Discrete Symmetries

by

Prof.dr Ing. J. F. J. van den Brand

Vrije Universiteit
Amsterdam, The Netherlands
www.nikhef.nl/ jo

Contents

1 INTRODUCTION

Symmetries and conservation laws play a fundamental role in physics. The invariance of a system under a continuous symmetry transformation leads to a conservation law by Noethers' theorem. For example, the invariance under space and time translations results in momentum and energy conservation. Besides these continuous symmetries one has discrete symmetries that play an important role. Particularly, three such discrete symmetries are a topic of interest in modern particle physics. The parity transformation $\mathcal{P}$ performs a reflection of the space coordinates at the origin ($\vec{r} \rightarrow -\vec{r}$). Position and momentum change sign, while spin is unaffected. The charge conjugation operator $\mathcal{C}$ transforms a particle into its antiparticle and vice versa. All intrinsic 'charges' change sign, but motion and spin are left unchanged. Time reversal $\mathcal{T}$ operates on the time coordinate. Now also spin changes sign, like momentum and velocity. Composed symmetries, such as $\mathcal{CP}$ and $\mathcal{CPT}$, can also be considered. It was long thought that $\mathcal{CP}$ was an exact symmetry in nature. In 1964 $\mathcal{CP}$-violation was discovered in the neutral kaon system by Christensen, Cronin, Fitch and Turlay. A few years later Kobayashi and Maskawa demonstrated that a third quark generation could accommodate $\mathcal{CP}$ violation in the Standard Model by a complex phase in the CKM matrix. Since then, however, $\mathcal{CP}$ (or $\mathcal{T}$) violation has not been observed in any other system and we do not understand its mechanisms and rare processes. The discovery of the b-quark in 1977 opened a new possibility to test the Standard Model in B-mesons studies. This might answer unresolved questions in the Standard Model or lead to new physics. With experiments that study B-meson decay one mainly addresses the following questions:

- Is the phase of the CKM-matrix the only source of $\mathcal{CP}$-violation?
- What are the exact values of the components of the CKM-matrix?
- Is there new physics in the quark region?

This introductory course is structured as follows. In chapter 2, an introduction to quark mixing is given. In our discussions we will follow a historic route. The present status of $\mathcal{CP}$ violation in the kaon system is discussed in chapter 3. Chapter 4 gives a brief overview of $\mathcal{CP}$ violation in the neutral B-meson system. The merit of various experiments for the study of $\mathcal{CP}$ violation in the B sector is discussed in chapter 5. The LHCb experiment is described in somewhat more detail. Appendix A provides a set of exercises. Solutions are presented in appendix B.

2 Symmetries

2.1 Symmetries in Quantum Mechanics

The concept of symmetries and their associated conservation laws has proven extraordinary useful in particle physics. From classical physics we know, for example, that the demand that laws need to be invariant under translation in time, leads to conservation of energy. In addition, one has that invariance with respect to spatial rotations leads to conservation of angular momentum. While the conservation laws for energy, momentum and angular momenta are always strictly valid, we know that other symmetries are broken in certain interactions. It was for example quite a surprise for physicists when it was demonstrated that mirror symmetry is violated in the weak interaction (and only in this interaction!); even maximally violated. Furthermore, we presently do not understand why this is the case, or why certain other symmetries ($\mathcal{CP}$, $\mathcal{T}$) are only 'slightly violated'.

Here, we first want to summarize the quantum mechanical basis, which we will need for our discussion of the various phenomena. A system is described by a wave function, ψ. A physical *observable* is represented by a quantum mechanical *operator*, $\mathbf{O}$, whose expectation values are given by the eigenvalues of this operator. The eigenvalues correspond to the results of measurements, and the expectation value of $\mathbf{O}$ in the state ψ_a is defined as[1]

$$< O >= \int \psi_a^* \mathbf{O} \psi_a dV. \tag{3}$$

Since the expectation values can be determined experimentally, they need to be real quantities, and consequently $\mathbf{O}$ needs to be *hermitian*. When $\mathbf{O}$ is an operator, then its hermitian conjugate operator $\mathbf{O}^\dagger$ is defined as

$$\int (\mathbf{O}\psi)^* \phi dV = \int \psi^* \mathbf{O}^\dagger \phi dV, \tag{4}$$

and the operator $\mathbf{O}$ is called hermitian when one has $\mathbf{O}^\dagger = \mathbf{O}$.

The time dependence of the wave function, ψ, is given by the Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = \mathbf{H}\psi. \tag{5}$$

In case the hamiltonian $\mathbf{H}$ is real, one also has

$$-i\hbar \frac{\partial \psi^*}{\partial t} = (\mathbf{H}\psi)^* = \psi^* \mathbf{H}. \tag{6}$$

[1]When we consider two states, one can write in analog fashion

$$O_{ba} = \int \psi_b^* \mathbf{O} \psi_a dV, \tag{1}$$

and O_{ba} is called the *transition matrix element* between the states a en b. The expectation value of $\mathbf{O}$ in state a is the diagonal element of O_{ba} for $b = a$,

$$< O >= O_{aa}. \tag{2}$$

The non-diagonal elements do not directly correspond to classical observables. However, the transitions between states a and b are related to O_{ba}.

For the time dependence of an observable, O, we then find

$$\begin{aligned} \frac{\partial}{\partial t} < O > &= \frac{\partial}{\partial t} \int \psi^* \mathbf{O} \psi dV \\ &= \int \left(\frac{\partial \psi^*}{\partial t} \mathbf{O} \psi + \psi^* \mathbf{O} \frac{\partial \psi}{\partial t} \right) dV \\ &= \frac{i}{\hbar} \int \psi^* (\mathbf{HO} - \mathbf{OH}) \psi dV. \end{aligned} \tag{7}$$

Thus, we conclude that $< O >$ does not chance, and corresponds to a constant of motion, in case the *commutator* $[\mathbf{H},\ \mathbf{O}]$ vanishes,

$$[\mathbf{H}, \mathbf{O}] \equiv \mathbf{HO} - \mathbf{OH} = 0. \tag{8}$$

Consequently, a wave function can be found, that is *simultaenously* an eigenfunction of $\mathbf{O}$ and of $\mathbf{H}$,

$$\mathbf{H}\psi = E\psi \qquad \text{and} \qquad \mathbf{O}\psi = o\psi, \tag{9}$$

where o is eigenvalue of $\mathbf{O}$ in state ψ.

To illustrate the way in which *conservation laws* can be found, we carry out a *unitary*[2], time independent symmetry transformation $\mathbf{U}$,

$$\psi'(\vec{r}, t) = \mathbf{U}\psi(\vec{r}, t). \tag{10}$$

Since ψ' needs to obey an *identical* Schrödinger equation, we obtain

$$\mathbf{H} = \mathbf{U}^{-1}\mathbf{HU} = \mathbf{U}^{\dagger}\mathbf{HU}, \tag{11}$$

and thus

$$[\mathbf{H},\ \mathbf{U}] = 0. \tag{12}$$

One observes that the operator for the symmetry transformation also commutes with the hamiltonian.

In case $\mathbf{U}$ is also hermitian, $\mathbf{U}^{\dagger} = \mathbf{U}$, then an observabel is associated with $\mathbf{U}$. When this is not the case, then it is possible, as we will demonstrate in the following examples, to define a variable associated with $\mathbf{U}$. We need to distinguish between the case that $\mathbf{U}$ represents a *continuous* or a *discrete* symmetry transformation. In the first case we generally will find an *additive conserved quantity* (such as momentum, angular momentum, energy), while in the second case a *multiplicative quantum number* (for example parity) will be found.

[2]A unitary transformation leads to a conserved normalization of the wave function; this means that $\int \psi^* \psi dV = \int (\mathbf{U}\psi)^* \mathbf{U}\psi dV = \int \psi^* \mathbf{U}^{\dagger}\mathbf{U}\psi dV$, and consequently $\mathbf{U}^{\dagger}\mathbf{U} = 1$. For a unitary operator one thus has that $\mathbf{U}^{\dagger} = \mathbf{U}^{-1}$. Unitary operators are generalisations of $e^{i\alpha}$, the complex numbers with absolute value 1. When the operator $\mathbf{M}$ is represented by a matrix with elements M_{ik}, then $\mathbf{M}^*$ is the complex conjugated matrix with elements M^*_{ik}, $\tilde{\mathbf{M}}$ with elements M_{ki} is the transposed matrix, and $\mathbf{M}^{\dagger}$ with elements M^*_{ki} is the hermitian conjugated matrix. Furthermore, one has that $(\mathbf{AB})^{\dagger} = \mathbf{B}^{\dagger}\mathbf{A}^{\dagger}$. $\mathbf{E} = \mathbf{I} = \mathbf{1}$ is the unit matrix with elements $E_{ik} = \delta_{ik}$. The matrix $\mathbf{H}$ is called hermitian when $H^*_{ki} = H_{ik}$. The matrix $\mathbf{U}$ is unitary when $U^*_{ki}U_{ik} = U_{ik}U^*_{ki} = \delta_{ik}$.

2.2 Continuous Symmetry Transformations

For an continuous transformation it is efficient to introduce an additional operator (a so-called *generator*) $\mathbf{G}$,

$$\mathbf{U} = e^{i\epsilon\mathbf{G}} = \mathbf{1} + i\epsilon\mathbf{G} + \frac{1}{2!}(i\epsilon\mathbf{G})^2 + .., \tag{13}$$

where ϵ represents a real quantity. From unitarity of $\mathbf{U}$ it follows that

$$\mathbf{U}^\dagger\mathbf{U} = e^{-i\epsilon\mathbf{G}^\dagger}e^{i\epsilon\mathbf{G}} = \mathbf{1} \quad \rightarrow \quad \mathbf{G}^\dagger = \mathbf{G}, \tag{14}$$

and we find that the hermitian operator $\mathbf{G}$ represents the conserved quantity discussed in section 2.1.

In case $\mathbf{U}$ corresponds to a symmetry transformation, $[\mathbf{H},\ \mathbf{U}] = \mathbf{0}$, we find in the limit of an infinitesimal transformation, $\mathbf{U} = \mathbf{1} + i\epsilon\mathbf{G}$, immediately the relation

$$\mathbf{H}(\mathbf{1} + i\epsilon\mathbf{G}) - (\mathbf{1} + i\epsilon\mathbf{G})\mathbf{H} = 0, \tag{15}$$

and thus

$$[\mathbf{H},\ \mathbf{G}] = 0. \tag{16}$$

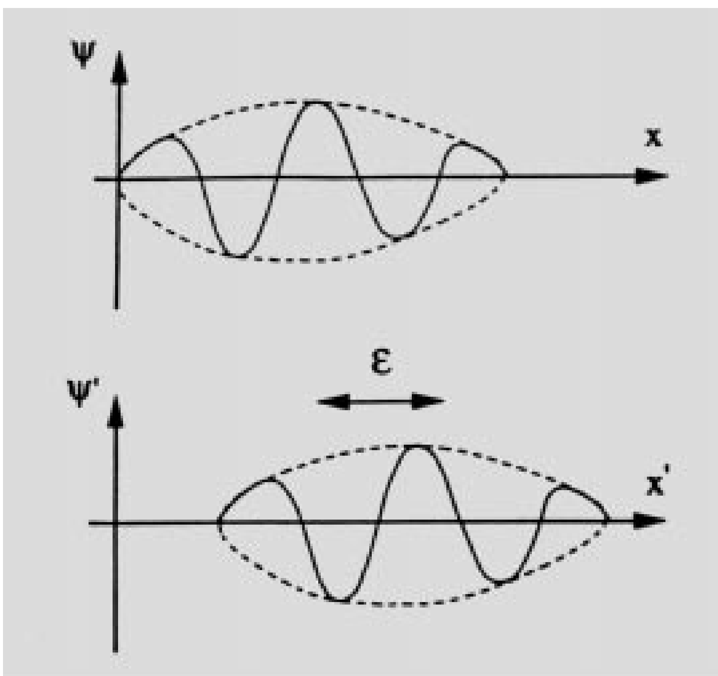

Figure 1: *Illustration of a continuous symmetry transformation through the translation of the wave function of a particle.*

2.2.1 Conservation of Momentum

We will elucidate the procedure with the help of a simple example. We consider in figure 1 the translation of the wave function of a particle in *one* dimension. We demand that for an observer in the translated reference system identical physical laws need to hold[3], and thus

$$\begin{aligned} \psi'(x') = \psi(x-\epsilon) &= \psi(x) - \epsilon \tfrac{d\psi(x)}{dx} + .. \\ &= (1 + i\epsilon \mathbf{G} + ..)\psi(x). \end{aligned} \tag{17}$$

With this we find

$$\mathbf{G} = i\frac{d}{dx} = -\frac{1}{\hbar}\mathbf{p_x}. \tag{18}$$

The operator $\mathbf{U}$ commutes with the hamiltonian $\mathbf{H}$, and consequently also with $\mathbf{G}$. The latter operator is proportional to the momentum operator $\mathbf{p_x}$. The corresponding observabele p_x is then the accompagning conserved quantity.

The translation operator

$$\mathbf{U}(\vec{a}) = \exp(-\frac{i}{\hbar}\vec{a}\cdot\vec{\mathbf{P}}) \tag{19}$$

generally corresponds to the following transformations ($\vec{\mathbf{P}}$ is the total momentum of the system),

$$\begin{aligned} \vec{r}' &= \mathbf{U}\,\vec{r}\,\mathbf{U}^{-1} &= \vec{r} - \vec{a} \quad &: \text{ spatial coordinates} \\ \vec{p}' &= \mathbf{U}\,\vec{p}\,\mathbf{U}^{-1} &= \vec{p} \quad &: \text{ momenta} \\ \vec{s}' &= \mathbf{U}\,\vec{s}\,\mathbf{U}^{-1} &= \vec{s} \quad &: \text{ spins.} \end{aligned} \tag{20}$$

2.2.2 Charge Conservation

When electrical charge would not be conserved, then the electron could decay, for example into a photon an an (electron) neutrino[4]

$$e \rightarrow \gamma + \nu_e. \tag{21}$$

Up to now this process has not been observed. The disappearance of a bound electron would, when the hole created in this manner is filled again by a neighboring electron, result in the emission of characteristic X rays. The relation between charge conservation and the Pauli principle is discussed for example by L.B. Okun[9]. Experiments show that the life time of the electron is larger than 4.3×10^{23} year. Moreover, there are many indications that charges are integer multiples of the elementary charge (for example Millekan's experiment, the neutrality of atoms)

$$q = Qe. \tag{22}$$

Therefore, we assume that the charge number is an additive conserved, discrete, quantity. In each reaction

$$a + b + .. + i \rightarrow c + d + .. + f \tag{23}$$

the sum of the corresponding charge numbers will be constant.

$$\sum Q_i = \sum Q_f. \tag{24}$$

[3] We could, of course, just as well assume that the system was translated over the same distance in the opposite direction.

[4] When we reverse the argument, then conservation of electrical charge guarantees the stability of the lightest charged particles.

However, what is the corresponding symmetry principle?

Assume that ψ_q represents the wave function of an object with charge q,

$$i\hbar \frac{\partial \psi_q}{\partial t} = \mathbf{H}\psi_q, \tag{25}$$

and $\mathbf{Q}$ is the *charge operator*. When $< \ Q \ >$ is conserved, one has

$$[\mathbf{H},\ \mathbf{Q}] = 0, \tag{26}$$

and ψ_q is simultaneously an eigenfunction of $\mathbf{Q}$ with eigenvalue q,

$$\mathbf{Q}\psi = q\psi. \tag{27}$$

The corresponding symmetry was discovered by Hermann Weyl[7]

$$\psi'_q = e^{i\epsilon\mathbf{Q}}\psi_q. \tag{28}$$

Such symmetry transformations are called *gauge transformations* and play an important role in particle physics. *Gauge invariance* again means that the transformed wave functions need to obey the same Schrödinger equation,

$$i\hbar \frac{\partial \psi'_q}{\partial t} = \mathbf{H}\psi'_q. \tag{29}$$

In the remainder of this section we will find several other conserved quantities (baryon number B, lepton numbers L_e, L_μ en L_τ, etc.).

2.2.3 Local Gauge Symmetries

We have seen that a *global gauge transformation*, $\epsilon =$ constant $\neq \epsilon(\vec{r}, t)$, leads to conservation of charge. However, note that we did not yet identify this charge as the electrical charge. Electrical charge is conserved at *each* space-time point. Thus we are dealing with a *local* conservation law. It is therefore necessary, but also esthetically attractive, to be able to chose the phase of the wave function, $e^{i\epsilon\mathbf{Q}}$, freely at *each* space-time point. We will generalize equation (28) for the wave function of a charged particle (for example a quark, or a charged lepton) to

$$\psi'_q = e^{i\epsilon(\vec{r},t)\mathbf{Q}}\psi_q = e^{i\epsilon(x)\mathbf{Q}}\psi_q. \tag{30}$$

The infinite set of phase transformations (30) constitutes a unitary group labeled U(1). Since $\epsilon(x)$ is a scalar quantity, the group U(1) is called Abelian[5]. The local gauge transformation (30) creates *different* phases for ψ_q at different locations in space-time. The description of a *free* charged particle is given by equation (25) and contains derivatives of $x = (t, \vec{x})$. However, these derivatives are *not* invariant under local gauge transformations. For example we find

$$\frac{\partial \psi'_q}{\partial t} = e^{i\epsilon(x)\mathbf{Q}}\frac{\partial \psi_q}{\partial t} + e^{i\epsilon(x)\mathbf{Q}}\frac{\partial \epsilon}{\partial t}\psi \neq e^{i\epsilon(x)\mathbf{Q}}\frac{\partial \psi_q}{\partial t}. \tag{31}$$

[5]More complex phase transformations are also possible, and may be specified by non-commuting operators. One then considers non-Abelian groups. Along these lines one has the group SU(2) as basis of the electroweak interaction, and the group SU(3) as basis of quantum chromodynamics.

The second term, with $\partial\epsilon/\partial t$, contains arbitrary functions of space-time and these functions prevent the invariance of the equations. We need to add dynamics to the system, if we are to maintain the principle of local symmetry. Local gauge invariance can be achieved by introducing a new dynamical field, and by allowing our particle (quark or charged lepton) to couple to this field. Before we carry out this procedure, we will make a brief excursion to electrodynamics[6].

We define the *vector* and *scalar* potentials, $\mathbf{A}$ and ϕ, which obey

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial \mathbf{t}}. \tag{36}$$

With these relations it is often possible to simplify the system of coupled equations. However, it has been known for a long time that the fields $\mathbf{B}$ en $\mathbf{E}$ are not uniquely defined by equation (36). In historic perspective this is the first manifestation of a gauge symmetry, and it appears in classical electrodynamics. We observe that $\mathbf{B}$ and $\mathbf{E}$ in equation (36) remain invariant when we replace $\mathbf{A}$ and ϕ by

$$\mathbf{A}' = \mathbf{A} + \nabla\epsilon, \quad \text{and} \quad \phi' = \phi - \frac{\partial\epsilon}{\partial t}. \tag{37}$$

The quantity $\epsilon(\vec{r},t) = \epsilon(x)$ represents an *arbitrary* scalar function of space-time. Each local change in the electric potential can be combined with a corresponding change in magnetic potential, in such a way that $\mathbf{E}$ and $\mathbf{B}$ are invariant. Such redefinitions are of no consequence for the classical fields $\mathbf{E}$ and $\mathbf{B}$, and we conclude that classical electrodynamics constitutes a local gauge invariant formalism. We often can exploit this freedom in the definition of the potentials in order to obtain decoupled (or at least simplified) differential equations for $\mathbf{A}$ and ϕ.

[6]Classical electrodynamics is described by Maxwell's equations. These yield the coupled partial differential equations between the electric, $\mathbf{E}$, and magnetic, $\mathbf{B}$, fields and one has

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \tfrac{\rho}{\epsilon_0} && \text{Coulomb's law,} \\
\nabla \times \mathbf{B} - \tfrac{1}{c^2}\tfrac{\partial \mathbf{E}}{\partial t} &= \mu_0 \mathbf{J} && \text{Ampere's law,} \\
\nabla \times \mathbf{E} + \tfrac{\partial \mathbf{B}}{\partial t} &= 0 && \text{Faraday's law,} \\
\nabla \cdot \mathbf{B} &= 0 && \text{absence of magnetic monopoles.}
\end{aligned} \tag{32}$$

We consider the fields in vacuum induced by the charge- and current densities ρ and $\mathbf{J}$. These quantities obey *local* conservation laws, that can be obtained by taking derivatives of Maxwell's equations. One has

$$\frac{\partial}{\partial t}\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\frac{\partial\rho}{\partial t}, \tag{33}$$

and

$$\nabla \cdot (\nabla \times \mathbf{B}) - \frac{1}{c^2}\nabla\cdot\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \nabla \cdot \mathbf{J}. \tag{34}$$

Next, we make use of the relations $\nabla \cdot (\nabla \times \mathbf{B}) = 0$ and $1/c^2 = \mu_0\epsilon_0$, and find the relation between charge and current,

$$\frac{\partial\rho}{\partial \mathbf{t}} + \nabla \cdot \mathbf{J} = 0. \tag{35}$$

This expression is valid at each arbitrary point in space and time.

This formal treatment of the electromagnetic potentials obtains a new and important meaning when we consider the quantum behavior of a charged particle in a gauge invariant theory. The probability to find a particle at a given location is determined by the wave function ψ_q. It is important to note that ψ_q does not represent the electric field of the particle (for example an electron), but its *matter* field. We have seen that by demanding local gauge symmetry of the wave function, *differences* in phase were created between different space-time coordinates. We can prevent these arbitrary effects from becoming observable by using the electromagnetic potentials as *gauge fields*. When we chose the function $\epsilon(x)$ in equation (31) identical to the function in equations (37), then the gauge transformation of $\mathbf{A}$ and ϕ exactly compensates the arbitrary changes in phase of the wave function ψ_q. Since these phase differences need to be compensated over arbitrary large distances, the gauge field $A_\mu = (A_0, \vec{A}) = (\phi, \mathbf{A})$ needs to have infinite range. The corresponding quantum, the photon, therefore needs to have a vanishing mass. In addition, the spin of the gauge particle needs to be equal to one, since the gauge field A_μ is a *vector field*. The proposed formalism for ψ_q, $\mathbf{A}$ and ϕ represents a theory that is locally gauge invariant.

To demonstrate this local gauge invariance, we proceed from the equation of motion of a free particle to that of a particle that interacts with a gauge field. For this we redefine the energy and momentum operators,

$$i\hbar\frac{\partial}{\partial t} \rightarrow i\hbar\frac{\partial}{\partial t} - q\phi, \quad \text{en} \quad \frac{\hbar}{i}\nabla \rightarrow \frac{\hbar}{i}\nabla - q\mathbf{A}, \tag{38}$$

and we now can write the Schrödinger equation for a charge particle that interacts with the gauge field A_μ as,

$$\left(i\hbar\frac{\partial}{\partial t} - q\phi\right)\psi_q = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q. \tag{39}$$

These substitutions are known as the *minimal substitution*, and lead to a locally gauge invariant formulation of the Schrödinger equation. We have

$$\begin{aligned} \left(i\hbar\tfrac{\partial}{\partial t} - q\phi'\right)\psi'_q &= e^{i\epsilon\mathbf{Q}}\left(i\hbar\tfrac{\partial}{\partial t} - q\phi\right)\psi_q, \\ \tfrac{1}{2m}\left(\tfrac{\hbar}{i}\nabla - q\mathbf{A}'\right)^2\psi'_q &= e^{i\epsilon\mathbf{Q}}\tfrac{1}{2m}\left(\tfrac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q. \end{aligned} \tag{40}$$

The structure of equation (39) is appearantly such that arbitrary phase changes of ψ_q are canceled by the gauge behavior of $\mathbf{A}$ and ϕ. The interpretation of the parameter q becomes apparant when we rewrite equation (39) as

$$i\hbar\frac{\partial\psi_q}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q + q\phi\psi_q. \tag{41}$$

We recover the familiar expression for the Schrödinger equation of a particle in an electromagnetic field, where the second term represents the electrostatic potential energy (Coulomb energy), $V = q\phi$. We can now identify q with the electrical charge.

In summary, we arrived at the remarkable conclusion that the demand of local gauge invariance dictates both the *existence* and *nature* of the interaction. From this formalism it follows that the mass of the gauge particle, the photon, vanishes. Furthermore, it follows that the spin of the gauge particle must be equal to one. The principle of gauge symmetry can also be applied to the relativistic wave equation for spin-$\frac{1}{2}$ particles, the Dirac equation. When local U(1) symmetry is imposed, this leads to a gauge invariant theory that is known as quantum electrodynamics.

2.2.4 Conservation of Baryon Number

Experiments show that also the proton is stable. Its life time has been determined for a large number of decay channels[7] and exceeds 10^{30} year. Some examples are

$$\begin{array}{lll} p & \to e^+\pi^0 & \tau > 5.5 \times 10^{32} \text{ year} \\ p & \to \mu^+\pi^0 & \tau > 2.7 \times 10^{32} \text{ year} \\ p & \to e^+\gamma & \tau > 4.6 \times 10^{32} \text{ year} \\ p & \to e^+ \text{ what ever} & \tau > 0.6 \times 10^{30} \text{ year.} \end{array} \tag{42}$$

Presently, a large effort is ongoing in the search for the decay of the proton, because various theoretical models predict a finite life time τ_p of the proton of about 10^{33} years. So far no proton decay has been demonstrated experimentally.

The above observation is one of the reasons why one, completely analogous to the charge number Q, introduces a baryon number B. However, we would like to point out *one* difference: in a field theory with local gauge symmetry one has that an exact conserved quantity (such as electrical charge) leads to the existence of a field with a longe range (a gauge field) that couples to this charge. However, up to now no interaction with longe range associated with baryon number could be identified: the equivalence principle demands that the ratio between inert and heavy masses must be equal for all objects. That this is indeed the case has been verified for the elements Al and Pt to a precision of about 10^{-12}. For these elements the ratio between mass and baryon number is considerably different, due to the differences in binding energies. From this it follows that the coupling to baryon number is certainly weaker than the gravitational coupling by a factor of 10^9.

