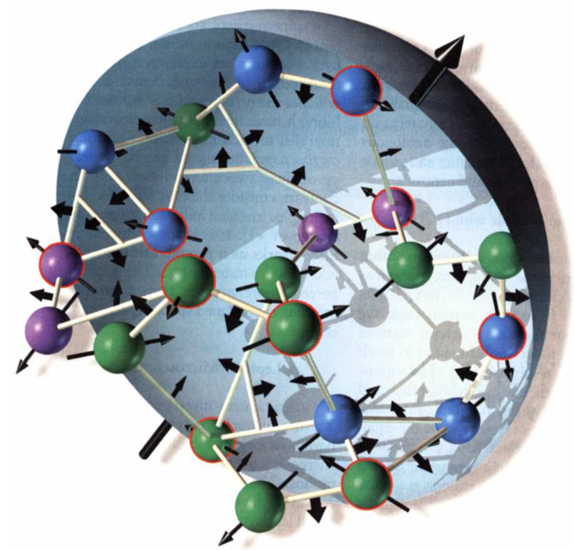
energy-momentum currents

$$T^q_{\mu\nu}(x) \sim \overline{q}(x)\gamma_{\{\mu}D_{\nu\}}q(x)$$

$$T^G_{\mu\nu}(x) \sim G_{\mu\alpha}(x)G^\alpha{}_\nu(x)$$

probed by gravitons

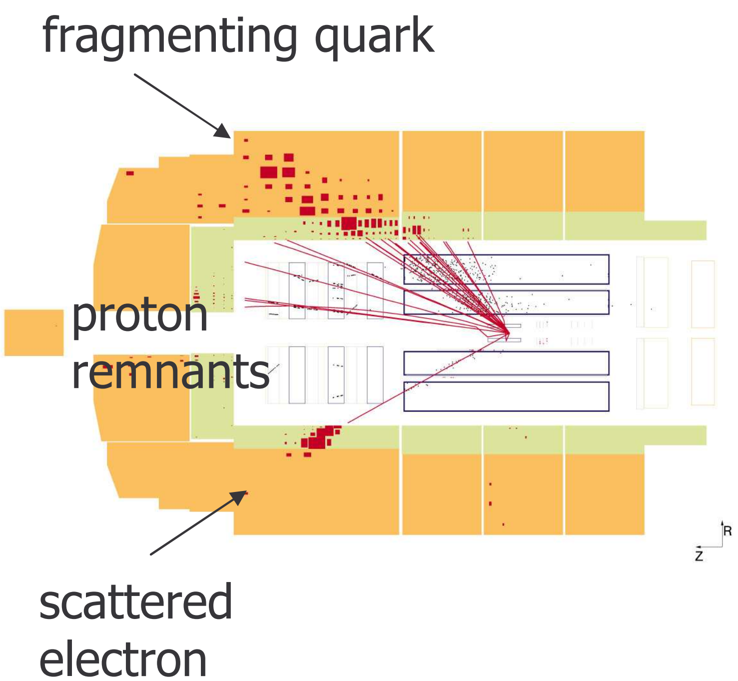
Towards the quarks themselves

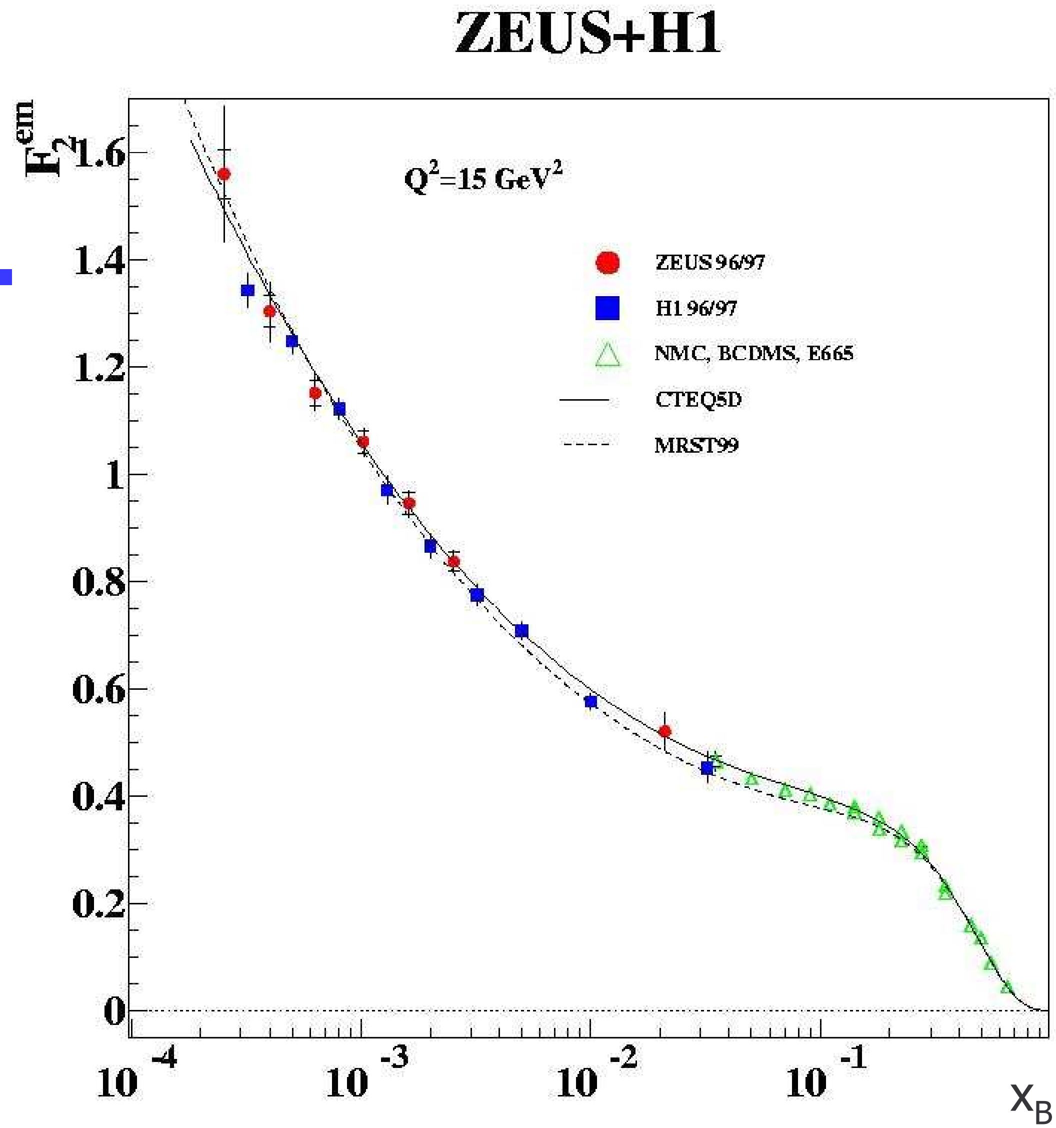


- The current provides the densities but only in specific combinations, e.g. *quarks minus antiquarks* and only flavor weighted
- No information about their correlations, (effectively) pions, or …
- Can we go beyond these global observables (which correspond to local operators)?
- Yes, in high energy (semi-)inclusive measurements we will have access to non-local operators!

- L_{QCD} (quarks, gluons) known!

Deep inelastic experiments



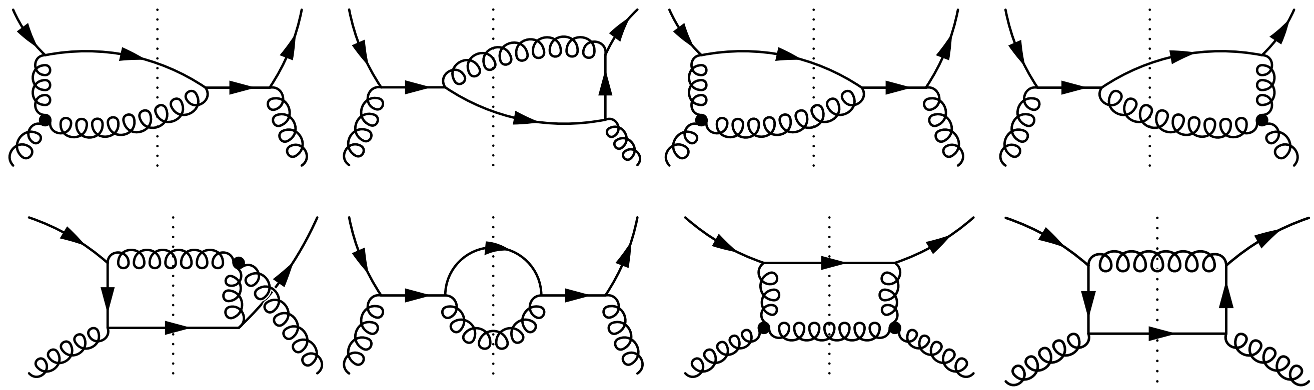


Results directly reflect *quark*, *antiquark* and *gluon* distributions in the proton

QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate cross sections, e.g. $\gamma^* q \to q$, $q\bar{q} \to \gamma^*$, $\gamma^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- E.g.

  $qg \to qg$



- Calculations work for plane waves

$$\left\langle 0 \middle| \psi_i^{(s)}(\xi) \middle| p, s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

Confinement in QCD

- Confinement limits us to hadrons as ‘quark sources’ or ‘targets’

$$\left\langle X \left| \psi_i^{(s)}(\xi) \right| P, S \right\rangle e^{+ip.\xi}$$

$$\left\langle X \left| \psi_i^{(s)}(\xi) A^\mu(\eta) \right| P, S \right\rangle e^{+i(p-p_1).\xi + ip_1.\eta}$$

- These involve nucleon states
- At high energies interference terms between different hadrons disappear as $1/P_1.P_2$
- Thus, the theoretical description/calculation involves for hard processes, a forward matrix element of the form

$$\Phi_{ij}(p,P) = \int \frac{d^3 P_X}{(2\pi)^3 2E_X} < P \,|\, \overline{\psi}_j(0) \,|\, X >< X \,|\, \psi_i(0) \,|\, P > \delta(P - P_X - p)$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{i\,p.\xi} < P \,|\, \overline{\psi}_j(0)\, \psi_i(\xi) \,|\, P >$$

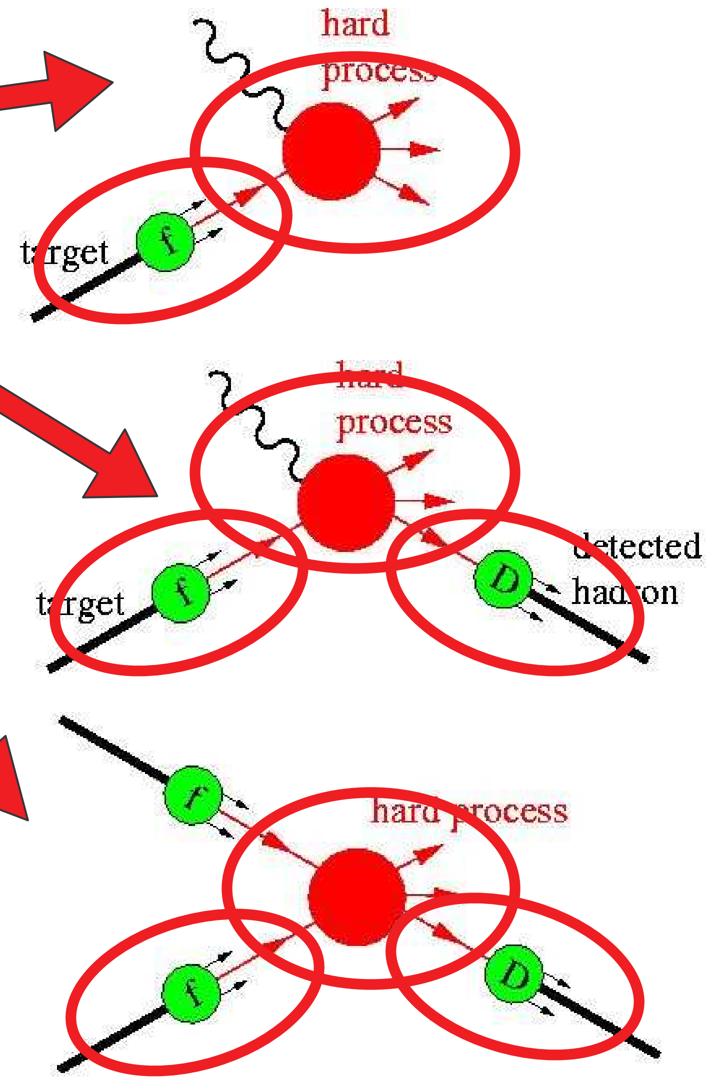
quark momentum

Partonic structure of hadrons

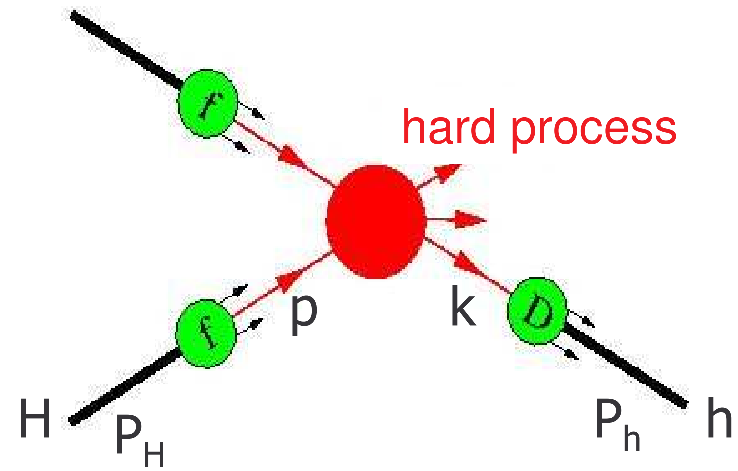
- Hard (high energy) processes
  - Inclusive leptoproduction
  - 1-particle inclusive leptoproduction
  - Drell-Yan
  - 1-particle inclusive hadroproduction

