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Spin Physics and Transverse Structure

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ABSTRACT

Spin Physics and Transverse Structure

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Spin is a welcome complication in the study of partonic structure that has led to new insights, even if experimentally not all dust has settled, in particular on quark flavor dependence and gluon spin. At the same time it opened new questions on angular momentum and effects of transverse structure, in particular the role of the transverse momenta of partons. This provides again many theoretical and experimental challenges and hurdles. But it may also provide new tools in high-energy scattering experiments linking polarization and final state angular dependence.

Parton distribution (PDF) and fragmentation (PFF) functions

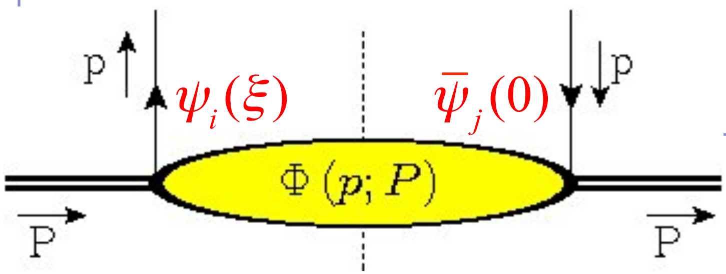
- Parton densities (PDFs) and decay functions (PFFs) are natural way of dealing with quarks/gluons in high energy processes (several PDF databases)
- PDFs can be embedded in a field theoretical framework via Operator Product Expansion (OPE), connecting Mellin moments of PDFs with particular QCD matrix elements of operators (spin and twist expansion)
- Hard process introduces necessary directionality with light-like directions, $P - M^2\, n$ and $n = P'/P.P'$ (satisfying $P.n = 1$)
- The lightcone momentum fraction x of the parton momentum $p = x\, P$ is linked to $x_B = Q^2/2P.q$ (DIS) or $x_1 = q.P_2/P_1.P_2$ and $x_2 = q.P_1/P_1.P_2$ (DY)
- Polarization, $MS = S_L\, P + MS_T - M^2\, S_L\, n$ (longitudinal or transverse) is a welcome complication, experimentally challenging but new insights emerged
- Consideration of partonic transverse momenta, $p = x\, P + p_T + (p^2\; p_T^{\,2})\, n$ opens new avenues to Transverse Momentum Dependent (TMD) PDFs, in short TMDs with experimental and theoretical challenges
- The measurement is mostly not for free, but requires dedicated final states (jets/flavor), polarized targets or polarimetry e.g. through decay orientation of final states (ρ or Λ)

Link to matrix elements → hadron correlators

- Structure of PDFs and PFFs as correlators built from nonlocal field configurations, extending on OPE and useful for interpretation, positivity bounds, modelling, …

- $$u_i(p,s)\bar{u}_j(p,s) \;\Rightarrow\; \Phi_{ij}(p\,|\,p) \;\sim \sum_X \langle P|\bar{\psi}_j(0)\,|\,X><X\,|\,\psi_i(0)|P\rangle\,\delta(p-P+P_X)$$

$$= \int \frac{d\xi}{2\pi}\, e^{i\,p.\xi} \langle P|\bar{\psi}_j(0)\,\psi_i(\xi)|P\rangle$$



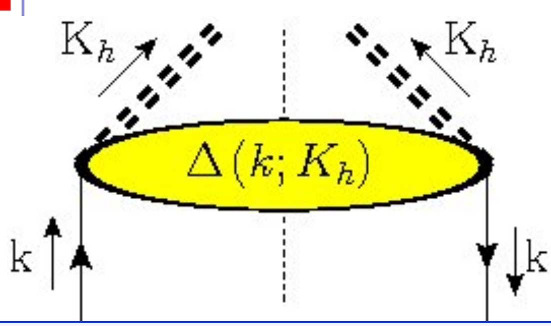
parametrized using symmetries (C, P, T)

|X><X| gateway to models: e.g. diquarks

WG6/12 Mao, Lu, Ma
AUT predictions (diquark model)

- $$u_i(k,s)\bar{u}_j(k,s) \;\Rightarrow\; \Delta_{ij}(p\,|\,p) \;\sim \sum_X \langle 0|\psi_i(0)\,|\,K_h X><K_h X\,|\,\bar{\psi}_j(0)|0\rangle\,\delta(k-K_h-K_X)$$

$$= \int \frac{d\xi}{2\pi}\, e^{i\,k.\xi} \langle 0|\psi_i(\xi)\,a_h^{\dagger} a_h\,\bar{\psi}_j(0)|0\rangle$$



no T-invariance!

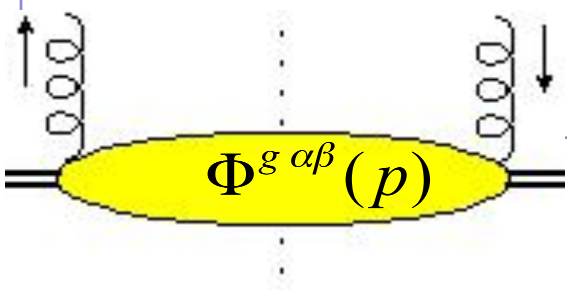
Collins & Soper, NP B 194 (1982) 445

Link to matrix elements → hadron correlators

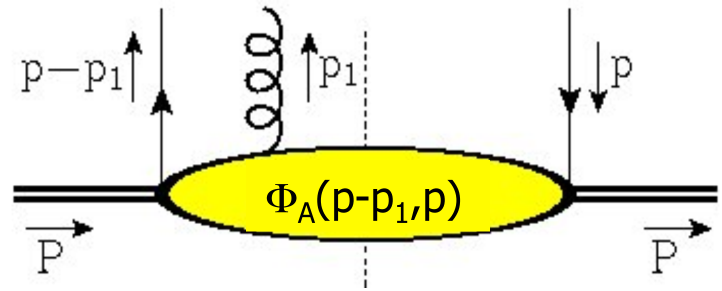
- Gluonic matrix elements

- $$\varepsilon^{\mu}(p,\lambda)\varepsilon^{\nu *}(p,\lambda) \;\Rightarrow\; \Phi^{g\,\alpha\beta}(p\,|\,p) \;\sim \sum_X \langle P | A^{\alpha}(0)\,|\,X >< X\,|\,A^{\beta}(\xi) | P \rangle \delta(p-P+P_X)$$

$$= \int \frac{d\xi}{2\pi} e^{i\,p.\xi} \langle P | F^{n\alpha}(0) F^{n\beta}(\xi) | P \rangle$$



