Course Manual

Advanced Quantum Mechanics

D. Boer and P.J. Mulders

Faculteit der Exacte Wetenschappen
Afdeling Natuurkunde en Sterrenkunde

Spring 2010

General information

Course:	Advanced Quantum Mechanics
course code:	420003
credit points:	6 ECTS
period:	4 (February-May 2010)
form:	lectures and tutorials
Lecturer:	D. Boer
E-mail:	dboer@kvi.nl
Lecturer:	P.J. Mulders
E-mail:	mulders@few.vu.nl
Office phone:	020 598 7863
Office location:	T2.22 in Building 1081

Introduction

This course offers advanced quantum-mechanical principles, techniques and methods that do not belong to the standard introductory topics taught at the bachelors level at the VU. The topics are selected on the basis of their relevance for a broad audience of physics and chemistry students intending to specialize in laser science, spectroscopy, advanced optical physics, condensed matter physics, biophysics and high-energy physics. Depending on participants, we may slightly deviate from the topics as mentioned in this manual.

The topics to be covered can be divided into four subjects:

1. angular momentum
2. systems of (many) identical particles
3. time dependent problems
4. relativistic quantum physics

Goals of the course

During the course the student will:

1. further develop the skills in performing quantum mechanical calculations.
2. learn about the role of symmetry in quantum mechanical calculations.
3. gain an understanding of the theoretical considerations that lead to the introduction of quantum fields.

In particular, at the end of the course the student will:

1. be able to perform calculations for systems of coupled angular momenta
2. be able to calculate energy levels of multi-electron atoms using various approximation methods.
3. have learned how to deal with the interaction of an electromagnetic field with atoms
4. be able to derive the relativistic corrections to the hydrogen atom Hamiltonian from the spin-1/2 relativistic quantum field equation, called the Dirac equation

Entry requirements

The course serves as an optional course for Bachelor students at the second half of their third year. It is intended to supplement the compulsory Quantum Mechanics course(s) and (in part) to provide a theoretical basis for results treated in "Struktuur der Materie: Atoom en Molekuulfysica". It is especially of relevance to students planning to enter the Master's programmes Theoretical Physics or Laser Sciences. In that respect, the course is also intended for students already taking a Master in physics (or chemistry), who require more background in the treated topics because of their chosen specialization.

The prerequisite knowledge is formed by the quantum mechanics courses belonging to the regular physics Bachelor curriculum at the VU ('Quantumfysica' and 'Quantum Mechanica' or 'Quantum Mechanica 1' and 'Quantum Mechanica 2'). Concerning the basic quantum mechanics knowledge, especially angular momentum and the quantum mechanical treatment of the hydrogen atom are important topics to be familiar with.

Set-up of the course

The set-up of the course of spring 2009 is as follows. The course runs for 14 weeks, consisting of 14 times 2 hours of lectures and in addition 2 hours of tutorial. The tutorial hours are exercise sessions, in which students can work on the provided exercises and discuss them collectively or individually with the assistant. A set of exercises will be handed out at the start of the course and additional exercises may be provided to the students during the course (see blackboard or webpage information).

Schedule

Approximate number of days per topic:

Angular momentum (3 days);
Systems of (many) identical particles (3 days);
Time dependent problems (3 days)
Relativistic quantum physics (5 days)

Preparation for the lectures and tutorials

Each of the four topics that is treated in the lectures will start from a very elementary level and move on quickly to an advanced level. In this way the generally quite different background levels of the students should not pose a big problem.

The course does not make use of one particular book or set of lecture notes, due to the choice and depth of topics to be covered. Instead, detailed references to various books are given. Hopefully this has as a positive side-effect that students will browse the library for literature best suited for them and to discover in passing other interesting quantum mechanical applications.

The literature lists at the end of this course manual indicate three types of references. The "Main references" are the ones that cover the mandatory topics of the course, that will all be treated in the lectures. Those topics can all be part of the exam, unless otherwise indicated. In total, there are 4 such main references (Cohen-Tannoudji Vol. II; Bransden & Joachain; Merzbacher; Bethe). The "Additional reading" references serve to facilitate the learning process. Often these references have a different level than the "Main references" and this may be of use for students who want to read about a topic at a more elementary/advanced level. The subjects indicated with "optional" are in general more advanced.