- Elementary hard processes

- Universal (?) soft parts
  - distribution functions f
  - fragmentation functions D



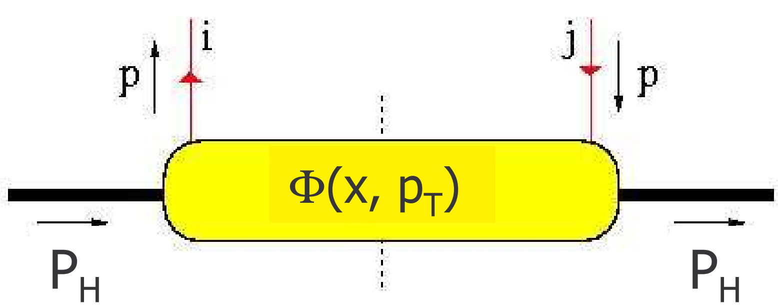
Partonic structure of hadrons



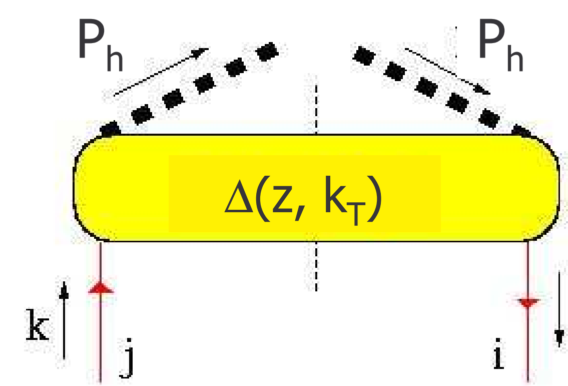
Need $P_H.P_h \sim s$ (large) to get separation of soft and hard parts

Allows $\int d\sigma\ldots = \int d(p.P)\ldots$

distribution correlator



fragmentation correlator



Intrinsic transverse momenta

- Hard processes: Sudakov decomposition for momenta:

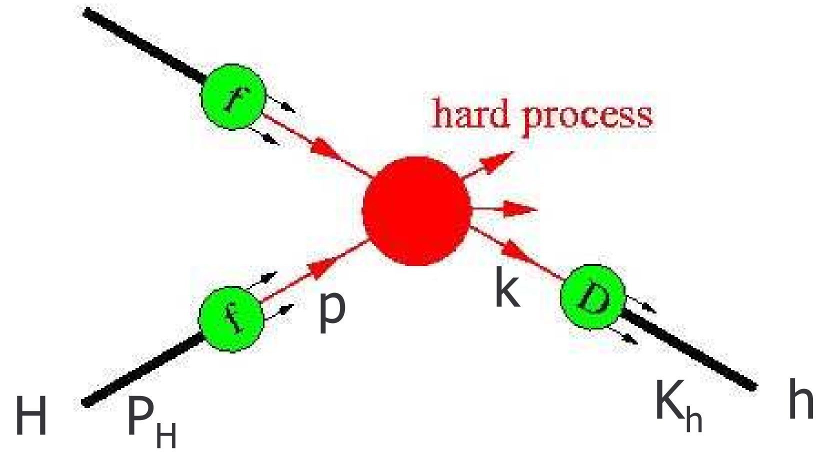
  $p = xP_H + p_T + \sigma\, n$

- zero: $p_T.P_H = n^2 = p_T.n$

  large: $P_H.n \sim \sqrt{s}$

  hadronic: $p_T{}^2 \sim P_H{}^2 = M_H{}^2$

  small: $\sigma \sim (p.P_H, p^2, M_H{}^2)/\sqrt{s}$
- Parton virtuality enters in σ and is integrated out à $\Phi^{H\to q}(x,p_T)$ describing quark distributions
- Integrating p_T à collinear $\Phi^{H\to q}(x)$
- Lightlike vector n enters in $\Phi(x,p_T)$, but is irrelevant in cross sections

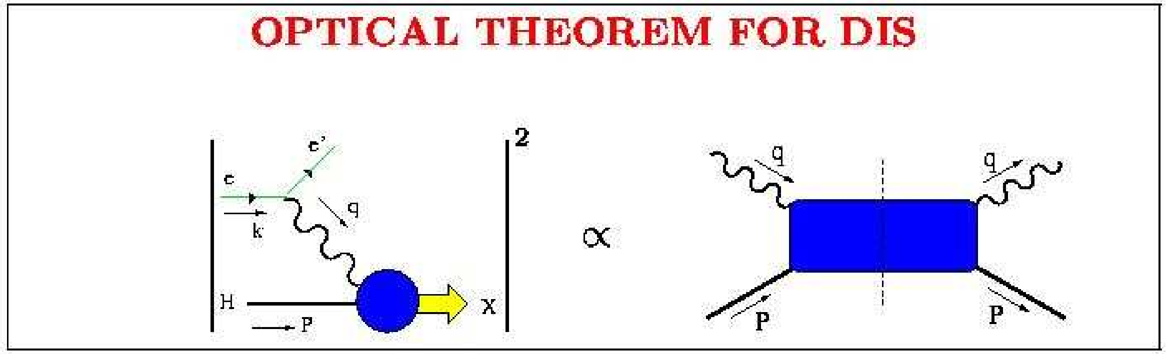


Similarly for quark fragmentation:

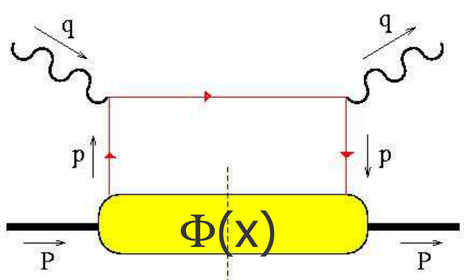
$k = z^{-1}K_h + k_T + \sigma'\, n'$

correlator $\Delta^{q\to h}(z,k_T)$

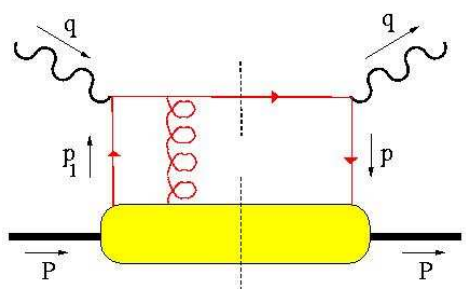
(calculation of) cross section in DIS



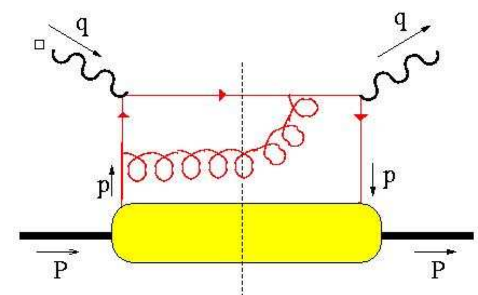
Full calculation



+



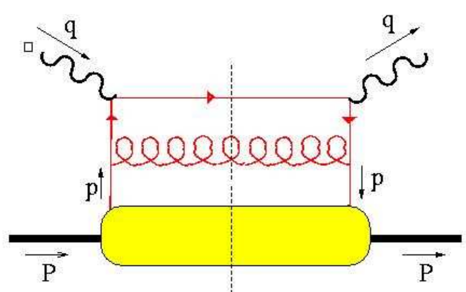
+



LEADING (in 1/Q)

$x = x_B = -q^2/P.q$

+

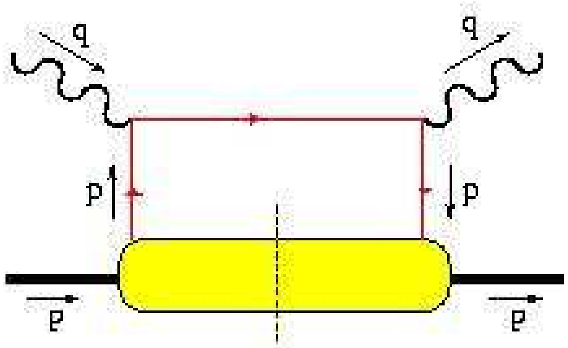


+ ...

LIGHTCONE DOMINANCE IN DIS

Large scale Q leads in a natural way to the use of lightlike vectors:
$n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \frac{x_B M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ \end{array}\right.$$



part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- d^2p_T \ldots$

Parametrization of lightcone correlator

DISTRIBUTION FUNCTIONS

Parameterization of p_T-integrated soft part including subleading order and including T-odd parts for a spin 1/2 hadron:

leading part

$$\Phi(x) = \boxed{\frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + S_L\, g_1(x)\gamma_5\,\not{n}_+ + h_1(x)\,\frac{[\not{S}_T, \not{n}_+]\gamma_5}{2}\right\}}$$

$$+ \frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not{S}_T + S_L\, h_L(x)\,\frac{[\not{n}_+, \not{n}_-]\gamma_5}{2}\right\}$$

T-odd

$$+ \frac{M}{2P^+}\left\{ \cancel{f_T(x)}\,\epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\, S_L\, \cancel{e_L(x)}\,\gamma_5 + \cancel{i\,h(x)}\,\frac{[\not{n}_+, \not{n}_-]}{2}\right\}$$

- M/P^+ parts appear as M/Q terms in cross section
- T-reversal applies to $\Phi(x)$ $\rightarrow$ no T-odd functions

Jaffe & Ji NP B 375 (1992) 527
Jaffe & Ji PRL 71 (1993) 2547

Basis of partons

- 'Good part' of Dirac space is 2-dimensional
- Interpretation of DF's

TWO 'SPIN' STATES FOR (GOOD) QUARK FIELDS

chiral eigenstates:

$$\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi : \qquad |R\rangle \quad \text{and} \quad |L\rangle$$

or

transverse spin eigenstates:

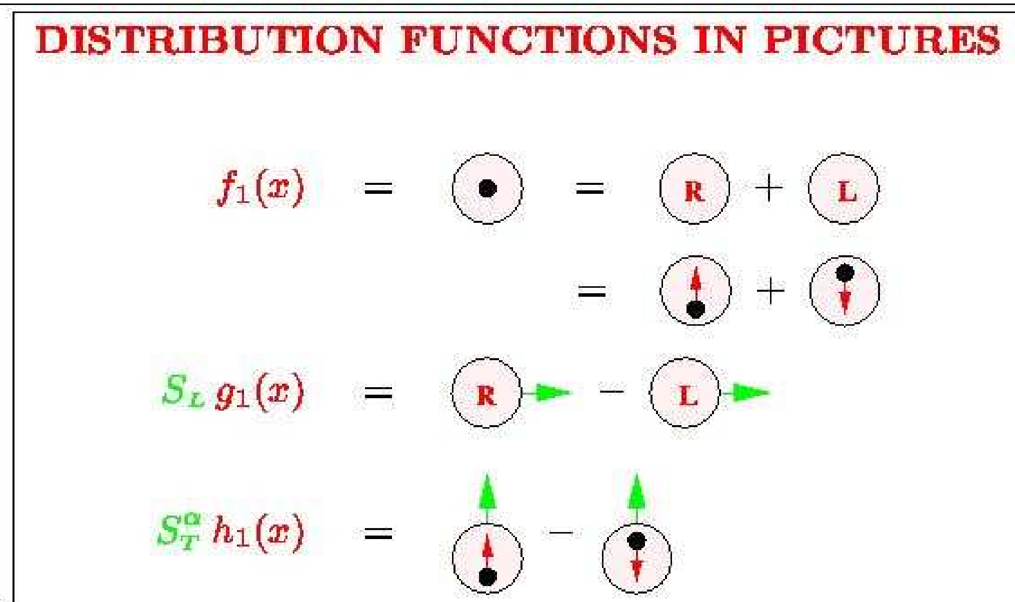
$$\psi_{\uparrow/\downarrow} \equiv \frac{1}{2}(1 \pm \gamma^\alpha \gamma_5)\psi : \qquad |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle$$

Note: $[\mathcal{P}_{R/L}, \mathcal{P}_+] = [\mathcal{P}_{\uparrow/\downarrow}, \mathcal{P}_+] = 0$

unpolarized quark distribution

helicity or chirality distribution

transverse spin distr. or transversity



$\frac{1}{2}$ $\Big|_{\xi^+=\xi_T=0}$

$= h_1(x)\, S_T^\alpha$

Matrix representation for M = $[\Phi(x)\gamma^+]^T$

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712

Quark production matrix, directly related to the helicity formalism

Anselmino et al.