- Also needed are multi-parton correlators



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8} e^{i(p-p_1).\xi + ip_1.\eta} \langle P | \bar{\psi}_j(0) A^{\alpha}(\eta)\, \psi_i(\xi) | P \rangle$$

- or better: $$\Phi^{\alpha}_{D;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8} e^{i(p-p_1).\xi + ip_1.\eta} \langle P | \bar{\psi}_j(0) D^{\alpha}(\eta)\, \psi_i(\xi) | P \rangle$$

$$\Phi^{\alpha}_{F;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8} e^{i(p-p_1).\xi + ip_1.\eta} \langle P | \bar{\psi}_j(0) F^{n\alpha}(\eta)\, \psi_i(\xi) | P \rangle$$

(color gauge invariance)

(Un)integrated correlators

$$\Phi(x,p_T,p.P)=\int\frac{d^4\xi}{(2\pi)^4}e^{i\,p.\xi}\left\langle P\left|\overline{\psi}(0)\,\psi(\xi)\right|P\right\rangle$$

- unintegrated

- TMD (light-front)

$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\left|\overline{\psi}(0)\,\psi(\xi)\right|P\right\rangle\Big/_{\boxed{\xi.n=\xi^+=0}}$$

- p^- = p.P integration makes time-ordering automatic.
  The soft part is simply sliced at a light-front instance

$$\Phi(x)=\int\frac{d(\xi.P)}{(2\pi)}e^{i\,p.\xi}\left\langle P\left|\overline{\psi}(0)\,\psi(\xi)\right|P\right\rangle\Big/_{\boxed{\xi.n=\xi_T=0}\ or\ \boxed{\xi^2=0}}$$

- collinear (light-cone)

- Is already equivalent to a point-like interaction

$$\Phi=\left\langle P\left|\overline{\psi}(0)\,\psi(\xi)\right|P\right\rangle\Big/_{\boxed{\xi=0}}$$

- local

- Local operators with calculable anomalous dimension

TMDs and color gauge invariance

- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

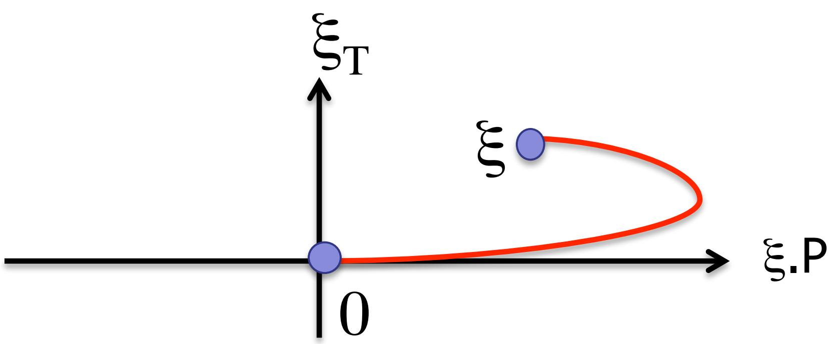
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\,\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$



Parametrization of $\Phi(x)$ → collinear PDFs

- Quarks in nucleon:

p = x P

compare

$$u(p,s)\bar{u}(p,s) = \not{p} + m$$

$$\boxed{\Phi^q(x) \;\propto\; x\, f_1^q(x)\, \not{P} + \;\ldots. \;=\; f_1^q(x)\, \not{p} + \;\ldots.}$$

unpolarized quarks

Parametrization of $\Phi(x)$ → collinear PDFs

- Quarks in nucleon:

compare

$$u(p,s)\overline{u}(p,s) = \not{p} + m$$

$$\Phi^q(x) \quad \propto \quad f_1^q(x)\,\not{p} \;+\; \ldots.$$

unpolarized quarks

- Operator connection through Mellin moments

$$x^{N-1}\,\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \left| \overline{\psi}(0)\,(\partial_\xi^n)^{N-1} U_{[0,\xi]}^{[n]}\,\psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p⁺ = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \left| \overline{\psi}(0)\,U_{[0,\xi]}^{[n]}\,(D_\xi^n)^{N-1}\,\psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$

- $\Phi^{(N)} = \int dx\, x^{N-1}\,\Phi(x) = \left\langle P \left| \overline{\psi}(0)(D^n)^{N-1}\,\psi(0) \right| P \right\rangle$

- Anomalous dimensions can be Mellin transformed into the splitting functions that govern the QCD evolution.

Collinear PDFs without polarization

- Quarks in nucleon:

p = x P

compare

$$u(p,s)\bar{u}(p,s) = \not{p} + m$$

$$\Phi^q(x) \quad \propto \quad x\, f_1^q(x)\, \not{P} \; + \; \ldots.$$

unpolarized quarks

Collinear PDFs with polarization

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T$ $\qquad 0 \le S_L^2 - S_T^2 \le 1$

$$\Phi^q(x) \;\propto\; x f_1^q(x)\,\not{P} + S_L\, x g_1^q(x)\,\not{P}\gamma_5 + x h_1^q(x)\,\not{S}_T\not{P}\gamma_5 + \ldots$$

unpolarized quarks

chiral quarks in L-polarized N

T-polarized quarks in T-polarized N

spin ←→ spin

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p} + m)(1 + \gamma_5\not{s})$$

- DIS/SIDIS programme to obtain
  - Flavor PDFs: $q(x) = f_1^q(x)$
  - Polarized PDFs: $\Delta q(x) = g_{1L}^q(x)$
    $\delta q(x) = h_{1T}^q(x)$

WG6/70
JAM global QCD analysis

WG6/142 Drachenberg (STAR)
Transversity in polarized pp

WG6/150 Gunarathne (STAR)
Longitudinal spin asymm. in W prod.