The tutorials are exercise sessions and the exercises generally follow the material covered in the lectures, but sometimes also contain new aspects not covered explicitly in the lectures. Therefore, it is important not to skip or postpone working out the exercises, as it may result in problems in understanding later lectures. At the start of each tutorial the relevant exercises will be indicated. The tutorial hour is meant for working on exercises and for asking questions about past exercises. It can also be used to request further clarification of topics covered in the lectures.

Examination

Depending on the number of participating students the exam will be either a written exam (in case of about 6 or more students) or an oral exam. The same applies to the second opportunity to take the exam (resit or 'hertentamen'). A written exam will generally consist of exercises similar to (but shorter than) the ones of the tutorials and can deal with any of the four main topics of this course: "angular momentum", "systems of (many) identical particles", "time dependent problems" and "relativistic quantum physics". The questions will deal only with the mandatory topics, and hence will be restricted to the subjects as indicated for the main references (or to a subset of those topics in case of time constraints). In general, if a student has attended all lectures and successfully made all exercises, then sufficient knowledge and insight should have been acquired to be able to pass the exam without problems.

Brief descriptions of the contents of the lectures

Angular momentum

Part 1:

- Motivation for addition of angular momentum
- Review of general theory of angular momentum
- Intrinsic and orbital angular momentum
- Properties of $|j, m\rangle$ space
- Addition of angular momenta
- Construction of $|j, m\rangle$ states from product states $|j_1, m_1\rangle \otimes |j_2, m_2\rangle$
- Definition of Clebsch-Gordan coefficients (CGC)
- Properties of CGC: triangular rule, orthogonality relations, recurrence relations
- Conventions of CGC: choice of phases; $3j$-symbols
- Addition of spherical harmonics

Part 2:

- Wigner-Eckart theorem for scalar and vector operators; projection theorem
- Applications: Zeeman splitting, calculation of Landé factor g_J, hyperfine structure, Stark effect.
- The role of the rotation group.

Literature:

- Cohen-Tannoudji, Quantum Mechanics Vol. II, Chapters: X; A_X–D_X
- Mandl, Quantum Mechanics, Chapters: 5; 7.5
- Bransden & Joachain, Quantum Mechanics, Chapters: 6.5–6.10; 12.3

Systems of (many) identical particles

Part 1:

- Introduction systems of many identical particles
- Exchange degeneracy and problems related to probability
- Spin and statistics
- Symmetrization postulate; permutation operator; Slater determinant
- Construction rules; physical state space; occupation number
- Direct and exchange processes
- Anyons

Part 2:

- Role of symmetrization postulate in spectrum of He
- Ground state in perturbation theory and variational method
- Excited states; role of exchange term; partial lifting of degeneracy

Part 3:

- Many electron atoms
- Considerations on inapplicability of perturbation theory
- Central field approximation; self-consistent field method
- Hartree and Hartree-Fock (HF) equations
- Koopman's theorem
- Corrections to HF: correlations; spin-orbit versus jj coupling

Literature:

- Cohen-Tannoudji, Quantum Mechanics Vol. II, Chapters: XIV; A_{XIV}; B_{XIV}
- Bethe, Intermediate Quantum Mechanics, Chapter 6
- Bransden & Joachain, Quantum Mechanics, Chapters: 10.1–10.5
- Mandl, Quantum Mechanics, Chapters: 7; 8
- Chaichian & Hagedorn, Symmetries in Quantum Mechanics, Chapter 9

Time dependent problems

Part 1:

- Evolution operator
- Schrödinger, Heisenberg & interaction pictures
- Time-dependent perturbation theory
- Periodic perturbations
- Rabi flopping frequency
- Magnetic resonance
- The adiabatic approximation
- Berry phase
- The sudden approximation

Part 2:

- Interactions of quantum systems with radiation
- Dipole approximation
- Absorption, stimulated emission, spontaneous emission
- Einstein coefficients
- Line intensities, widths & shapes

Part 3:

- Forced harmonic oscillator
- Coherent states
- Quantization of the electromagnetic field
- Photon distributions

Literature:

- Bransden & Joachain, Quantum Mechanics, Chapters: 9, 11 & 12.4
- Merzbacher, Quantum Mechanics, Chapters 10.7, 14 & 19

Relativistic quantum physics

Part 1:

- Klein-Gordon equation (KGE); charge & current densities; continuity equation
- Negative energy solutions; antiparticles
- Localization problems; from wave functions to fields
- Dirac equation (DE); continuity equation

Part 2:

- Coupling to an external electromagnetic field
- Nonrelativistic limit: Pauli equation; corrections to hydrogen atom Hamiltonian

Part 3:

- Plane wave solutions of DE
- Klein's paradox and graphene
- Zitterbewegung
- Supercritical atoms
- Comments on QED: Lamb shift, $g-2$
- Comments on second quantization

Literature:

- Bethe, Intermediate Quantum Mechanics, Chapters: 16, 17, 18
- Bransden & Joachain, Quantum Mechanics, Chapter 15
- Holstein, Topics in Advanced Quantum Mechanics, Chapters: VI, VII

Detailed references to the literature

Angular Momentum - Literature

Main reference: Cohen-Tannoudji, Quantum Mechanics Vol. II, Chapter X

- Section A. Introduction [p.999-1002]
- B. Addition of 2 spin $\frac{1}{2}$'s [p.1003-1008] (relevant for exercises 1 & 3)
- C. Addition of 2 arbitrary angular momenta [p.1009-1024]
- A_X. Examples [p.1027-1034] (relevant for exercises 1 & 3)
- B_X. Clebsch-Gordan coefficients [p.1035-1042] (relevant for exercise 2)
- C_X. [Optional] Additional of spherical harmonics [p.1043-1047]
- D_X. Wigner-Eckart theorem. Theory [p.1048-1055] & application [p.1055-1058] (relevant for exercise 4)

Additional reading: Mandl, Quantum Mechanics, Chapters 5 & 7

- § 5.1–5.8 Basics of angular momentum [p.119-141]
- § 7.5 Zeeman effect [p.177-183] (not § 7.5.3)

Additional reading: Bransden & Joachain, Quantum Mechanics, Chapters 6 & 12

- § 6.5–6.10 General theory of angular momentum [p.292-322] In particular:
- § 6.9 Total angular momentum and rotations [p.312-315]
- § 6.10 Mind mistake in Table 6.3, p.320: $\sqrt{m^2}$ should be m in $\langle j_1, 1; m_1, 0 | j_1 m_1 \rangle$, see exercise 1
- § 12.3 One-electron atoms in external magnetic fields [p.574-576]

More advanced literature (generally beyond level of the course):
Chaichian & Hagedorn, Symmetries in Quantum Mechanics, Chapters 4 & 5

- § 5.3.7 [Optional] $3j$-symbols [p.147-149]
- § 5.4 [Optional] Addition of 3 angular momenta [p.159-162] (relevant for ex. 3)

Systems of (many) identical particles - Literature

Main reference 1: Cohen-Tannoudji, Quantum Mechanics Vol. II, Chapter XIV

- Section A. Introduction [p.1371-1377]
- B. Permutation operators [p.1377-1386]
- C. § 1–3 Symmetrization postulate [p.1386-1393]
- D. Discussion [p.1396-1402] (relevant for exercise 7)
- A_{XIV}.1 Many electron atoms [p.1410-1414]
- B_{XIV}. Energy levels of the He atom [p.1418-1431] (note: K=direct term, J=exchange term)

Main reference 2: Bethe, Intermediate Quantum Mechanics, Chapters 3–6

- Chapter 3. Identical particles & symmetry [p.13-21]
- 4. Two electron atoms: perturbation calculation [p.22-32] (note: J=direct term, K=exchange term)
- 5. Two electron atoms: variational calculation [p.33-40]
- 6. Self-consistent field [p.41-52] (relevant for exercise 8)

Additional reading: Mandl, Quantum Mechanics, Chapters 7 & 8

- § 7.1–7.4 Bound state perturbation theory [p.162-177] (relevant for exercise 6)
- § 8.1–8.3 Variational method [p.186-193] (relevant for exercise 6)