MATRIX REPRESENTATION FOR SPIN 1/2

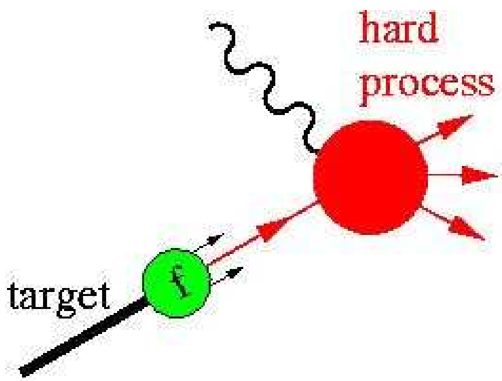
p_T-integrated distribution functions:

For a spin 1/2 hadron (e.g. nucleon) the quark production matrix in quark$\otimes$nucleon spin space is given by

$$M^{(\mathrm{prod})} = \begin{array}{cc} \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix} & \begin{array}{c} R\rightarrow \\ \leftarrow R \\ L\rightarrow \\ \leftarrow L \end{array} \\ \begin{array}{cccc} R\rightarrow & \leftarrow R & L\rightarrow & \leftarrow L \end{array} & \end{array}$$

- Off-diagonal elements (RL or LR) are chiral-odd functions
- Chiral-odd soft parts must appear with partner in e.g. SIDIS, DY

Results for deep inelastic processes



$$\sigma_{\gamma^* N \to X} = f_1^{N \to q} \otimes \hat{\sigma}_{\gamma^* q \to q}$$

$$\Delta\sigma_{\gamma^* \vec{N} \to X} = g_1^{N \to q} \otimes \Delta\hat{\sigma}_{\gamma^* \vec{q} \to \vec{q}}$$

Ø This has resulted in a good knowledge of $u(x) = f_1^{\text{pà u}}(x)$, $d(x)$, $\bar{u}(x)$, $\bar{d}(x)$ and (through evolution equations) also $G(x)$

Ø For example in proton $\bar{d} > \bar{u}$, in neutron $\bar{u} > \bar{d}$ (naturally explained by a $\pi^- p$ component in neutron, providing additional insight beyond G_E)

Ø Polarized experiments (double spin asymmetries) have provided spin densities $\Delta u(x) = g_1^{\text{pà u}}(x)$, etc.

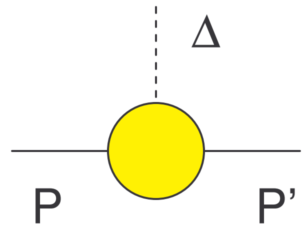
End lecture 1

Lecture 2

Local – forward and off-forward

Local operators (coordinate space densities):

$$< P' | O(\xi) | P > = e^{i\Delta.\xi}\left[G_1(t) - i\Delta_\mu G_2^\mu(t)\right]$$



$t = \Delta^2$

Form factors

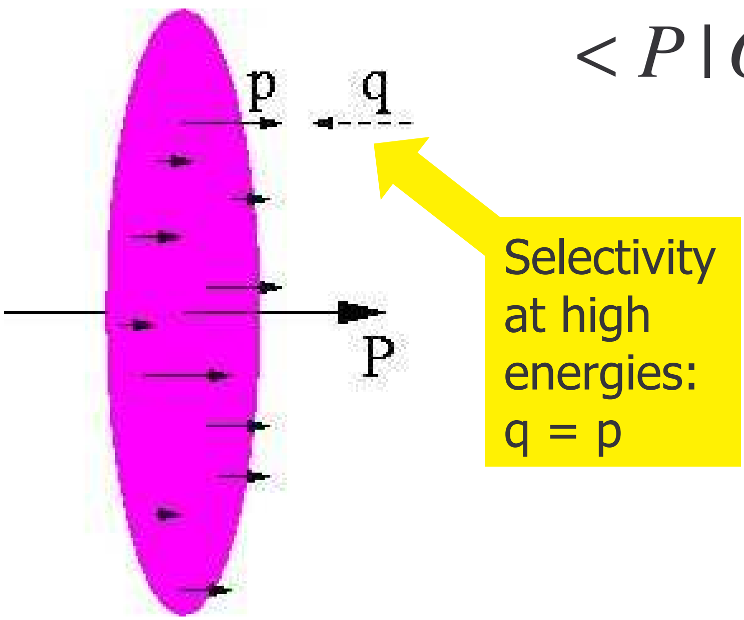
Static properties:

$$G_1(0) = < P | O(\xi) | P >$$

$$G_2^\mu(0) = < P | \xi^\mu O(\xi) | P >$$

<u>Examples</u>:
(axial) charge
mass
spin
magnetic moment
angular momentum

Nonlocal - forward



Nonlocal forward operators (correlators):

$$< P \,|\, O(\eta,\eta+\xi) \,|\, P > \;=\; < P \,|\, O(0,\xi) \,|\, P >$$

Specifically useful: 'squares'

$$O(0,\xi) = \phi^{\dagger}(0)\ldots\phi(\xi)$$

Momentum space densities of ϕ-ons:

$$\int d\xi\, e^{ip.\xi} < P \,|\, \phi^{\dagger}(0)\phi(\xi) \,|\, P > \;=\; \left| < P-p \,|\, \phi(0) \,|\, P > \right|^{2} = f(p)$$

Sum rules à form factors

$$\int dp\;\, f(p) = G_1(0)$$

Quark number

- Quark distribution and quark number

$$x = \frac{p.n}{P.n} = \frac{p^+}{P^+}$$

$$< P \,|\, \overline{\psi}(0)\gamma^\mu \psi(0) \,|\, P > \;=\; (n - \overline{n})\, 2P^\mu$$

$$\int d(\xi.P)\; e^{i\,p.\xi} < P \,|\, \overline{\psi}(0)\gamma^\mu \psi(\xi) \,|\, P >|_{LC} \;=\; f_1(x)\; 2P^\mu$$

- Sum rule:

$$\int_{-1}^{1} dx\; f_1^{\,q}(x) = \int_{0}^{1} dx\; f_1^{\,q}(x) - \int_{0}^{1} dx\; f_1^{\,\overline{q}}(x) = n_q - n_{\overline{q}}$$

- Next higher moment gives ‘momentum sum rule’

$$< P \,|\, \underbrace{\overline{\psi}(0)\gamma^{\{\mu}\partial^{\nu\}}\psi(0)}_{T_q^{\mu\nu}} \,|\, P > \;=\; (\varepsilon_q + \varepsilon_{\overline{q}})\, 2P^{\{\mu}P^{\nu\}}$$

$$\int_{-1}^{1} dx\; x f_1^{\,q}(x) = \int_{0}^{1} dx\; x f_1^{\,q}(x) + \int_{0}^{1} dx\; x f_1^{\,\overline{q}}(x) = (\varepsilon_q + \varepsilon_{\overline{q}})$$

Quark axial charge/spin sum rule

- Quark chirality distribution and quark spin/axial charge

$$< P,S \,|\, \overline{\psi}(0)\gamma^{\mu}\gamma_5\psi(0) \,|\, P,S > = \; g_A \, 2M \, S^{\mu}$$

$$\int d(\xi.P) \; e^{i\,p\cdot\xi} < P,S \,|\, \overline{\psi}(0)\gamma^{\mu}\gamma_5\psi(\xi) \,|\, P,S >|_{LC} = \; g_1(x) 2M \, S^{\mu}$$

- Sum rule:

$$\int_{-1}^{1} dx \; g_1^q(x) = \int_0^1 dx \; g_1^q(x) + \int_0^1 dx \; g_1^{\overline{q}}(x) = g_A^q + g_A^{\overline{q}} = \Delta q + \Delta \overline{q}$$

- This is one part of the spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_Q + \Delta G + L_G$$

Full spin sum rule

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma + L_Q}_{J_Q} + \underbrace{\Delta G + L_G}_{J_G}$$

- The angular momentum operators in this spin sum rule

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho} + \Sigma^{\mu\rho\sigma} = x^\rho \Theta^{\mu\sigma} - x^\sigma \Theta^{\mu\rho}$$

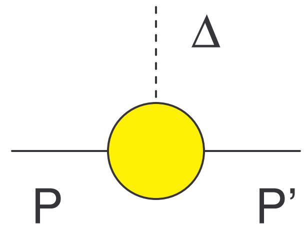
$$T^{\mu\nu} = T_Q^{\mu\nu} + T_G^{\mu\nu}$$

- The off-forward matrix elements of the (symmetric) energy momentum tensor give access to J_Q and J_G

Local – forward and off-forward

Local operators (coordinate space densities):

$$< P' | O(\xi) | P > = e^{i\Delta.\xi}\left[G_1(t) - i\Delta_\mu G_2^\mu(t) \right]$$



$$t = \Delta^2$$

Form factors

Static properties:

$$G_1(0) = < P | O(\xi) | P >$$

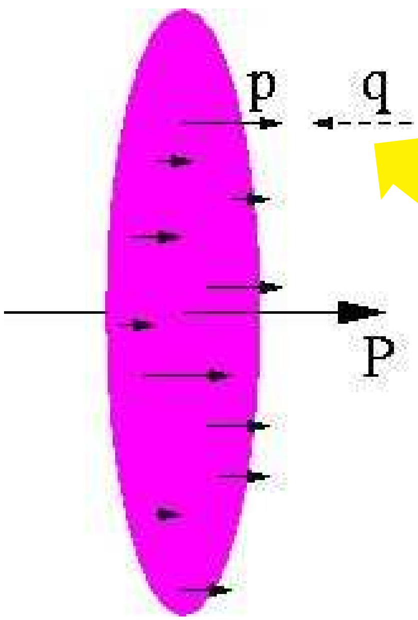
$$G_2^\mu(0) = < P | \xi^\mu O(\xi) | P >$$

reminder

Nonlocal - forward

Nonlocal forward operators (correlators):

$$< P \,|\, O(\eta, \eta+\xi) \,|\, P > \; = < P \,|\, O(0,\xi) \,|\, P >$$



Selectivity at high energies: q = p

Specifically useful: 'squares'

$$O(0,\xi) = \phi^{\dagger}(0) \ldots \phi(\xi)$$

Momentum space densities of ϕ-ons:

$$\int d\xi \, e^{ip.\xi} < P \,|\, \phi^{\dagger}(0)\phi(\xi) \,|\, P > \; = \; \left| < P - p \,|\, \phi(0) \,|\, P > \right|^2 = f(p)$$

Sum rules à form factors

$$\int dp \;\; f(p) = G_1(0)$$

reminder

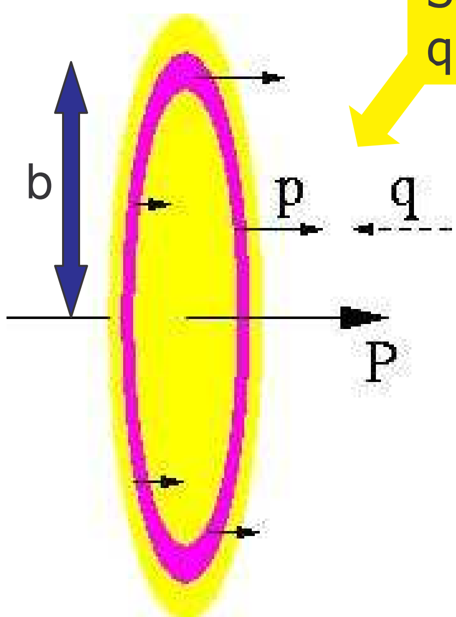
Nonlocal – off-forward

Nonlocal off-forward operators (correlators AND densities):

$$\int dx\, e^{ip.x} < P'|\phi^\dagger(y)\phi(y+x)|P> \; = \; e^{i\Delta.y}\left[f_1(t,p) - i\Delta_\mu f_2^\mu(t,p)\right]$$

GPD's

Selectivity
q = p



Sum rules à form factors

$$\int dp\; f_1(t,p) = G_1(t)$$

$$\int dp\; f_2^\mu(t,p) = G_2^\mu(t)$$

$$t = \Delta^2$$

Forward limit à correlators

$$\int dx\, e^{ip.x} < P|\phi^\dagger(y)\phi(y+x)|P> \; = \; f_1(0,p)$$

$$\int dx\, e^{ip.x} < P|\, y^\mu\, \phi^\dagger(y)\phi(y+x)|P> \; = \; f_2^\mu(0,p)$$

Quark tensor charge

- Quark chirality distribution and quark spin/axial charge

$$< P,S \,|\, \overline{\psi}(0)\gamma^{\mu}\gamma_T^{\alpha}\gamma_5\psi(0) \,|\, P,S > = g_T \, 2P^{\mu} S_T^{\alpha}$$

$$\int d(\xi.P)\; e^{i\,p.\xi} < P,S \,|\, \overline{\psi}(0)\gamma^{\mu}\gamma_T^{\alpha}\gamma_5\psi(\xi) \,|\, P,S >|_{LC} = h_1(x)\, 2P^{\mu} S_T^{\alpha}$$

- Sum rule:

$$\int_{-1}^{1} dx\; h_1^q(x) = \int_0^1 dx\; h_1^q(x) - \int_0^1 dx\; h_1^{\bar{q}}(x) = g_T^q - g_T^{\bar{q}} = \delta q - \delta\overline{q}$$

- Note that this is not a 'spin' measure, even if $h_1(x)$ is the distribution of transversely polarized quarks in a transversely polarized nucleon!