In the decay of the neutron both charge and baryon number (and lepton number) are conserved.

$$\begin{array}{ccccccccccc} & & n & \to & p & + & e^- & + & \bar{\nu}_e \\ \\ Q & : & 0 & = & 1 & - & 1 & + & 0 \\ B & : & 1 & = & 1 & + & 0 & + & 0 \\ L_e & : & 0 & = & 0 & + & 1 & - & 1 \end{array} \tag{43}$$

The proton and neutron are the only 'normal' particles that carry baryon charge. However, there exists a series of resonances and excited states that also have $B = 1$, such as N(1440), N(1550), ..; $\Delta(1232)$, $\Delta(1620)$, ..; Λ_0, $\Sigma^\pm$, Σ^0, Ξ^0, Ξ^-, Ω^-, and so on. All these particles are, as we presently assume, composed of three quarks, that each carry a baryon number $B = \frac{1}{3}$. The corresponding antibaryons, that are composed of antiquarks, have $B = -1$.

[7]It will be clear that the decay $p \to 3\nu$, which also violates charge conservation, will be difficult to determine experimentally. In addition, one can ask the question whether the decay $p \to$ NOTHING, which also violates energy conservation is 'conceivable'.

For all nuclei we have that the baryon number is equal to the number of nucleons (A), and thus

$$B = A = N + Z. \tag{44}$$

In the case of leptons, $e^{\pm}$, ν_e, $\bar{\nu}_e$, $\mu^{\pm}$, etc. and mesons, that are built from a quark-antiquark pair, we have that $B = 0$. In all observed decay processes and reactions, baryon number is always conserved. We do not know why[8].

2.2.5 Conservation of Lepton Number

Also in reactions with light particles one has discovered, analogous to the case of baryons, that these particles are created and annihilated in pairs. One has for example the reaction

$$\gamma \rightarrow e^+ e^- \qquad \text{in the field of a nucleus.} \tag{45}$$

Furthermore, certain reactions are allowed while others are forbidden. To be able to ‘explain’ these observations, one has introduced a lepton number L and postulates that this number is conserved in all interactions. To elucidate this we first consider two ordinary β-decays,

$$\begin{array}{lcccccccc} & n & \rightarrow & p & + & e^- & + & \bar{\nu}_e \\ & {}^3\mathrm{H} & \rightarrow & {}^3\mathrm{He} & + & e^- & + & \bar{\nu}_e \\ & & & & & & & \\ L_e \; : & 0 & = & 0 & + & 1 & - & 1. \end{array} \tag{46}$$

When we assign $L_e = 1$ to the electron[9], then it follows that for the simultaneously emitted neutrino $\bar{\nu}_e$ we have $L_e = -1$. Therefore, we denote this particle as an antineutrino. Later we will see that the quantum numbers, related to charge, obtain an opposite sign for antiparticles (the charge itself for example is for a positron, the antiparticle of the electron, positive; for an antiproton negative). With these definitions it seems natural to introduce the following lepton numbers in the case of β-decay,

$$\begin{array}{lccccccccl} & p & \rightarrow & n & + & e^+ & + & \nu_e & & \text{in nuclei} \\ & & & & & & & & & \\ \text{for example} & {}^{35}\mathrm{Ar} & \rightarrow & {}^{35}\mathrm{Cl} & + & e^+ & + & \nu_e & & \text{typical } \beta^+ - \text{decay} \\ L_e \;\; : & 0 & = & 0 & - & 1 & + & 1 & & \\ & & & & & & & & & \\ \text{or} & {}^{37}\mathrm{Ar} & + & e^- & \rightarrow & {}^{37}\mathrm{Cl} & + & \nu_e & & \\ L_e \;\; : & 0 & + & 1 & = & 0 & + & 1 & . & \end{array} \tag{47}$$

From the kinematics of β-decay (kurie plot) we know that the masses of ν_e and $\bar{\nu}_e$ are zero (or at least that they are small, $m_{\bar{\nu}_e} < 10 - 15$ eV/c^2). From conservation of angular momentum we conclude that the spin of the neutrino is equal to $\frac{1}{2}$. The charges vanish and both particles have only little interaction with matter. They can for example penetrate the earth without being absorbed.

The question that naturally arises is: in what respect are the electron-neutrino and electron-antineutrino different? - In their lepton number! We can experimentally demonstrate that the lepton numbers (and their associated conservation laws) form a meaningful

[8]An answer like: ‘because there exists a corresponding gauge invariance’, only shifts the question!

[9]Here, we write L_e for the electronic lepton number, since two more lepton numbers (L_μ, and L_τ) will be introduced (we will be forced to do this later).

concept. One possibility is the study of neutrino reactions. However, it is not easy to study reactions with neutrinos. Because of their extraordinary small cross section it took more than twenty years before the existence of the (anti)neutrino, postulated by Wolfgang Pauli in 1930, could be discovered by Cowan and Reines[8]

In the following we describe the basic ideas of this experiment. Antineutrinos can induce in a substance that contains hydrogen, the following reactions,

$$\begin{array}{rccccccccc} & & \bar{\nu}_e & + & p & \to & n & + & e^+ \\ L_e & : & -1 & + & 0 & \to & 0 & - & 1. \end{array} \tag{48}$$

As a source with sufficient intensity for $\bar{\nu}_e$ one can employ a nuclear reactor[10]. In the fission of heavy nuclei primarily elements with a neutron excess are produced, which in turn leads to various β^--decay chains. On average about six $\bar{\nu}_e$'s are emitted in a decay with energies ranging from 0 to 8 MeV.

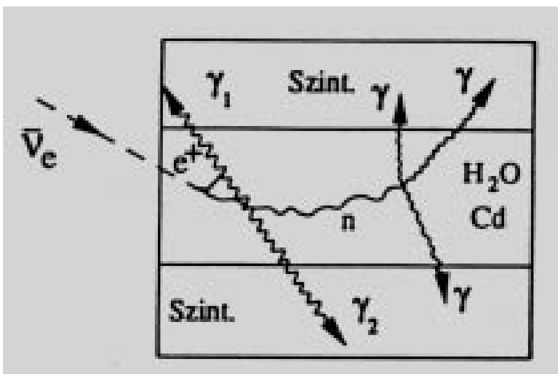

Figure 2: *Schematic representation of the experimental set up used by Cowan and Reines to demonstrate the existence of the antineutrino.*

Figure 2 shows the detector, consisting of a vessel filled with 200 liters of water (with some $CdCL_2$ added). The vessel is placed between three liquid scintillators each with a content of 1400 liters (at that time a gigantic experiment!). The positron will rapidly slow down and annihilate with an electron,

$$e^+ + e^- \to 2\gamma. \tag{49}$$

Both annihilation quanta are measured in coincidence with the help of the scintillators. The produced neutrons are slowed down to thermal energies by collisions in the water, and are finally captured[11] in the ^{113}Cd. The γ-quanta produced in this reaction are registered in a (delayed) coincidence, which yields a clear signature of the real events. With the reactor switched on (700 MW) an increase in the counting rate of 3.0 ± 0.2 events per hour is measured. From this an average cross section of

$$< \sigma >= (12^{+7}_{-4}) \times 10^{-44} \text{ cm}^2 \tag{50}$$

[10]At first Cowan and Reines contemplated an atomic explosion as a source for the electronic antineutrinos

[11]The element ^{113}Cd is an effective neutron absorber: the cross section has a resonance at $T_n = 0.0253$ eV with a maximum of $\sigma_{n\gamma} = 2450 \pm 30$ barn.

is deduced, which is in agreement with the theoretical predictions. At almost the same time, and at the same reactor, R. Davis demonstrated that antineutrinos cannot induce the reaction

$$\begin{array}{lcccccccc} & \bar{\nu}_e & + & {}^{37}\mathrm{Cl} & \rightarrow & {}^{37}\mathrm{Ar} & + & e^- \\ L_e \;: & -1 & + & 0 & \rightarrow & 0 & + & 1! \end{array} \tag{51}$$

However, later it could be shown that neutrinos originating from the sun, can indeed, as we would expect on the basis of the lepton numbers, induce this reaction,

$$\begin{array}{lcccccccc} & \nu_e & + & {}^{37}\mathrm{Cl} & \rightarrow & {}^{37}\mathrm{Ar} & + & e^- \\ L_e \;: & 1 & + & 0 & \rightarrow & 0 & + & 1! \end{array} \tag{52}$$

However, the number of ^{37}Ar atoms collected in this reaction over the last several decades is about a factor 2 - 3 smaller that we would expect on the basis of solar calculations. This is the famous problem of the solar neutrinos[10], which constitutes one of the principal mysteries in modern nuclear and particle physics.

Also the measurements of double β-decay and several other experimental facts indicate that the ν_e and $\bar{\nu}_e$ are different particles, and that they can be characterized by $L_e = +1$ or $L_e = -1$, respectively[12].

In reactions, where the 'heavy' electrons $\mu^\pm$ and $\tau^\pm$ participate, often neutrinos are produced, absorbed, or scattered. This directly poses the question whether these particles behave in the same fashion as the now familiar electronic neutrinos ν_e and $\bar{\nu}_e$. For example, the positively charged pion mostly decays into a μ^+ and only rarely into an e^+,

$$\begin{array}{ll} \pi^+ & \rightarrow \mu^+ + \nu_\mu \qquad \mathrm{B.R.} \;=\; 0.999878 \\ \pi^+ & \rightarrow e^+ + \nu_e \qquad \mathrm{B.R.} \;=\; 1.2 \times 10^{-4}. \end{array} \tag{53}$$

The antiparticles, with identical life time and identical decay probabilities, decay as follows,

$$\begin{array}{ll} \pi^- & \rightarrow \mu^- + \bar{\nu}_\mu \\ \pi^- & \rightarrow e^- + \bar{\nu}_e. \end{array} \tag{54}$$

Also the neutrinos that play a role in the muon decay channel have a spin $\frac{1}{2}$, a charge 0 and most probably a rest mass that is equal to zero ($m_\mu < 0.17$ MeV/c^2). In spite of all this, they distinguish themselves from the electronic neutrinos ν_e and $\bar{\nu}_e$ (that is the reason why we used different symbols to begin with). We can demonstrate the formalism

[12] At this point we cannot address the question whether both particles 'only' differ in their helicity. When the particles would have a, however small, mass then both these states can be transformed to each other (by a Lorentz transformation with sufficient speed).

by investigating the following reactions.

$$
\begin{array}{lcccccccccl}
 & & \nu_e & + & n & \longrightarrow & p & + & e^- & & \\
L_e & : & 1 & + & 0 & = & 0 & + & 1 & \} & \text{occurs} \\
L_\mu & : & 0 & + & 0 & = & 0 & + & 0 & & \\
 & & & & & & & & & & \\
 & & \nu_\mu & + & n & \longrightarrow & p & + & e^- & & \\
L_e & : & 0 & + & 0 & \neq & 0 & + & 1 & \} & \text{does not occur} \\
L_\mu & : & 1 & + & 0 & \neq & 0 & + & 0 & & \\
 & & & & & & & & & & \\
 & & \nu_\mu & + & n & \longrightarrow & p & + & \mu^- & & \\
L_e & : & 0 & + & 0 & = & 0 & + & 0 & \} & \text{occurs} \\
L_\mu & : & 1 & + & 0 & = & 0 & + & 1 & &
\end{array}
\tag{55}
$$

Two experiments demonstrated to high precision that the lepton families are essentially different and that L_e and L_μ are conserved separately.

- The reaction
$$\mu^- \longrightarrow e^- + e^+ + e^- \tag{56}$$
was studied[11] happens *not* to occur. The branching ratio is smaller than 10^{-12} and was measured the so-called SINDRUM experiment at the Paul Scherrer Institute in Villingen, Zwitzerland.

- Also the reactions[12]
$$
\begin{array}{ll}
\mu^- +^{32}\mathrm{S} & \to e^- +^{32}\mathrm{S}, \quad \sigma/\sigma_{\nu,\ \mathrm{capture}} < 7\times 10^{-11} \\
\mu^- +^{32}\mathrm{S} & \to e^+ +^{32}\mathrm{Si}, \quad \sigma/\sigma_{\nu,\ \mathrm{capture}} < 9\times 10^{-10}
\end{array}
\tag{57}
$$
show no indication for violation of lepton number conservation. Note that the second process also violated conservation of total lepton number.

2.3 Discrete Symmetry Transformations

2.3.1 Parity or Symmetry under Spatial Reflections

The unitary parity transformation $\mathcal{P}$ inverts all spatial coordinates (by reflection through the origin) and momenta,

$$\begin{aligned} \vec{\mathbf{r}}' &= \mathcal{P}\,\vec{\mathbf{r}}\,\mathcal{P}^{-1} &= -\vec{\mathbf{r}} \\ \vec{\mathbf{p}}' &= \mathcal{P}\,\vec{\mathbf{p}}\,\mathcal{P}^{-1} &= -\vec{\mathbf{p}}. \end{aligned} \tag{58}$$

Angular momenta and spins do not reverse sign,

$$\begin{aligned} \vec{L}' &= (-\vec{r} \times (-\vec{p})) &= \vec{L} \\ \vec{s}' &= \vec{s}. \end{aligned} \tag{59}$$

We assume that all internal quantum numbers of the particle (charge, baryon number, etc.) do not change under this transformation.

Until the year 1956 it was taken for granted that all physical laws would obey mirror symmetry[13],

$$[\mathbf{H},\ \mathcal{P}] = 0. \tag{60}$$

With this assumption we can again find wave functions that are simultaneously eigenstates of both $\mathbf{H}$ and $\mathcal{P}$,

$$\begin{aligned} \mathbf{H}\psi &= E\psi \\ \mathcal{P}\psi &= \pi\psi. \end{aligned} \tag{61}$$

For *non-degenerate* systems one then has that $\pi = \pm 1$. for degenerate systems one needs to be more cautious: the hydrogen atom serves as an example. In case we consider a spherically symmetric potential,

$$\mathbf{H}(\vec{r}) = \mathbf{H}(-\vec{r}) = \mathbf{H}(r), \tag{62}$$

one consequently finds $[\mathbf{H}, \mathcal{P}] = 0$. The wave functions

$$\psi(r, \vartheta, \varphi) = \chi(r) Y_l^m(\vartheta, \varphi) \tag{63}$$

have a well defined parity, given by $(-1)^l$. In case we neglect the fine structure, then the levels are degenerate in the hydrogen atom (with the ground state as the only exception: $n = 1$, $l = 0$). The first excited state for example, with principal quantum number $n = 2$, then has the same energy for both angular momenta $l = 0$ and $l = 1$. We immediately can write down a linear combination of wave functions, that *do not have* a well defined parity, $\psi(-\vec{r}) \neq \pm\psi(\vec{r})$.

The state of a nucleon (n or p) *is* an eigenstate of $\mathcal{P}$, since no other object exists with the same charge, mass, etc. The relative parity between states with different quantum numbers Q and B is arbitrary. Due to conservation of baryon number and charge, we can fix the eigenparity of the electron π_e, the proton π_p, and that of the neutron π_n at $+1$[14]. It was an incredible surprise when Lee and Yang[13] pointed out in 1956, that it is not at all evident that parity is conserved in all interactions. A short time later it became possible to demonstrate that parity is violated in the weak interaction (even maximally violated)[15]. Next, we will explain some of these experiments in more detail.

[13] The fact that only a single form of vitamin C exists, that helps against a cold and not the other form, is not a counter example! No more than the fact that in all bars in the world one only finds righthanded corkscrews.

[14] Since the proton and neutron form an isospindoublet, a different normalization would be unfortunate.

[15] Experimental physicists could have found earlier evidence for this if they would not have taken this symmetry for granted.

2.3.2 Parity Violation in β-decay

Parity violation was demonstrated for the first time by Wu and her collaborators[14]. The concept of that experiment is represented in a schematic manner in figure 3.

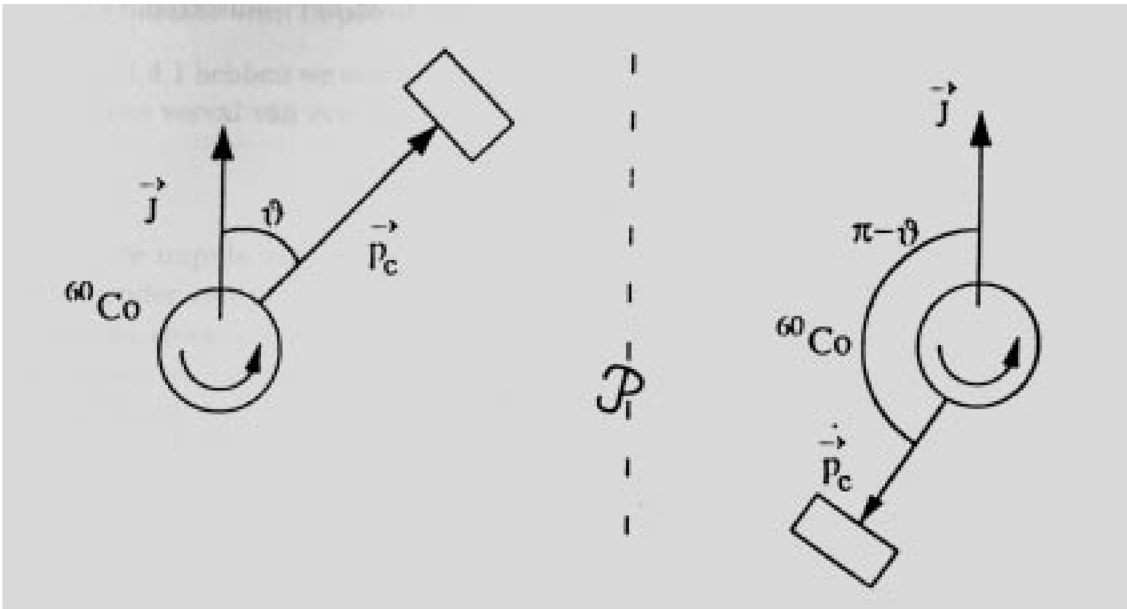

Figure 3: *Schematic representation of the experimental set up used by Wu and collaborators to demonstrate the violation of parity in the β-decay of ^{60}Co.*

A polarized nucleus with spin $\vec{J}$ emits electrons with a momentum $\vec{p}_e$. On the right-hand side of figure 3, the parity transformed experimental situation is sketched. Parity invariance demands both situations to be indistinguishable, and the counting rates $I(\vartheta)$ and $I(\pi - \vartheta)$ to be identical.

The experiment was carried out with the isotope ^{60}Co. This nucleus has a spin $J^{\pi} = 5^{+}$ and decays with a half life time of $\tau_{\frac{1}{2}} = 5.2$ year preferably (> 99 %) to an excited state (with $J^{\pi} = 4^{+}$) of ^{60}Ni. The ^{60}Co nuclei can be polarized by placing them in a strong magnetic field $\vec{B}$ and by lowering the temperature. The reason is that states with different magnetic quantum number M, where $-J \leq M \leq J$, have different energies in a magnetic field,

$$E(M) = E_0 - g\mu_N BM. \tag{64}$$

The relative occupancy of state M' is given by the Boltzmann-factor,

$$\frac{n(M')}{n(M)} = \frac{e^{-E(M')/kT}}{e^{-E(M)/kT}} = e^{\frac{(M-M')g\mu_N B}{kT}}. \tag{65}$$

Only the lowest level will be occupied for $kT \ll g\mu_N B$ and the nuclei become completely polarized (dependent on the sign of g, the vector $\vec{J}$ is then aligned parallel or antiparallel to $\vec{B}$). The ^{60}Co source is placed in a crystal of cerium magnesium nitrate. When this material is placed in a relatively weak external magnetic field (≈ 0.05 T), a local magnetic field in the order of 10 - 100 T will be generated by the electronic moments. The ^{60}Co then becomes polarized due to the hyperfine interaction at a temperature of about 10 mK[16].

[16]This technique is known as adiabatic nuclear demagnetization of a paramagnetic salt.

The nuclear polarization can be measured by detecting the radiation from the decay of ^{60}Ni to its ground state. For an E2 transition the angular distribution is given by $W(\theta) = \sum_{n=0}^{2} a_{2n} \times \cos^{2n}\theta$. One measures the γ-anisotropy coefficient $[W(\pi/2) - W(0)]/W(\pi/2)$ with two NaI detectors.

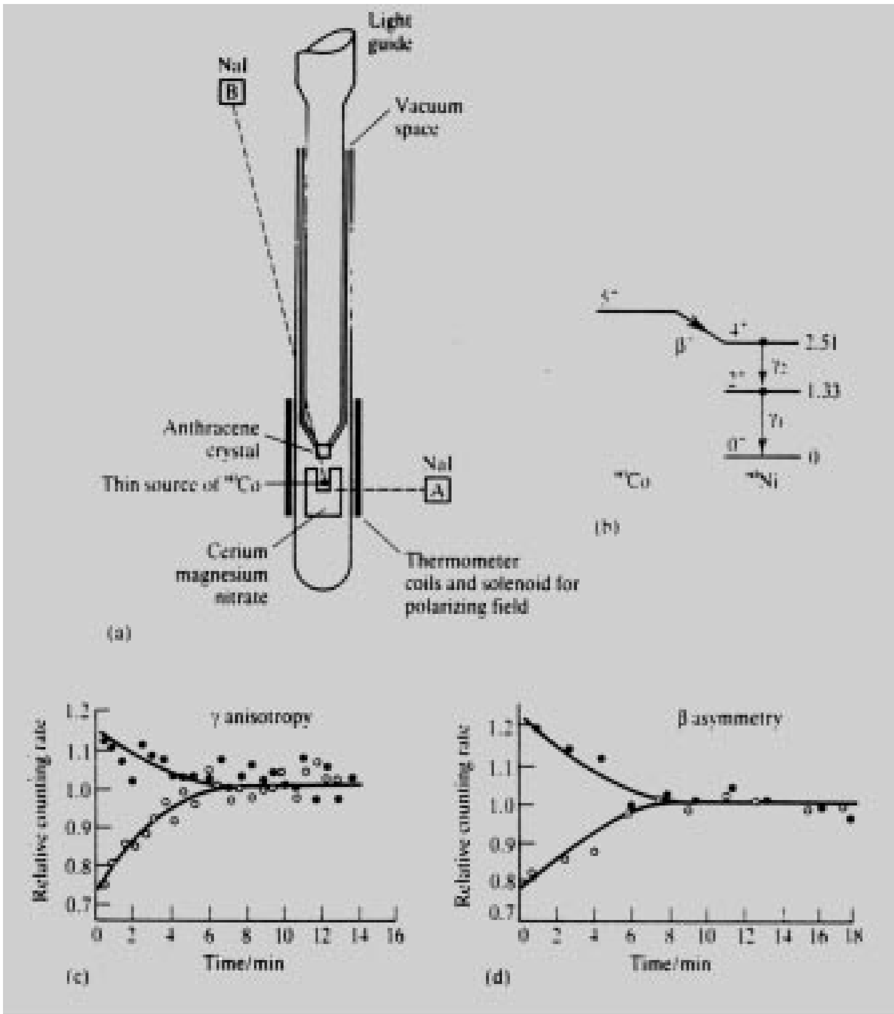

Figure 4: *(a) Experimental set up used bu Wu and collaborators to demonstrate the violation of parity in β-decay of ^{60}Co; (b) Schematic representation of ^{60}Co Gamow-Teller decay; (c) Photon asymmetry measured with detector A ($\bullet$) and detector B ($\circ$) as function of time as the crystal warms; the difference between the curves is a measure of the net polarization of the nuclei; (d) β-asymmetry in the counting rates measured with the anthracene crystal for two directions of the magnetic field ($\bullet$, down $\downarrow$; $\circ$, up $\uparrow$).*

Figure 4 shows the principle of the experimental set up and the result for the β-asymmetry. One measures the counting rate of the emitted electrons with an anthracene crystal for two different orientations of the applied external magnetic field. At sufficiently low temperatures one indeed observes an asymmetry that proves the existence of parity violation. As the radioactive material heats up, the asymmetry vanishes, since the polarization decreases (this latter fact constitutes an important systematic check).

2.3.3 Helicity of Leptons

In section 2.3.2 we have analyzed the asymmetry in electron emission for the weak decay of *polarized* nuclei. The pseudoscalar quantity

$$A = < \vec{p}_e \cdot \vec{J} >, \tag{66}$$

where $\vec{p}_e$ represents the momentum of the electron (or positron) and $\vec{J}$ is the spin of the mother nucleus, changes sign under a parity transformation. Therefore, in the case of mirror symmetry, observables cannot depend on A. However, we have seen that mirror symmetry does not hold in the weak interaction (we will discuss the electromagnetic and strong interactions later).

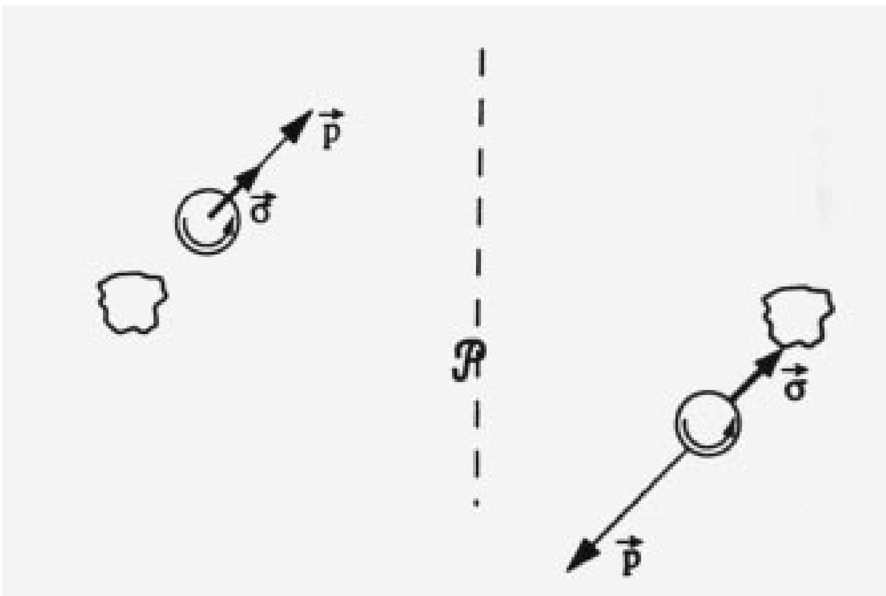

Figure 5: *Helicity of particles that are emitted by an unpolarized source. The figure on the right-hand side shows the situation after a parity transformation.*

Figure 5 shows another pseudoscalar quantity that should vanish for particle decay, in case parity is conserved: the helicity of particles that are emitted by a *non polarized* source,

$$h = < \hat{p} \cdot \hat{\sigma} > . \tag{67}$$

Here, $\hat{p}$ represents a unit vector in the direction of motion of the particle and $\hat{\sigma}$ represents the spin direction of the particle. For spins aligned along the direction of motion (righthanded circularly polarized), one has $< h >= +1$. For completely lefthanded circularly polarized particles one has $< h >= -1$.