TMDs with polarization

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T$ $\quad 0 \le S_L^2 - S_T^2 \le 1$

$$\Phi^q(x,p_T) \propto x f_1^q(x,p_T^2)\,\not{P} + S_L x g_{1L}^q(x,p_T^2)\,\not{P}\gamma_5 + x h_{1T}^q(x,p_T^2)\,\not{S}_T\not{P}\gamma_5 + \ldots$$

unpolarized quarks

chiral quarks in L-polarized N

T-polarized quarks in T-polarized N

compare

$$u(p,s)\overline{u}(p,s) = \tfrac{1}{2}(\not{p} + m)(1 + \gamma_5 \not{s})$$

- … but also

$$\Phi^q(x,p_T) \propto \ldots + \frac{(p_T \cdot S_T)}{M} x g_{1T}^q(x,p_T^2)\,\not{P}\gamma_5 + \ldots$$

spin ←→ spin

chiral quarks in T-polarized N (worm-gear)

TMDs with polarization

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T$ $\quad 0 \le S_L^2 - S_T^2 \le 1$

$$\Phi^q(x,p_T) \propto x f_1^q(x,p_T^2)\,\not{P} + S_L x g_{1L}^q(x,p_T^2)\,\not{P}\gamma_5 + x h_{1T}^q(x,p_T^2)\not{S}_T\not{P}\gamma_5 + \ldots$$

unpolarized quarks

chiral quarks in L-polarized N

T-polarized quarks in T-polarized N

compare

$$u(p,s)\overline{u}(p,s) = \tfrac{1}{2}(\not{p}+m)(1+\gamma_5\not{s})$$

- … but also

$$\Phi^q(x,p_T) \propto \ldots + \frac{(p_T \cdot S_T)}{M} x h_{1T}^{\perp q}(x,p_T^2)\frac{\not{p}_T}{M}\not{P}\gamma_5 + \ldots$$

spin ←→ spin

T-polarized quarks in T-polarized N (pretzelocity)

WG6/103 Prokudin, Lefky
Extraction pretzelosity distribution

TMDs with polarization

- … and T-odd functions

$$\Phi^q(x, p_T) \propto \;\ldots + i h_1^{\perp q}(x, p_T^2)\frac{\not{p}_T}{M}\not{P} + i\frac{(p_T \times S_T)}{M} x f_{1T}^{\perp q}(x, p_T^2)\not{P} + \ldots$$

T-polarized quarks in unpolarized N (Boer-Mulders)

unpolarized quarks in T-polarized N (Sivers)

spin ←→ orbit

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p} + m)(1 + \gamma_5 \not{s})$$

- spin-orbit correlations are T-odd correlations. Because of T-conservation they show up in T-odd observables, such as single spin asymmetries, e.g. left-right asymmetry in $p\, p_\uparrow \to \pi X$

TMD structures for quark PDFs

QUARKS	$\not{p}$	$\not{p}\gamma_5$	$\not{p}\gamma^\alpha\gamma_5$
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

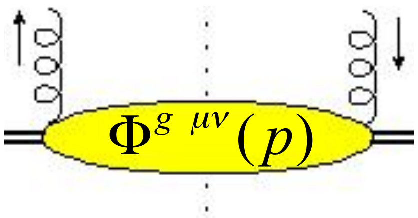
gluon TMD's without polarization

- Also for gluons there are new features in TMD's

circularly polarized gluons in L-pol. N

$$\Phi^{g\;\mu\nu}(x,p_T) \;\propto\; -g_T^{\mu\nu}\,x f_1^{g}(x,p_T^2) \;+\; iS_L\,\varepsilon_T^{\mu\nu}\,x g_{1L}^{g}(x,p_T^2)$$
$$+\left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\,\frac{p_T^\mu}{2M^2}\right) x h_1^{\perp g}(x,p_T^2) \;+\; \ldots$$

unpolarized gluons in unpol. N quarks



compare

$$\varepsilon^{\mu}(p,\lambda)\,\varepsilon^{\nu *}(p,\lambda) \;=\; -g_T^{\mu\nu} + \ldots$$

linearly polarized gluons in unpol. N (Rodrigues, M)

- Collinear situation:

  Unpolarized gluon PDFs: $g(x) = f_1^{g}(x)$

  Polarized gluon PDF: $\Delta g(x) = g_{1L}^{g}(x)$

- Some of the TMDs also show up in resummation. An example are the linearly polarized gluons (Catani, Grazzini, 2011)

WG6/75 Li (STAR)
Jet and di-jet in pol. pp

WG6/63 Guragain (PHENIX)
A_{LL} results, impact on Δg

WG6/316 Pisano
Linear polarization Higgs + jet prod

TMD structures for quark and gluon PDFs

QUARKS	$\not{p}$	$\not{p}\gamma_5$	$\not{p}\gamma^{\alpha}\gamma_5$
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T},\ h_{1T}^{\perp}$

GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}$
U	f_1^{g}		$h_1^{\perp g}$
L		g_{1L}^{g}	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^{g}	$h_{1T}^{g},\ h_{1T}^{\perp g}$

TMD structures for quark and gluon PFFs

QUARKS	$\not{p}$	$\not{p}\gamma_5$	$\not{p}\gamma^\alpha\gamma_5$
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T}, H_{1T}^\perp$

GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}$
U	D_1^g		$H_1^{\perp g}$
L		G_{1L}^g	$H_{1L}^{\perp g}$
T	$D_{1T}^{\perp g}$	G_{1T}	$H_{1T}^g, H_{1T}^{\perp g}$

Requirements and possibilities to measure Q_T

- Parton collinear momentum (lightcone fractions) $\leftarrow\rightarrow$ scaling variables

- Parton transverse momenta appear in noncollinear phenomena (azimuthal asymmetries)
  - Jet fragments: $k = K_h/z + k_T$ or $k_T = k - K_h/z = k_{jet} - K_h/z$
  - SIDIS: $q + p = k$ but $q + x\,P_H \neq K_h/z$
  - Hadrons: $p_1 + p_2 = k_1 + k_2$ but $x_1\,P_1 + x_2\,P_2 \neq k_1 + k_2$

- Two 'separated' hadrons involved in a hard interaction (with scale Q), e.g. SIDIS-like ($\gamma^*H \rightarrow h\,X$ or $\gamma^*H \rightarrow$ jet X), DY-like ($H_1H_2 \rightarrow \gamma^*\,X$ or $H_1H_2 \rightarrow$ dijet X or $H_1H_2 \rightarrow$ Higgs X), annihilation ($e^+e^- \rightarrow h_1h_2\,X$ or $e^+e^- \rightarrow$ dijet X)

- Number of p_T's in parametrization $\leftarrow\rightarrow$ rank of TMDs $\leftarrow\rightarrow$ azimuthal sine or cosine $m\phi$ asymmetry $\rightarrow$ Need for dedicated facilities at low and high energies

Rich phenomenology

WG6/151 Seder (COMPASS)
Charged kaon multiplicities in SIDIS

WG6/161 Vossen
FFs at Belle

WG6/135 Sbrizzai (COMPASS)
Transverse spin asymmetries SIDIS

WG6/84 Anulli
Collins Asymmetry with BaBar

WG6/22 Lyu
Collins Asymmetry at BESIII

WG6/75 Li (STAR)
Jet and di-jet in pol. pp

WG6/136 Diefenthaler (SeaQuest)
Polarized DY measurements

WG6/153 Barish (PHENIX)
Transverse Single-spin asymm.

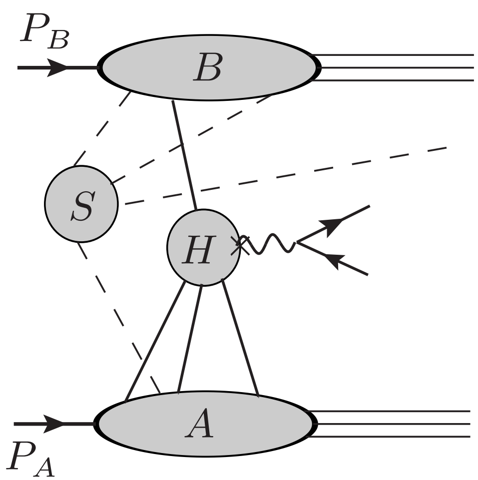
WG6/139 Allada
Recent SIDIS results at JLab

WG6/318 Liyanage
Recent polarized DIS results at JLab

TMDs and factorization

- Collinear PDFs involve operators of a given twist
- x ‘measurable’

$$d\sigma \sim F(x_1;\mu)F(x_2;\mu)H(Q,\mu)$$



- TMDs involve operators of all twist
- p_T in a convolution (scale and rapidity cutoffs)

$$d\sigma \sim \int dp_{1T}dp_{2T}\delta(p_{1T}+p_{2T}-q_T).....$$
$$.....F(x_1,p_{1T};\varsigma_1,\mu)F(x_2,p_{2T};\varsigma_2,\mu)$$

- Impact parameter representation including also large b_T

$$d\sigma \sim \int db_T \exp(iq_T b_T)....$$
$$...F(x_1,b_T;\varsigma_1,\mu)F(x_2,b_T;\varsigma_2,\mu)$$
$$+.........$$

- At small b_T, large k_T collinear physics (Collins-Soper-Sterman)

WG6/33 Collins
TMD factorization and evolution

WG6/312 Kang
TMD evolution and global analysis

J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

Non-universality because of process dependent gauge links

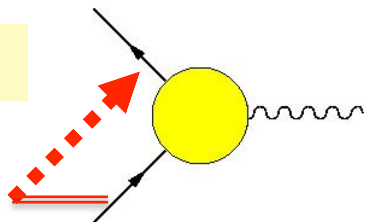
$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}_j(0)\, U^{[C]}_{[0,\xi]}\, \psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

TMD

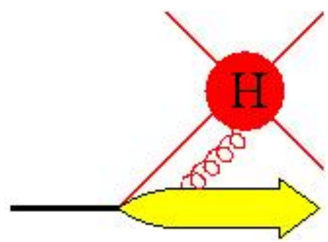
path dependent gauge link

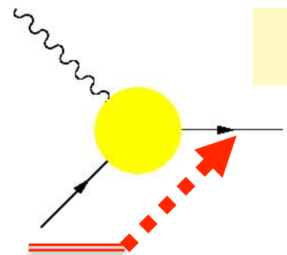
- Gauge links associated with dimension zero (not suppressed!) collinear $A^n = A^+$ gluons, leading for TMD correlators to process-dependence:

DY



... A^+ ...
(resummation)

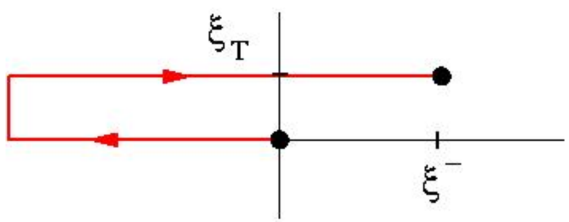




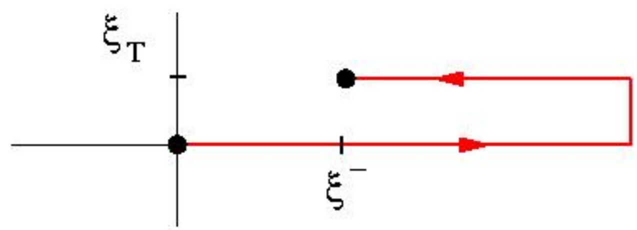
SIDIS

... A^+ ...