'Transverse spin' ~ $\int_0^1 dx\; h_1^q(x) + \int_0^1 dx\; h_1^{\bar{q}}(x) = \delta q + \delta\overline{q}$

(no decent operator!)

A transverse spin rule

- One can write down a 'transverse spin' sum rule
- It was first discussed by Teryaev and Ratcliffe, but it involves the twist-3 funtion $g_T = g_1 + g_2$

$$\int_{-1}^{1} dx\ g_T^q(x) = \int_{-1}^{1} dx\ g_1^q(x) = \Delta q + \Delta \overline{q}$$

  (Burkhardt-Cottingham sumrule)
- … and a similar gluon sumrule

- It does not involve the 'transverse spin'. This appears in the Bakker-Leader-Trueman sumrule (which involves the assumption of having 'free' quarks).

(my version of Trieste meeting)

End lecture 2

Lecture 3

Issues

- Knowledge of partonic structure can be extended by looking at the ‘transverse structure’
- Time reversal invariance provides a nice discriminator for ‘special effects’
- Example is the color flow in hard processes, which is reflected in the nonlocal structure of matrix elements and shows up in single spin asymmetries
- Single spin asymmetries are being measured (HERMES@DESY, JLAB, COMPASS@CERN, KEK, RHIC@Brookhaven)

The partonic structure of hadrons

The cross section can be expressed in hard squared QCD-amplitudes and distribution and fragmentation functions entering in forward matrix elements of nonlocal combinations of quark and gluon field operators (ϕ à ψ or G)

$$p^\mu = x P^\mu + p_T^\mu + \frac{p.P - xM^2}{P.n} n^\mu$$

$$\Phi(x, p_T) = \int \frac{d(\xi.P) d^2\xi_T}{(2\pi)^3} e^{i p.\xi} \langle P | \phi^\dagger(0) \phi(\xi) | P \rangle_{\xi.n=0}$$

TMD

lightfront: $\xi^+ = 0$

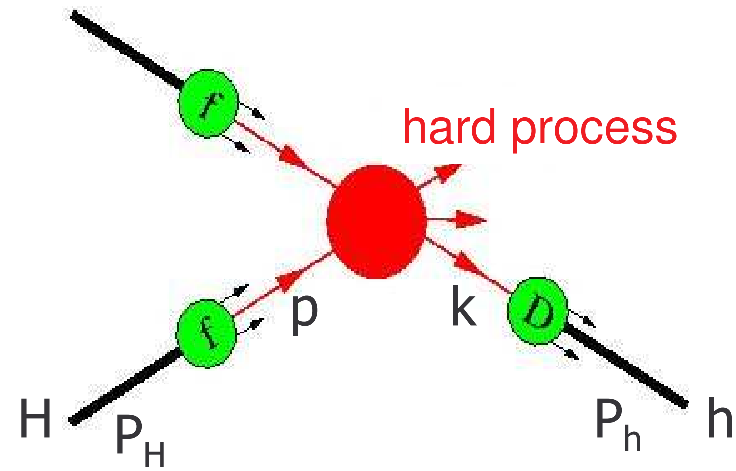
$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i p.\xi} \langle P | \phi^\dagger(0) \phi(\xi) | P \rangle_{\xi.n=\xi_T=0}$$

lightcone

FF $$\Delta(z, k_T) = \int \frac{d(\xi.P) d^2\xi_T}{(2\pi)^3} e^{-ik.\xi} \langle 0 | \phi(0) | P, X \rangle \langle P, X | \phi^\dagger(\xi) | 0 \rangle_{\xi.n=0}$$

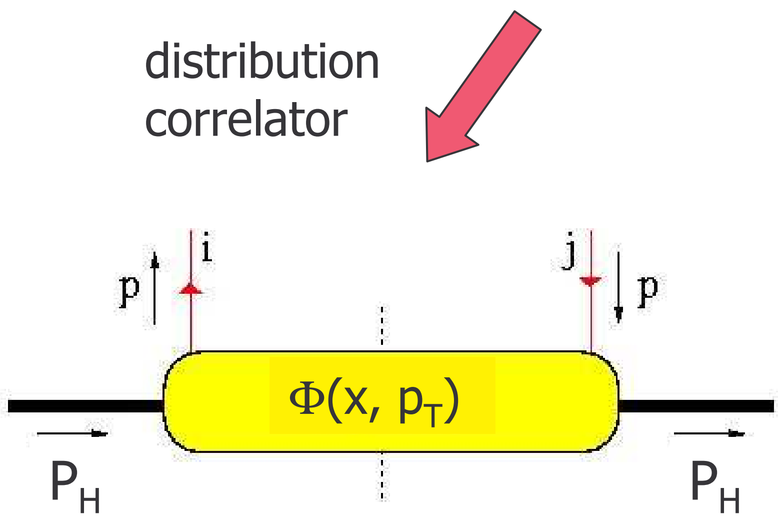
Partonic structure of hadrons

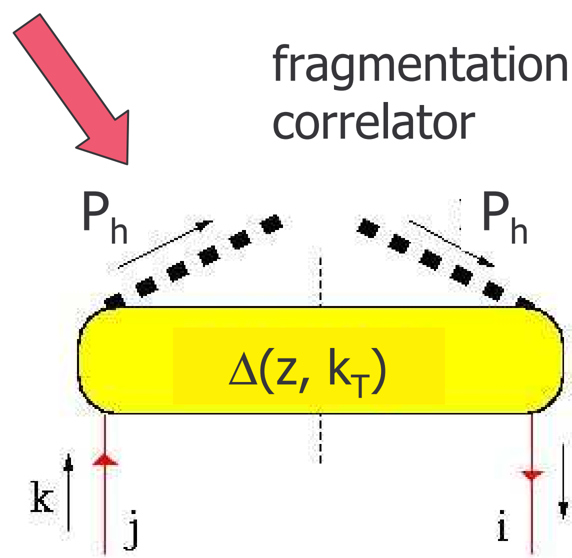
reminder



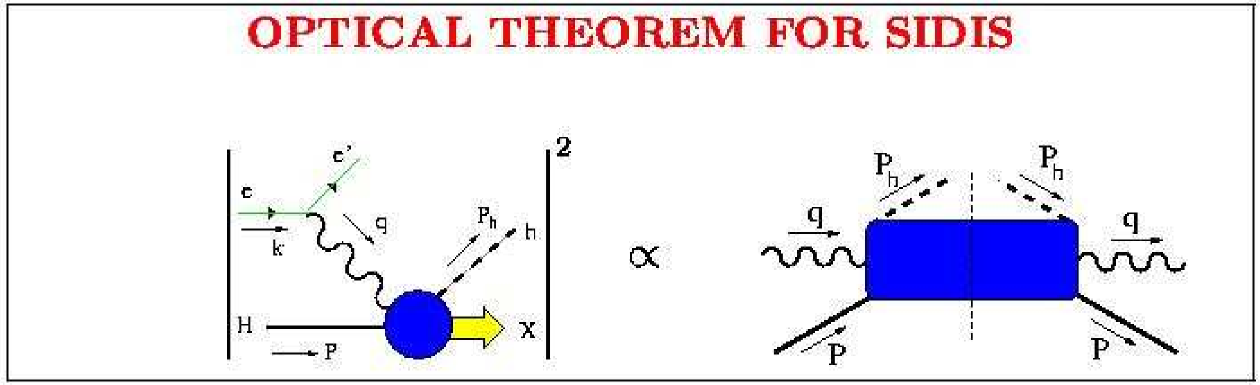
Need $P_H.P_h \sim s$ (large) to get separation of soft and hard parts

Allows $\int d\sigma\ldots = \int d(p.P)\ldots$

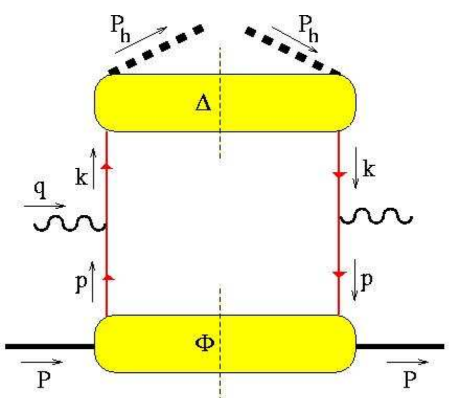




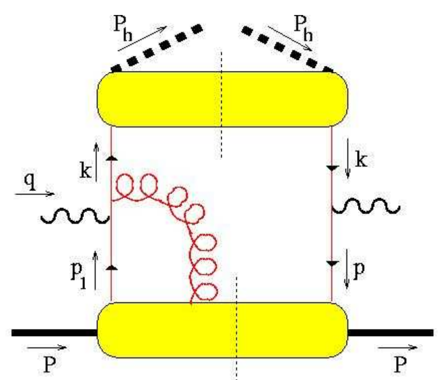
(calculation of) cross section in SIDIS



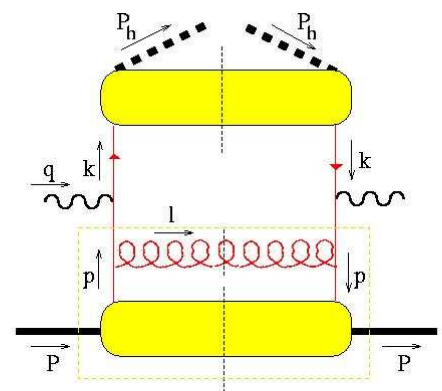
Full calculation



+

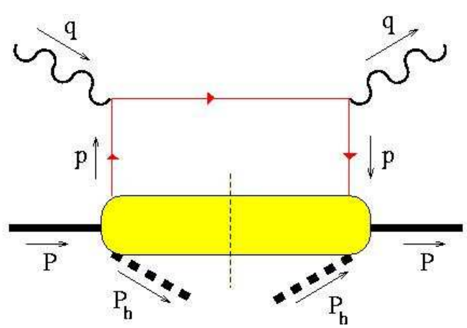


+



LEADING
(in 1/Q)

+

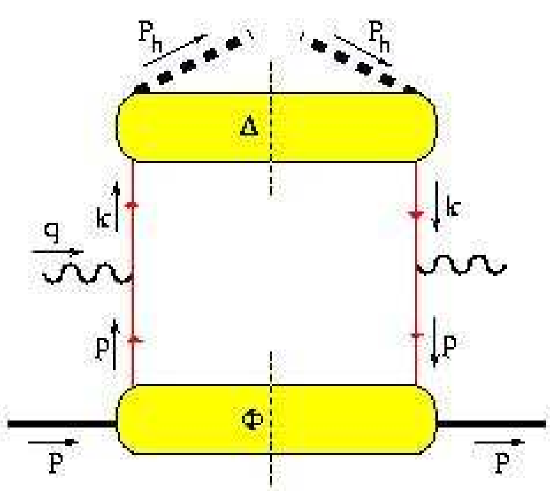


+ ...