Additional reading: Bransden & Joachain, Quantum Mechanics, Chapter 10

- § 10.1–10.2 Systems of many particles [p.469-477]
- § 10.4–10.5 Two- and many-electron atoms [p.485-498]

Additional reading: Chaichian & Hagedorn, Symmetries in Quantum Mechanics, Chapter 9

- § 9.1–9.2 [Optional] Anyons [p.227-231]
- § 9.3–9.4 [Optional] Application: particle-flux system [p.232-237]

Time dependent problems - Literature

Main reference 1: Bransden & Joachain, Quantum Mechanics, Chapters 9, 11 & 12

- § 9.1 Time-dependent perturbation theory [p.431-435]
- § 9.2 Time-independent perturbation [p.435-443] (relevant for exercise 11)
- § 9.3 Periodic perturbation [p.443-447] (relevant for exercise 11)
- § 9.4 The adiabatic approximation (incl. the Berry phase) [p.447-458]
- § 9.5 The sudden approximation [p.458-464]
- § 11.1 Interactions of quantum systems with radiation, including the dipole approximation [p.515-522]
- § 11.2 Absorption & stimulated emission [p.522-527]
- § 11.3 Spontaneous emission & Einstein coefficients [p.527-529]
- § 11.5 Line intensities, widths & shapes [p.538-544]
- § 12.2 [Optional] Berry phase [p.566-567]
- § 12.4 Magnetic resonance [p.576-581]

Comment: Page numbers refer to the Second Edition; the same topics may also be found (except for the Berry phase) in the older version called "Introduction to Quantum Mechanics".

Main reference 2: Merzbacher, Quantum Mechanics, Chapters: 10.7, 14 & 19

- § 10.7 Coherent states [p.225-227]
- § 14.1 Evolution operator [p.315-319]
- § 14.2 Schrödinger, Heisenberg & interaction pictures [p.319-323]
- § 14.6 Forced harmonic oscillator [p.335-342]
- § 19.1–19.2 Time-dependent perturbation theory [p.482-487]
- § 19.4 The atom in a radiation field [p.491-495]
- § 19.7 Golden rule for constant transition rates [p.503-510]
- § 19.8 Exponential decay [p.510-515]

Comment: Page numbers refer to the Third Edition; the same topics may also be found in the earlier editions, but presented in a different order. For instance, in the Second Edition the relevant sections are: § 15.7, 15.9, 15.10, 18.1, 18.2, 18.4, 18.7, 18.8 & 18.9

Relativistic quantum physics - Literature

Main reference 1: Bethe, Intermediate Quantum Mechanics, Chapters 16–18

- Chapter 16. Klein-Gordon equation [p.181-184]
- 17. Dirac equation [p.187-202]
- 18. Solutions to Dirac eq. [p.203-214;p.219-220]

Main reference 2: Bransden & Joachain, Quantum Mechanics, Chapter 15

- § 15.1–15.3 Klein-Gordon and Dirac eq., incl. relativistic covariance [p.679-696]
- § 15.4 Plane wave solutions [p.696-701]
- § 15.6 Nonrelativistic limit of Dirac eq. [p.711-715] (relevant for exercise 14)
- § 15.7 Negative energy states [p.715-717]

Additional reading: Holstein, Topics in Advanced Quantum Mechanics, Chapters: VI, VII

- § VI.1 Klein-Gordon eq. [p.257-266]
- § VI.2 Klein's paradox and Zitterbewegung [optional] [p.267-272]
- § VII.-1 Dirac eq. [p.279-289]
- § VII.-2 Bilinear forms [optional] [p.289-293]
- § VII.-3 Nonrelativistic reduction [p.293-299]
- § VII.-5 Plane wave solutions [p.306-317]
- § VII.-6 Negative energy solutions and antiparticles [p.319-322]

More advanced literature Itzykson & Zuber, Quantum Field Theory, Chapter 2 [optional]

- § 2.1 Dirac eq., incl. relativistic covariance [p.45-55]
- § 2.2 Physical content (plane wave solutions, Zitterbewegung, electromagnetic coupling) [p.55-69]
- § 2.3 Hydrogen-like atoms (comprehensive overview of corrections & discussion of supercritical atoms) [p.72-84]