In a brilliant experiment[15] of Goldhaber, Grodzins and Sunyar it could already in 1958 be demonstrated that the helicity of the neutrino, emitted in the weak decay of ^{152}Eu, is negative. It was found that $< h_{\nu_e} >= -1.0 \pm 0.3$.

Figure 6 shows the experimental set up and the data. After the capture of a K-electron in ^{152}Eu, first a neutrino ν_e with energy $E_\nu = 840$ keV is emitted. The decay goes to an excited state of ^{152}Sm with a life time of about $\tau_{\frac{1}{2}} = 2 \times 10^{-14}$ s. This state decays through the emission of a γ-quantum to the ground state. In the discussion

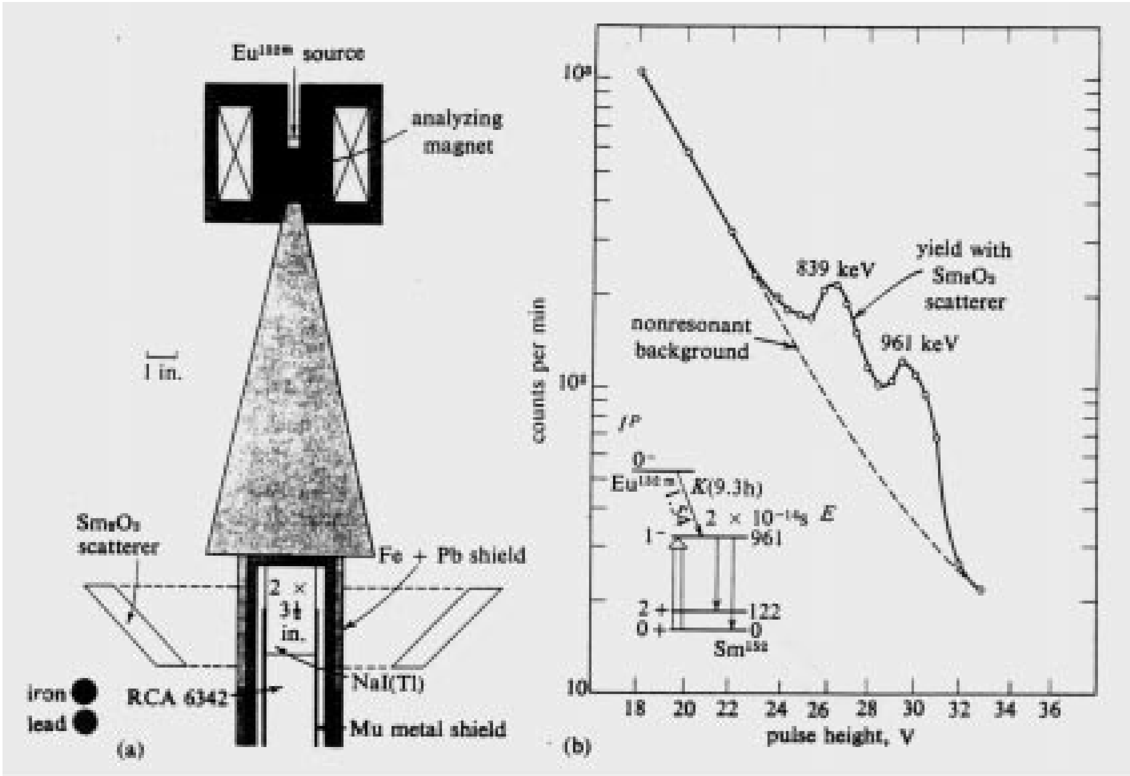

Figure 6: *Experimental set up used by Goldhaber, Grodzins and Sunyar to demonstrate that the helicity of neutrinos, emitted in the decay of* ^{152}*Eu, is negative. The analyzer magnet selects the circular polarization of the photons, The* Sm_2O_3 *scatters through nuclear fluoresence radiation to the NaI detector.*

of this experiment, we first make the assumption that the neutrino is emitted in the 'upward' direction (positive z-axis) and that the γ-quantum is emitted 'downwards'. From conservation of angular momentum in the z-direction it then follows that the γ-quantum must be lefthanded circularly polarized, when the helicity of the ν_e is negative (and vice versa). The γ-quantum traverses a piece of magnetized iron (with the magnetic field direction of $\vec{B}$ parallel or antiparallel to the z-direction). The absorption is different for right- and lefthanded circularly polarized γ-quanta. It turns out that indeed $\sigma_\gamma = -1$ and consequently that $\sigma_\nu = -\frac{1}{2}$. However, how can we ascertain our initial assumption that the neutrino was emitted in the 'upward' direction and that its helicity is negative? This is possible by resonance scattering from a ^{152}Sm scatterer. Only when the neutrino is emitted in the upward direction, and the excited nucleus travels downward, the energy of the γ-quantum has exactly the right value to excite the 961 keV level.

Since 1958 a large number of experiments has been carried out, that all reveal the helicity of the leptons emitted in β-decay of nuclei to be always as follows:

- all neutrinos (ν_e, but also ν_μ and ν_τ) have a helicity -1, and all antineutrinos ($\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) have a helicity +1.
- The charged leptons (e^-) emitted in β-decay have a helicity $-v/c$, while the antiparticles (e^+) have helicity $+v/c$.

These observations are in agreement with the standard model of the electroweak interaction. Every deviation would be a sensation, because it would be an indication that besides the usual lefthanded vector bosons ($W_L^\pm$) also righthanded particles ($W_R^\pm$) would exist. Because of their larger mass these particles would not have been produced so far with the existing particle accelerators.

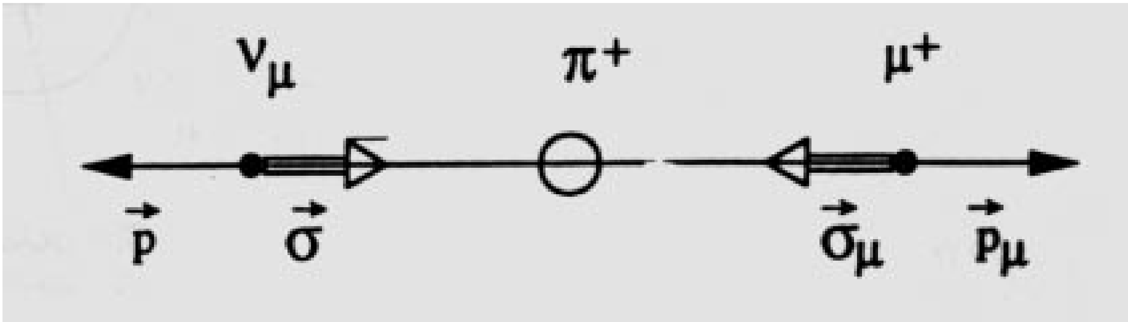

Figure 7: *Schematic representation of the helicities in the decay of a positively charged pion into a muon and muon neutrino.*

Next, we want to discuss the interesting case of the helicity suppressed decay of charged pions, for example

$$\begin{array}{lll} \pi^+ & \to \mu^+ + \nu_\mu, & \text{B.R.} = 0.999878 \\ \pi^+ & \to e^+ + \nu_e, & \text{B.R.} = 1.2 \times 10^{-4}. \end{array} \tag{68}$$

The charged leptons have, because of conservation of angular momentum, in a sense the 'wrong' helicity (see figure 7). Next, we calculate the decay rates while only taking the phase space factors into account. We assume that the matrix elements are equal in both cases. However, in this way we obtain an incorrect result,

$$\frac{\lambda_e}{\lambda_\mu} = \frac{1 + (m_e/m_\pi)^2}{1 + (m_\mu/m_\pi)^2} \cdot \frac{1 - (m_e/m_\pi)^2}{1 - (m_\mu/m_\pi)^2} \simeq 3.5(\quad \text{incorrect!}). \tag{69}$$

Only when we multiply the above expression with the correction factor

$$f = \frac{1 - v_e/c}{1 - v_\mu/c} = \frac{m_e^2}{m_\mu^2} \frac{1 + (m_\mu/m_\pi)^2}{1 + (m_e/m_\pi)^2} = 3.7 \times 10^{-5}, \tag{70}$$

do we obtain the correct result. This implies that this exception confirms the rule, or formulated more precisely, the fact that the corrected result agrees so well with the measured value, indeed represents and important test for the nature of the interaction (a pure V - A coupling, as demanded by the standard model with only lefthanded $W^\pm$) that lies at the basis of the decay.

2.3.4 Conservation of Parity in the Strong Interaction

Conservation of parity in the strong interaction has been verified in a great number of experiments. One of the most precise experiments[16] was performed using the experimental set up outlined in figure 8.

The injector cyclotron delivers a transverse polarized proton beam with an energy of T_p = 50 MeV, an intensity of about 5 μA and a polarization P_y of 0.8. With spin

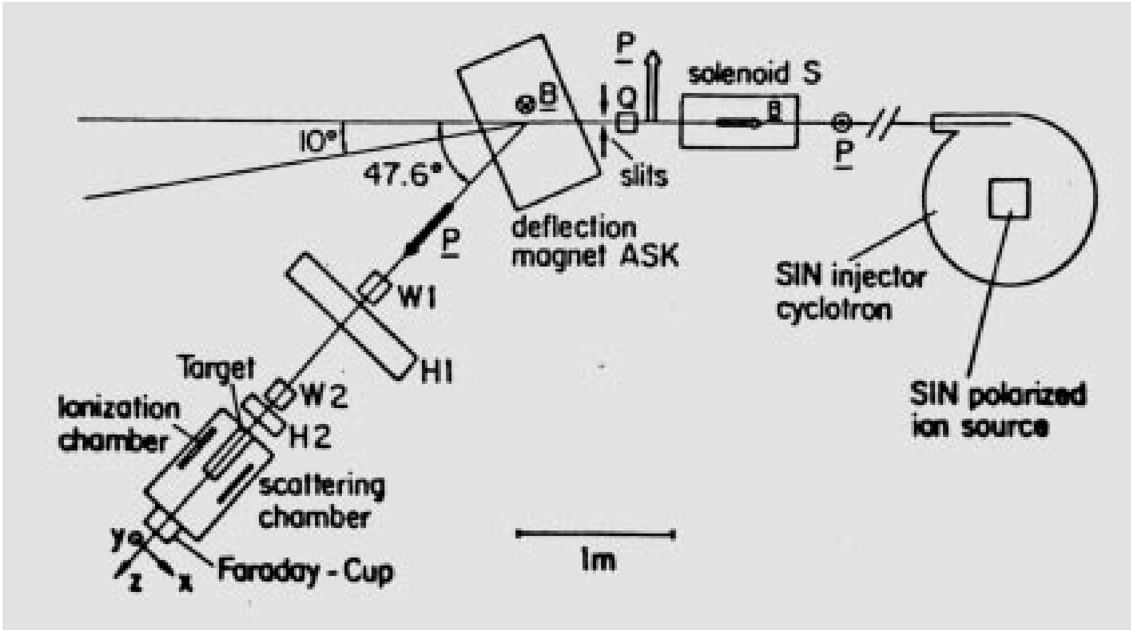

Figure 8: *Experimental set up for the measurement of parity violation in proton-proton scattering. Longitudinal polarized protons with an energy of 50 MeV are scattered from hydrogen.*

precession in various magnetic fields one obtains a longitudinally polarized beam, that is subsequently scattered from a hydrogen target. Conservation of parity demands that the cross section for protons with positive helicity σ^+ is equal to that of protons with negative helicity σ^-. The experiment yields the result

$$\frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = (-1.5 \pm 0.2) \times 10^{-7}. \tag{71}$$

The small deviation from zero is of the same order as we would expect on theoretical grounds. The quarks and thus also the nucleons experience *in addition to the strong force* also a weak interaction, and this latter interaction maximally violates parity. The corresponding strength is about 10^{-7} weaker compared to the dominant strong interaction.

Turning the argument around we sometimes can exploit the fact that parity is conserved in the strong interaction in order to determine the *intrinsic parity* of a particle. As an example we discuss the manner in which the parity of the negatively charged pion, $\mathcal{P}_\pi$, can be determined using the reaction

$$\pi^- + d \to n + n. \tag{72}$$

We assume that we know the spins of all particles participating in the reaction,

$$J_\pi = 0, \quad J_d = 1, \quad J_n = \frac{1}{2}. \tag{73}$$

In addition, we know the intrinsic parity of the deuteron[17], $\mathcal{P}_d = +1$.

[17] The deuteron consists of a proton and a neutron, that are bound mainly in an S-state with orbital angular momentum $l_{pn} = 0$.

When the π^- is captured by a deuterium nucleus, then at first states with orbital angular momentum $l_{\pi d} \neq 0$ will be occupied. However, the pionic deuterium will rapidly decay to a state with $l_{\pi d} = 0$, where characteristic Röntgen radiation will be emitted. It is possible to detect these photons and in this way to experimentally determine that after the capture of a negative pion in an S-state, the above discussed reaction indeed occurs. The total angular momentum then amounts to

$$|\vec{J}_{\rm tot}| = |\vec{l}_{\pi d} + \vec{J}_\pi + \vec{J}_d| = 1 = |\vec{l}_{nn} + \vec{J}_n + \vec{J}_n|, \tag{74}$$

and the parity is $\mathcal{P}_{\rm tot} = \mathcal{P}_\pi \cdot \mathcal{P}_d \cdot (-1)^{l_{\pi d}} = \mathcal{P}_\pi = (\mathcal{P}_n)^2\ (-1)^{l_{nn}}$.

Since the wave function of both neutrons needs to be antisymmetric, the reaction only proceeds through a $^3P_1$-state with $l_{nn} = 1$. Consequently, we find that $\mathcal{P}_\pi = -1$. Also both other partner pions, π^+ en π^0, of the same isospintriplet ($T_\pi = 1$) have a negative intrinsic parity.

2.4 Charge Symmetry

2.4.1 Particles and Antiparticles, $\mathcal{CPT}$-Theorem

Starting from the relativistic relation between energy and momentum of a particle, $E^2 = p^2c^2 + m^2c^4$, we can write the corresponding relativistic wave equation[18] by substituting the following operators

$$p \rightarrow \frac{\hbar}{i}\nabla \quad \text{en} \quad E \rightarrow i\hbar\frac{\partial}{\partial t}. \tag{75}$$

We find the so-called Klein-Gordon equation

$$\left[\Box + \left(\frac{mc}{\hbar}\right)^2\right]\psi(\vec{r},t) = 0, \tag{76}$$

where

$$\Box \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta, \quad \text{met} \quad \Delta = \nabla\cdot\nabla. \tag{77}$$

We find two solutions for the energy,

$$E^\pm = \pm\sqrt{m^2c^4 + p^2c^2}. \tag{78}$$

It turns out that the solution with the negative sign cannot just be wiped 'unter den Teppich', as was originally planned by for example Schrödinger. Paul Dirac[18] had already in 1930 postulated that this second solution represents the motion of an antiparticle. The existence of the antiparticle of the electron, the positron, was demonstrated experimentally in 1933 by Anderson[19].

Next, we discuss a simplified heuristic train of thought with the intention to show how one can arrive at an interpretation. The wave function with positive energy,

$$\psi^+(x,t) = \exp\left\{\frac{i}{\hbar}(px - E^+t)\right\}, \tag{79}$$

[18] This yields the so-called Klein-Gordon equation which is valid for spinless particles. Note that the fundamental building blocks of the subatomic world all have spin-$\frac{1}{2}$.

represents a wave with positive phase velocity

$$v_p^+ = \frac{E^+}{p}. \tag{80}$$

The maximum of the wave (we mean here the particle)[19] travels along the positive x-axis for increasing time t.
For the solution with negative energy we can write

$$\begin{aligned} \psi^-(x,t) &= \exp\left\{\tfrac{i}{\hbar}(px - E^- t)\right\} \\ &= \exp\left\{\tfrac{i}{\hbar}(px - E^+(-t))\right\}. \end{aligned} \tag{81}$$

When we reverse the direction of time[20], $t \to -t$, we can easily convince ourselves, that the same result would be obtained in case we would reverse the charge. Therefore, it seems that second solution represents the normal motion of an antiparticle.

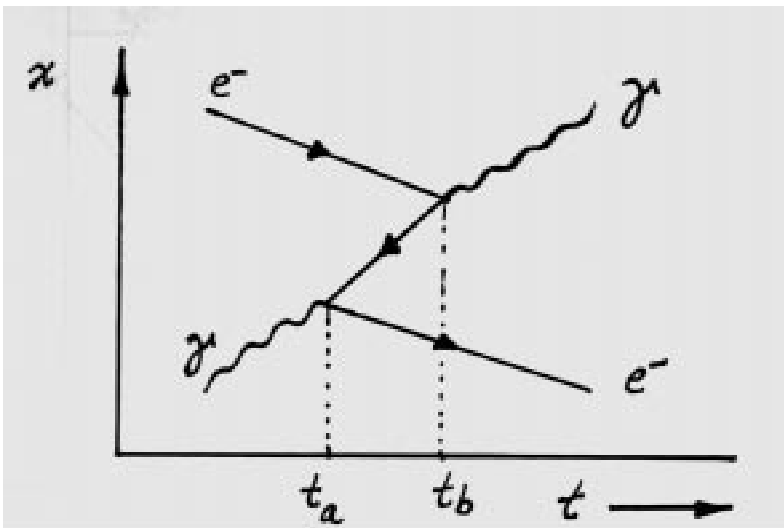

Figure 9: *Motion of a particle and antiparticle in the Feynman diagram for photon-electron scattering.*

For the scattering of a γ-quantum from an electron, we can interpret the Feynman diagram sketched in figure 9 as follows.

- For $t < t_a$ we see an electron and photon approaching each other.
- At time $t = t_a$ an electron-positron pair is created.
- During the period $t_a < t < t_b$ two electrons and one positron exist. Note that the total charge did not change.
- At time $t = t_b$ the electron-positron pair annihilates.
- For $t > t_a$ we see the scattered electron and photon moving apart.

[19] In order to make the discussion more explicit, one should introduce a wave packet at this point and work with the group velocity.

One can easily verify that when only electromagnetic forces are acting, always the same law of motion is valid (because it is invariant for the operations $\mathcal{P}$, $\mathcal{C}$, and $\mathcal{T}$). However, we know that the operation $\mathcal{P}$ is violated in the weak interaction. Also for this interaction we can obtain 'an almost' invariant law of motion after carrying out a $\mathcal{CP}$-transformation[20]. It can be shown that under quite general assumptions the combined operation $\mathcal{CPT}$ always commutes with $\mathbf{H}$, for *all* interactions. From this it follows that a particle and an antiparticle always need to have exactly the same mass and life time. However, for all additive quantum numbers (see tabel 1), a reverse of sign occurs.

Table 1: *Relation between the most important properties and quantum numbers for particles and their associated antiparticles.*

Quantity	Particle	Antiparticle
Mass	m	m
Life time	τ	τ
Spin	J	J
Isospin	T	T
Isospin (z-component)	T_z	$-T_z$
Charge	Q	$-Q$
Strangeness	S	$-S$
Charm, etc.	$\tilde{C}$	$-\tilde{C}$
Intrinsic parity (fermion)	π	$-\pi$
Intrinsic parity (boson)	π	π
Baryon number	B	$-B$
Lepton number	$L_e,\ L_\mu,\ L_\tau$	$-L_e,\ -L_\mu,\ -L_\tau$

Presently, for almost all particles the corresponding antiparticle is known. The existence of the antiproton was demonstrated in 1955 in Berkeley, after searching in vain in cosmic radiation. Also it proved possible (see figure 10) to demonstrate[21] the existence of exotic particles like the anti-Ω. The mechanism of the production is as follows (the difficulty is hopefully clear: one needs to produce a particle with strangeness $S = 3$!)

$$\begin{array}{rcccccccccccccccccc} & & K^+ & + & d & \rightarrow & \overline{\Omega}^+ & + & \Lambda & + & \Lambda & + & p & + & \pi^+ & + & \pi^- \\ B & : & 0 & + & 2 & \rightarrow & -1 & + & 1 & + & 1 & + & 1 & + & 0 & + & 0 \\ S & : & 1 & + & 0 & \rightarrow & 3 & - & 1 & - & 1 & + & 0 & + & 0 & + & 0. \end{array} \tag{82}$$

The Ω^- mainly decays into a Λ and a K^-. In this decay the strangeness changes by $\Delta S = 1$. The decay of the $\overline{\Omega}^+$, as indicated in figure 10, proceeds as follows

$$\overline{\Omega}^+ \rightarrow \overline{\Lambda} + K^+. \tag{83}$$

Both decays are induced by the weak interaction. The strangeness changes and the life time of the anti-Ω is remarkably large, $\tau \sim 8.2 \times 10^{-11}$ s.

[20] We will discuss this small violation of $\mathcal{CP}$ in great detail later. It has been observed in the decay of neutral K-mesons.

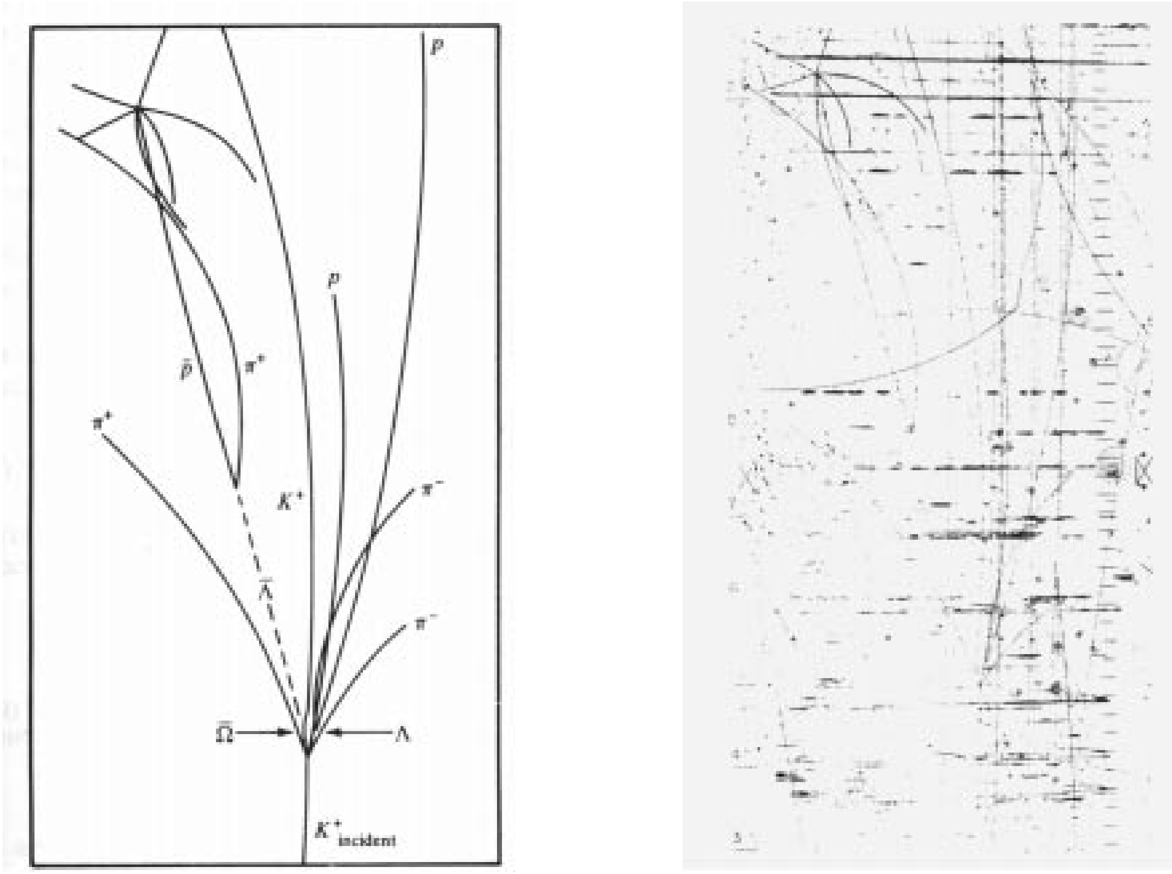

Figure 10: *Sketch (left) and bubble chamber picture (right) for the reaction $K^+d \rightarrow \overline{\Omega}\Lambda\Lambda p\pi^+\pi^-$ observed in the study of K^+d-interactions at a momentum of 12 GeV/c using the two-mile linear accelerator of the Stanford Linear Accelerator Center.*

Each of these examples shows how fruitful the concept of antiparticle is. In the following we use the operator $\mathcal{C}$ to transform a particle into its corresponding antiparticle (note that now not only the charge changes sign). One has

$$\mathcal{C}|u>=|\bar{u}>, \quad \mathcal{C}|d>=|\bar{d}> . \tag{84}$$

For a small number of particles (for example γ and π^0) all values, that under a $\mathcal{C}$-operation would change sign, are equal to zero. In these cases one cannot distinguish particle and antiparticle, and consequently they are the same object. When we act with the operator for charge conjugation on a charged pion,

$$\mathcal{C}|\pi^+>\rightarrow|\pi^->\neq\eta|\pi^+>, \tag{85}$$

we clearly do *not* obtain an eigenstate. However, for the neutral pion the situation is different

$$\mathcal{C}|\pi'>=\eta|\pi'> . \tag{86}$$

When we act a second time with $\mathcal{C}$, we again obtain the initial state. From this it follows that $\eta^2=1$ and thus $\eta=\pm1$. Therefore, the quantity η is denoted as $\mathcal{C}$-parity in analogous fashion to the normal parity. Also there we did already conclude that some states have a well defined parity, while other do not.