LIGHTCONE DOMINANCE IN SIDIS

Large scale Q leads in a natural way to the use of lightlike vectors:
$n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \\ 2\,P_h\cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \frac{z_h\,Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ + q_T \\ P = \frac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$



Three external momenta

$P \quad P_h \quad q$

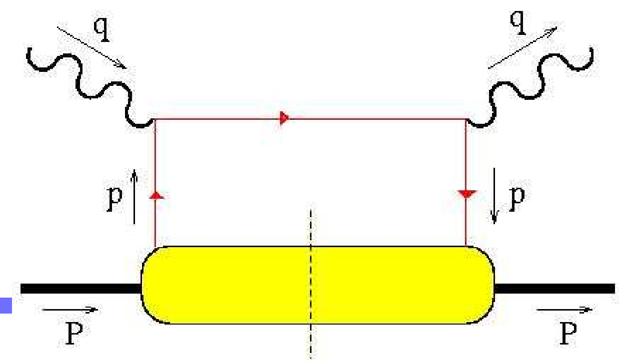
transverse directions relevant

$q_T = q + x_B\,P - P_h/z_h$

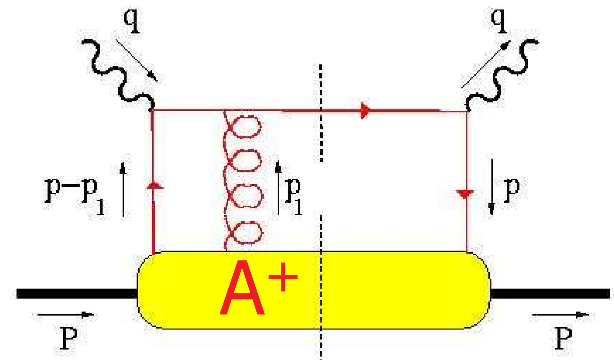
or

$q_T = -P_{h\perp}/z_h$

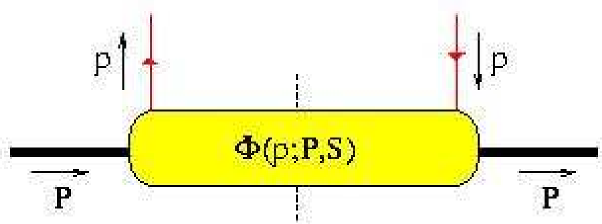
Gauge link in DIS



- In limit of large Q^2 the result of 'handbag diagram' survives
- … + contributions from A^+ gluons ensuring color gauge invariance



SOFT PARTS IN DIS



p^+/P^+

$\downarrow$

$$\Phi_{ij}(x) = \int dp^-\, d^2p_T\; \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle \Big|_{\xi^+=\xi_T=0}$$

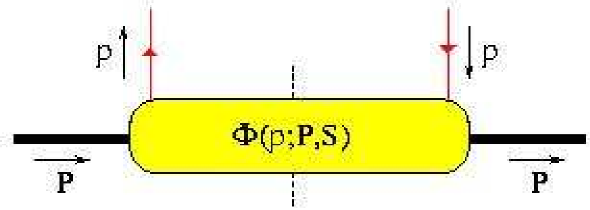
Ellis, Furmanski, Petronzio
Efremov, Radyushkin

A^+ gluons
à gauge link

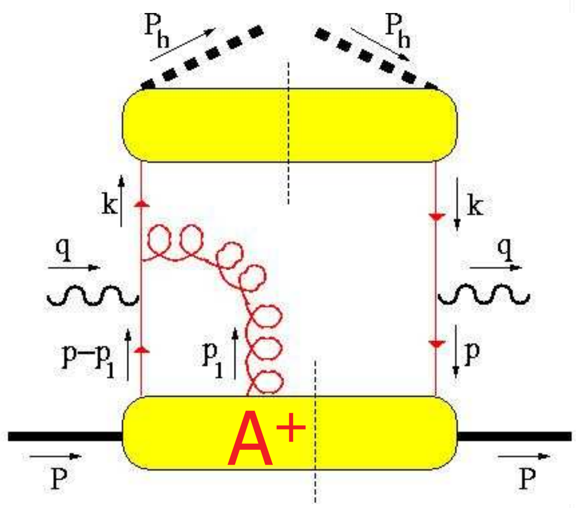
LIGHTFRONT QUARK-QUARK CORRELATOR

Distribution

$x = p^+/P^+$



$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi} \langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}$$



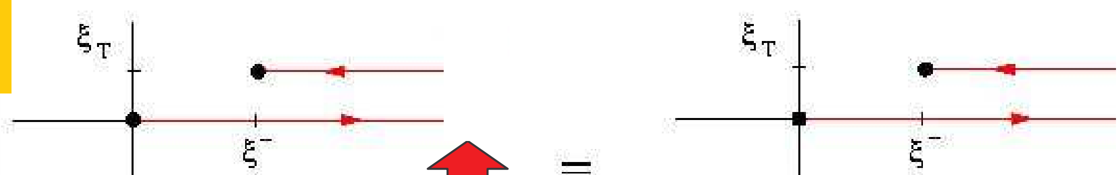
including the gauge link (in SIDIS)

$$\Phi^{[+]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{ip\cdot\xi} \langle P,S|\overline{\psi}(0)\, U^-_{[0,\infty]} U^T_{[0_T,\infty_T]} U^T_{[\infty_T,\xi_T]} U^-_{[\infty,\xi]}\, \psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

One needs also A_T

$G^{+\alpha} = \partial^+ A_T^\alpha$

$A_T^\alpha(\xi) = A_T^\alpha(\infty) + \int d\eta\, G^{+\alpha}$



From $<\psi(0)A_T(\infty)\psi(\xi)>$ m.e.

Belitsky, Ji, Yuan, hep-ph/0208038
Boer, M, Pijlman, hep-ph/0303034

Parametrization of $\Phi(x,p_T)$

- Link dependence allows also T-odd distribution functions since $T\,U[0,\infty]\,T^\dagger = U[0,-\infty]$
- Functions $h_1^\perp$ and $f_{1T}^\perp$ (Sivers) nonzero!
- Similar functions (of course) exist as fragmentation functions (no T-constraints) $H_1^\perp$ (Collins) and $D_{1T}^\perp$

DISTRIBUTION FUNCTIONS

Parameterization of p_T-dependent soft part at leading order and including T-odd parts for polarized hadrons:

$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\,h_1^\perp(x,p_T^2)\,\frac{\not p_T}{M} \right\} \not n_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\, g_{1L}(x,p_T^2)\,\gamma_5 + S_L\, h_{1L}^\perp(x,p_T^2)\gamma_5\,\frac{\not p_T}{M} \right\} \not n_+$$

$$\Phi_T(x,p_T) = \left\{ g_{1T}(x,p_T^2)\,\frac{p_T\cdot S_T}{M}\,\gamma_5 + f_{1T}^\perp(x,p_T^2)\,\frac{\epsilon_{T\,\rho\sigma}\,p_T^\rho S_T^\sigma}{M} \right.$$
$$\left. + h_{1T}(x,p_T^2)\,\gamma_5\,\not S_T + h_{1T}^\perp(x,p_T^2)\,\frac{p_T\cdot S_T}{M}\,\frac{\gamma_5\,\not p_T}{M} \right\} \not n_+$$

$$\Phi_{LL}(x,p_T) = \ldots$$

Interpretation

unpolarized quark distribution

need p_T

T-odd

helicity or chirality distribution

need p_T

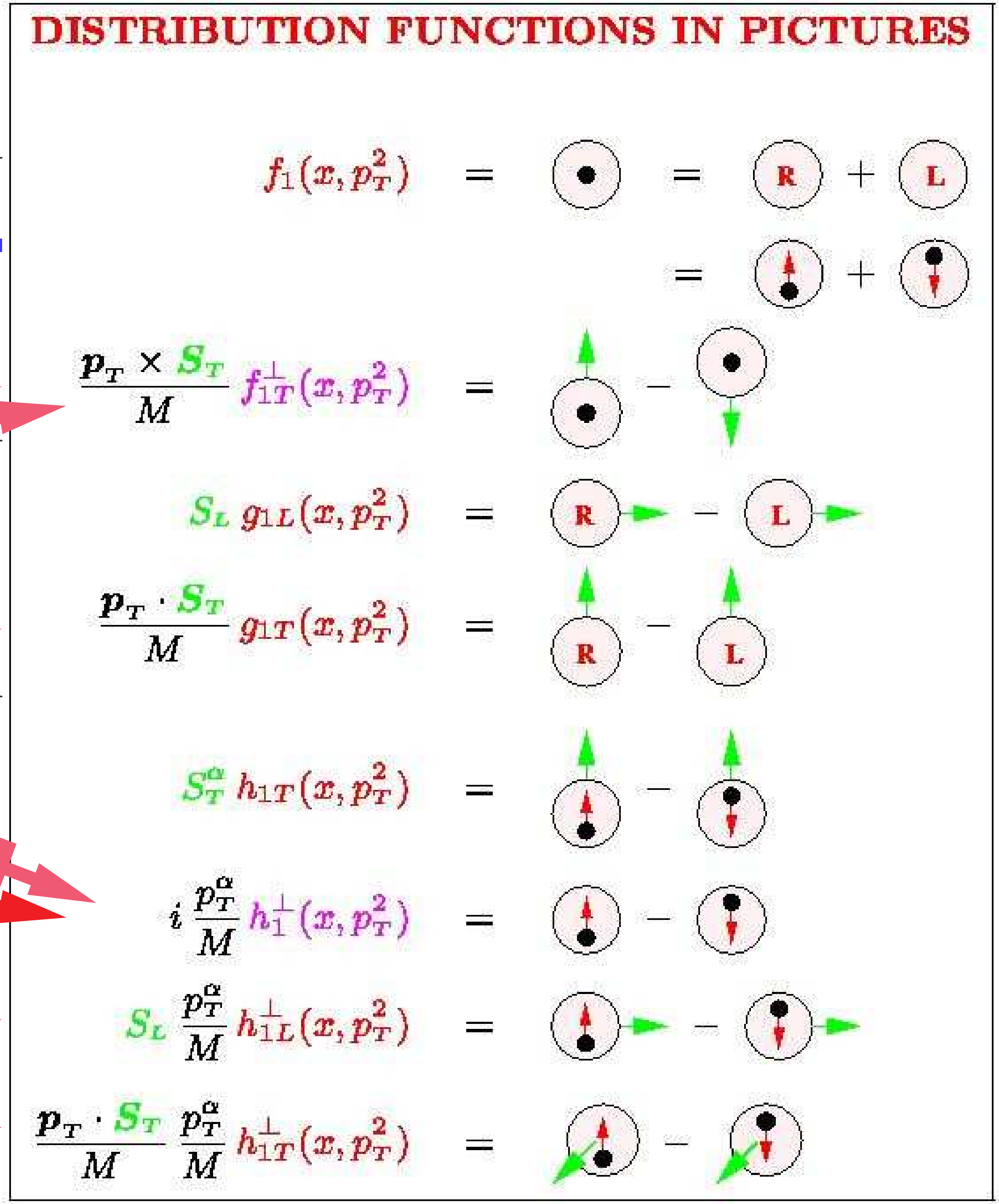
T-odd

need p_T

transverse spin distr. or transversity

need p_T

need p_T



Matrix representation for M = $[\Phi^{[\pm]}(x,p_T)\gamma^+]^T$

§ p_T-dependent functions

MATRIX REPRESENTATION FOR SPIN 1/2

p_T-dependent quark distributions:

$$
\begin{array}{cccc}
\mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L}
\end{array}
$$

$$
\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}
$$

T-odd: g_{1T} à $g_{1T} - i\, f_{1T}^{\perp}$ and $h_{1L}^{\perp}$ à $h_{1L}^{\perp} + i\, h_1^{\perp}$ (imaginary parts)

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712

T-odd ↔ single spin asymmetry

- $W_{\mu\nu}(q;P,S;P_h,S_h) = -W_{\nu\mu}(-q;P,S;P_h,S_h)$ — symmetry structure
- $W^*_{\mu\nu}(q;P,S;P_h,S_h) = W_{\nu\mu}(q;P,S;P_h,S_h)$ — hermiticity
- $W_{\mu\nu}(q;P,S;P_h,S_h) = \overline{W}_{\mu\nu}(\bar{q};\bar{P}, -\bar{S};\bar{P}_h, -\bar{S}_h)$ — parity
- $W^*_{\mu\nu}(q;P,S;P_h,S_h) = \overline{W}_{\mu\nu}(\bar{q};\bar{P},\bar{S};\bar{P}_h,\bar{S}_h)$ — time reversal

- with time reversal constraint only even-spin asymmetries
- the time reversal constraint cannot be applied in DY or in ≥ 1-particle inclusive DIS or e^+e^-
- In those cases single spin asymmetries can be used to measure T-odd quantities (such as T-odd distribution or fragmentation functions)

Lepto-production of pions

MATRIX REPRESENTATION FOR SPIN 0

p_T-dependent quark fragmentation functions:

$$M^{(\mathrm{dec})} = \begin{array}{cc} \left[\begin{array}{cc} D_1 & i\,\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^{\perp} \\ -i\,\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^{\perp} & D_1 \end{array}\right] & \begin{array}{c} \mathrm{R} \\ \mathrm{L} \end{array} \\ \begin{array}{cc} \mathrm{R} & \mathrm{L} \end{array} & \end{array}$$

SIDIS: $\ell + H^{\uparrow} \to \ell + h + X$

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell - \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|S_T|\left(1 - y + \frac{1}{2}\,y^2\right)\sum_{a,\bar{a}} e_a^2\,x_B\,f_{1T}^{\perp(1)a}(x_B)\,D_1^a(z_h)$$

$$\left\langle \frac{Q_T}{M_h}\,\sin(\phi_h^\ell + \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|S_T|\,2(1-y)\sum_{a,\bar{a}} e_a^2\,x_B\,h_1^a(x_B)\,H_1^{\perp(1)a}(z_h)$$

$H_1^{\perp}$ is T-odd and chiral-odd

End lecture 3

Lecture 4

Quarks

$$\Phi_{ij}^{q}(x, p_T; n, C) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}_j(0)\, U_{[0,\xi]}^{[C]}\, \psi_i(\xi) \right| P \right\rangle_{\xi.n=0}$$

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}_j(0)\, U_{[0,\xi]}^{[n]}\, \psi_i(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

- Integration over $\xi^- = \xi.P$ allows 'twist' expansion
- Gauge link essential for color gauge invariance

$$U_{[0,\xi]}^{[C]} = P \exp\left(-ig \int_0^{\xi} ds^{\mu} A_{\mu} \right)$$

- Arises from all 'leading' matrix elements containing $\psi\, A^+ \ldots A^+ \psi$

Sensitivity to intrinsic transverse momenta

$p \approx x\, P + p_T$

$k \approx z^{-1}\, K + k_T$

- In a hard process one probes partons (quarks and gluons)
- Momenta fixed by kinematics (external momenta)

  DIS $\quad x = x_B = Q^2/2P.q$

  SIDIS $\quad z = z_h = P.K_h/P.q$
- Also possible for transverse momenta

  SIDIS $\quad q_T = q + x_B P - K_h/z_h \approx -K_{h\perp}/z_h$

  $= k_T - p_T$

  2-particle inclusive hadron-hadron scattering

  $q_T = K_1/z_1 + K_2/z_2 - x_1 P_1 - x_2 P_2 \approx K_{1\perp}/z_1 + K_{2\perp}/z_2$

  $= p_{1T} + p_{2T} - k_{1T} - k_{2T}$
- Sensitivity for transverse momenta requires ≥ 3 momenta

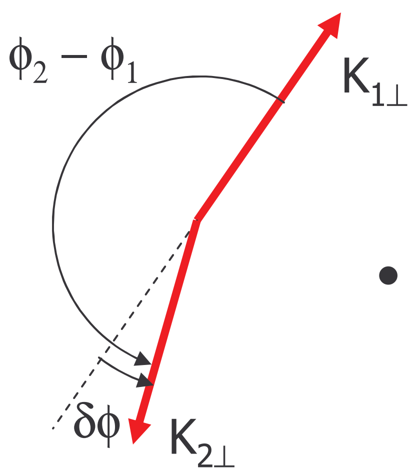
  SIDIS: $\gamma^* + H$ à $h + X$

  DY: $H_1 + H_2$ à $\gamma^* + X$

  e+e-: γ^* à $h_1 + h_2 + X$

  hadronproduction: $H_1 + H_2$ à $h_1 + h_2 + X$

  à $h + X$ (?)

