

E Exam questions 2011

Inleveren uiterlijk op woensdagochtend 26 oktober 2011 in T224, T238 of postvak Mulders

Exam exercise 1

In classical mechanics total energy, total momentum and total angular momentum are conserved quantities.

- What is the underlying principle to find conserved quantities?
- What are the properties of the corresponding operators in quantum mechanics?
- Are there more conserved quantities than the ones mentioned above? Discuss the issue considering both classical and quantum mechanics.

(Limit your answer/discussion to at most half a page.)

Exam exercise 2

Consider the three-nucleon atomic nuclei ${}^3\text{H}$ (one proton, two neutrons) and ${}^3\text{He}$ (two protons, one neutron). In the groundstate the wave functions of protons and neutrons presumably are (dominantly) s-waves.

- Construct the (spin part of the) wave function of these nuclei in terms of proton and neutron spin states, $|p \uparrow\rangle$, $|p \downarrow\rangle$, $|n \uparrow\rangle$ and $|n \downarrow\rangle$. You can best start with writing down the spin wave function for the two protons or two neutrons and then add the third nucleon.
- The magnetic moment of the nucleons is given by

$$\boldsymbol{\mu}_p = g_p \frac{e}{2M} \mathbf{s}_p \quad \text{and} \quad \boldsymbol{\mu}_n = g_n \frac{e}{2M} \mathbf{s}_n,$$

where M is the nucleon mass, \mathbf{s}_p and \mathbf{s}_n are the nucleon spins and $g_p \approx 5.6$ and $g_n \approx -3.8$. The magnetic moment of the nucleus is found as the sum of the nucleon magnetic moments,

$$\boldsymbol{\mu} = \sum_i g_i \frac{e}{2M} \mathbf{s}_i.$$

What do you find for ${}^3\text{He}$ and ${}^3\text{H}$ in terms of the total spin \mathbf{S} .

Exam exercise 3

We now go one level down and consider the three-quark description for protons p (two up quarks, one down quark) and n (two down quarks, one up quark). Again for the groundstate the wave functions of the quarks presumably are (dominantly) s-waves. An important difference is that there is an additional degree of freedom, namely the color of the quarks. The color part of the wave function is a singlet wave function (being completely antisymmetric),

$$|\text{color}\rangle = \frac{1}{\sqrt{6}} (|rgb\rangle - |grb\rangle + |gbr\rangle - |rbg\rangle + |brg\rangle - |bgr\rangle).$$

- (a) Construct the (spin part of the) wave function of the nucleons, that have spin 1/2, in terms of quark spin states, $|u \uparrow\rangle$, $|u \downarrow\rangle$, $|d \uparrow\rangle$ and $|d \downarrow\rangle$. You can best start with writing down the spin wave functions for the two up quarks or two down quarks, respectively and then add the third quark (using appropriate Clebsch-Gordan coefficients).
- (b) The magnetic moment of the quarks (being elementary fermions with g -factors being 2) are given by

$$\boldsymbol{\mu}_q = g \frac{e_q}{2m_q} \mathbf{s}_q = \frac{e_q}{m_q} \mathbf{s}_q,$$

where m_q is a quark (constituent) mass, which we as a first guess take to be $m_u = m_d = M/3$. The charges are $e_u = +2/3 e$ and $e_d = -1/3 e$. What do you find for the nucleon magnetic moments in this description, i.e. calculate g_p and g_n in

$$\boldsymbol{\mu}_N = g_N \frac{e}{2M} \mathbf{s}_N = \sum_q \frac{e_q}{m} \mathbf{s}_q.$$

Exam exercise 4

Look at the exact solution of a two-level problem where the unperturbed Hamiltonian H_0 has eigenvalues $E_1^{(0)}$ and $E_2^{(0)}$ and the perturbation is of the form

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}.$$

Show in the limit that the matrix elements of V are much smaller than the difference of the unperturbed energies that the results for first and second order perturbation theory arise after expanding the exact results up to the appropriate order.

Exam exercise 5

The form factor of a proton is approximately given by

$$F(\mathbf{q}) = \frac{1}{(1 + \hbar^2 \mathbf{q}^2 / \Lambda^2)^2},$$

with $\Lambda^2 \approx 0.71 \text{ GeV}^2/c^2$. What is the form of the charge distribution of a proton and what is its size (root mean square radius of the charge distribution).

(Note $\hbar c \approx 0.2 \text{ GeV fm}$, $1 \text{ fm} = 10^{-15} \text{ m}$).

Exam exercise 6

One can exactly solve the Klein-Gordon equation for stationary solutions with energy E in a central electric field. The solutions are of the form

$$\psi(\mathbf{r}, t) = \frac{u(r)}{r} Y_\ell^m(\theta, \varphi) e^{-iEt/\hbar},$$

Show that they satisfy

$$\left(E + \frac{Z\alpha\hbar c}{r} \right)^2 u(r) = \left(\hbar^2 c^2 \left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} \right) + m^2 c^4 \right) u(r). \quad (520)$$

By mapping this equation into a 'generalized Laguerre polynomial' form show that one finds solutions with energies

$$E_{n\ell} = \frac{mc^2}{\sqrt{1 + \frac{Z^2\alpha^2}{(n-\delta_\ell)^2}}}, \quad (521)$$

where $\delta_\ell = \ell + 1/2 - \sqrt{(\ell + 1/2)^2 - Z^2\alpha^2}$.