For a situation with only electromagnetic fields a change of sign occurs under a $\mathcal{C}$-transformation: therefore the photon has negative $\mathcal{C}$-parity,

$$\mathcal{C}|\gamma>= -|\gamma> . \tag{87}$$

The neutral pion decays with a life time of 8.4×10^{-6} s into two photons. The decay $\pi^0 \rightarrow 3\gamma$ does not occur (B.R. $< 3.1 \times 10^{-8}$). This indicates that the electromagnetic interaction is $\mathcal{C}$-invariant. Since $\mathcal{C}$-parity yields a multiplicative quantum number, the $\mathcal{C}$-parity of the π^0-meson needs to be positive, $\mathcal{C}\ |\pi^0>= +|\pi^0>$. An entire series of follow-up experiments reveal that the strong and the electromagnetic interactions are $\mathcal{C}$-invariant. In contrary, we find that the weak interaction is *almost* invariant under the combined operation $\mathcal{CP}$ (apart from small deviations that we will discuss later). Since $\mathcal{P}$ is maximally violated in the weak interaction, also $\mathcal{C}$ needs to be maximally violated.

2.4.2 Charge Symmetry of the Strong Interaction

It has been known for a long time in nuclear physics that the proton and the neutron are similar particles, when one neglects the electromagnetic interaction. First it strikes us that both masses are almost equal,

$$\frac{m_n - m_p}{m_n + m_p} = 7 \times 10^{-4}. \tag{88}$$

Furthermore, we know that so-called mirror nuclei (these are nuclei where all neutrons are transformed into protons and vice versa) have similar properties, such as similar level schemes, good agreement between binding energies after we correct for the electromagnetic interaction, and so on.

In addition, the cross sections for mirror reactions, such as

$$\begin{array}{llllll} \sigma(n+^3\mathrm{He} & \rightarrow & ..) & \simeq \sigma(p+^3\mathrm{H} & \rightarrow & ..) \\ \sigma(d+d & \rightarrow & n+^3\mathrm{He}) & \simeq \sigma(d+d & \rightarrow & p+^3\mathrm{H}) \end{array} \tag{89}$$

are almost equal. It seems natural to introduce an operator $\mathbf{C_s}$ (for charge symmetry), that changes a proton into a neutron and the other way around. However, it is more efficient to define the actions of $\mathbf{C_s}$ for quark states.

$$\begin{array}{llll} \mathbf{C_s}|u> & = -|d\ >, & \mathbf{C_s}|d> & = +|u\ > \\ \\ \mathbf{C_s}|\bar{u}> & = -|\bar{d}\ >, & \mathbf{C_s}|\bar{d}> & = +|\bar{u}\ > . \end{array} \tag{90}$$

If $\mathbf{C_s}$ is indeed a good symmetry of the strong interaction, then it should be possible, on the basis of the above assumptions, to predict the equality of decay rates and cross sections for reactions involving exotic particles. To illustrate this concept we start with the reaction

$$\pi^- + p \rightarrow \Lambda + K^0. \tag{91}$$

When we carry out a charge symmetry transformation, we obtain

$$\begin{array}{llllll} \mathbf{C_s}|\pi^-> & = \mathbf{C_s}|\bar{u}d> & = - & |\bar{d}u> & = - & |\pi^+> \\ \mathbf{C_s}|p> & = \mathbf{C_s}|uud> & = & |ddu> & = & |n> \\ \mathbf{C_s}|\Lambda> & = \mathbf{C_s}|uds> & = - & |dus> & = - & |\Lambda> \\ \mathbf{C_s}|K^0> & = \mathbf{C_s}|d\bar{s}> & = & |u\bar{s}> & = & |K^+>, \end{array} \tag{92}$$

and expect that the cross section for the following reaction is exactly equal,

$$\pi^+ + n \rightarrow \Lambda + K^+. \tag{93}$$

Obviously, the electromagnetic interaction breaks this symmetry! However, after we carry out all electromagnetic corrections, as good as we can calculate these at present, there still remains a small discrepancy. This is especially clear when we look at the masses,

$$\begin{array}{ll} m_n & > m_p \\ m_{K^0} & > m_{K^+} \\ m_{\Sigma^+} & < m_{\Sigma^0} < m_{\Sigma^-} (!) \end{array} \tag{94}$$

In itself it is remarkable that the charged particles are often *lighter* than the corresponding neutral particles. This small *symmetry breaking* is attributed[17] to the difference in mass[21] of the up and down quarks,

$$m_d - m_u = 3.3 \pm 0.3 \;\; \mathrm{MeV/c^2}. \tag{95}$$

For completeness we mention here **G**-conjugation, which is closely connected to charge symmetry[22],

$$\mathbf{G} \equiv \mathbf{C_s C}, \tag{96}$$

where **C** represents the charge symmetry operator.

2.5 Invariance under Time Reversal

In classical physics we have determined that Newton's law of motion,

$$\vec{F} = m \frac{d^2 \vec{r}}{dt^2}, \tag{97}$$

is invariant under reflection of the time axis, $t \rightarrow -t$. All particle orbits can just as well be traversed in opposite direction. When we observe a few particles in a microscopic system (see figure 11), we would not be able to distinguish whether a movie, on which all orbits and collisions are recorded, is projected in the forward or backward direction. When the probability for the two processes shown in figure 11 is equal, we talk about invariance under time reversal, also know as *microscopic reversibility*. All this directly changes when we study a macroscopic system, where irreversible processes occur, such as friction, heat conductivity, or diffusion (in the equations that describe these processes also first-order derivates of time occur). Clearly, in nature there exists a preferred direction for time[23] - only outgoing waves - a violation of elementary time reversal. In the following we will not discuss such phenomena, but instead consider whether also in elementary collision processes there is a preferred direction for time ($\mathcal{T}$-invariance of the interactions).

[21]It is non trivial to determine the masses of quarks, since there are no free quarks. In general, one finds in the literature the values of the so-called *current masses*. These are the values that one should use in the QCD Lagrangian. Since quarks are surrounded by a cloud of gluons and quark-antiquark pairs, the mass depends on the energy of the reaction, and can be calculated in QCD. Usual values at a momentum transfer of about 1 GeV/c, amount to $m_u \simeq 5.5 \pm 0.8$ MeV and $m_d \simeq 9.0 \pm 1.2$ MeV.

[22]This is also relevant for the discussion of the weak interaction. It turns out that no strangeness conserving semileptonic decays exist that violate G-parity (so-called second-order currents). These do not exist in the standard model and have also not been seen in experiment.

[23]However, it is not at all obvious whether and how the different factors that determine the direction of time are connected: increase in entropy - expansion of the universe. '*It is a poor memory that remembers only backwards*' (Alice in Wonderland).

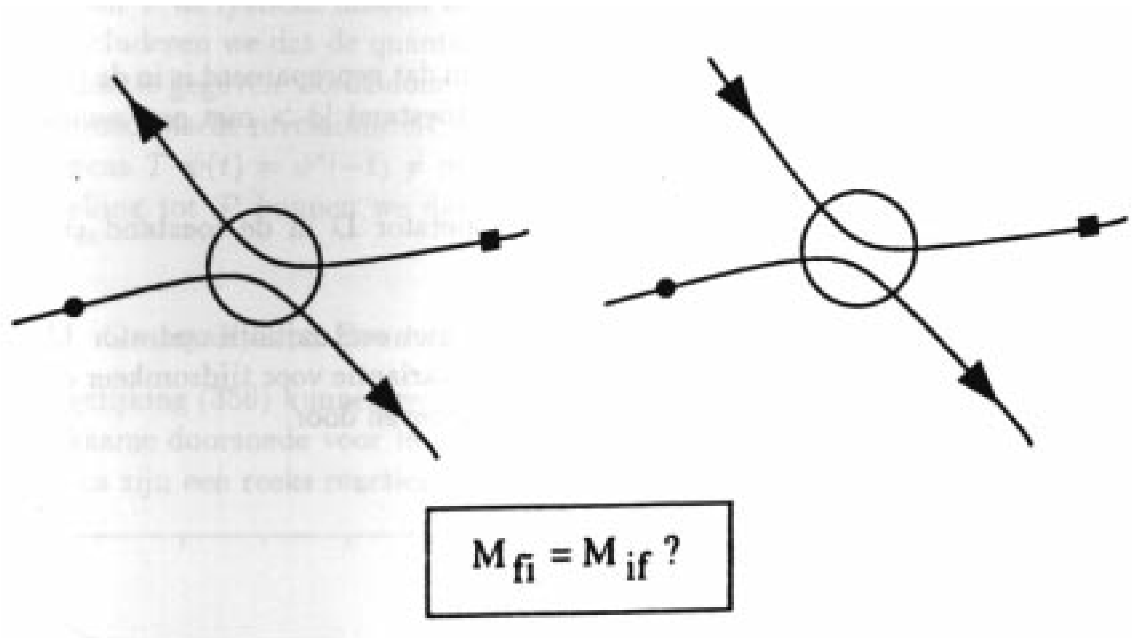

Figure 11: *Schematic representation of time reversal invariance in a two-particle collision.*

Shortly after Lee and Yang expressed in 1956 their presumption that parity $\mathcal{P}$ might not be conserved in some processes, it was shown in many experiments that this fundamental symmetry (together with charge conjugation) is maximally broken in *all* transitions that are induced by the weak interaction. With time reversal we are dealing with a completely different situation. Small violations of $\mathcal{CP}$ (or $\mathcal{T}$) were only discovered in 1964 in the decay of neutral K-mesons. Since then one did not succeed in finding a single other system where a violation under time reversal occurs, in spite of the significant effort in looking for such effects[24]. Although violation of time reversal can be accomodated in the standard model, one is completely in the dark with respect to the mechanism of this small symmetry reaking. Also at this moment various experiments are performed (or prepared) to determine the boundaries of the violation under time reversal[25]. In the following we will consider a few nuclear physics experiments more closely. Modern particle physics experiments will be discussed later.

In a time reversal transformation the position, momentum and spin of a particle change as follows,

$$\begin{aligned} \vec{\mathbf{r}}' &= \mathcal{T}\,\vec{\mathbf{r}}\,\mathcal{T}^{-1} &&= \quad \vec{\mathbf{r}} \\ \vec{\mathbf{p}}' &= \mathcal{T}\,\vec{\mathbf{p}}\,\mathcal{T}^{-1} &&= - \ \vec{\mathbf{p}} \\ \vec{\mathbf{J}}' &= \mathcal{T}\,\vec{\mathbf{J}}\,\mathcal{T}^{-1} &&= - \ \vec{\mathbf{J}}. \end{aligned} \tag{98}$$

When we want to describe this transformation, we need to remember that in quantum mechanics there are two types of observables.

- When we perform a measurement on a system that has been prepared in state $|\psi>$,

[24]There is one other 'experiment', which however is somewhat difficult to repeat: the Big Bang. Presently, one assumes that equal amounts of particles and antiparticles were created in the beginning. The presently observed asymmetry (probably no galaxies exist that are composed entirely of antimatter) can be 'explained' by the mechanism of $\mathcal{CP}$-violation.

[25]This scientific question determines for an important part the future research program at NIKHEF. The B physics group participates in the LHCb experiment at CERN, with the goal to investigate $\mathcal{CP}$-violation in the decay of B-mesons.

we can find the system in state $|\phi>$ with a probability $|<\phi|\psi>|^2$.

- The expectation value of a dynamical operator $\mathbf{D}$ in state ψ is given by the matrix element $<D>=<\psi|\mathbf{D}|\psi>$.

The operation $t \to -t$ cannot be represented by a unitary operator $\mathbf{U}$. This can be shown with the following argument. Time reversal invariance demands that $\mathbf{U}^\dagger\mathbf{H}\mathbf{U} = \mathbf{H}$. The time evolution of the wave function is given by

$$\mathbf{H}|\psi>= i\hbar\frac{\partial}{\partial t}|\psi> . \tag{99}$$

After the transformation we find

$$\mathbf{U}\mathbf{H}|\psi>= \mathbf{U}i\hbar\frac{\partial}{\partial t}|\psi>= -i\hbar\frac{\partial}{\partial t}\mathbf{U}|\psi>, \tag{100}$$

because $\mathbf{U}$ represents the change $t \to -t$. Since $\mathbf{UH} = \mathbf{HU}$ we find

$$\mathbf{H}\mathbf{U}|\psi>= -i\hbar\frac{\partial}{\partial t}\mathbf{U}|\psi> . \tag{101}$$

One has $|\psi'>= \mathbf{U}|\psi>$ and thus we see that $|\psi'>$ does not obey the same Schrödinger equation, but instead

$$\mathbf{H}|\psi'>= -i\hbar\frac{\partial}{\partial t}|\psi'> . \tag{102}$$

However, note that the wave function is not an observable and that the symmetry can be restored by using a different definition. It is possible to represent the transformation for time reversal, $\mathcal{T}$, as the product of a unitary operator $\mathbf{U}_\mathcal{T}$ and an operator $\mathbf{K}$, where $\mathbf{K}$ implies the transition to the complex geconjugated quantity,

$$\mathcal{T} = \mathbf{U}_\mathcal{T} \cdot \mathbf{K}. \tag{103}$$

For such an antiunitary transformation one has that when $|\psi(t)>$ is a solution of the Schrödinger equation, then $|\psi^*(-t)>$ will also be a solution. This can be shown as follows

$$\mathcal{T}\mathbf{H}|\psi>= \mathbf{H}\mathcal{T}|\psi>= -i\hbar\mathcal{T}\frac{\partial}{\partial t}|\psi> = i\hbar\frac{\partial}{\partial t}\mathcal{T}|\psi> . \tag{104}$$

Wigner noted that the operator $\mathcal{T}$ is antilinear,

$$\mathcal{T}(C_1|\psi> + C_2|\phi>) = C_1^*\mathcal{T}|\psi> + C_2^*\mathcal{T}|\phi>, \tag{105}$$

and antiunitary,

$$<\psi'|\phi'>=<\psi\mathcal{T}^\dagger|\mathcal{T}\phi>=<\psi|\phi>^*=<\phi|\psi> . \tag{106}$$

However, note that, because of

$$|<\psi'|\phi'>| = |<\phi|\psi>| = |<\psi|\phi>|, \tag{107}$$

the operator $\mathcal{T}$ leaves the physical content of quantum mechanics untouched. In summary, we conclude that the quantum mechanical equivalent of the classical time reversal transformation is given by $t \to -t$ en $i \to -i$. The quantum mechanical principle of microscopic reversibility obeys this transformation.

Since $\mathcal{T}\ \psi(t) = \psi^*(-t) \neq \eta\psi(t)$ there are no observable eigenvalues of $\mathcal{T}$. Contrary to $\mathcal{P}$ we cannot search for $\mathcal{T}$-allowed or forbidden transitions[22].

2.5.1 Principle of Detailed Balance

From equation (106) we can directly deduce the *principle of detailed balance*. The cross section for inverse reactions is equal, when we correct for differences in phase space factors. In nuclear physics a series of reactions and their corresponding reverse reactions has been carefully measured.

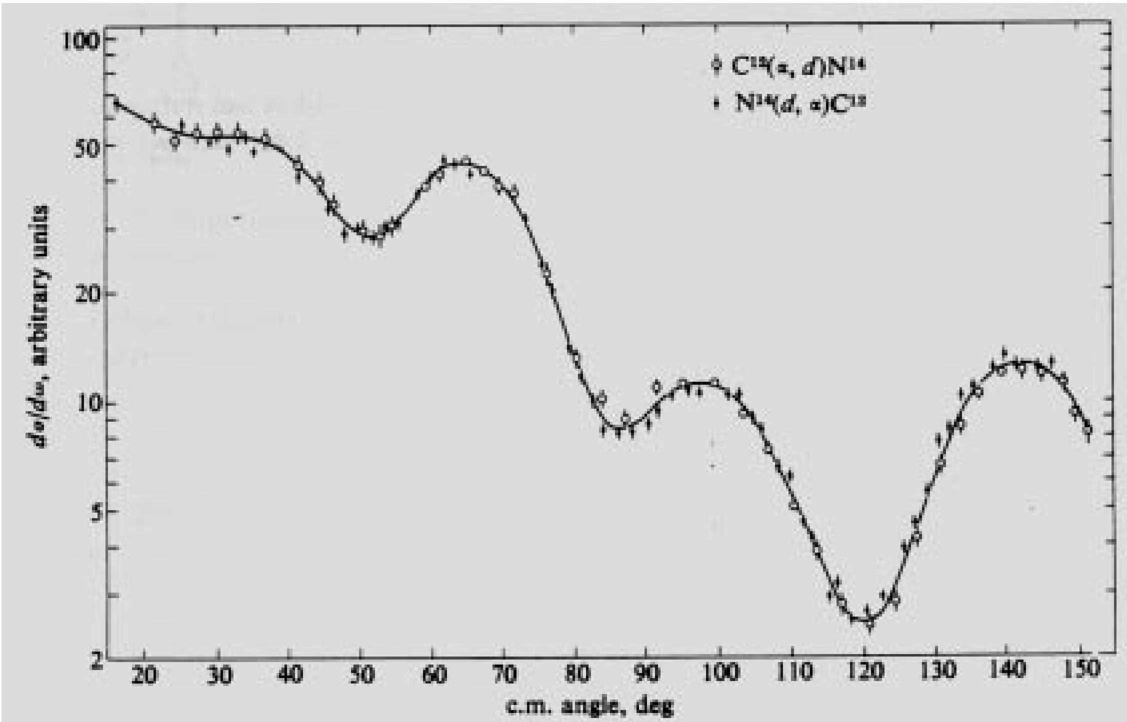

Figure 12: *Comparison of the cross section for the reaction* $^{14}N(d,\alpha)^{12}C$ *and its inverse reaction* $^{12}C(\alpha,d)^{14}N$.

In figure 12 we give an example of this. One has always found that within the precision of the measurements one has

$$\frac{d\sigma/d\Omega(a+b\rightarrow A+B)}{d\sigma/d\Omega(A+B\rightarrow a+b)}=\underbrace{\frac{p_{AB}^2}{p_{ab}^2}\ \frac{(2J_A+1)(2J_B+1)}{(2J_a+1)(2J_b+1)}}_{\text{Phase space factor}}\ \underbrace{\frac{|T_{ab\rightarrow AB}|^2}{|T_{AB\rightarrow ab}|^2}}_{=1\ !}. \qquad (108)$$

2.5.2 Electric Dipole Moment of the Neutron

It has been discovered[23] that the charge distribution of a neutron does not vanish (there is no need for it to vanish since baryons are composed of charged quarks). Therefore, one can speculate that the neutron not only has a magnetic dipole moment (this is the case, $\mu_n = (1.041\ 875\ 6 \pm 0.000\ 000\ 3) \times 10^{-3}\ \mu_{\mathrm{Bohr}}$), but also a static electric dipole moment. The orientation of a particle can be specified by the orientation of its spin with respect to an axis. When we chose the z-as for this, the electric dipole moment is given by

$$\mu_e = \int \rho z dV, \qquad (109)$$

where ρ represents the charge distribution of the particle.

For the neutron one can think a priori about the following order of magnitude

$$d_n = \mu_e^n \sim e \times 10^{-13} \ \ \text{cm.} \tag{110}$$

However, one can easily contemplate[26], that the dipole moment vanishes, when the interactions are parity invariant. One then has $\phi_n(x, y, z) = \pm\phi_n(-x, -y, -z)$, resulting in an expectation value of $\mathbf{z}$,

$$< z >= z_{nn} = \int \phi_n^* z \phi_n dV, \tag{111}$$

that vanishes. However, this is not always the case, since we need to account for the contribution of the weak interaction, which is approximately 10^{-7} times smaller than that of the strong interaction. This leaves the possibility that

$$d_n \sim e \times 10^{-20} \ \ \text{cm.} \tag{112}$$

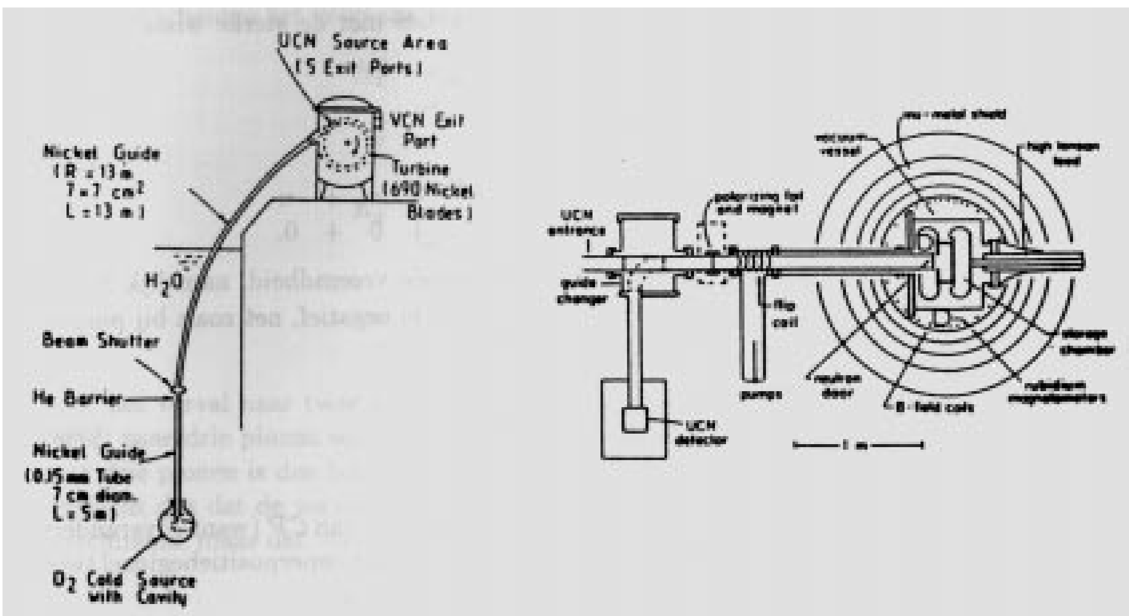

Figure 13: *Experimental set up used for the determination of the electric dipole moment of the neutron.*

Moreover, the dipole moment also vanishes due to time reversal invariance. A measurement of d_n with the required high precision will yield indications about the magnitude of a possible violation under invariance of reversal of time and parity. Various theoretical models predict values in the order of

$$d_n \sim e \times 10^{-25} \ \ \text{cm ... } e \times 10^{-33} \ \ \text{cm.} \tag{113}$$

[26]We want to stress here that these arguments do not hold in case we are dealing with a system that has several degenerate energy states (something that is not the case for a neutron). In such a system it is possible that an antisymmetric charge distribution develops, which in turn, when placed in an electric field, leads to an alignment (in the interplay of collisions with other molecules) and an energy splitting $\vec{\mu}_e \cdot \vec{E}$. Many molecules indeed have a large electric dipole moment.

The most precise experiments yield

$$d_n < 1.1 \times 10^{-25}\ e \cdot \mathrm{cm},\ \text{conf. level} = 95\%. \tag{114}$$

To reach this incredible precision a series of cunning tricks is applied. Next, we describe a few aspects of an experiment at ILL Grenoble[24].

From a high-flux reactor ($f_{\text{therm}} = 4.5{\times}10^{14}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$), neutrons are extracted vertically from a cold source (liquid deuterium at 25 K). Through total reflection and a Doppler-shift turbine, one obtains ultra-cold neutrons ($v < 8$ m/s!) with a density of about 60 n·cm^{-3}, which can be 'stored' in a bottle for a limited time. These polarized neutrons have an interaction hamiltonian in an electromagnetic field, given by

$$H_{\text{int}} = \mu_n\ \vec{\sigma} \cdot \mathbf{B} + d_n\ \vec{\sigma} \cdot \mathbf{E}, \tag{115}$$

where $\vec{\sigma}$ represents the spin, and μ_n the magnetic and d_n the electric dipole moment of the neutron. Consequently, the neutrons precess with a frequency

$$h\nu = -\mu_n^n|\vec{B}| \pm 2d_n|\vec{E}|. \tag{116}$$

Note that an electric field of $E = 10$ kV/cm with a hypothetic $d_n = 10^{-25}$ e·cm would result in less than one revolution per week! This small effect can be measured using Ramsey-resonance techniques[25]. However, up to now all results are compatible with zero within the uncertainties of the measurements.

2.5.3 Triple Correlations in β-Decay

In the β-decay of a polarized nucleus one has for the decay rate

$$W = W_0 \left\{ 1 + A\vec{v}_e \cdot J + D\vec{J} \cdot (\vec{v}_e \times \vec{v}_\nu) + R\vec{\sigma}_e(\vec{J} \times \vec{v}_e) \right\}. \tag{117}$$

Here, $\vec{J}$ represents the nuclear polarization, $\vec{v}_e$ and $\vec{v}_\nu$ are the velocities of the emitted electron and neutrino (in units of c) and $\vec{\sigma}_e$ represents the direction in which the polarization of the electron is measured.

The asymmetry is characterized by the asymmetry parameter A, that plays a role in parity violation (see 2.3.3). One can show that for $\mathcal{T}$-invariance both D and R need to vanish[27]. The most precise measurements[26, 27] yield results that are again equal to zero within the uncertainties of the experiments:

$$\begin{aligned} D &< 10^{-3}, \\ R &= 0.004 \pm 0.014. \end{aligned} \tag{118}$$

[27]Neglecting small corrections that are induced by the interaction of the particles in the final state.

3 SU(2) × U(1) Symmetry or Standard Model

In this section we closely follow the book "CP-violation" of Branco *et al.* [41] and the course on Particle Astrophysics by Henk Jan Bulten (Vrije Universiteit - Amsterdam). Note that the book contains an excellent overview of the current experimental and theoretical status on $\mathcal{CP}$ violation.

3.1 Notes on Field Theory

This section outlines a few issues of quantum field theory. A thorough treatment of field theory is not possible, and the interested reader is refered to the literature.