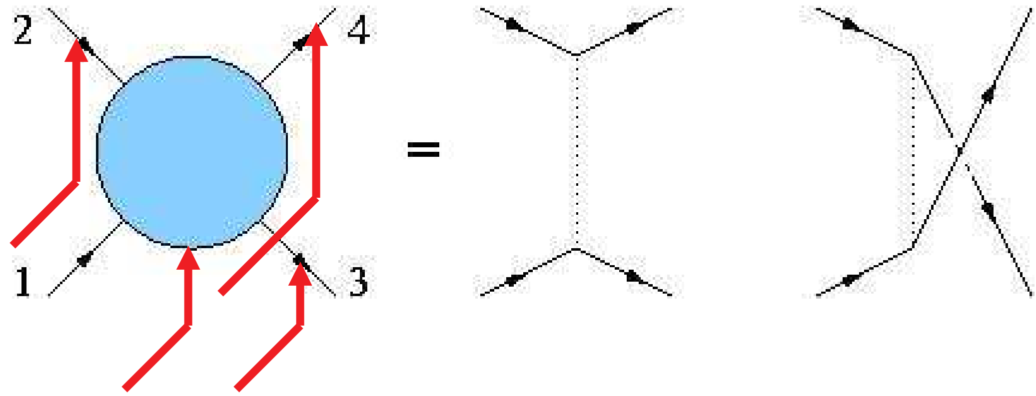


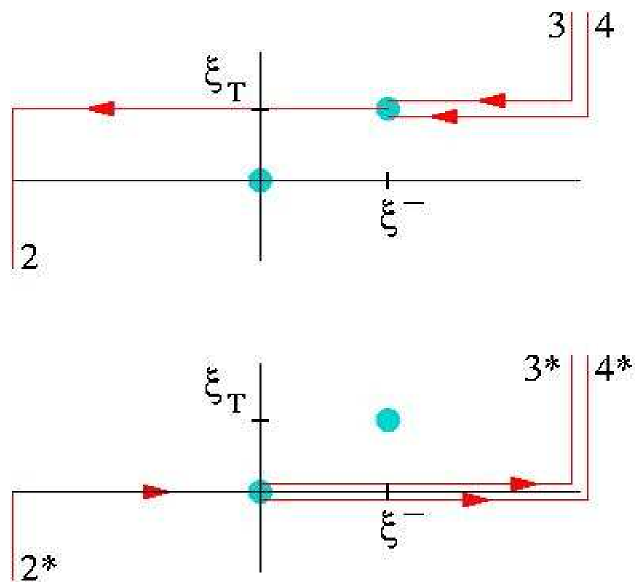
pp-scattering

Knowledge of hard process(es)!

Generic hard processes

- E.g. qq-scattering as hard subprocess
- Matrix elements involving parton 1 and additional gluon(s) $A^+ = A.n$ appear at same (leading) order in 'twist' expansion
- insertions of gluons collinear with parton 1 are possible at many places
- this leads for correlator involving parton 1 to gauge links to lightcone infinity





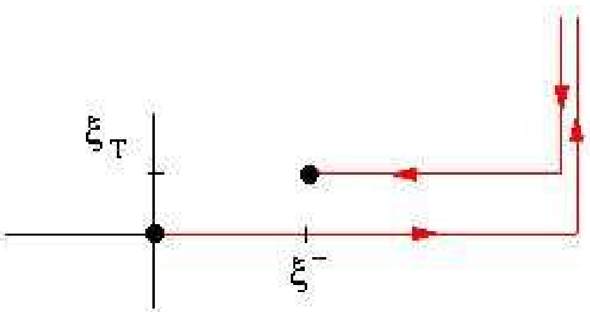
Link structure for fields in correlator 1

C. Bomhof, P.J. Mulders and F. Pijlman, PLB 596 (2004) 277 [hep-ph/0406099]; EPJ C 47 (2006) 147 [hep-ph/0601171]

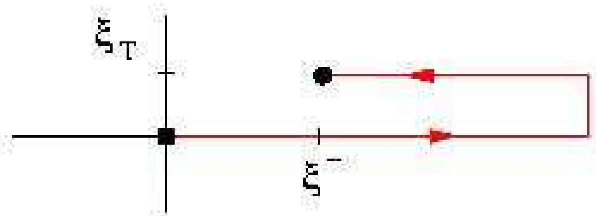
SIDIS

SIDIS à $\Phi^{[U^+]} = \Phi^{[+]}$

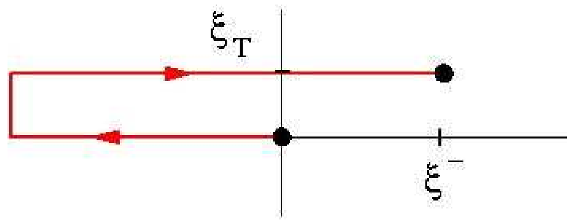
$$\Phi^{[+]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{ip\cdot\xi} \langle P,S|\overline{\psi}(0)\, U^-_{[0,\infty]} U^T_{[0_T,\infty_T]} U^T_{[\infty_T,\xi_T]} U^-_{[\infty,\xi]}\, \psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$



$\equiv$

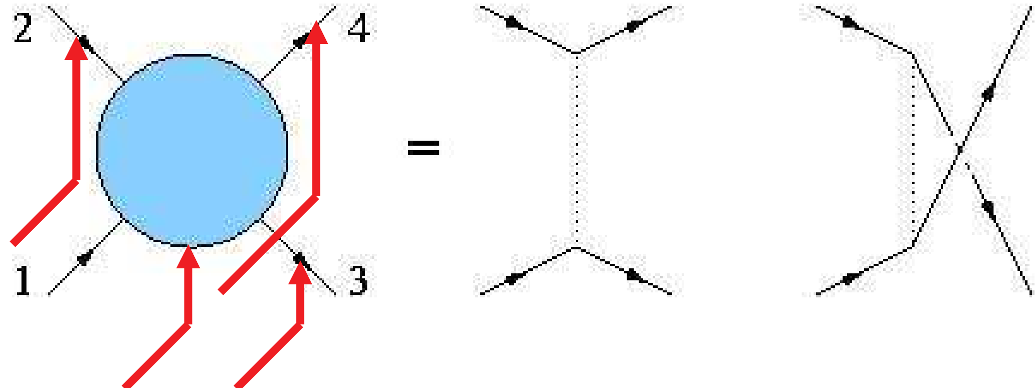


DY à $\Phi^{[U^-]} = \Phi^{[-]}$

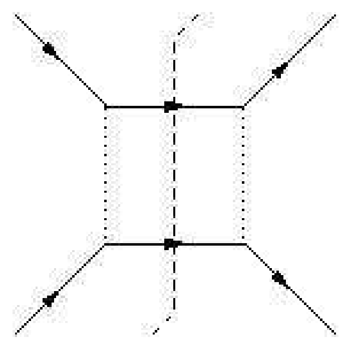


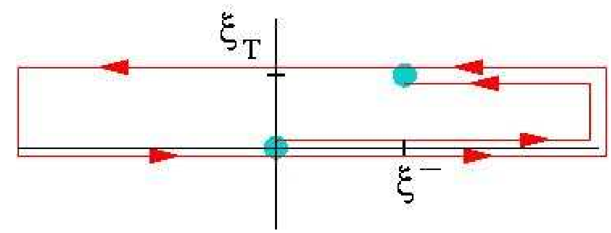
A 2 à 2 hard processes: qq à qq

- E.g. qq-scattering as hard subprocess
- The correlator $\Phi(x,p_T)$ enters for each contributing term in squared amplitude with specific link

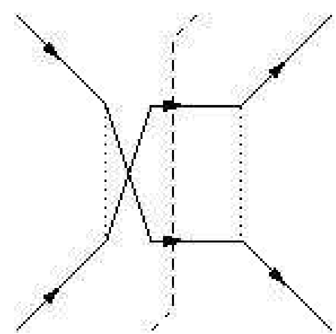


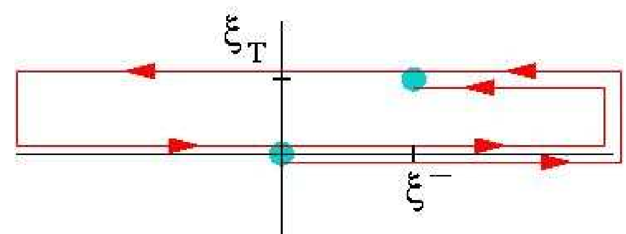
$U^{\square} = U^{+}U^{-\dagger}$





$\Phi^{[\mathrm{Tr}(U^{\square})U^{+}]}(x,p_T)$





$\Phi^{[U^{\square}U^{+}]}(x,p_T)$

Gluons

$$\Phi_g^{\alpha\beta}(x, p_T; C, C') = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \left| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \right| P \right\rangle_{\xi.n=0}$$

- Using 3x3 matrix representation for U, one finds in gluon correlator appearance of two links, possibly with different paths.
- Note that standard field displacement involves C = C'

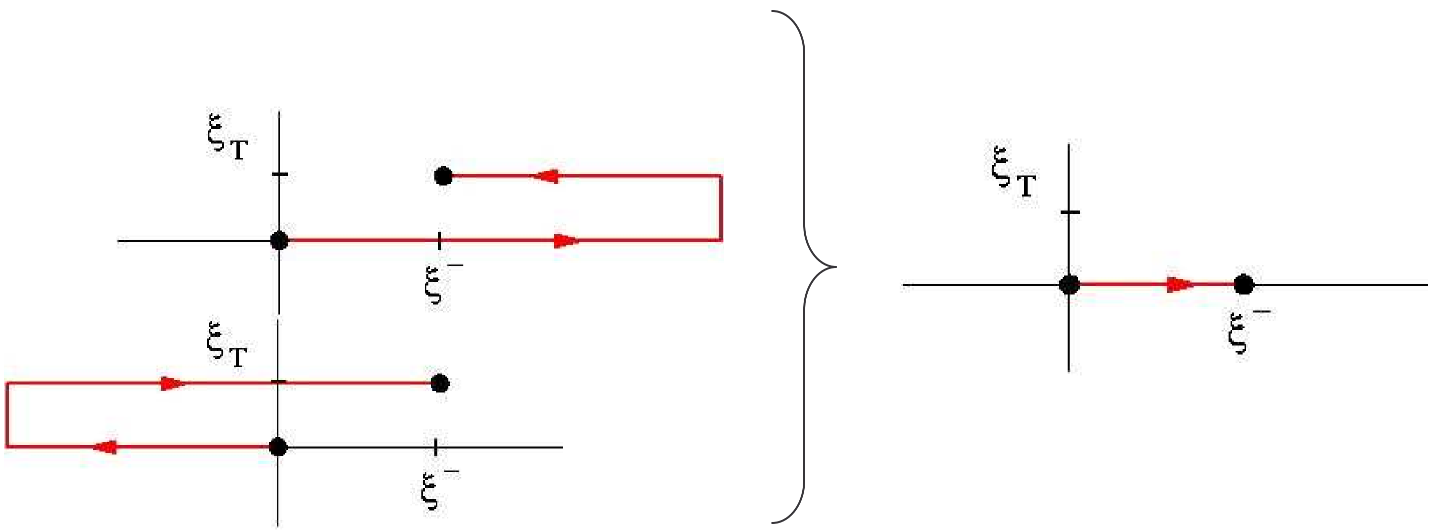
$$F^{\alpha\beta}(\xi) \rightarrow U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