$\Phi^{[-]}$



Time reversal



$\Phi^{[+]}$

Belitsky, Ji, Yuan, 2003; Boer, M, Pijlman, 2003

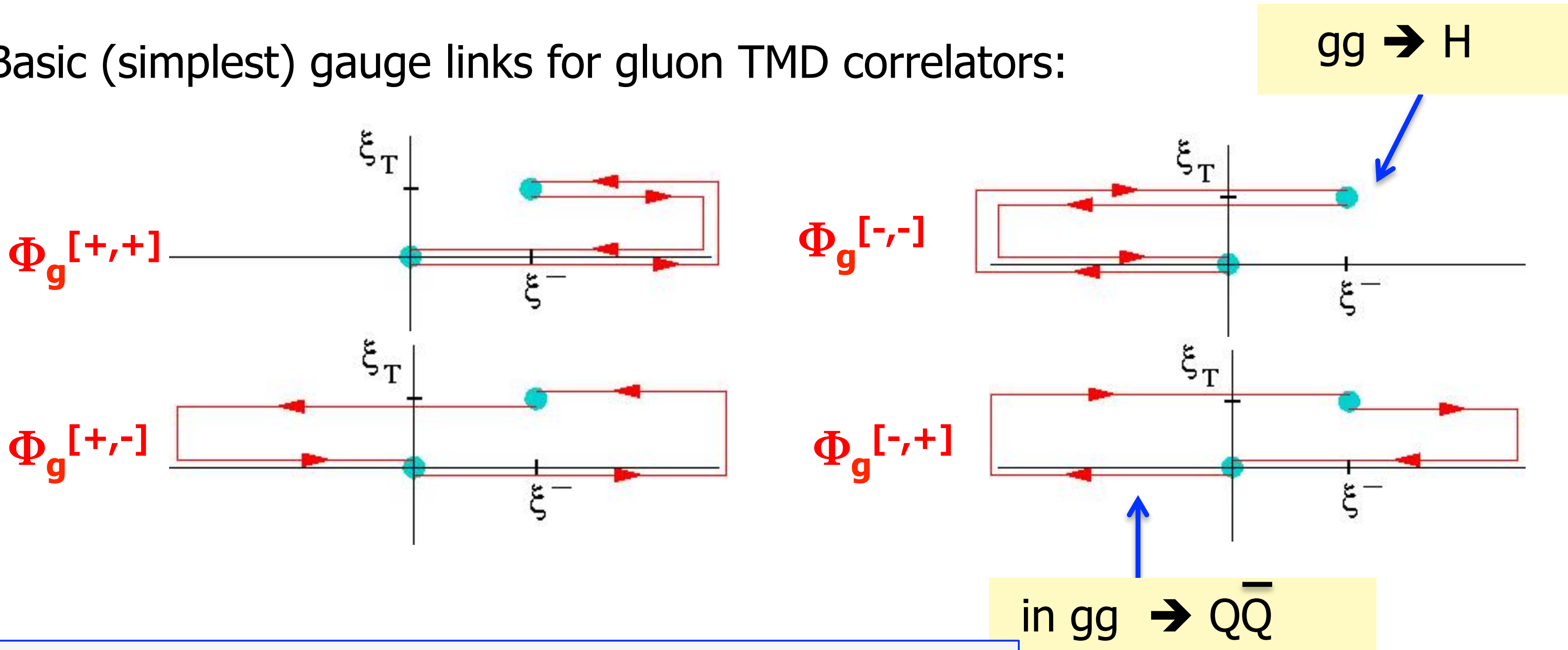
Non-universality because of process dependent gauge links

$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \left\langle P \middle| U^{[C]}_{[\xi,0]} F^{n\alpha}(0) U^{[C']}_{[0,\xi]} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \to U^{[C]}_{[\eta,\xi]} F^{\alpha\beta}(\xi) U^{[C]}_{[\xi,\eta]}$$

- Basic (simplest) gauge links for gluon TMD correlators:



Bomhof, M, Pijlman, 2006; Dominguez, Xiao, Yuan, 2011

Consequences for parametrizations of TMDs

- Gauge link dependence

- Leading quark TMDs

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

- Leading gluon TMDs

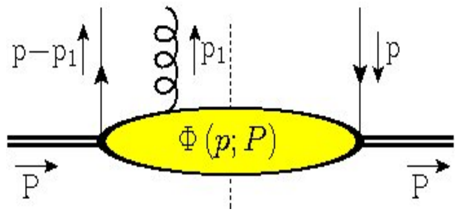
$$2x\,\Gamma^{\mu\nu[U]}(x,p_T) = -g_T^{\mu\nu}\,f_1^{g[U]}(x,p_T^2) + g_T^{\mu\nu}\,\frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp g[U]}(x,p_T^2) + i\epsilon_T^{\mu\nu}\,g_{1s}^{g[U]}(x,p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\,\frac{p_T^2}{2M^2}\right)h_1^{\perp g[U]}(x,p_T^2) - \frac{\epsilon_T^{p_T\{\mu}\,p_T^{\nu\}}}{2M^2}\,h_{1s}^{\perp g[U]}(x,p_T) - \frac{\epsilon_T^{p_T\{\mu}\,S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\,p_T^{\nu\}}}{4M}\,h_{1T}^{g[U]}(x,p_T^2).$$

Transverse moments → operator structure of TMD PDFs

- Operator analysis for TMD functions: in analogy to Mellin moments consider transverse moments involving gluonic operators

$$p_T^\alpha \Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\pm\infty]} iD_T^\alpha U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

calculable



- $$\boxed{\int dp_T\ p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\Phi_G^\alpha(x)}$$

T-even

T-odd

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

T-even (gauge-invariant derivative)

$$\Phi_A^\alpha(x) = PV\int \frac{dx_1}{x_1}\,\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

$$\Phi_D^\alpha(x) = \int dx_1\ \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\boxed{\Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0\,|\,x)}$$

T-odd (soft-gluon or gluonic pole, ETQS m.e.)