The Schrödinger equation is a non-relativistic equation, it can therefore not be a consistent description of particles. We want to generalize the Schrödinger equation. As a first step, we consider the relativistic equation for kinetic energy, $m^2 = E^2 - p^2$. Using $E = i\hbar\frac{d}{dt}$, $p = i\hbar\frac{d}{dx}$, and $\hbar = 1$ we can write down the Klein-gordon equation for a particle with no spin (a scalar particle, described by a complex wave function ϕ) in free fall,

$$(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2)\phi = 0. \tag{119}$$

In the Schrödinger formalism we had for the probability density and - current

$$\rho = \phi^*\phi, \quad j = \frac{-i}{2m}\phi^*\nabla\phi - \phi\nabla\phi^*$$

and the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\dot{j} = \phi^*\left(\frac{\partial\phi}{\partial t} - \frac{i}{2m}\nabla^2\phi\right) + \phi\left(\frac{\partial\phi^*}{\partial t} + \frac{i}{2m}\nabla^2\phi^*\right) = 0, \tag{120}$$

where the Schrödinger equation and its complex conjugate have been used.

The relativistic generalization of the density ρ should transform like a time-like component of a four-vector; the current density reads

$$j^\mu = (\rho, \mathbf{j}) = \frac{i}{2m}[\phi^*(\partial^\mu\phi) - \phi(\partial^\mu\phi^*)]. \tag{121}$$

The field ϕ^* also obeys the Klein-Gordon equation, hence we obtain again

$$\partial_\mu j^\mu = \phi^*(\partial_0{}^2 - \nabla^2)\phi - \phi(\partial_0{}^2 - \nabla^2)\phi^* = 0 \tag{122}$$

The density here however is *not positive-definite*. This is due to the fact that the Klein-Gordon equation is of second order and ϕ and $\partial\phi/\partial t$ can be chosen freely. Also, the Klein-Gordon equation has solutions with negative energy E. The interpretation of ϕ as a quantum-field operator instead of a single-particle wave function resolves this problem.

3.1.1 Dirac Equation

In order to obtain a linear first-order relativistic equation that can be boosted and rotated in space, *i.e.* that commutes with the Poincaré group, one arrives at the Dirac equation. This equation reads

$$i\gamma^\mu\partial_\mu - m\psi(x) = 0, \tag{123}$$

where the spinor ψ has four components. There are infinite representations of the gamma matrices, we introduce here the Dirac representation.

$$\gamma_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad ; \quad \gamma_i = \begin{pmatrix} 0 & \sigma_{\mathbf{i}} \\ -\sigma_{\mathbf{i}} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}_{i=1} , \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}_{i=2} , \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}_{i=3} . \tag{124}$$

The Dirac equation for free particles at rest is described by a plane wave solution $\psi(x) = u(0)e^{-imt}$ for the positive energy states, and $\psi(x) = v(0)e^{+imt}$ for the negative energy states. Both u and v have two components, one for spin up and one for spin down. The spinors read

$$u^{(1)}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, u^{(2)}(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, v^{(1)}(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, v^{(2)}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{125}$$

By boosting the spinors one arrives at the description for particles with momentum p,

$$u^{(1)}(p) = \frac{E+m}{2m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_+}{E+m} \end{pmatrix}, u^{(2)}(p) = \frac{E+m}{2m}\begin{pmatrix} 0 \\ 1 \\ \frac{p_-}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix},$$

$$v^{(1)}(p) = \frac{E+m}{2m}\begin{pmatrix} \frac{p_-}{E+m} \\ \frac{-p_z}{E+m} \\ 1 \\ 0 \end{pmatrix}, v^{(2)}(p) = \frac{E+m}{2m}\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_+}{E+m} \\ 0 \\ 1 \end{pmatrix}. \tag{126}$$

where $p_\pm = p_x \pm ip_y$. The spinors form an orthonormal basis. Apart from γ^μ one frequently encounters the matrix γ^5 defined by $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The definition of a field operator $\bar\psi$ is $\bar\psi = \gamma^0\psi^\dagger$.

3.1.2 Quantum Fields

The quantity $\bar\psi\psi$ transforms as a scalar, $\bar\psi\gamma^\mu\psi$ as a vector, $\bar\psi\gamma^5\psi$ as a pseudo-scalar, $\bar\psi\gamma^5\gamma^\mu\psi$ as a pseudo-vector or axial vector, and $\bar\psi(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi$ as a rank-2 antisymmetric tensor, giving all the possibilities for an object in 4 dimensions to transform under parity, rotations and Lorentz transformations.

In quantum field theory, ϕ and ψ are interpreted as field operators. They act on the vacuum, creating and destroying field quanta. For instance the Klein-Gordon field $\phi(x)$ is written

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2k_0}[a(k)e^{-ikx} + b^\dagger(k)e^{ikx}],$$
$$\phi^\dagger(x) = \int \frac{d^3k}{(2\pi)^3 2k_0}[b(k)e^{-ikx} + a^\dagger(k)e^{ikx}], \tag{127}$$

where a(k) annihilates a boson with momentum k, $a^\dagger(k)$ creates a boson with momentum k, $a^\dagger a$ counts the number of bosons, and the annihilation and creation operators b work on the antiparticles of the complex Klein-Gordon field. Furthermore, the classical Lagrangian is quantized by replacing the Poisson brackets with (anti-) commutation operators. Whereas one classically has $[\pi, x] = 0$ (i.e. the momentum and the coordinate commutate), in quantum field theory this is replaced by $[\pi, x] = i\hbar$. The classical Lagrangian depends in quantum field theory on the fields. We will consider Lagrangian densities $L = L(\phi, \partial_\mu\phi)$ that do not explicitly depends on the absolute position in space, and only depends on the fields and the first-order derivatives of the fields. The action reads $S = \int L(\phi, \partial_\mu\phi)d^4x$. The Klein-Gordon Lagrangian is given by

$$L_{KG} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2. \tag{128}$$

This may be verified by using the action principle, leading to the Euler-Lagrange equations.

The variation $\Delta\phi$ is constituted from two parts, the functional variation $\delta\phi$ from the change in the internal field parameters in ϕ at the same coordinate and by a variation in coordinate,

$$\phi'(x') - \phi(x) = \Delta\phi(x) = \phi'(x') - \phi(x') + \phi(x') - \phi(x) = \delta\phi + (\partial_\mu\phi)\delta x^\mu. \tag{129}$$

The variation in the action is now

$$\delta S = \int_R \frac{\partial L}{\partial\phi}\delta\phi + \frac{\partial L}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi)d^4x, \tag{130}$$

where $\delta(\partial_\mu\phi) = \partial_\mu(\delta\phi)$ since the variation is chosen to disappear at the three-dimensional boundary ∂R of the region R in space-time over which we integrate the Lagrangian density. With

$$\frac{\partial L}{\partial(\partial_\mu\phi)}\partial_\mu(\delta\phi) = \partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu\phi)}(\delta\phi)\right] - \partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu\phi)}\right](\delta\phi), \tag{131}$$

we obtained a total divergence which can be converted using Gauss's theorem to

$$\delta S = \int_R \left[\frac{\partial L}{\partial\phi} - \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\phi)}\right)\right]\delta\phi d^4x + \int_{\partial R}\frac{\partial L}{\partial(\partial_\mu\phi)}\delta\phi d^3\sigma, \tag{132}$$

where the surface integral disappears since the variation disappears at the boundary. We obtained the Euler-Lagrange equation for a field ϕ,

$$\frac{\partial L}{\partial\phi} - \partial_\mu(\frac{\partial L}{\partial(\partial_\mu\phi)}) = 0. \tag{133}$$

.
This yields the Klein-Gordon equation, as can be found from inspection,

$$\frac{\partial L}{\partial \phi} = -m^2\phi, \quad \frac{\partial L}{\partial(\partial_\mu \phi)} = \partial^\mu \phi,$$

and inserting these equations into the Euler-Lagrange equation leads to

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

for a stationary action.

Finally we mention gauge transformations. A scalar field with 2 real components may also be written as a complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

$$\phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \tag{134}$$

for which we put

$$L = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi.$$

From the Euler-Lagrange equations we derive that both ϕ and ϕ^* satisfy the Klein-Gordon equation. The Lagrangian L is clearly invariant under the global gauge transformation $\phi \to e^{-i\Lambda}\phi, \quad \phi^* \to \phi^* e^{i\Lambda}$. The infinitesimal change $\delta\phi$ under this transformation reads $\delta\phi = -i\Lambda\phi, \quad \delta\phi^* = i\Lambda\phi^*$. The invariance of the Lagrangian yields a conserved current (Noether's theorem, equivalent to the derivation of conservation of energy and momentum from translations in time and space in classical mechanics)

$$J^\mu = \frac{\partial L}{\partial(\partial_m \phi)}(-i\phi) + \frac{\partial L}{\partial(\partial_m \phi^*)}(i\phi^*) = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*). \tag{135}$$

The current J^μ has a vanishing divergence, as it should have, and the corresponding conserved quantity is

$$Q = \int J^0 dV = i \int ((\phi^* \partial^0 \phi - \phi \partial^0 \phi^*) dV. \tag{136}$$

Now Q can be identified with the total electrical charge.

By making a local gauge transformation, *i.e.* by allowing Λ to take an arbitrary value at any coordinate x, we loose the invariance of the Lagrangian. For an infinitesimal variation we have

$$\delta\phi = i\Lambda(x)\phi, \quad \partial_\mu \phi \to \partial_\mu \phi - i(\partial_\mu \Lambda)\phi - i\Lambda \partial_\mu \phi.$$

We see that the derivatives of the field transform in a different manner than the field itself. Furthermore, the action is now no longer invariant. We obtain

$$\delta L = \frac{\partial L}{\partial \phi}\delta\phi + \frac{\partial L}{\partial(\partial_\mu \phi}\delta(\partial_\mu \phi) + (\phi \to \phi^*), \tag{137}$$

which yields, using the Euler-Lagrange equation,

$$\delta L = \partial_\mu \left[\frac{\partial L}{\partial(\partial_\mu \phi} \right] (-i\Lambda\phi) + \frac{\partial L}{\partial(\partial_\mu \phi}(-i\Lambda\partial_\mu\phi - i\phi\partial_\mu\Lambda) + (\phi \to \phi^*). \tag{138}$$

The first term is a total derivative, so the change in the action is zero and we may ignore it. The second term gives, using the explicit form of the Lagrangian,

$$\delta L = i\partial_\mu\Lambda(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) = J^\mu\partial_\mu\Lambda. \tag{139}$$

To render the Lagrangian invariant under local gauge transformations one introduces an additional term in the Lagrangian, consisting of a new 4-vector A_μ which couples to the current J^μ:

$$L_1 = -eJ^\mu A_\mu. \tag{140}$$

Furthermore, we demand that under local gauge transformation A transforms as

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\Lambda \tag{141}$$

leading to the condition that the variation in L_1 equals

$$\delta L_1 = -e(\delta J^\mu)A_\mu - J^\mu\partial_\mu\Lambda.$$

This last term cancels the variation in L we saw before. However, now we have to cancel the first term. We have

$$\delta J^\mu = i\delta(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) = 2\phi^*\phi\partial^\mu\Lambda$$

If we introduce a term L_2

$$L_2 = e^2 A_\mu A^\mu \phi^*\phi \tag{142}$$

we obtain an invariant action $\delta L + \delta L_1 + \delta L_2 = 0$.

The field A_μ which was introduced, should contribute also to the Lagrangian. We need a term that is invariant under gauge transformations, but will describe the contributions of A_μ. This term is found in L_3.

$$L_3 = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad , F_\mu\nu = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{143}$$

We have now derived the Lagrangian for the electromagnetic field!

$$L_{EM} = (\partial_\mu\phi)(\partial^\mu\phi^*) - ie(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)A_\mu + e^2A_\mu A^\mu\phi^*\phi - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} =$$

$$(\partial_\mu\phi + ieA_\mu\phi)(\partial^\mu\phi^* - ieA^\mu\phi^*) - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{144}$$

From classical electrodynamics we recognize the electromagnetic field tensor $F_{\mu\nu}$. Furthermore, we see that the derivative ∂_μ is replaced by a *covariant derivative* $D_\mu = \partial_\mu + ieA_\mu$. It transforms as the field ϕ,

$$\delta(D_\mu\phi) = \delta(\partial_\mu\phi) + ie(\delta A_\mu)\phi + ieA_\mu\delta\phi = -i\Lambda(D_\mu\phi). \tag{145}$$

Just like in the case of classical mechanics, one may replace the momentum $\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$. The Hamiltonian of a charged particle in the presence of electromagnetic fields H reads

$$H = \frac{1}{2m}(\mathbf{P} - e\mathbf{A})^2 + e\phi,$$

which is also obtained here with the covariant derivative.

The field ϕ describes a field with charge +e, ϕ^* the field with quanta -e. We see, that the electromagnetic field arises naturally from a gauge principle. To make this explicit, one can state that *it is necessary to introduce the electromagnetic field, if nature is invariant under the local U(1)-gauge symmetry $\phi \rightarrow \phi e^{-i\Lambda(x)}$.*

3.2 Electroweak Interaction in the Standard Model

This section gives an overview of the electroweak interaction in the standard model. It is assumed that the reader is familiar with quantum field theory; for an introduction to quantum field theory see for example references [42] and [43].

3.2.1 Electroweak Bosons

In the standard model, the electroweak interaction is described in terms of the SU(2)*U(1) symmetry groups. The group SU(2) has $n^2 - 1 = 3$ generators, T_1, T_2, and T_3. The generators follow the anti-commutation relation

$$[T_i, T_j] = i\epsilon_{ijk}T_k \quad . \tag{146}$$

In the fundamental representation, the generators are given by the Pauli spin matrices,

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad . \tag{147}$$

Furthermore, one defines

$$T_{\pm} = \frac{T_1 \pm iT_2}{\sqrt{2}} \quad , \tag{148}$$

leading to

$$[T_+, T_-] = T_3 \ , \quad [T_3, T_{\pm}] = \pm T_{\pm} \ , \quad T_- = T_+^{\dagger} \quad . \tag{149}$$

The covariant derivative in the electroweak interaction is

$$D^{\mu} \ = \ \partial^{\mu} - ig(W_1^{\mu}T_1 + W_2^{\mu}T_2 + W_3^{\mu}T_3) - ig'B^{\mu}Y \quad . \tag{150}$$

Here, g is the SU(2) coupling constant and g' the U(1) coupling constant. The U(1) charge Y is called the weak hypercharge. B^{μ} is the U(1) field, and the three W^{μ} fields are the SU(2) fields (for which one also defines $W^{\pm} = -(\pm W_2 - W_1)/\sqrt{2}$). Instead of g and g' one usually applies the electromagnetic coupling constant e and the angle θ_w, defined through

$$g = \frac{e}{\sin(\theta_w)} \ , \quad g' = \frac{e}{\cos(\theta_w)} \quad . \tag{151}$$

The gauge fields A and Z result from the orthogonal rotation of B and W_3,

$$\begin{pmatrix} B \\ W_3 \end{pmatrix} = \begin{pmatrix} \cos(\theta_w) & \sin(\theta_w) \\ -\sin(\theta_w) & \cos(\theta_w) \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix} \quad . \tag{152}$$

The covariant derivative may then be written as

$$D^\mu \; = \; \partial^\mu + ie(A^\mu Q - ig(W^{+\mu}T_+ W^{-\mu}T_-) + i\frac{g}{\cos\theta_w} Z(T_3 - Q\sin^2\theta_w) \quad . \tag{153}$$

Due to the presence of SU(2), a non-abelian group, self-interactions occur between the gauge bosons. The field tensors can be written as:

$$\begin{aligned} F_1^{\mu\nu} &= \partial^\mu W_1^\nu - \partial^\nu W_1^\mu + g(W_2^\mu W_3^\nu - W_2^\nu W_3^\mu) \quad , \\ F_2^{\mu\nu} &= \partial^\mu W_2^\nu - \partial^\nu W_2^\mu + g(W_3^\mu W_1^\nu - W_3^\nu W_1^\mu) \quad , \\ F_3^{\mu\nu} &= \partial^\mu W_3^\nu - \partial^\nu W_3^\mu + g(W_1^\mu W_2^\nu - W_1^\nu W_2^\mu) \quad , \\ F_Y^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \quad , \end{aligned} \tag{154}$$

yielding the gauge-kinetic Lagrangian

$$\begin{aligned} -\frac{1}{4}(F_1^{\mu\nu}F_{\mu\nu 1} + F_2^{\mu\nu}F_{\mu\nu 2} + F_3^{\mu\nu}F_{\mu\nu 3} + F_Y^{\mu\nu}F_{\mu\nu Y} = \\ -(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu -}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{\mu -}) \\ -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2}(\partial_\mu A_\nu)(\partial^\nu A^\nu) \\ -\frac{1}{2}(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) + \frac{1}{2}(\partial_\mu Z_\nu)(\partial^\nu Z^\nu) \\ +\mathrm{non-quadratic\ terms} \quad . \end{aligned} \tag{155}$$

The non-quadratic terms give rise to the vertices that are shown in Fig. 14.

In the standard model, there is a scalar doublet, denoted Φ, with hypercharge $Y = 2$. The term in the Lagrangian $\mu\Phi^\dagger\Phi$ leads, with negative μ, to the symmetry breaking of SU(2)*U(1) into U(1) of the electromagnetic interaction (the Higgs mechanism). We write

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ v + \frac{(H+i\chi)}{\sqrt{2}} \end{pmatrix} . \tag{156}$$

Here, H and χ are Hermitian Klein-Gordon fields, H is the Higgs boson, χ the Goldstone boson that is absorbed into the longitudinal component of the massive Z_0-boson, and the bosons $\phi^\pm$ are absorbed into the longitudinal components of the $W^\pm$-bosons. The conjugate doublet field $\tilde{\Phi}$ with hypercharge Y= -1/2 is defined through

$$\tilde{\Phi} = i\tau_2 {\Phi^\dagger}^T = \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix} = \begin{pmatrix} v + \frac{(H-i\chi)}{\sqrt{2}} \\ -\phi^- \end{pmatrix} . \tag{157}$$

The gauge-kinetic Lagrangian for this field becomes

$$\left[\partial\phi^- - ieA\phi^- + i\frac{g}{\sqrt{2}}W^-\phi^{0\dagger} + i\frac{g}{2\cos\theta_w}Z(\cos^2\theta_w - \sin^2\theta_w)\phi^-\right]$$

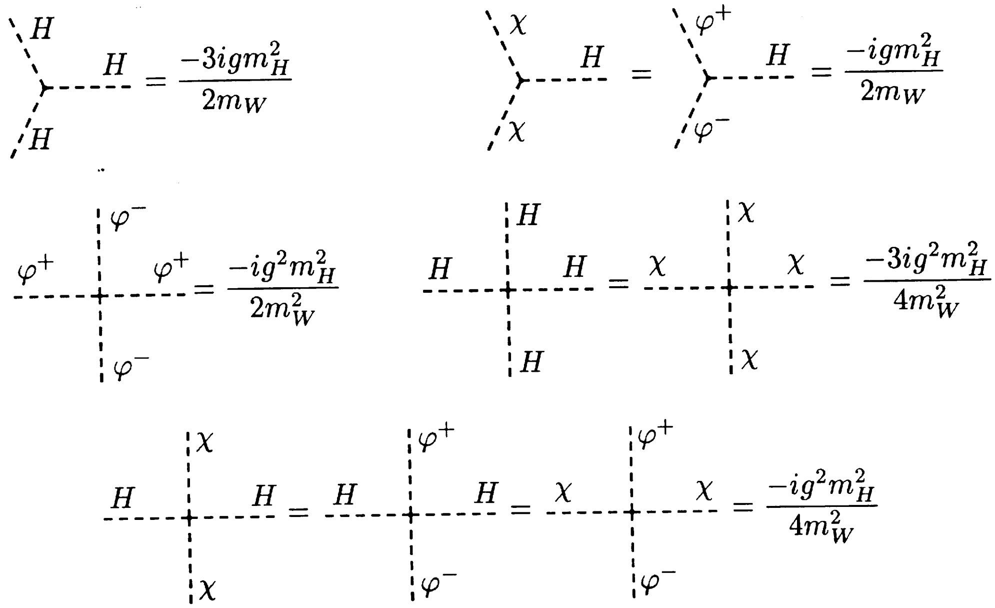

Figure 14: *Self-interactions of the Electroweak gauge bosons.*

$$
\begin{aligned}
&\times \left[\partial\phi^+ + ieA\phi^+ - i\frac{g}{\sqrt{2}}W^+\phi^0 - i\frac{g}{2\cos\theta_w}Z(\cos^2\theta_w - \sin^2\theta_w)\phi^+\right] \\
&+ \left[\partial\phi^{0\dagger} + i\frac{g}{\sqrt{2}}W^+\phi^- - i\frac{g}{2\cos\theta_w}Z\phi^{0\dagger}\right] \\
&\times \left[\partial\phi^0 - i\frac{g}{\sqrt{2}}W^-\phi^+ + i\frac{g}{2\cos\theta_w}Z\phi^0\right].
\end{aligned}
\tag{158}
$$

With the relation

$$
v = \frac{\sqrt{2}}{g}m_W = \frac{\sqrt{2}}{g}\cos\theta_w m_Z, \tag{159}
$$

this can be converted to

$$
\begin{aligned}
&(\partial\phi^-)(\partial\phi^+) + \frac{1}{2}\left[(\partial H)^2 + (\partial\chi)^2\right] + {m_W}^2W^+W^- + \frac{1}{2}{m_Z}^2Z^2 \\
&+im_W(W^-\partial\phi^+ - W^+\partial\phi^-) + m_Z Z\partial\chi \\
&+\mathrm{non-quadratic\ terms.}
\end{aligned}
\tag{160}
$$

The contributions from the second line are removed by the gauge-fixing part of the Lagrangian, the terms in the third part are shown in Fig. 15.

The group SU(2) is a non-abelian group. It contains infinite contributions to the propagator arising from certain diagrams. 't Hooft has proven that SU(2) can be renormalized. In order to do so one needs to choose a specific gauge. The gauge-fixing part in the Lagrangian and the associated ghost fields are discussed next.
The quadratic parts in the Z and χ fields in the Lagrangian reads

$$
-\frac{1}{2}(\partial_\mu Z_\nu)\left[(\partial^\mu Z^\nu) - (\partial^\nu Z^\mu)\right] + \frac{1}{2}{m_Z}^2 Z_\mu Z^\mu + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + m_Z Z_\mu(\partial^\mu\chi). \tag{161}
$$

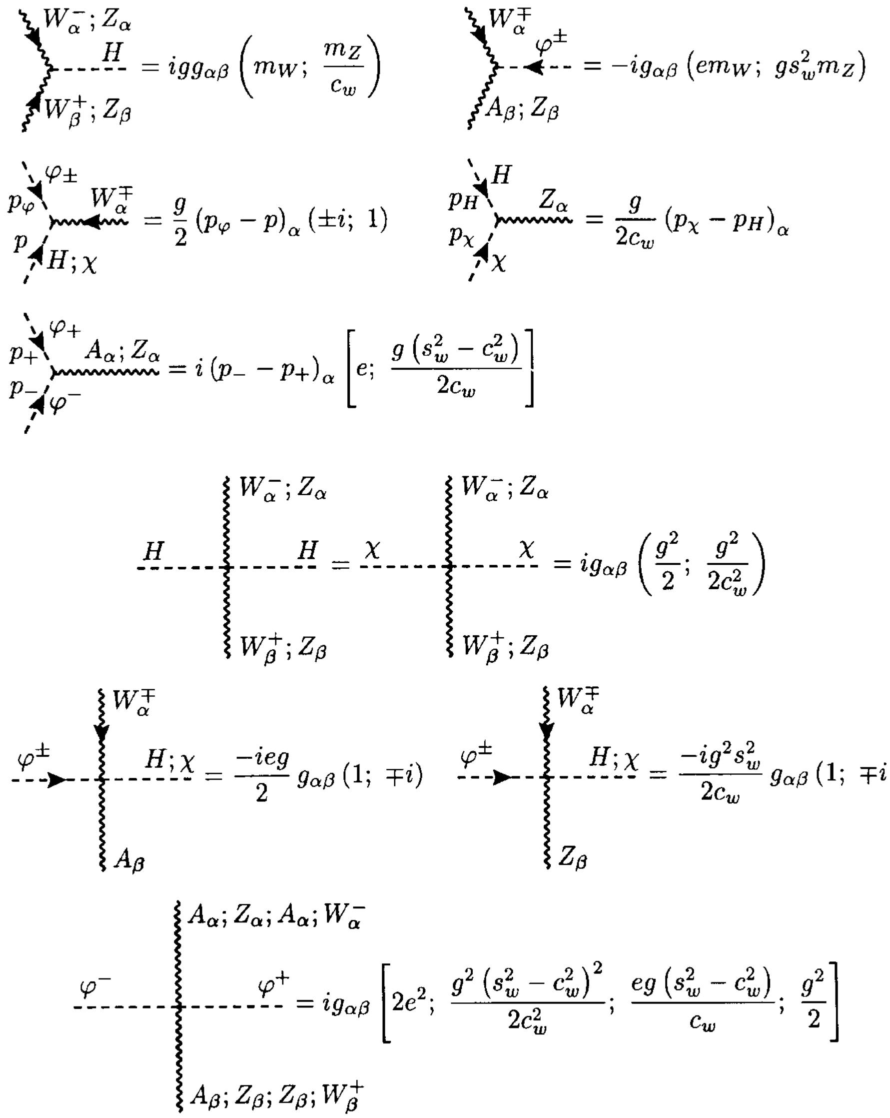

Figure 15: *Gauge-interactions of the SU(2)*U(1) scalars.*

One now fixes the gauge by using the term in the Lagrangian

$$-\frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z m_Z \chi)^2 = -\frac{1}{2\xi_Z}(\partial_\mu Z^\nu)(\partial_\nu Z^\mu) + m_Z\chi(\partial_\mu Z^\mu) - \frac{\xi_Z}{2}{m_Z}^2\chi^2. \tag{162}$$

Here, the real positive parameter ξ_Z relates different gauges. In the Landau gauge it is chosen 0, in the Feynman gauge it is 1. Measureable quantities should be independent on the choice of gauge. Adding this gauge-fixing term to the Lagrangian, we see that the terms for χ yield the usual propagator for a scalar boson with mass $\sqrt{\xi_Z {m_Z}^2}$. The mixed terms with both χ and Z are removed by integration in parts.