Integrating $\Phi^{[\pm]}(x,p_T) \rightarrow \Phi^{[\pm]}(x)$

$$\Phi^{[\pm]}(x,p_T) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \psi^\dagger(0) U^n_{[0,\pm\infty]} U^T_{[0_T,\xi_T]} U^n_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

collinear correlator

$$\Phi^{\cancel{[\pm]}}(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \psi^\dagger(0) U^n_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$



Integrating $\Phi^{[\pm]}(x,p_T) \rightarrow \Phi_\partial^{\alpha[\pm]}(x)$

transverse moment

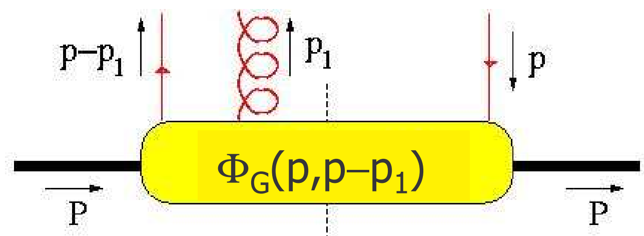
$$\Phi_\partial^{\alpha[\pm]}(x) = \int d^2p_T\; p_T^\alpha\, \Phi^{[\pm]}(x,p_T)$$

$$\Phi_\partial^{\alpha[\pm]}(x) = \int d^2p_T \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \psi^\dagger(0)\, U^n_{[0,\pm\infty]}\, i\partial_T^\alpha\, U^T_{[0_T,\xi_T]}\, U^n_{[\pm\infty,\xi]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$= \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \Big[\left\langle P \middle| \psi^\dagger(0)\, U^n_{[0,\xi]}\, iD_T^\alpha\, \psi(\xi) \middle| P \right\rangle$$

$$- \left\langle P \middle| \overline{\psi}(0)\, U^n_{[0,\pm\infty]} \int_{\pm\infty}^{\xi} d(\eta.P)\, U^n_{[\pm\infty,\eta]}\, gG^{n\alpha}(\eta)\, U^n_{[\eta,\xi]}\, \psi(\xi) \middle| P \right\rangle \Big]_{LC}$$

$$= \Phi_D^\alpha(x) + \int dx_1\, \frac{i}{x_1 \mp i\varepsilon}\, \Phi_G^\alpha(x, x - x_1)$$



$$= \underbrace{\Phi_D^\alpha(x) + \int dx_1\, P\frac{i}{x_1}\, \Phi_G^\alpha(x, x - x_1)}_{\Phi_\partial^\alpha(x)} \pm \pi \Phi_G^\alpha(x,x)$$

T-even

T-odd

Gluonic poles

- Thus: $\Phi^{[U]}(x) = \Phi(x)$

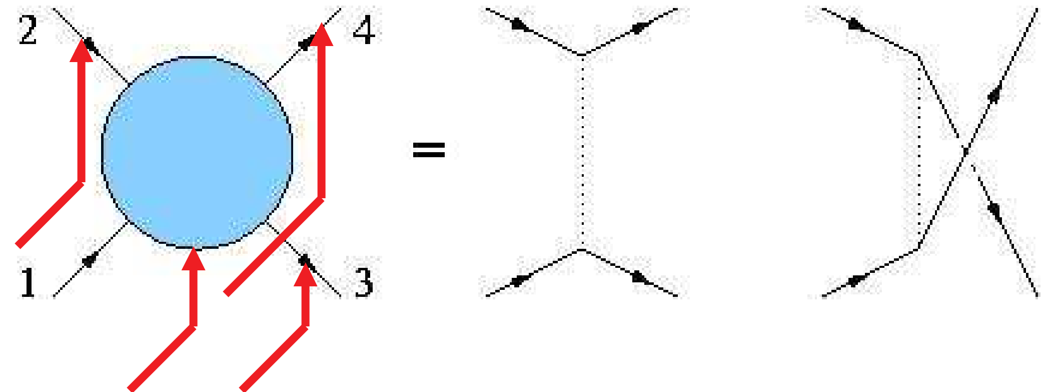
$$\Phi_\partial^{[U]\alpha}(x) = \tilde{\Phi}_\partial^{\alpha}(x) + C_G^{[U]}\, \pi\Phi_G^{\alpha}(x,x)$$

- Universal gluonic pole m.e. (T-odd for distributions)
- $\pi\Phi_G(x)$ contains the weighted T-odd functions $h_1^{\perp(1)}(x)$ [Boer-Mulders] and (for transversely polarized hadrons) the function $f_{1T}^{\perp(1)}(x)$ [Sivers]
- $\tilde{\Phi}_\partial(x)$ contains the T-even functions $h_{1L}^{\perp(1)}(x)$ and $g_{1T}^{\perp(1)}(x)$
- For SIDIS/DY links: $C_G^{[U\pm]} = \pm 1$
- In other hard processes one encounters different factors: $C_G^{[U^\square\, U^+]} = 3$, $C_G^{[Tr(U^\square)U^+]} = N_c$

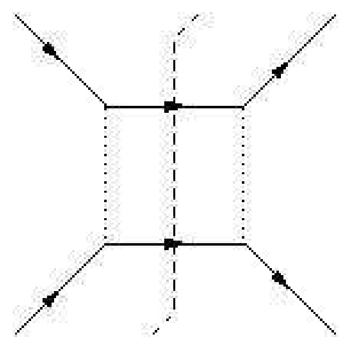
Efremov and Teryaev 1982; Qiu and Sterman 1991
Boer, Mulders, Pijlman, NPB 667 (2003) 201

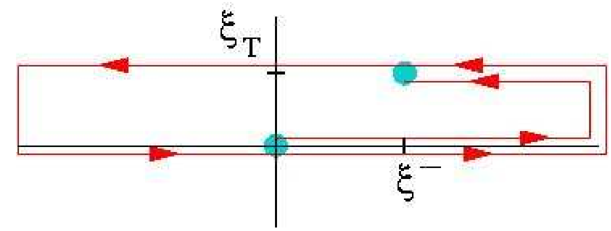
A 2 à 2 hard processes: qq à qq

- E.g. qq-scattering as hard subprocess
- The correlator $\Phi(x,p_T)$ enters for each contributing term in squared amplitude with specific link



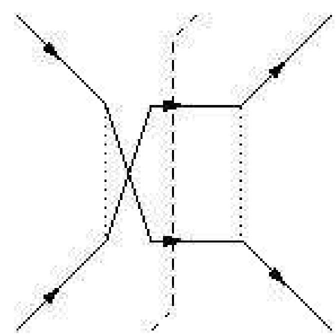
$U^{\square} = U^{+}U^{-\dagger}$

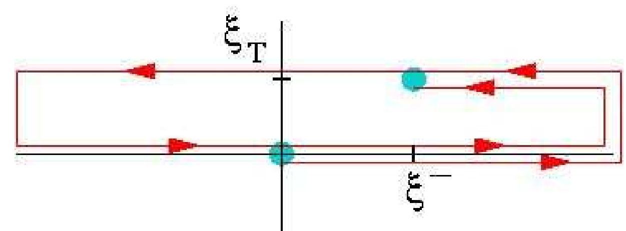




$\Phi^{[Tr(U^{\square})U^{+}]}(x,p_T)$

$\Phi^{[U^{\square}U^{+}]}(x,p_T)$



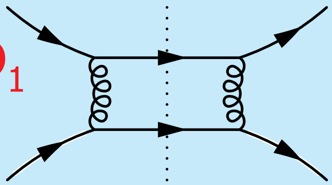


Bacchetta, Bomhof, Pijlman, Mulders, PRD 72 (2005) 034030; hep-ph/0505268

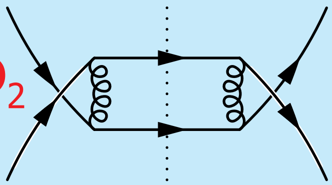
examples: qqà qq in pp

$$\frac{Tr(U^{\Box})}{N_c}U^{+} \qquad U^{\Box}U^{+}$$

D1



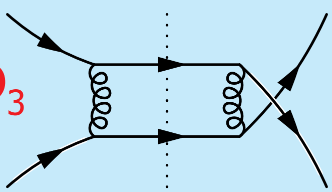
D2



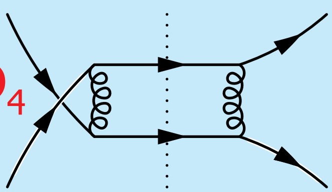
$$\Phi_q = \frac{N_c^2+1}{N_c^2-1}\Phi^{[(\Box)+]} - \frac{2}{N_c^2-1}\Phi^{[\Box+]} \rightarrow \tilde{\Phi}_{\partial} + \frac{N_c^2-5}{N_c^2-1}\pi\Phi_G$$

$C_G^{[D_1]} = C_G^{[D_2]}$

D3



D4

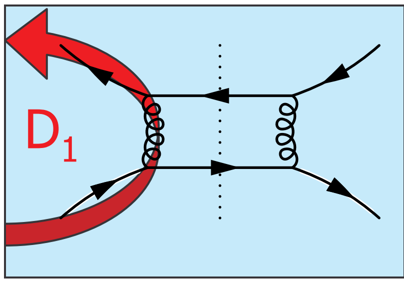


$$\Phi_q = \frac{2N_c^2}{N_c^2-1}\Phi^{[(\Box)+]} - \frac{N_c^2+1}{N_c^2-1}\Phi^{[\Box+]} \rightarrow \tilde{\Phi}_{\partial} - \frac{N_c^2+3}{N_c^2-1}\pi\Phi_G$$

$C_G^{[D_3]} = C_G^{[D_4]}$

Bacchetta, Bomhof, D'Alesio,Bomhof, Mulders, Murgia, hep-ph/0703153

examples: $q\bar{q} \to q\bar{q}$ in pp

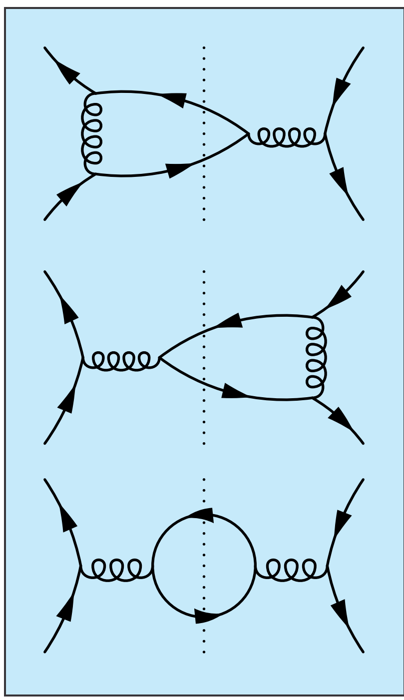


$$\Phi_q = \frac{1}{N_c^2-1}\Phi^{[(\Box^\dagger)+]} + \frac{N_c^2-2}{N_c^2-1}\Phi^{[-]} \quad \to \quad \tilde{\Phi}_\partial - \frac{N_c^2-3}{N_c^2-1}\pi\Phi_G$$

For $N_c \to \infty$:

$C_G^{[D_1]} \longrightarrow -1$

(color flow as DY)



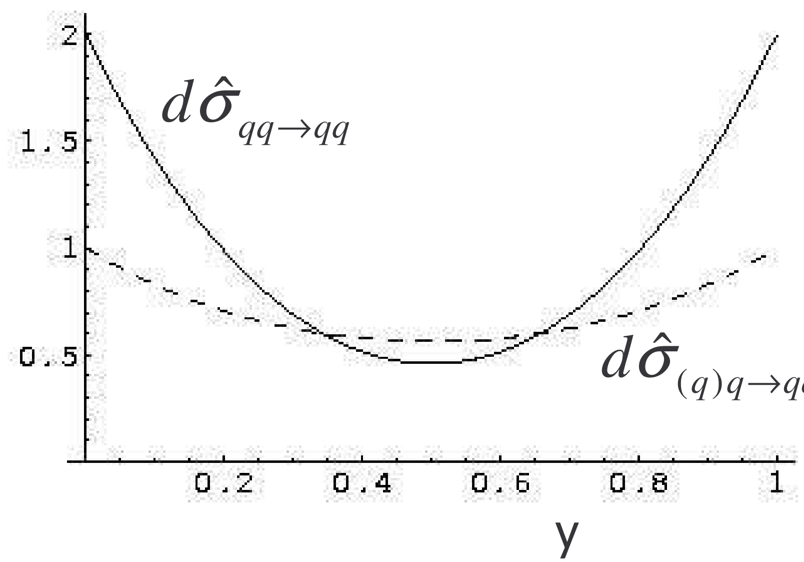
$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box^\dagger)+]} - \frac{1}{N_c^2-1}\Phi^{[-]} \quad \to \quad \tilde{\Phi}_\partial + \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

Gluonic pole cross sections

- In order to absorb the factors $C_G^{[U]}$, one can define specific hard cross sections for gluonic poles (which will appear with the functions in transverse moments)
- for pp: $$\hat{\sigma}_{qq\to qq} = \sum_{[D]} \hat{\sigma}^{[D]} \qquad \hat{\sigma}_{[q]q\to qq} = \sum_{[D]} C_G^{[U(D)]} \hat{\sigma}^{[D]}$$

  (gluonic pole cross section)

  etc.