Efremov, Teryaev; Qiu, Sterman;
Brodsky, Hwang, Schmidt; Boer, Teryaev; Bomhof, Pijlman, M

TMD and collinear twist 3

- There are many more links between TMD factorized results, pQCD resummation results and collinear twist 3 results:

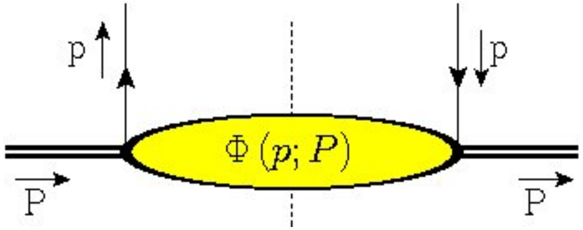
WG6/57 Pitonyak et al. Transverse SSA in pp → π/γ X
WG6/120 Koike Collinear twist 3 for spin asymm.
WG6/309 Metz Twist-3 spin observables
WG6/108 Dai Lingyun Sivers asymmetry in SIDIS
WG6/104 Granados Left-right asymm. in chiral dynamics
WG6/315 Burkardt Transverse Force on Quarks in DIS

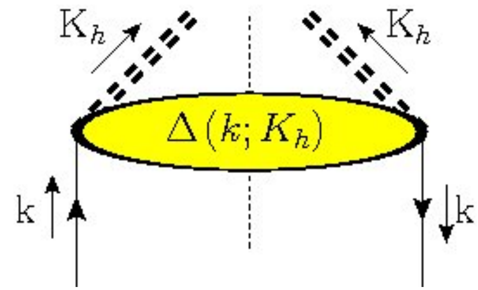
Operator classification of TMDs according to rank

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

Buffing, Mukherjee, M

Distribution versus fragmentation functions





- Operators:

$$\Phi^{[U]}(p\,|\,p) \sim \langle P\,|\,\bar{\psi}(0)U_{[0,\xi]}\psi(\xi)\,|\,P\rangle$$

$$\int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\,\Phi_G^\alpha(x)$$

T-even T-odd (gluonic pole)

$$\boxed{\Phi_G^\alpha(x) = \pi\Phi_F^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \langle 0\,|\,\psi(\xi)\,|\,K_h X\rangle\langle K_h X\,|\,\bar{\psi}(0)\,|\,0\rangle$$

$$\boxed{\Delta_G^\alpha(x) = \pi\Delta_F^{n\alpha}\left(\tfrac{1}{Z},0\,\middle|\,\tfrac{1}{Z}\right) = 0}$$

$$\int d^2k_T\; k_T^\alpha \Delta^{[U]}(x,k_T) = \tilde{\Delta}_\partial^\alpha(x)$$

T-even operator combination,
but still T-odd functions!

Collins, Metz; Meissner, Metz;
Gamberg, Mukherjee, M

Operator classification of TMDs

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

factor	TMD PFF RANK			
	0	1	2	3
1	$\Delta(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial}(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial\partial}(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial\partial\partial}(z^{-1},k_T^2)$

Buffing, Mukherjee, M

Operator classification of quark TMDs (unpolarized nucleon)

factor	QUARK TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Example: quarks in an unpolarized target are described by just 2 TMD structures; in general rank is limited to to 2(S_{hadron}+s_{parton})

$$\tilde{\Phi}(x,p_T^2) = \Big(f_1\,(x,p_T^2)\Big)\frac{\not{P}}{2}$$

T-even

$$\tilde{\Phi}_G^\alpha(x,p_T^2) = \left(i h_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

T-odd

[B-M function]

- Gauge link dependence: $h_1^{\perp[U]}(x,p_T^2) = C_G^{[u]} h_1^\perp(x,p_T^2)$ (coupled to process!)

Operator classification of quark TMDs (polarized nucleon)

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_{1L},\ h_{1T}$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

Three pretzelocities:

$$A:\ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Process dependence also for (T-even) pretzelocity,

$$h_{1T}^{\perp[U]} = h_{1T}^{\perp(1)(A)} + C_{GG,1}^{[U]}\,h_{1T}^{\perp(B1)} + C_{GG,2}^{[U]}\,h_{1T}^{\perp(1)(B2)}$$

Buffing, Mukherjee, M

Operator classification of TMDs (including trace terms)

factor	GLUON TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1^g		$h_1^{g\perp(A)}$	
$C_{GG,c}^{[U]}$			$h_1^{g\perp(Bc)}$	

- Note also process dependence of (T-even) linearly polarized gluons

$$h_1^{g\perp[U]} = h_1^{\perp(A)} + \sum_c C_{GG,c}^{[U]} h_1^{\perp(Bc)}$$

Operator classification of TMDs (including trace terms)

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$	$\Phi_{G.G,c}(x,p_T^2)$		$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

- Trace terms affect p_T width (note $\Phi_{G.G}(x) = 0$)

Boer, Buffing, M, arXiv:1503.03760

Operator classification of TMDs (including trace terms)

factor	GLUON TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1^{g}		$h_1^{g\perp(A)}$	
$C_{GG,c}^{[U]}$	$\delta f_1^{g(Bc)}$		$h_1^{g\perp(Bc)}$	