The remaining terms yield the propagator for the Z-boson. The second part contains a pole on the unphysical squared mass $\xi_z {m_Z}^2$. Its effects must cancel out with the effects of the propagator of χ and of the ghost fields.

For the W-sector the same procedure yields also the usual charged scalar propagator for the $\phi^\pm$ particles with a mass-squared of $\xi_w {m_W}^2$, and a W propagator with an unphysical part with mass $\xi_w m_W{}^2$, which has to cancel in all observeables. For the photon, one has the gauge-fixing term

$$-\frac{1}{2\xi_A}(\partial^\mu A_\mu)(\partial^\nu A_\nu). \tag{163}$$

The gauges for the Z, W, and A fields may be chosen independently (the three ξ-factors are three, possibly different, real non-negative numbers). The propagators of the gauge bosons and of the scalars are shown in figures 16 and 17, respectively.

$$\underset{\mu \quad A \quad \nu}{\overset{k}{\rightsquigarrow}} = \frac{-ig_{\mu\nu}}{k^2} + (1-\xi_A)\,\frac{ik_\mu k_\nu}{k^4}$$

$$\underset{\mu \quad Z \quad \nu}{\overset{k}{\rightsquigarrow}} = \frac{-ig_{\mu\nu}}{k^2-m_Z^2} + \frac{ik_\mu k_\nu}{m_Z^2}\left(\frac{1}{k^2-m_Z^2} - \frac{1}{k^2-\xi_Z m_Z^2}\right)$$

$$\underset{\mu \quad W \quad \nu}{\overset{k}{\rightsquigarrow}} = \frac{-ig_{\mu\nu}}{k^2-m_W^2} + \frac{ik_\mu k_\nu}{m_W^2}\left(\frac{1}{k^2-m_W^2} - \frac{1}{k^2-\xi_W m_W^2}\right)$$

Figure 16: *Propagators of the gauge bosons.*

$$\underset{H}{\overset{k}{\dashrightarrow}} = \frac{i}{k^2-m_H^2} \qquad \underset{\chi}{\overset{k}{\dashrightarrow}} = \frac{i}{k^2-\xi_Z m_Z^2} \qquad \underset{\varphi^\pm}{\overset{k}{\dashrightarrow}} = \frac{i}{k^2-\xi_W m_W^2}$$

Figure 17: *Propagators of the scalars.*

The second step in renormalizing the theory leads to the introduction of ghost fields. These are mathematical constructs obeying a Grassmann algebra. The procedure of fixing the gauge and introducing ghost fields is necessary to remove contributions to the action of

$$\int DADWDZe^{-i\int L_{eff}dx}$$

that are only connnected via gauge transformations. The action is given by integrating the Lagrangian density for all possible paths and all possible field configurations. In the formula above, $\int DA$, $\int DW$, $\int DZ$ stand for the functional integrals over the gauge fields. Since a gauge transformation leaves the Lagrangian invariant, one wants to remove these infinite contributions.

The ghost fields are non-physical fields since they follow Fermi-Dirac statistics but have boson propagators; they can only enter inside loops in Feynman diagrams. The propagators of the ghost fields are shown in Fig. 18, and the vertices that they contribute to in Fig. 19. Note, that they contain strengths depending on the arbitrary gauge-fixing parameters ξ; these contributions should cancel in the calculations of all physical observeables.

The scalars in the Lagrangian also have self-interaction via the potential V, with

$$V = \mu\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2,$$

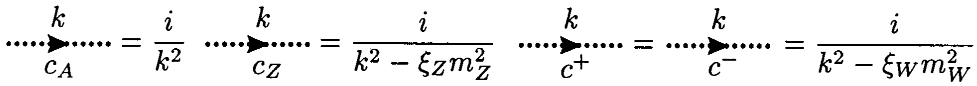

Figure 18: *Propagators of the ghost fields.*

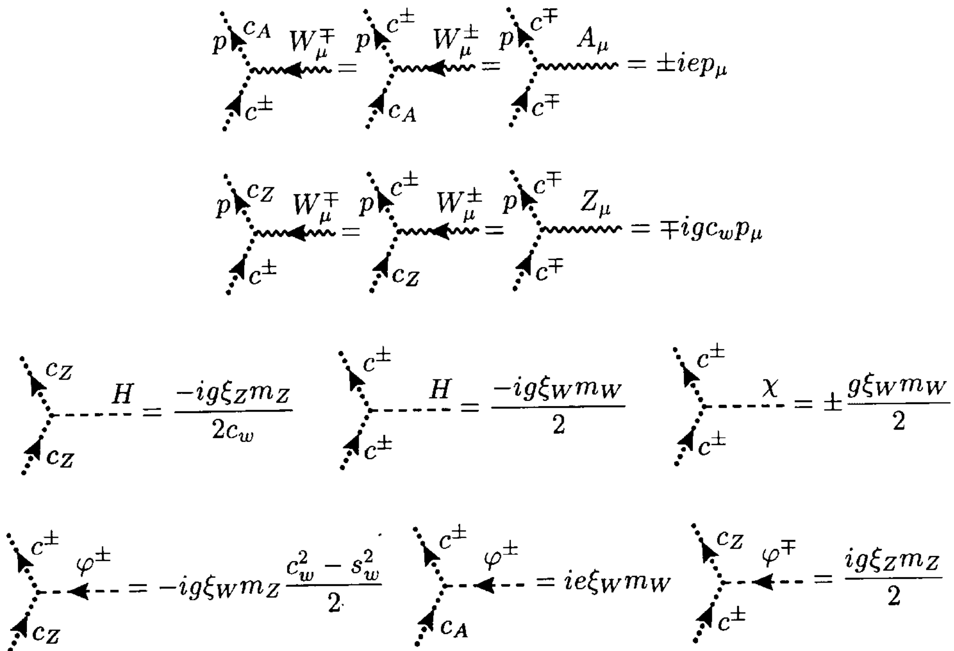

Figure 19: *Vertices with a ghost field involved.*

$$\phi^\dagger \phi = v^2 + \sqrt{2} v H + \frac{H^2 + \chi^2}{2} + \phi^- \phi^+. \tag{164}$$

In order to have a stable vacuum one requires that the linear term in the Higgs field disappears: $\mu = -2\lambda v^2$. Furthermore, we use $\lambda = {m_H}^2/(4v^2)$ and Eq. 159 to obtain

$$\begin{aligned} V = &-\frac{{m_W}^2 {m_H}^2}{2g^2} + \frac{{m_H}^2}{2} H^2 + \frac{g {m_H}^2}{2 m_W} \left[\frac{H^2 + \chi^2}{2} + \phi^- \phi^+ \right] H \\ &+ \frac{g^2 {m_H}^2}{8 {m_W}^2} \left[\frac{H^2 + \chi^2}{2} + \phi^- \phi^+ \right]^2 . \end{aligned} \tag{165}$$

We can recognize a vacuum-energy term $-{m_W}^2 {m_H}^2 / 2g^2$, the mass term for the Higgs field, and terms with third and fourth powers of the fields, which give rise to the vertices in Fig. 20.

3.2.2 Fermions in the Standard Model

We discussed the full bosonic section of the electroweak theory in the standard model. We proceed with the discussion of the fermions. The electroweak interaction acts on left-handed doublets and right-handed singlets. The quarks form 3 doublets with hypercharge

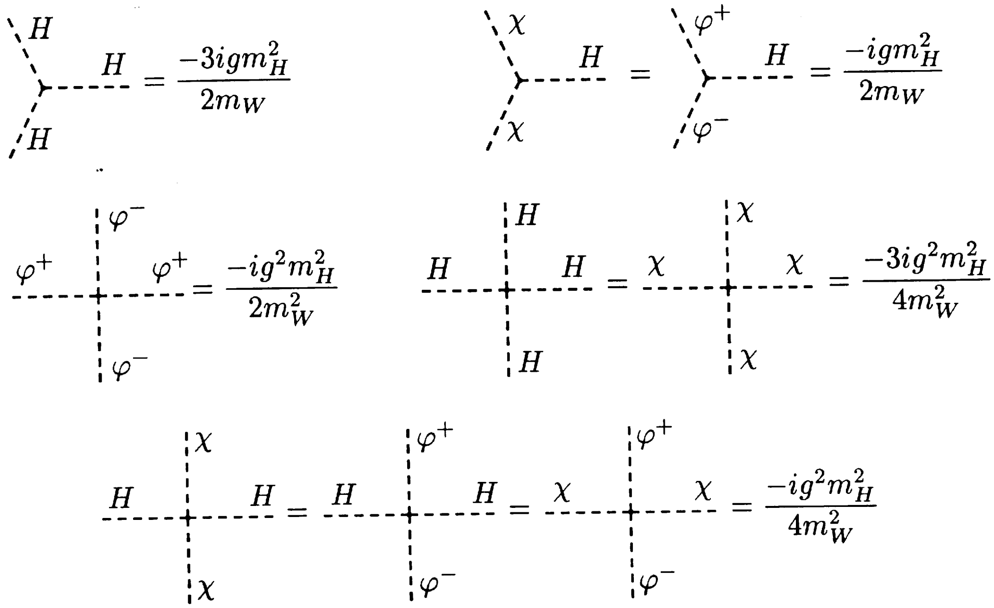

Figure 20: *Vertices with a ghost field involved.*

Y = -1/6 and 6 singlets.

$$Q_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}^{Y=+1/6} \qquad {p_R}^{Y=Q=+2/3} \qquad {n_R}^{Y=Q=-1/3}. \tag{166}$$

The gauge-kinetic Lagrangian for the quark sector reads for a fermion multiplet f

$$\bar{f}\gamma^\mu \left[i\partial_\mu - eA_\mu Q + g(W_\mu^+ T_+ + W_\mu^- T_-) + \frac{g}{\cos\theta_w} Z_\mu (T_3 - Q\sin^2\theta_w) \right]. \tag{167}$$

This can be separated in the electromagnetic interaction terms $L_A = -eA_\mu J^\mu_{em}$ with the electromagnetic current J^μ_{em} equal to

$$J^\mu_{em} = \frac{2}{3}(\bar{p_L}\gamma^\mu p_L + \bar{p_R}\gamma^\mu p_R) - \frac{1}{3}(\bar{n_L}\gamma^\mu n_L + \bar{n_R}\gamma^\mu n_R) \quad , \tag{168}$$

a neutral-current part L_Z

$$L_Z = \frac{g}{2\cos\theta_w} Z_\mu (\bar{p_L}\gamma^\mu p_L - \bar{n_L}\gamma^\mu n_L - 2\sin^2\theta_w J^\mu_{em}) \quad , \tag{169}$$

and the charged-current part that interchanges members of the doublet

$$L_W = \frac{g}{\sqrt{2}}(W_\mu^+ \bar{p_L}\gamma^\mu n_L + W_\mu^- \bar{n_L}\gamma^\mu p_L) \quad . \tag{170}$$

The lepton sector looks similar. It contains lefthanded doublets with weak hypercharge -1/2 and righthanded singlets with Q=Y=-1:

$$L_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}^{Y=-1/2} \qquad l_R^{Q=Y=-1} \quad . \tag{171}$$

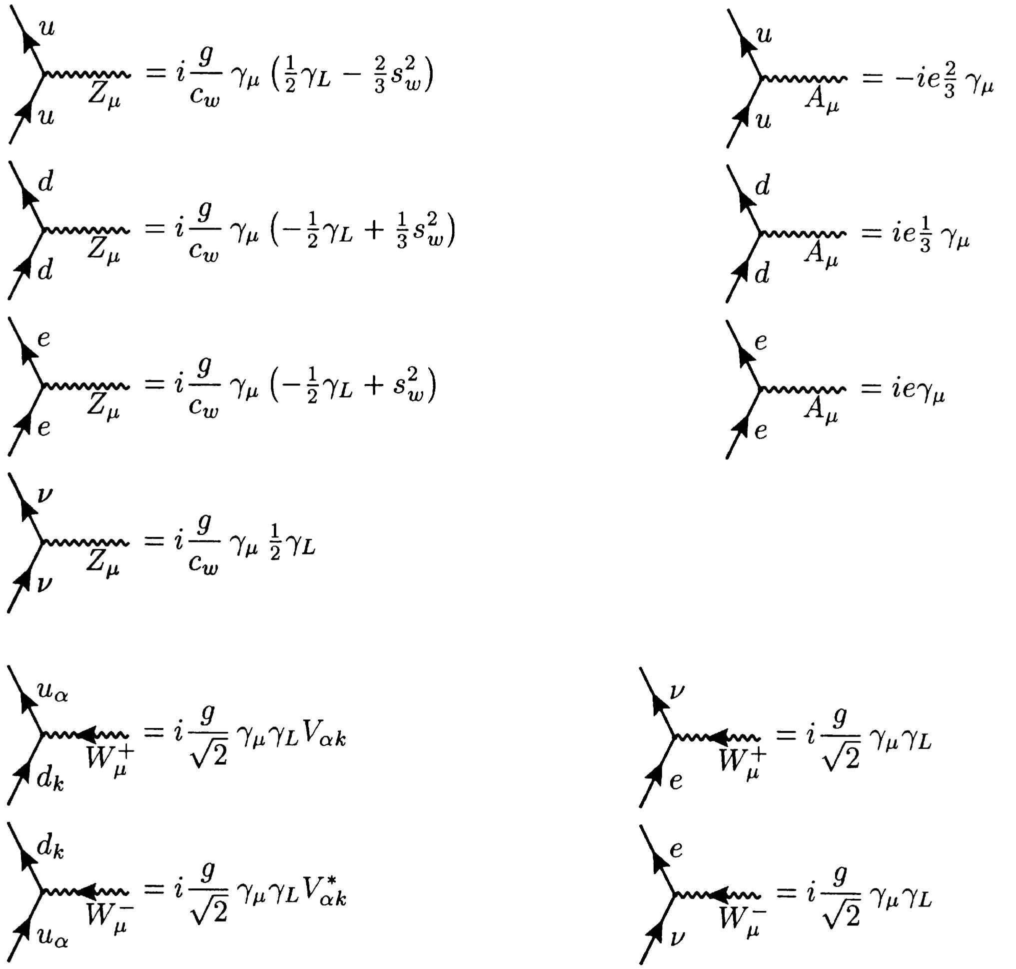

Figure 21: *Gauge interactions of the fermions in the standard model.*

No righthanded neutrino's are introduced in the standard model.

The electromagnetic current reads

$$L_A = -eA_\mu\left(\bar{l_L}\gamma^\mu l_L + (\bar{l_R}\gamma^\mu l_R\right) \quad , \tag{172}$$

and the neutral current

$$L_Z = Z_\mu\left[\frac{g}{2\cos\theta_w}\left(\bar{\nu_L}\gamma^\mu\nu_L - \bar{l_L}\gamma^\mu l_L\right) + \frac{g\sin^2\theta_w}{\cos\theta_w}\left(\bar{l_L}\gamma^\mu l_L + \bar{l_R}\gamma^\mu l_R\right)\right]. \tag{173}$$

The charged current is given by

$$L_W = \frac{g}{\sqrt{2}(W_\mu^+\bar{\nu_L}\gamma^\mu l_L + W_\mu^-\bar{l_L}\gamma^\mu\nu_L} \tag{174}$$

Furthermore, we have the Yukawa interactions involving the fermions and the Higgs doublet:

$$L_Y = -\left(\bar{Q_L}\Gamma\phi n_R + \bar{Q_L}\Delta\tilde{\phi}p_R + \bar{L_L}\Pi l_R\right) + h.c. \tag{175}$$

with Γ, Δ, and Π arbitrary complex numbers (and as usual h.c. stands for the Hermitian conjugate). By replacing ϕ^0 with its vacuum-expectation value v we can extract the mass terms for the fermions:

$$L_{mass} = -\bar{n_L} M_n n_R - \bar{p_L} \frac{M_p}{\sqrt{2}v} p_R + \bar{l_L} M_l l_R + h.c. \tag{176}$$

with the particle masses squared $M_n = v\Gamma$, $M_p = v\Delta$, and $M_l = v\Pi$. The remaining

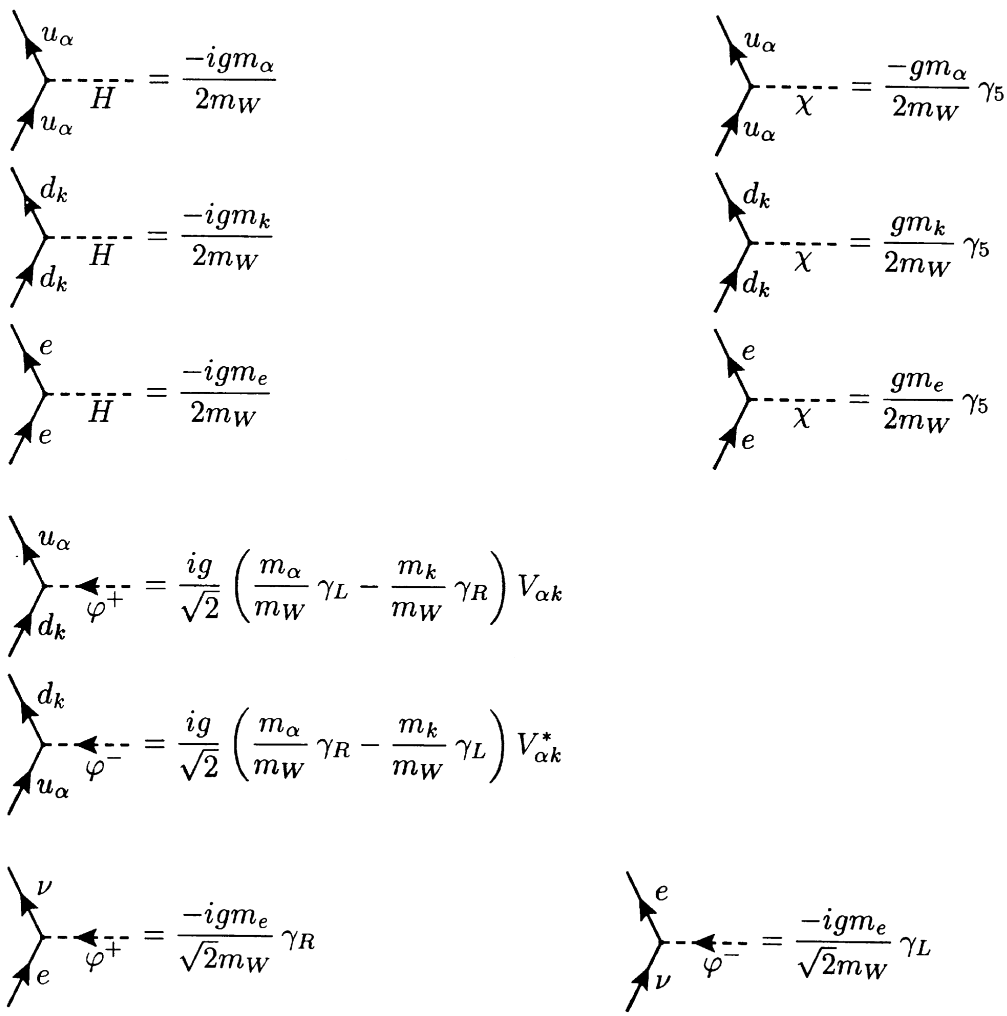

Figure 22: *Yukawa couplings of the fermions.*

terms are the Yukawa interactions between the fermions and the Higgs fields:

$$L_Y - L_{Mass} = -\bar{n_L}\frac{M_n}{\sqrt{2}v}(H+i\chi)n_R - \bar{p_L}\frac{M_p}{\sqrt{2}v}(H-i\chi)p_R - \bar{l_L}\frac{M_l}{\sqrt{2}v}(H+i\chi)p_R$$
$$-\bar{p_L}\frac{M_n}{v}\phi^+ n_R + \bar{n_L}\frac{M_p}{v}\phi^- p_R - \bar{\nu_L}\frac{M_l}{v}\phi^+ l_R + h.c. \tag{177}$$

We know from observation that there are three light families both in the lepton sector and in the hadron sector. The terms Γ, Δ, and Π, leading to the masses of the fermions, are represented by $n_g \times n_g$ matrices in generation space, where $n_g = 3$ in the standard model. This fact is not understood from a general principle (*e.g.* from a gauge theory), and also the matrices Γ, Δ, and Π do not have to be Hermitian (but they are unitary; else there would be fermion production/decay from/to the vacuum).

The Yukawa couplings M_p, M_n are not necessarily Hermitian, but they can be diagonalized by a unitary transformation

$$p_L = U_L^P u_L, \quad p_R = U_R^P u_R, \quad n_L = U_L^n d_L, \quad n_L = U_R^n d_R,$$

with the mass matrices

$$U_L^{p\dagger} M_p U_R^p = M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad U_L^{n\dagger} M_n U_R^n = M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \quad (178)$$

The charged-current interaction written in terms of the *mass eigenstates* reads then

$$L_W = \frac{g}{\sqrt{2}}(W_\mu^+ \bar{u_L} \gamma^\mu V d_L + (W_\mu^- \bar{d_L} \gamma^\mu V^\dagger u_L), \quad (179)$$

with V the CKM matrix

$$V_{CKM} = U_L^{p\dagger} U_L^n = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (180)$$

In the standard model, there are no right-handed neutrinos and the left-handed neutrinos are massless. There is no neutrino mixing and lepton family is conserved. The gauge interactions of the fermions are shown in Fig. 21 and the Yukawa interactions in Fig. 22. In these figures, u and a greek index α refer to the member of the quark multiplet with charge $+2/3$, d and a latin index k to the multiplet with $Q = -1/3$. In the following section we will discuss the CKM-mixing matrix at lenght.

4 Quark Mixing

4.1 Introduction

The decays $n \to pe^-\overline{\nu}_e$, $\mu^+ \to e^+\overline{\nu}_\mu\nu_e$ and $\pi^+ \to \mu^+\nu_\mu$ all take place through the exchange of charged currents. The decay rates for these processes are vastly different. However, when we correct these rates for the differences in phase space, one observes that the charged currents couple with approximately a universal Fermi coupling constant $G = \sqrt{2}g^2/M_W^2$. This is called the *quark-lepton universality* of the weak interaction. In these decay processes, shown in Fig. 23, the hadrons involved all have the same strangeness ($S = 0$) and the hadronic current is a strangeness conserving, or $\Delta S = 0$ current. Experimentally, it is observed that the $\Delta S = 0$ hadronic current is slightly weaker the purely leptonic current (a few percent).

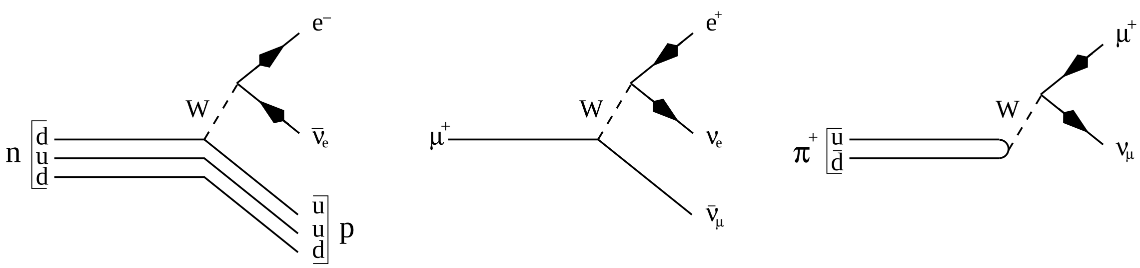

Figure 23: *(a) Charged-current decay $n \to pe^-\overline{\nu}_e$, $\mu^+ \to e^+\overline{\nu}_\mu\nu_e$, and $\pi^+ \to \mu^+\nu_\mu$.*

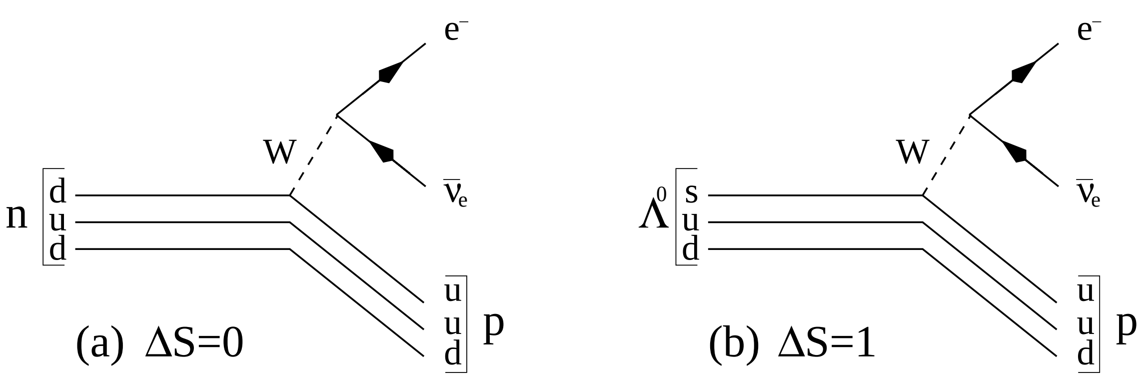

Figure 24: *(a) Strangeness conserving charged-current decay $n \to pe^-\overline{\nu}_e$; (b) Strangeness changing charged-current decay $\Lambda^0 \to pe^-\overline{\nu}_e$.*

Fig. 24 shows that when instead of $n \to pe^-\overline{\nu}_e$, one studies decays such as $\Lambda^0 \to pe^-\overline{\nu}_e$ the hadronic current is a strangeness changing, or $\Delta S = 1$, weak current. Measurements show that the strangeness changing hadronic current is about twenty times weaker than the $\Delta S = 0$ hadronic current and it seems that universality is breaking down.

4.2 Cabibbo Formalism

In order to maintain quark-lepton universality and to avoid the introduction of additional coupling constants, Cabibbo [1] proposed in 1963 to modify the quark doublets[28]. He assumed that the weak interaction couples to 'rotated' quark states and involves only u and d' quark states. The weak interaction eigenstates become linear superpositions of the strong interaction eigenstates and the quarks can be assigned to a 'weak isospin' doublet,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}, \tag{181}$$

where θ_C is the Cabibbo angle and d' the Cabibbo rotated quark.