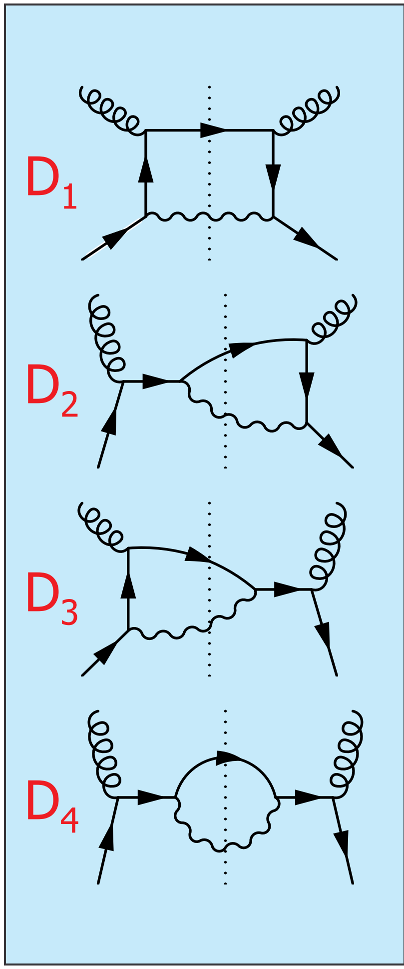
- for SIDIS: $$\hat{\sigma}_{\ell[q]\to\ell q} = +\hat{\sigma}_{\ell q\to\ell q}$$

  for DY: $$\hat{\sigma}_{[q]\bar{q}\to\ell\bar{\ell}} = -\hat{\sigma}_{q\bar{q}\to\ell\bar{\ell}}$$

Bomhof, Mulders, JHEP 0702 (2007) 029 [hep-ph/0609206]

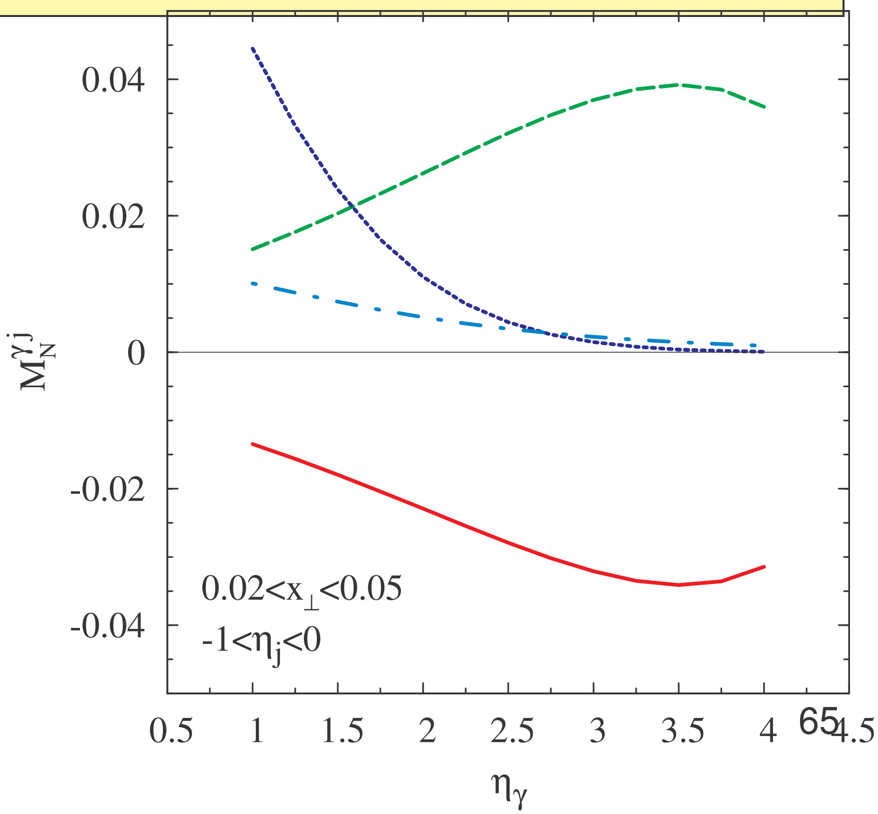
examples: qgà qγ in pp

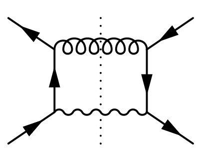
Transverse momentum dependent | collinear



$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\square)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad\rightarrow\quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

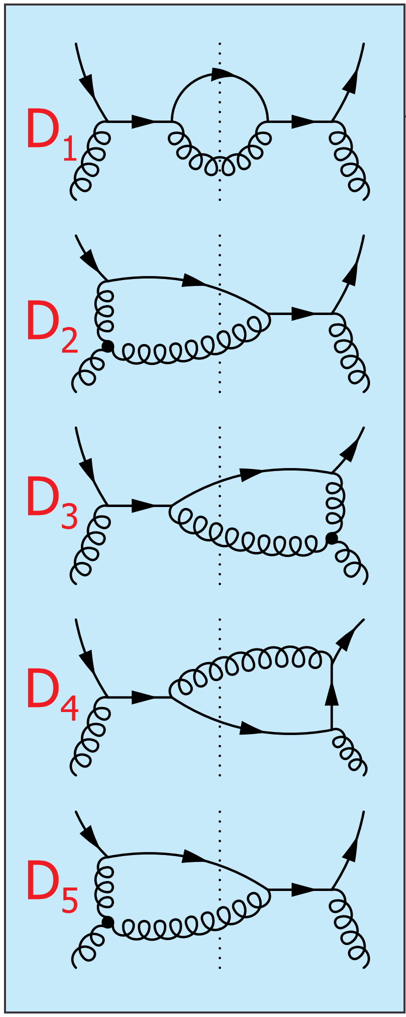
Only one factor, but more DY-like than SIDIS



Note: also  etc.

e.g. relevant in Bomhof, Mulders, Vogelsang, Yuan, PRD 75 (2007) 074019

examples: qgà qg



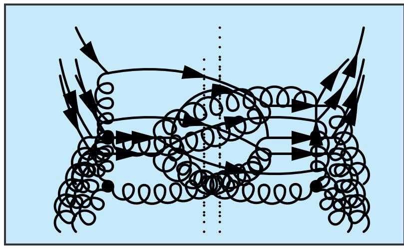
Transverse momentum dependent | collinear

$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\square)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad \rightarrow \quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

examples: qgà qg

Transverse momentum dependent

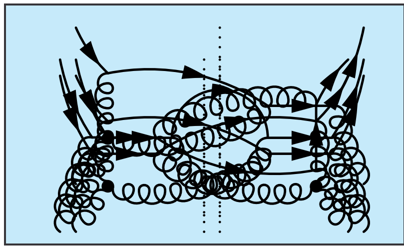
collinear



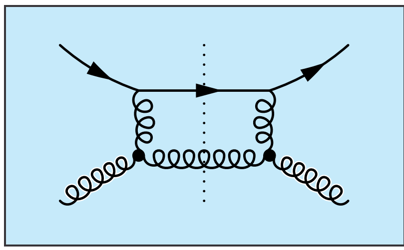
$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\square)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad \rightarrow \quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

examples: qgà qg

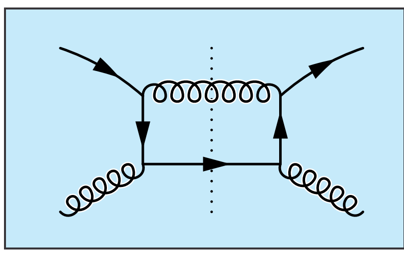
Transverse momentum dependent | collinear



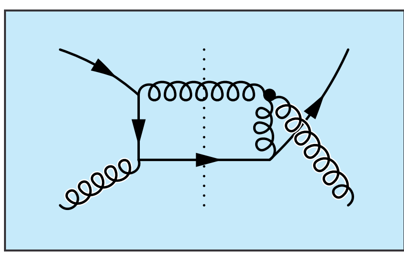
$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad\to\quad \tilde{\Phi}_\partial - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$



$$\Phi_q = \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)(\Box^\dagger)+]} + \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad\to\quad \tilde{\Phi}_\partial - \frac{1}{N_c^2-1}\pi\Phi_G$$



$$\Phi_q = \frac{N_c^4}{(N_c^2-1)^2}\Phi^{[(\Box)(\Box^\dagger)+]} - \frac{N_c^2}{(N_c^2-1)^2}\Phi^{[(\Box)-]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad\to\quad \tilde{\Phi}_\partial + \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$$

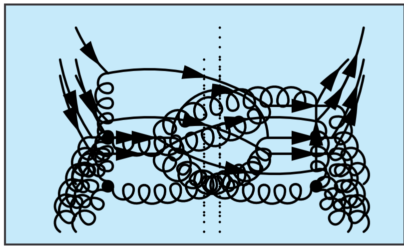


$$\Phi_q = \frac{N_c^2}{N_c^2-1}\Phi^{[(\Box)(\Box^\dagger)+]} - \frac{1}{N_c^2-1}\Phi^{[+]} \quad\to\quad \tilde{\Phi}_\partial + \pi\Phi_G$$

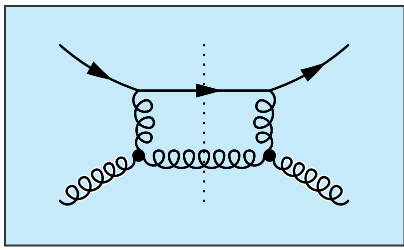
examples: qgà qg

Transverse momentum dependent

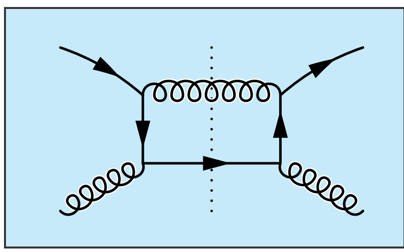
collinear



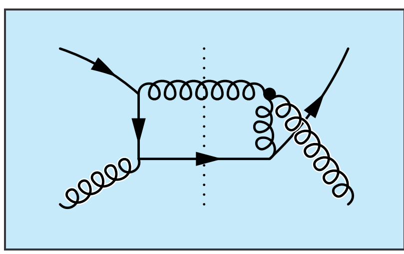
$\rightarrow \quad \tilde{\Phi}_{\partial} - \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$



$\rightarrow \quad \tilde{\Phi}_{\partial} - \frac{1}{N_c^2-1}\pi\Phi_G$



$\rightarrow \quad \tilde{\Phi}_{\partial} + \frac{N_c^2+1}{N_c^2-1}\pi\Phi_G$



$\rightarrow \quad \tilde{\Phi}_{\partial} + \pi\Phi_G$

It is also possible to group the TMD functions in a smart way into two!
(nontrivial for nine diagrams/four color-flow possibilities)

$$\tilde{\Phi}_{\partial} = \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\square)(\square^\dagger)+]} + \frac{N_c^2}{2(N_c^2-1)}\Phi^{[(\square)-]} - \frac{1}{N_c^2-1}\Phi^{[+]}$$

$$\pi\Phi_G = \frac{N_c^2}{N_c^2-1}\Phi^{[(\square)(\square^\dagger)+]} - \frac{1}{N_c^2-1}\Phi^{[+]}$$

But still no factorization!

‘Residual’ TMDs

- We find that we can work with basic TMD functions $\Phi^{[\pm]}(x,p_T)$ + ‘junk’
- The ‘junk’ constitutes process-dependent residual TMDs

$$\Phi^{[(\Box)(\Box^\dagger)+]}(x,p_T) = \Phi^{[+]} + \underbrace{\left[\Phi^{[(\Box)(\Box^\dagger)+]}(x,p_T) - \Phi^{[+]}(x,p_T)\right]}_{\Phi_{\text{int}}^{[(\Box)(\Box^\dagger)+]}(x,p_T)}$$

$$\Phi^{[\Box+]}(x,p_T) = 2\Phi^{[+]}(x,p_T) - \Phi^{[-]}(x,p_T) + \Phi_{\text{int}}^{[\Box+]}(x,p_T)$$

no definite T-behavior

definite T-behavior

- The residuals satisfies $\Phi_{\text{int}\,\partial}(x) = 0$ and $\pi\Phi_{\text{int}\,G}(x,x) = 0$, i.e. cancelling k_T contributions; moreover they most likely disappear for large k_T

Conclusions

- Appearance of single spin asymmetries in hard processes is calculable
- For integrated and weighted functions factorization is possible
- For TMDs one cannot factorize cross sections, introducing besides the normal ‘partonic cross sections’ some ‘gluonic pole cross sections’
- Opportunities: the breaking of universality can be made explicit and be attributed to specific matrix elements

Related:
Qiu, Vogelsang, Yuan, hep-ph/0704.1153
Collins, Qiu, hep-ph/0705.2141
Qiu, Vogelsang, Yan, hep-ph/0706.1196
Meissner, Metz, Goeke, hep-ph/0703176