- Note process dependence, not only of linearly polarized gluons but also affecting the p_T width of unpolarized gluons

$$h_1^{g\perp[U]} = h_1^{\perp(A)} + C_{GG,c}^{[U]} h_1^{\perp(Bc)}$$

$$f_1^{g[U]} = f_1^{g} + C_{GG,c}^{[U]} \delta f_1^{g(Bc)} \quad \text{with } \delta f_1^{g(Bc)}(x) = 0$$

Boer, Buffing, M, arXiv:1503.03760

Operator classification of TMDs (including trace terms)

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_{1L},\ h_{1T}$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$\delta f_1,\ \delta g_1,\ \delta h_1$		$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

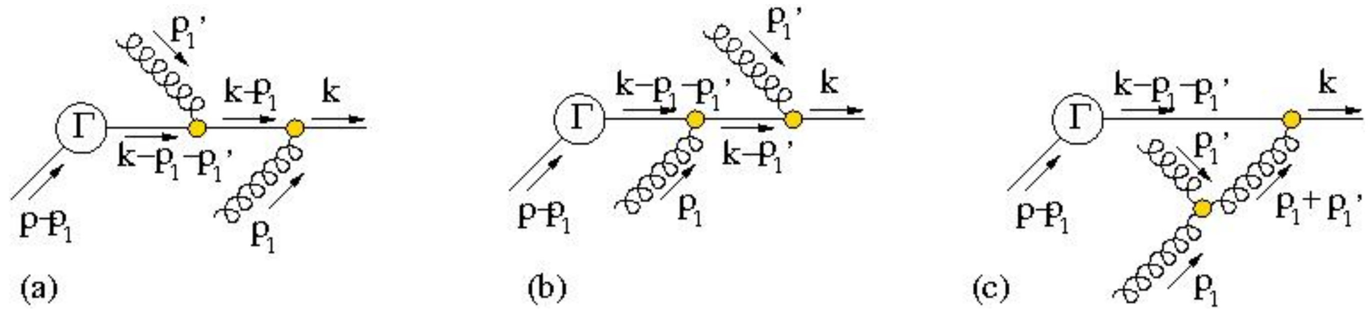
Process dependence in p_T width of TMDs is due to gluonic pole operator (e.g. Wilson loops)

$$f_1^{[U]} = f_1 + C_{GG,c}^{[U]}\,\delta f_1^{(Bc)} \quad \text{with } \delta f_1^{(Bc)}(x) = 0$$

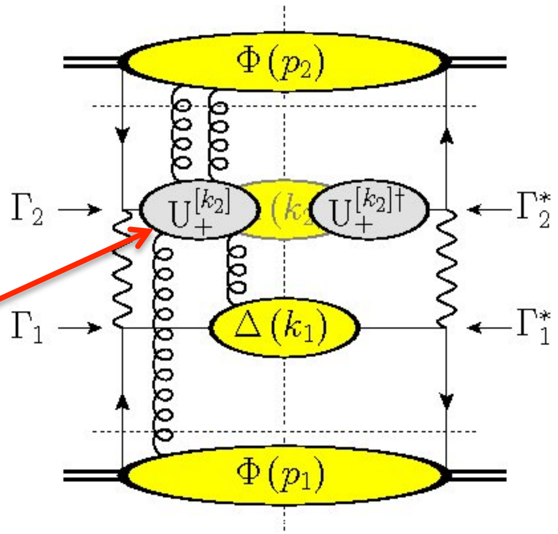
Boer, Buffing, M, arXiv:1503.03760

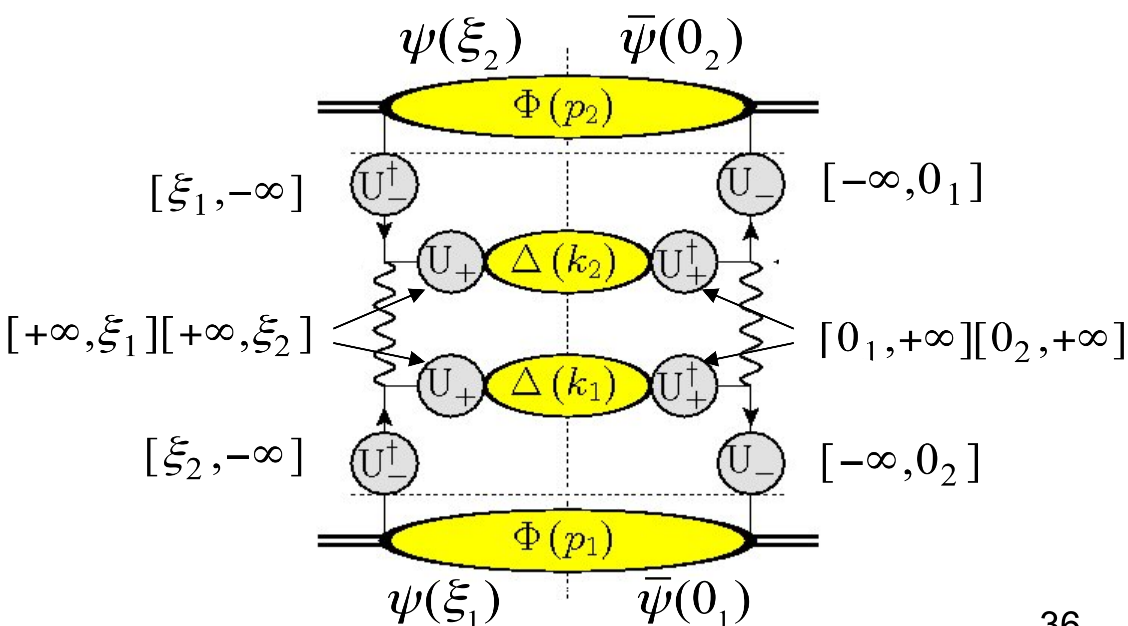
Entanglement in processes with two initial state hadrons