With this formalism several decay rates turned out to be consistent when $\theta_C \approx 13°$ was chosen. Not only is the difference between $\Delta S = 0$ and $\Delta S = 1$ transitions explained, but also the slight difference between $\Delta S = 0$ and pure leptonic decays. The transition rates can then be written as

$$\begin{array}{lll} \Gamma(\mu^+ \to e^+\overline{\nu}_\mu\nu_e) \propto g^4 & \text{purely leptonic} & \\ \Gamma(n \to pe^-\overline{\nu}_e) \propto g^4\cos^2\theta_C & \Delta S = 0 & \text{semileptonic} \\ \Gamma(\Lambda \to pe^-\overline{\nu}_e) \propto g^4\sin^2\theta_C & \Delta S = 1 & \text{semileptonic} \end{array} \tag{182}$$

With this formalism there are now 'Cabibbo favored' (proportional to $\cos\theta_C$) and 'Cabibbo suppressed' transitions (proportional to $\sin\theta_C$). One can compare $\Delta S = 1$ and $\Delta S = 0$ decays, for example

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} \sim \sin^2\theta_C \sim \frac{1}{20}, \tag{183}$$

and the predicted decay rates are in good agreement with the data.

4.3 GIM Scheme

A major problem with Cabibbo's theory was the experimental observation of the absence of strangeness-changing neutral currents. To explain this difficulty, Fig. 25 shows the first-order Feynman diagrams for the reactions $K^+ \to \mu^+\nu_\mu$ and $K^0_L \to \mu^+\mu^-$. The branching fractions have been measured and one has $\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma = 63.51 \pm 0.18\%$ and $\Gamma(K^0_L \to \mu^+\mu^-)/\Gamma = (7.2 \pm 0.5) \times 10^{-9}$.

Glashow, Iliopoulos and Maiani (GIM) proposed in 1970 [5] the existence of the c quark[29]. In the GIM scheme there is a second doublet, consisting of the lefthanded c quark and a combination of s and d quarks that is orthogonal to d'. Thus we have

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_C - d\sin\theta_C \end{pmatrix}. \tag{184}$$

Neutral current occurs for d' and s', not for d and s. We need to calculate the sum of the

[28] At that time only the u, d and s quark flavors were known.

[29] Note that the c quark was discovered in 1974.

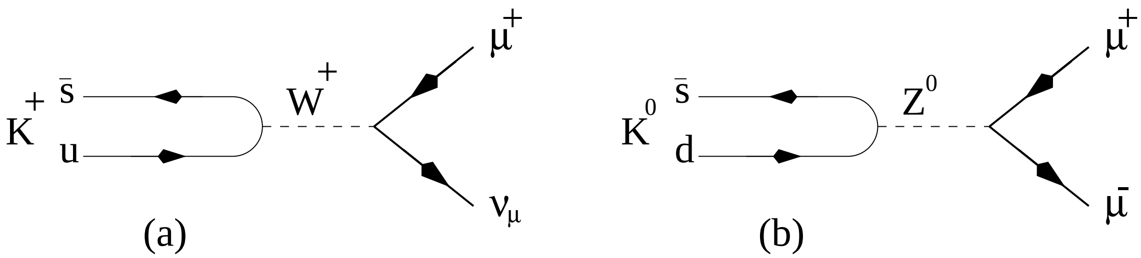

Figure 25: *(a) Charged-current decay $K^+ \to \mu^+\nu_\mu$; (b) neutral-current decay $K_L^0 \to \mu^+\mu^-$.*

matrix elements for the two families that contribute. For the neutral current we find

$$\begin{aligned}
< d'|J^{nc}|d' > \quad & + < s'|J^{nc}|s' > \\
& =< d\cos\theta_C + \ s\sin\theta_C|J^{nc}|d\cos\theta_C + s\sin\theta_C > \\
& + < s\cos\theta_C - d\sin\theta_C|J^{nc}|s\cos\theta_C - d\sin\theta_C > \\
& = \ \ < d|J^{nc}|d > (\cos^2\theta_C + \sin^2\theta_C) + < s|J^{nc}|s > (\cos^2\theta_C + \sin^2\theta_C) \\
& + < s|J^{nc}|d > (\sin\theta_C\cos\theta_C - \sin\theta_C\cos\theta_C) \\
& + < d|J^{nc}|s > (\cos\theta_C\sin\theta_C - \sin\theta_C\cos\theta_C) \\
& = \ \ < d|J^{nc}|d > + < s|J^{nc}|s > .
\end{aligned} \tag{185}$$

One sees that with the introduction of the second lefthanded quark doublet the main problem with Cabibbo's theory is solved. There is no contribution of a neutral current connecting the down and strange quarks. The Z^0 couples directly only to $u\overline{u}$, $d\overline{d}$, $s\overline{s}$ and $c\overline{c}$. Consequently, the first-order diagram (b) of Fig. 25 does not contribute to the transition rate for the decay $K_L^0 \to \mu^+\mu^-$.

Fig. 26 shows possible second-order contributions to the decay $K_L^0 \to \mu^+\mu^-$ involving two intermediate W bosons. In the absence of a c quark the process would proceed according to diagram (a) of Fig. 26. The matrix element for diagram (a) is proportional to $\mathcal{M} \sim g^4\cos\theta_C\sin\theta_C$ and the calculated rate significantly exceeds the measured value. The matrix element for diagram (b) corresponds to $\mathcal{M} \sim -g^4\cos\theta_C\sin\theta_C$ and this contribution cancels with diagram (a). Note that this cancellation is not perfect, $\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma = (7.2 \pm 0.5) \times 10^{-9}$, due to the mass difference between the u and c quarks.

4.3.1 Summary of Quark Mixing in Two Generations

The GIM model describes the flavor-changing neutral current in the four-quark case. In this model the weak interaction couples to lefthanded $u \leftrightarrow d'$ and $c \leftrightarrow s'$ quark states. The electroweak interaction states d' and s' are orthogonal combinations of the quark mass eigenstates of definite flavor, d and s. The quark mixing is described by a single parameter θ_C. One has

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}. \tag{186}$$

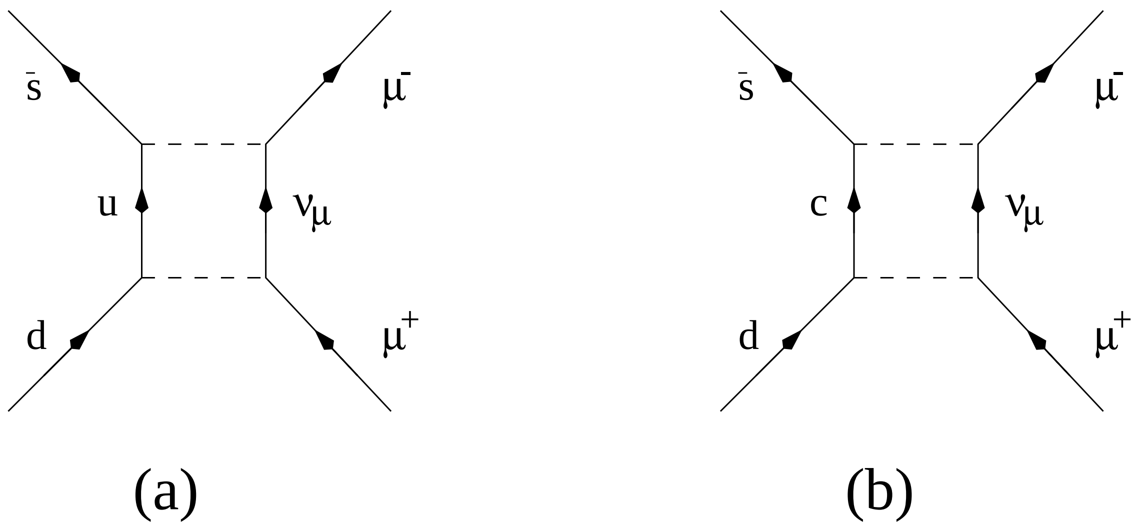

Figure 26: *Second-order contributions to the neutral-current decay $K^0_L \to \mu^+\mu^-$. (a) Feynman diagram with an intermediate u quark; (b) intermediate c quark.*

We can rewrite the above expression in the form

$$d'_i = \sum_j V_{ij} d_j, \tag{187}$$

with $d_1 \equiv d_L$ and $d_2 \equiv s_L$ where L denotes a lefthanded quark state. The matrix V is unitary. One has

$$\sum_i \overline{d}'_i d'_i = \sum_{i,j,k} \overline{d}_j V^\dagger_{ji} V_{ik} d_k = \sum_j \overline{d}_j d_j, \tag{188}$$

and flavor-changing transitions are forbidden. Next, we discuss the procedure for three generations.

4.4 The Cabibbo-Kobayashi-Maskawa Mixing Matrix

The SU(2) $\times$ U(1) Standard Model describes the electroweak interactions between the fermionic building blocks of nature, the quarks and leptons. The lefthanded fermions for the weak interaction can be considered as elementary weak-isospin doublets, while the righthanded fermions are isosinglets.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \text{and} \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{189}$$

$$e_R^- \qquad \mu_R^- \qquad \tau_R^- \qquad\qquad u_R,\ d_R \quad c_R,\ s_R \quad t_R,\ b_R$$

The quark mass eigenstates differ from the weak eigenstates and the mixing between different quarks is given by the complex Cabibbo-Kobayashi-Maskawa matrix. The matrix elements relate eigenstates of mass and weak interaction, as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{190}$$

Fig. 27 shows that the element V_{ij} specifies the coupling of the charged currents to the quarks with flavor i and j (for example $d \rightarrow u + W^-$).

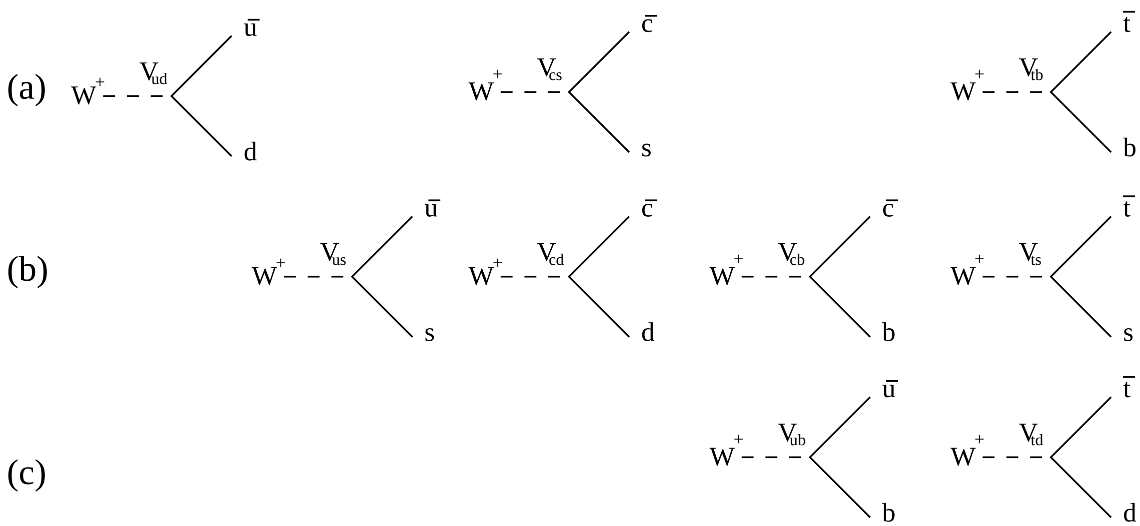

Figure 27: *The CKM matrix elements are related to the strength of the coupling of the electroweak charged currents to different quark flavors. The diagrams can be organized as follows: a) Cabibbo allowed, b) Cabibbo suppressed, and c) almost forbidden transitions.*

It is assumed that the CKM matrix is unitary,

$$\mathbf{U}^\dagger \mathbf{U} = \mathbf{U}\mathbf{U}^\dagger = 1 \quad \text{and} \mathbf{U}^\dagger = \mathbf{U}^{-1}. \tag{191}$$

Consequently, we have

$$V^\dagger = \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} = V^{-1}. \tag{192}$$

Our present knowledge[2] of the 90 % confidence level on the magnitude of the matrix elements is as follows

$$\begin{pmatrix} 0.9745 \text{ to } 0.9757 & 0.219 \text{ to } 0.224 & 0.002 \text{ to } 0.005 \\ 0.218 \text{ to } 0.224 & 0.9736 \text{ to } 0.9750 & 0.036 \text{ to } 0.046 \\ 0.004 \text{ to } 0.014 & 0.034 \text{ to } 0.046 & 0.9989 \text{ to } 0.9993 \end{pmatrix}. \tag{193}$$

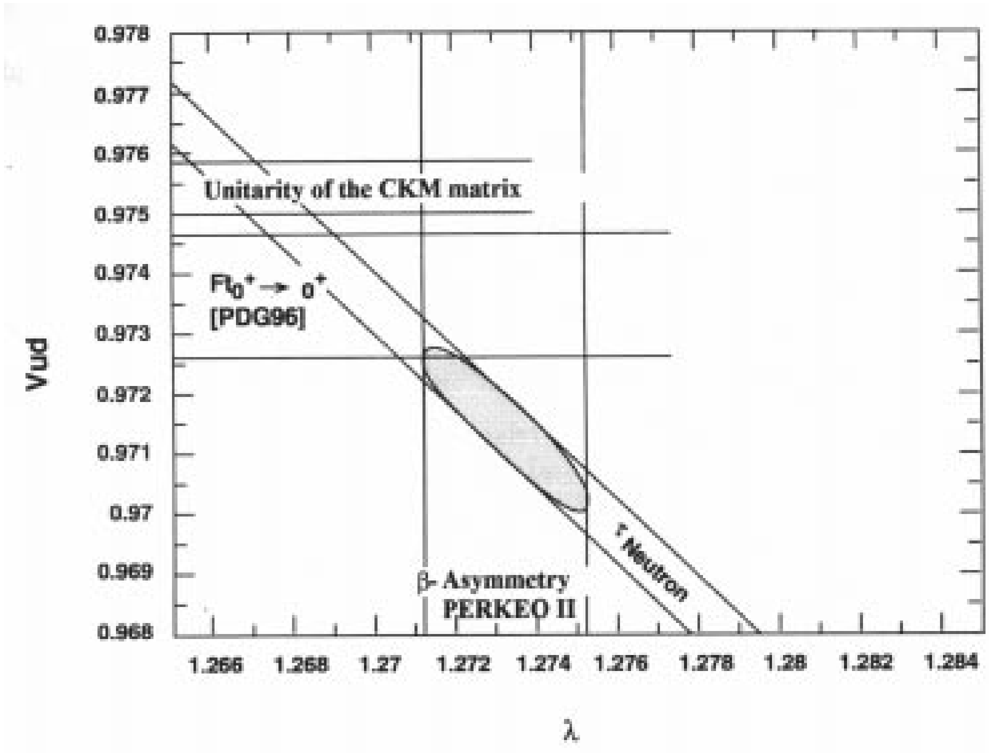

Figure 28: *Determination of the CKM matrix element V_{ud} from measurement of the neutron life time.*

The parameters in the CKM matrix are not predicted by theory, but are fundamental parameters of the Standard Model with three families. By measuring each element independently, we can test unitarity and determine whether all couplings are consistent with the CKM matrix for three families. Note that our present knowledge of the 2×2 CKM matrix is insufficient to predict a third generation. Fig. 28 shows results for V_{ud} from precision measurements of the life time of the neutron.

There are several representations of the CKM matrix. It can be reduced to a form where there are only three generalized Cabibbo angles, θ_1, θ_2 and θ_3, which one recognizes as so-called Euler angles, and a phase factor δ.

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}, \tag{194}$$

where c_i represents $\cos\theta_i$ and s_i represents $\sin\theta_i$. The matrix is again unitary,

$$V^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} = V^{-1}. \tag{195}$$

The product is

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}. \tag{196}$$

In the limit $\theta_2 = \theta_3 = 0$, the third generation decouples, and the usual Cabibbo mixing of the first two generations is recovered. One can identify θ_1 with the Cabibbo angle.

Wolfenstein noted[29] that an empirical hierarchy in the magnitudes of the matrix elements can be expressed as follows

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{197}$$

where $\lambda = \sin\theta_c$, with θ_c the Cabibbo angle, and $A \approx 1$.

For an $n \times n$ unitary matrix there are $n(n-1)/2$ real angles and $(n-1)(n-2)/2$ phases. For the three generation CKM matrix we thus have 3 angles and 1 phase. Exploiting unitarity,

$$\begin{pmatrix} V^*_{ud} & V^*_{cd} & V^*_{td} \\ V^*_{us} & V^*_{cs} & V^*_{ts} \\ V^*_{ub} & V^*_{cb} & V^*_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{198}$$

we can obtain six relations,

$$\begin{gathered} V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0 \\ V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \\ V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0 \\ V_{ud}V^*_{cd} + V_{us}V^*_{cs} + V_{ub}V^*_{cb} = 0 \\ V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0 \\ V_{cd}V^*_{td} + V_{cs}V^*_{ts} + V_{cb}V^*_{tb} = 0. \end{gathered} \tag{199}$$

The above expressions each represent a sum of three complex numbers.

Next, we consider the orthogonality condition between the first and third columns of V,

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0. \tag{200}$$

It is possible to represent this expression by a triangle in the complex plane. This is shown in Fig 44 together with the representation of a second useful relation,

$$V_{tb}V^*_{ub} + V_{ts}V^*_{us} + V_{td}V^*_{ud} = 0. \tag{201}$$

Note that the other four relations would result in topological strange triangles.

The angles α, β and γ are defined by

$$\alpha \equiv \arg\left(-\frac{V_{td}V^*_{tb}}{V^*_{ub}V_{ud}}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right), \quad \gamma \equiv \arg\left(-\frac{V^*_{ub}V_{ud}}{V_{cd}V^*_{cb}}\right). \tag{202}$$

The angles $(\alpha',\ \beta',\ \gamma')$ of the second useful triangle can be defined in a similar way.

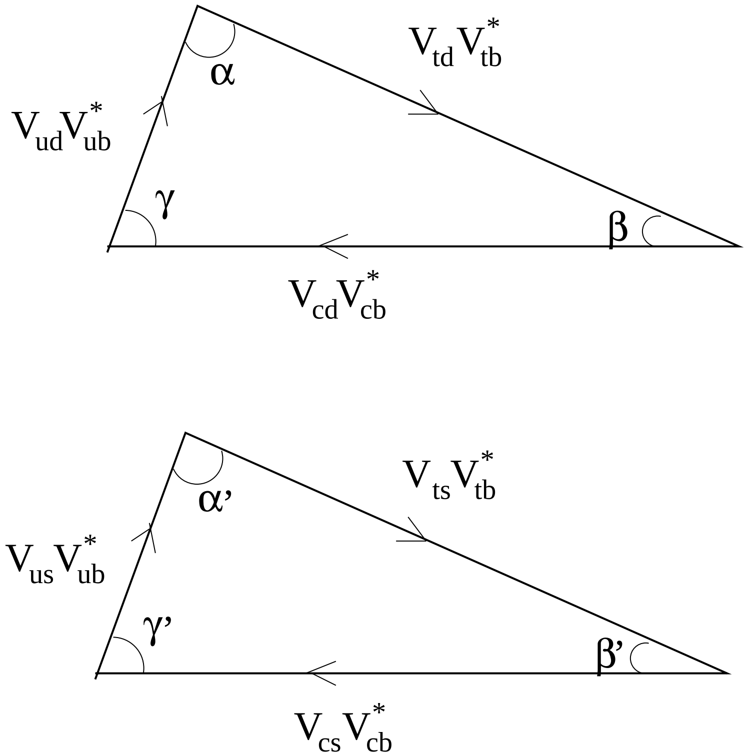

Figure 29: *Representation in the complex plane of the triangle formed by the CKM matrix elements of the first and third columns.*

It is useful to rescale expression (200) by dividing by its second term. We obtain

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0. \tag{203}$$

In this way we choose to orient the triangle such that $V_{cd}V_{cb}^*$ lies along the horizontal. In our parametrization V_{cb} is real, while V_{cd} is real to a good approximation in any case. Furthermore, we set the cosines of small angles to unity and obtain

$$V_{ub}^* + V_{td} = \sin\theta_C V_{cb}^*. \tag{204}$$

Fig 30 shows the corresponding triangle in the complex plane.

After rescaling the triangle with the factor $1/|\sin\theta_C V_{cb}|$, the base is of unit length and the coordinates of the vertices become

$$A(\mathrm{Re}(V_{ub}/|\sin\theta_C V_{cb}|, -\mathrm{IM}(V_{ub}/|\sin\theta_C V_{cb}|),\ B(1,0),\ C(0,0). \tag{205}$$

In the Wolfenstein parametrization the coordinates of the vertex A of the unitary triangle are simply (ρ, η), as shown in Fig. 30. All amplitudes can be measured, due to hermiticity of $VV^\dagger$.

Different authors remarked that all $\mathcal{CP}$ violating observables are proportional to a quantity J, which is independent of quark phases

$$J = \mathrm{Im}\{V_{us}V_{cb}V_{ub}^*V_{cs}^*\} = \mathrm{Im}\{V_{ud}V_{tb}V_{ub}^*V_{td}^*\}. \tag{206}$$

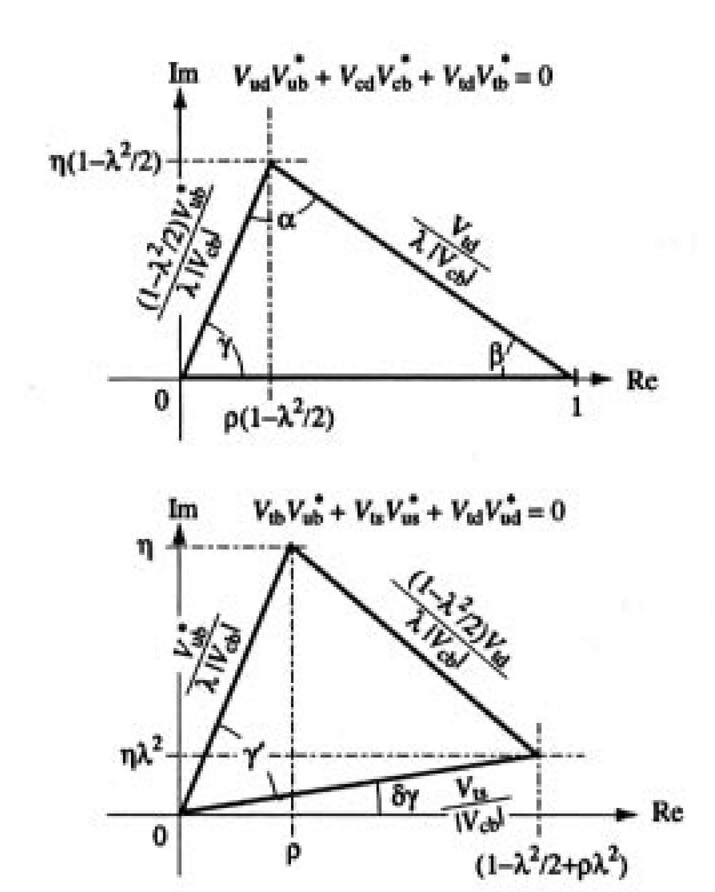

Figure 30: *Representation in the complex plane of the triangle formed by the CKM matrix elements of the first and third columns. The triangle has been rescaled.*

In Wolfenstein's parametrization one has $J \approx A^2\lambda^6\eta$, where J is equal to $2A\lambda^6$ times the area of the unitary triangle.

Without prove we state that for a theory that contains only four quarks, the symmetry $\mathcal{CP}$ is conserved. In a theory with six quarks it is possible that the short- and long-lived neutral kaons are not exact eigenstates of $\mathcal{CP}$. It turned out to be possible to perform such experiments that demonstrate the violation of $\mathcal{CP}$ (or $\mathcal{T}$) in K^0 decay. This will be the subject of the next section.

5 The neutral Kaon system

5.1 Particle Mixing for the Neutral K - Mesons

The action of symmetry principles will be discussed first for the creation and decay of neutral K-mesons. Neutral K-mesons are produced in reactions with the strong interaction, for example

$$\begin{array}{lccccccccccc} & \pi^- & + & p & \to & \Lambda & + & K^0 & & & & \\ S: & 0 & + & 0 & \to & -1 & + & 1 & & & & \\ & & & & & & & & & & & \\ & \pi^- & + & p & \to & \overline{\Lambda} & + & \overline{K}^0 & + & n & + & n \\ S: & 0 & + & 0 & \to & 1 & - & 1 & + & 0 & + & 0. \end{array} \tag{207}$$

Together with the charged kaons they form an isospin doublet (K^0, K^+) and an antiparticle doublet (K^-, $\overline{K}^0$). These particles are strong interaction eigenstates and have a well defined strangeness, $S = 1$ for the K^0, and $S = -1$ for the $\overline{K}^0$. Neutral kaons can be obtained by associated production and the pion energy threshold for the reaction $\pi^- + p \to \Lambda + K^0$ is 0.91 GeV, whereas the threshold for the reactions $\pi^+ + p \to K^+ + \overline{K}^0 + p$ and $\pi^- + p \to \overline{\Lambda} + \overline{K}^0 + n + n$ amounts to 1.5 and 6.0 GeV, respectively. Consequently, it is possible to produce a pure K^0 beam through a suitable choice of pion energy. The intrinsic parity of the neutral kaons is negative, just like in the case of pions.

Under the combined operation $\mathcal{CP}$ one has[30]

$$\begin{aligned} CP|K^0> &= -|\overline{K}^0> \\ CP|\overline{K}^0> &= -|K^0> . \end{aligned} \tag{208}$$

Clearly both the K^0 and the $\overline{K}^0$ are no eigenstates of $\mathcal{CP}$ (because S changes with two units). However, with the help of the superposition principle we can construct two new states, that are eigenstates of $\mathcal{CP}$,

$$\begin{aligned} |K_1^0> \equiv \tfrac{1}{\sqrt{2}}(|K^0> -|\overline{K}^0>) \quad &; \quad CP|K_1^0> = +|K_1^0> \\ |K_2^0> \equiv \tfrac{1}{\sqrt{2}}(|K^0> +|\overline{K}^0>) \quad &; \quad CP|K_2^0> = -|K_2^0> . \end{aligned} \tag{209}$$

The particle K^0 is the antiparticle of $\overline{K}^0$ and both mesons have, according to the $\mathcal{CPT}$-theorem, the same mass and lifetime. This does not hold for the K_1^0 and K_2^0, since they are not each other's antiparticles.