- Resummation of collinear gluons coupling onto external lines contribute to gauge links



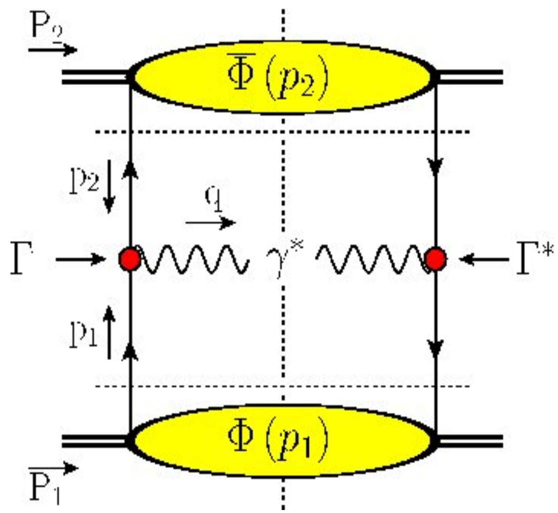
- …. leading to entangled situation (Rogers, M), breaking universality

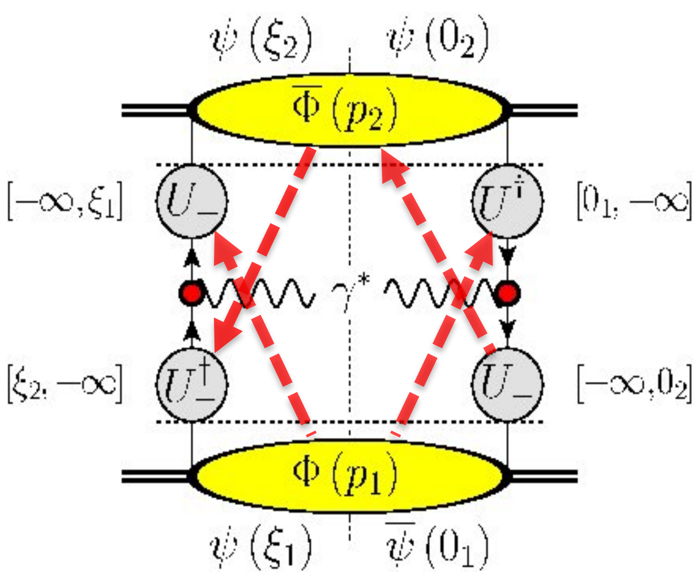




- Gauge knots (Buffing, M)

Two initial state hadrons (e.g. DY)





$$
\begin{aligned}
d\sigma_{\mathrm{DY}} &\sim \xcancel{\mathrm{Tr}_c\left[\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma\right]} \\
&\xcancel{= \frac{1}{N_c}\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma,}
\end{aligned}
$$

- Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in

$$
\begin{aligned}
d\sigma_{\mathrm{DY}} &= \mathrm{Tr}_c\Big[U_-^\dagger[p_2]\Phi(x_1,p_{1T})U_-[p_2]\Gamma^* \\
&\qquad \times U_-^\dagger[p_1]\overline{\Phi}(x_2,p_{2T})U_-[p_1]\Gamma\Big] \\
&\neq \frac{1}{N_c}\Phi^{[-]}(x_1,p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2,p_{2T})\Gamma,
\end{aligned}
$$

- This leads to color factors just as for twist-3 squared in collinear DY

$$
\begin{aligned}
\sigma_{DY}(x_1,x_2,q_T) &= \frac{1}{N_c} f_1(x_1,p_{1T})\otimes \bar{f}_1(x_2,p_{2T}) \\
&\quad - \frac{1}{N_c}\frac{1}{N_c^2-1} h_1^\perp(x_1,p_{1T})\otimes \bar{h}_1^\perp(x_2,p_{2T})\cos(2\varphi)
\end{aligned}
$$

Buffing, M, PRL 112 (2014), 092002

Attempts to provide a useful database have started

TMDlib project

http://tmdlib.hepforge.org

- Home
- TMDplotter
- TMD-Project
- Source Code Download
- CCFM uPDF evolution code
- Tracker
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TMDlib 1.0.2

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TMDLIB manual

TMDlib and TMDplotter: library and plotting tools for Transverse Momentum Dependent parton distributions

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Table of Contents

Conclusions and outlook

- TMDs extend collinear PDFs (including polarization 3 for quarks and 2 for gluons) to novel TMD PDF and PFF functions
- Although operator structure includes ultimately the same operators as collinear approach, it is a physically relevant combination/ resummation of higher twist operators that governs transverse structure (definite rank linked to azimuthal structure)
- Knowledge of operators for transverse structure is important for interpretation, relations to twist 3 and lattice calculations.
- Transverse structure for PDFs (in contrast to PFFs) requires careful study of process dependence linked to color flow in hard process
- Like spin, the transverse structure does offer valuable tools, but you need to know how to use them!

Some other (related) contributions

WG6/21 Lowdon (ArXiv:1408.3233) Spin operator decompositions	WG6/58 Courtoy, Bacchetta, Radici Collinear Dihadron FFs
WG6/102 Prokudin et al. Tensor charge from Collins asymm.	WG2/87 Yuan TMDs at small x
WG6/304 Kasemets Polarization in DPS	WG6/315 Burkardt Transverse Force on Quarks in DIS
WG2/112 Tarasov Rapidity evolution of gluon TMD	WG2/87 Yuan TMDs at small-x
WG2/326 Stasto Matching collinear and small-x fact.	WG4/88 Yuan Soft gluon resumm for dijets
WG7/297 Zhihong SoLID-SIDIS: future of T-spin, TMD	WG1/325 Kasemets TMD (un)polarized gluons in H prod