All this is important since the K-mesons are the lightest mesons with $S \neq 0$ and therefore decay *weakly* (both in the electromagnetic and in the strong interaction the quantum number S is conserved). We first study the decay of two pions. We start with the $\pi^+\pi^-$ system. In het center of mass of both pions the operation $\mathcal{P}$ interchanges the π^+ and the π^-. Because the particles are bosons, their wave function must be symmetric under particle exchange. Charge conjugation also interchanges the π^+ and π^-, and the combined operation $\mathcal{CP}$ results in the initial state, independent of orbital angular momentum l.

[30] Here, one encounters different notations in the literature since the overall phase of the states is arbitrary.

The same is true for a system with two neutral pions, where Bose symmetry excludes the antisymmetric odd-l states. Therefore, one has

$$CP|2\pi> = +|2\pi> . \qquad (210)$$

Assuming that $\mathcal{CP}$ is conserved in the weak interaction[31], the state K_1^0 can decay into two pions and three pions, but the three-pion state cannot exist in the lowest angular momentum state. On the other hand[32]

$$CP|3\pi> \simeq -|3\pi> . \qquad (211)$$

Thus, the K_2^0-meson can decay into three pions, but not two.

We differentiate between the following observed decays of the neutral kaons [2].

$$\begin{array}{rlll}
K_S \approx K_1^0 & \to \pi^+ + \pi^- & \text{B.R.} & (68.61 \pm 0.28)\% \\
& \to \pi^0 + \pi^0 & \text{B.R.} & (31.39 \pm 0.28)\% \\
& \to \pi^+ + \pi^- + \gamma & \text{B.R.} & (1.78 \pm 0.05) \times 10^{-3} \\
& \to \gamma + \gamma & \text{B.R.} & (2.4 \pm 0.9) \times 10^{-6} \\
& \to \pi^\pm + e^\mp \nu & \text{B.R.} & (6.70 \pm 0.07) \times 10^{-4} \\
& \to \pi^\pm + \mu^\mp \nu & \text{B.R.} & (4.69 \pm 0.06) \times 10^{-4} \\
\tau & = (8.927 \pm 0.009) \times 10^{-11}\ \text{s} & & \\
& & & \\
K_L \approx K_2^0 & \to \pi^0 + \pi^0 + \pi^0 & \text{B.R.} & (21.12 \pm 0.27)\% \\
& \to \pi^0 + \pi^+ + \pi^- & \text{B.R.} & (12.56 \pm 0.20)\% \\
& \to \pi^\pm + \mu^\mp \nu & \text{B.R.} & (27.17 \pm 0.25)\% \\
& \to \pi^\pm + e^\mp \nu & \text{B.R.} & (38.78 \pm 0.27)\% \\
& \to \gamma + \gamma & \text{B.R.} & (5.92 \pm 0.15) \times 10^{-4} \\
& \to \pi^0 + \gamma + \gamma & \text{B.R.} & (1.70 \pm 0.28) \times 10^{-6} \\
& \to \pi^0 + \pi^\pm + e^\mp \nu & \text{B.R.} & (5.18 \pm 0.29) \times 10^{-5} \\
& \to \pi^\pm + e^\mp \nu + \gamma & \text{B.R.} & (1.3 \pm 0.8)\% \\
& \to \pi^+ + \pi^- + \gamma & \text{B.R.} & (4.61 \pm 0.14) \times 10^{-5} \\
& \to \pi^+ + \pi^- & \text{B.R.} & (2.067 \pm 0.035) \times 10^{-3} \\
& \to \pi^0 + \pi^0 & \text{B.R.} & (9.36 \pm 0.20) \times 10^{-4} \\
& \to \mu^+ + \mu^- & \text{B.R.} & (7.2 \pm 0.5) \times 10^{-9} \\
& \to \mu^+ + \mu^- + \gamma & \text{B.R.} & (3.23 \pm 0.30) \times 10^{-7} \\
& \to e^+ + e^- + \gamma & \text{B.R.} & (9.1 \pm 0.5) \times 10^{-6} \\
& \to e^+ + e^- + \gamma + \gamma & \text{B.R.} & (6.5 \pm 1.2) \times 10^{-7} \\
& \to e^+ + e^- + e^+ + e^- & \text{B.R.} & (4.1 \pm 0.8) \times 10^{-8} \\
\tau & = (5.17 \pm 0.04) \times 10^{-8}\ \text{s} & &
\end{array} \qquad (212)$$

For the decay into two pions there is an energy available of about 215 MeV, and for the decay into three pions of about 78 MeV. The phase space for three-pion decay is consequently significantly smaller than that for the decay into two pions. One thus expects

[31] This is only an approximation. In section 5.4 we will discuss $\mathcal{CP}$ violation in the decay of neutral kaons. Consequently, we will change notation: K_1^0 and K_2^0 will be the $\mathcal{CP}$ eigenstates, while K_S^0 and K_L^0 will be used to denote the short-lived and long-lived decay states.

[32] In the 3π-decay of the K_2^0 there is only a relatively small amount of kinetic energy available for the three pions ($Q \approx 78$ MeV). Their orbital angular momentum is therefore mainly $l = 0$. From this it follows that $CP = -1$.

that the decay times will be significantly different. Also their massa's are different, but that difference is small

$$m_{K_2^0} - m_{K_1^0} = m_2 - m_1 = (3.491 \pm 0.009) \times 10^{-6} \text{ eV}. \tag{213}$$

The system is almost completely degenerate.

$$\frac{\Delta E}{E} = \frac{\Delta m}{m_K} \simeq 7 \times 10^{-15}, \tag{214}$$

and this leads to a series of interesting phenomena.

The wave functions of both particles have in vacuum the following time dependence,

$$|\psi_1(\tau) >= |\psi_1(0) > \ e^{-im_1\tau - \frac{1}{2}\Gamma_1\tau} \quad \text{and} \quad |\psi_2(\tau) >= |\psi_2(0) > \ e^{-im_2\tau - \frac{1}{2}\Gamma_2\tau}, \tag{215}$$

with $m_{1,2}$ and $\Gamma_{1,2}$ the masses and decay rates of K_1^0 and K_2^0. Here τ is the eigentime, $\tau = t\sqrt{1 - v^2}$, and t is the time measured in the laboratory.

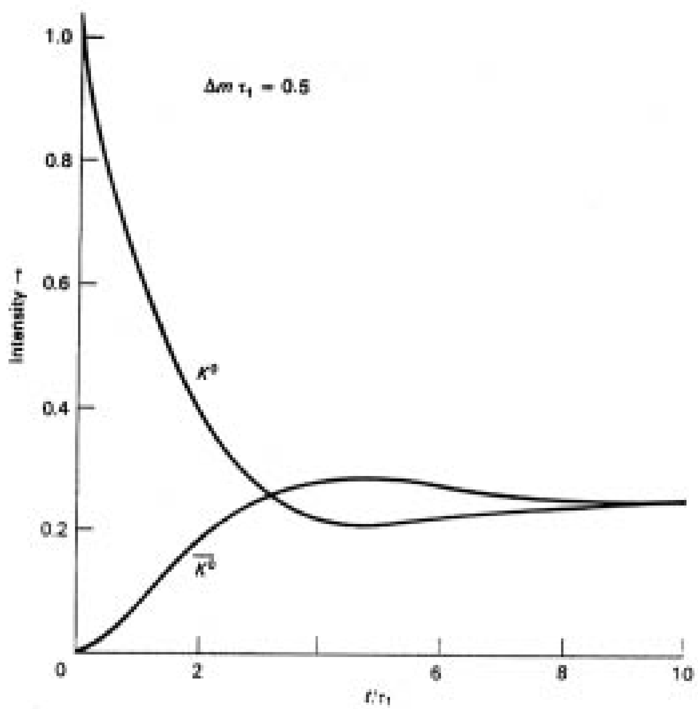

Figure 31: *A beam initially consisting of only K^0-mesons transforms after a certain time into a K_2^0 beam, that contains equal amounts of K^0 and $\overline{K}^0$.*

We consider what happens when a system composed of only K^0's, as produced in the reaction $\pi^- + p \rightarrow \Lambda + K^0$, decays in vacuum. The probability to find a K^0 must then, according to the superposition principle of quantum mechanics, have the following time dependence,

$$I(\tau) = | < K^0|\psi(\tau) > |^2 = \frac{1}{4}\left\{e^{-\Gamma_1\tau} + e^{-\Gamma_2\tau} + 2e^{-\frac{\Gamma_1+\Gamma_2}{2}\tau}\cos(m_2 - m_1)\tau\right\}. \tag{216}$$

The probability to find a $\overline{K}^0$ amounts to

$$\overline{I}(\tau) = | < \overline{K}^0 | \psi(\tau) > |^2 = \frac{1}{4} \left\{ e^{-\Gamma_1 \tau} + e^{-\Gamma_2 \tau} - 2 e^{-\frac{\Gamma_1 + \Gamma_2}{2} \tau} \cos(m_2 - m_1) \tau \right\}. \tag{217}$$

The K^0 first decays fast, during which it preferentially emits two pions. After a certain time ($\tau \gg \Gamma_S^{-1}$) we obtain an almost pure K_2^0 state, that among others decays by three-pion emission. Fig. 31 shows that in the intermediate region oscillations occur, that can be readily observed. Data on oscillations are presented in Fig. 32.

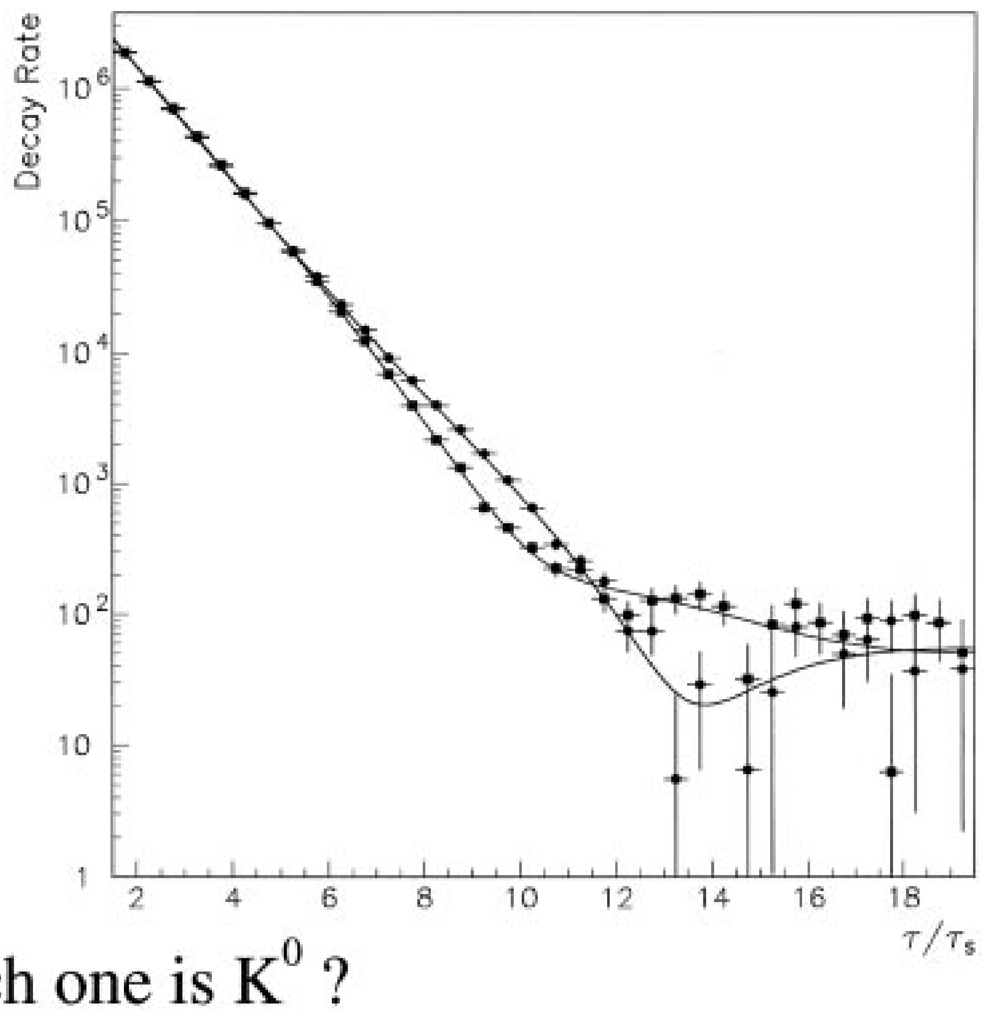

Figure 32: *Number of $\pi^+\pi^-$ decays from K^0 and $\overline{K^0}$ as function of eigentime. The best fit demands the existence of an interference term between K_1^0 and K_2^0.*

We can define the asymmetry $A(\tau)$ as

$$A(\tau) = \frac{I(\tau) - \overline{I}(\tau)}{I(\tau) + \overline{I}(\tau)} = \frac{2 e^{-\frac{\Gamma_1 + \Gamma_2}{2} \tau}}{e^{-\Gamma_1 \tau} + e^{-\Gamma_2 \tau}} \cos(m_2 - m_1) \tau. \tag{218}$$

This asymmetry represents the beam strangeness $< S >$ which oscillates with a frequency $\Delta m / 2\pi$.

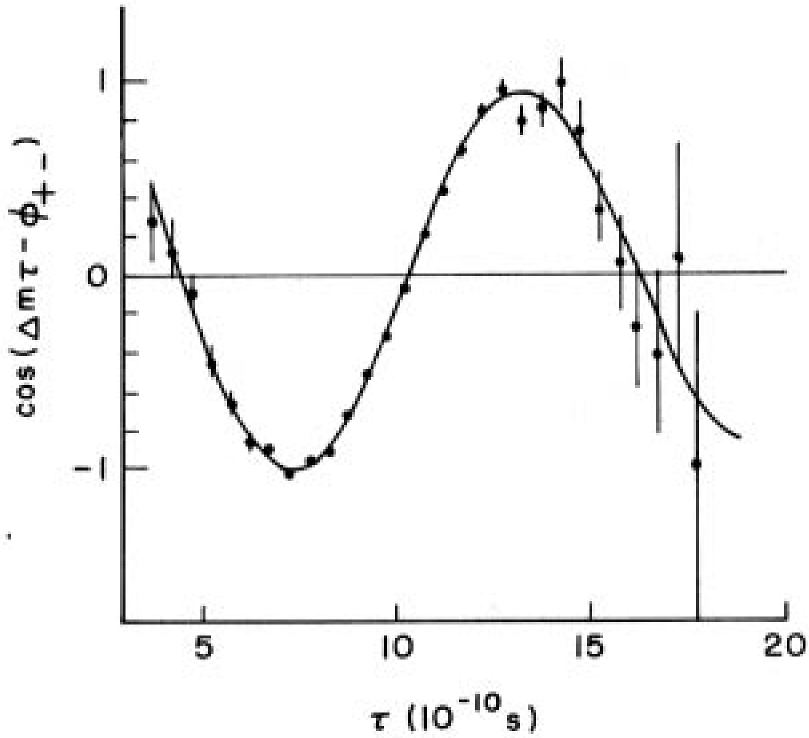

Figure 33: *Strangeness oscillations and the interference term. From the fit one can determine the $K_2^0 - K_1^0$ mass difference Δm and the phase ϕ_{+-} between both amplitudes.*

Fig. 33 shows that it can be measured, because in semileptonic decay one has the $\Delta S = \Delta Q$ rule, where Q is the charge of the hadron. The K^0 predominantly decays into

$$K^0 \to \pi^- + e^+ + \nu_e; \quad \Delta S = -1,\ \Delta Q = -1, \tag{219}$$

while the $\overline{K}^0$ decays into *the experimentally different channel*

$$\overline{K}^0 \to \pi^+ + e^- + \bar{\nu}_e; \quad \Delta S = +1,\ \Delta Q = +1. \tag{220}$$

The emperical rule $\Delta S = \Delta Q$ can be understood by considering for example the first-order Feynman diagrams for the reactions $K^0 \to \pi^- + e^+ + \nu_e$ and $\overline{K}^0 \to \pi^+ + e^- + \bar{\nu}_e$ shown in Fig. 34. The basic process is the decay of the $\overline{s}$ (s) quark with the d ($\overline{d}$) quark acting as a spectator.

The ratio of the number of leptons with the 'wrong' sign (e.g. electrons) and leptons with the 'right' sign (e.g. positrons) for a state that started as K^0 integrated over time is

$$r_K = \frac{(\Gamma_1 - \Gamma_2)^2 + 4(\Delta m)^2}{2(\Gamma_1 + \Gamma_2)^2 - (\Gamma_1 - \Gamma_2)^2 + 4(\Delta m)^2}, \tag{221}$$

with $\Delta m = m_1 - m_2$. Since Γ_1, the decay rate of the K_1^0 is much larger than Γ_2, the decay rate of K_2^0, the value for r_K with be close to unity.

5.2 Mass and Decay Matrices

The hamiltonian that determines the time evolution of a particle state ψ is

$$H = H_{\text{strong}} + H_{\text{em}} + H_{\text{weak}} \tag{222}$$

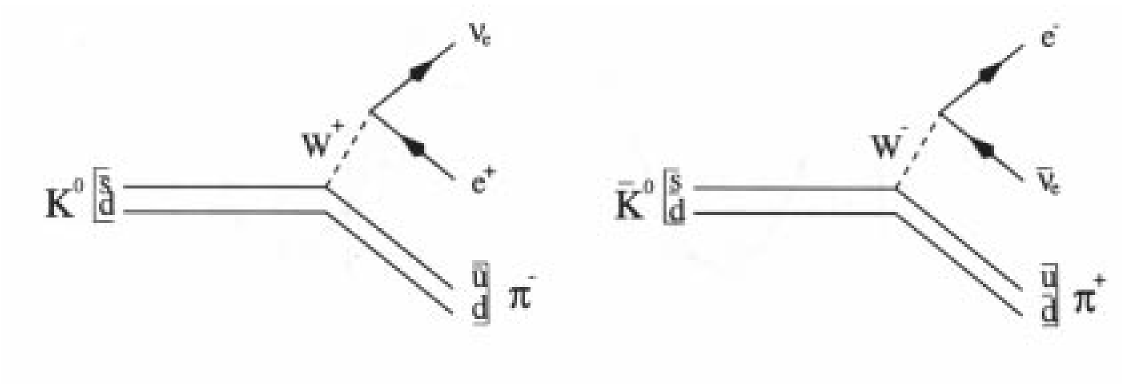

Figure 34: *First-order Feynman diagrams for the reactions* $K^0 \to \pi^- + e^+ + \nu_e$ *and* $\overline{K}^0 \to \pi^+ + e^- + \bar{\nu}_e$.

and obeys

$$i\frac{\partial\psi}{\partial\tau} = H\psi. \tag{223}$$

For an unstable particle of mass M and lifetime $\tau_{\rm life} = 1/\Gamma$, the wave function in the rest frame of the particle has time dependence $\exp\{-i(M - i\Gamma/2)\tau\}$ and the Schrödinger equation is

$$i\frac{\partial\psi}{\partial\tau} = H\psi = \left(M - i\frac{\Gamma}{2}\right)\psi, \tag{224}$$

where M and Γ are positive numbers.

Next we consider a *mixed system* of unstable particles, e.g. K^0 and $\overline{K}^0$, and assume that at time τ we have an arbitrary superposition of states K^0 and $\overline{K}^0$, with time dependent coefficients c_1 and c_2,

$$\psi(\tau) = c_1(\tau)|K^0> + c_2(\tau)|\overline{K}^0> . \tag{225}$$

The state can be represented by a two-component wave function (see Eq. (225))

$$\psi(\tau) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \tag{226}$$

where $c_1(\tau)$ and $c_2(\tau)$ are the amplitudes for finding K^0 and $\overline{K}^0$ at time τ, respectively, when the initial state was $\psi(0)$. When we insert

$$|K^0> = \frac{1}{\sqrt{2}}(|K_1^0(0)> + |K_2^0(0)>), \quad \text{and} \quad |\overline{K}^0> = \frac{1}{\sqrt{2}}(|K_1^0(0)> - |K_2^0(0)>), \tag{227}$$

in Eq. (225), we find

$$\psi(\tau) = |K_1^0(\tau)> + |K_2^0(\tau)> = \left[\frac{c_1(\tau) + c_2(\tau)}{\sqrt{2}}\right]|K_1(0)> + \left[\frac{c_1(\tau) - c_2(\tau)}{\sqrt{2}}\right]|K_2(0)> . \tag{228}$$

From Eq. (215) we derive

$$i\frac{\partial\psi_{1,2}}{\partial\tau} = \left(m_{1,2} - i\frac{\Gamma_{1,2}}{2}\right)\psi_{1,2}, \tag{229}$$

and find with Eq. (228)

$$\begin{aligned} i\left(\tfrac{\partial c_1}{\partial\tau} + \tfrac{\partial c_2}{\partial\tau}\right) &= \left(m_1 - i\tfrac{\Gamma_1}{2}\right)(c_1 + c_2), \\ i\left(\tfrac{\partial c_1}{\partial\tau} - \tfrac{\partial c_2}{\partial\tau}\right) &= \left(m_2 - i\tfrac{\Gamma_2}{2}\right)(c_1 - c_2). \end{aligned} \tag{230}$$

This can be written as follows

$$\begin{aligned} i\tfrac{\partial c_1}{\partial\tau} &= \left[\tfrac{m_1+m_2}{2} - \tfrac{i(\Gamma_1+\Gamma_2)}{4}\right]c_1 + \left[\tfrac{m_1-m_2}{2} - \tfrac{i(\Gamma_1-\Gamma_2)}{4}\right]c_2 \\ i\tfrac{\partial c_2}{\partial\tau} &= \left[\tfrac{m_1-m_2}{2} - \tfrac{i(\Gamma_1-\Gamma_2)}{4}\right]c_1 + \left[\tfrac{m_1+m_2}{2} - \tfrac{i(\Gamma_1+\Gamma_2)}{4}\right]c_2. \end{aligned} \tag{231}$$

In matrix notation this can be expressed in the K^0 - $\overline{K}^0$ basis as

$$i\frac{\partial\psi}{\partial\tau} = H\psi = (M - i\frac{\Gamma}{2})\psi = \begin{pmatrix} \overline{m} - i\frac{\overline{\Gamma}}{2} & \Delta m - i\frac{\Delta\Gamma}{2} \\ \Delta m - i\frac{\Delta\Gamma}{2} & \overline{m} - i\frac{\overline{\Gamma}}{2} \end{pmatrix}\psi, \tag{232}$$

with $\overline{m} = (m_1 + m_2)/2$, $\overline{\Gamma} = (\Gamma_1 + \Gamma_2)/2$, $\Delta m = (m_1 - m_2)/2$ and $\Delta\Gamma = (\Gamma_1 - \Gamma_2)/2$.

The matrices M and Γ are called the mass and decay matrix, respectively. Both M and Γ are hermitian ($M^*_{ij} = M_{ji}$, $\Gamma^*_{ij} = \Gamma_{ji}$), whereas H is not hermitian since the particles decay and probability is not conserved[33]. From $\mathcal{CPT}$ invariance it follows that a particle and antiparticle have identical mass and life time; thus $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.

$\mathcal{CP}$ is violated if the off-diagonal elements of the hamiltonian are different, $H_{12} \neq H_{21}$. This implies that Im M_{12} and Im Γ_{12} contribute to $\mathcal{CP}$ violation. This can be easily demonstrated by considering the case Im $M_{12} =$ Im $\Gamma_{12} = 0$, since then the off-diagonal elements are equal and the eigenvectors are the $\mathcal{CP}$ eigenstates

$$|K_1^0 > \equiv \frac{1}{\sqrt{2}}(|K^0 > - |\overline{K}^0 >) \;\text{ and }\; |K_2^0 > \equiv \frac{1}{\sqrt{2}}(|K^0 > + |\overline{K}^0 >), \tag{234}$$

with eigenvalues $CP = +1$ and -1, respectively. The eigenvalues of the mass matrix corresponding to these eigenstates are

$$m_1 = \overline{m} + M_{12}, \quad \Gamma_1 = \overline{\Gamma} + \Gamma_{12} \rightarrow m_1 - i\frac{\Gamma_1}{2} \tag{235}$$

and

$$m_2 = \overline{m} - M_{12}, \quad \Gamma_2 = \overline{\Gamma} - \Gamma_{12} \rightarrow m_2 - i\frac{\Gamma_2}{2} \tag{236}$$

[33]Note that in case one assumes $H_{\rm weak} = 0$, both K^0 and $\overline{K}^0$ cannot decay, and consequently one has $\Gamma = 0$. In that case H reduces to

$$H \rightarrow M = \begin{pmatrix} m_{K^0} & 0 \\ 0 & m_{\overline{K}^0} \end{pmatrix}, \tag{233}$$

where $m_{K^0} =< K^0|H_{\rm strong} + H_{\rm em}|K^0 >$ and $m_{\overline{K}^0} =< \overline{K}^0|H_{\rm strong} + H_{\rm em}|\overline{K}^0 >$.

and the mass and width differences are

$$\Delta m = \frac{m_1 - m_2}{2} = M_{12} \;\text{ and }\; \Delta\Gamma = \frac{\Gamma_1 - \Gamma_2}{2} = \Gamma_{12}. \tag{237}$$

Thus even without $\mathcal{CP}$ violation flavor oscillations still occur, because K_1^0 and K_2^0 evolve differently in time. For a discussion of the more general case, with Im $M_{12} =$ Im $\Gamma_{12} \neq 0$, it is useful to study the diagonalization of the mass and decay matrix[34]. From diagonalization (see Eq. (238)) of Eq. (232) we find that

$$\cos\beta = 0,\; \alpha = 0,\; D = \overline{m} - i\frac{\overline{\Gamma}}{2},\; E = \Delta m - i\frac{\Delta\Gamma}{2}. \tag{241}$$

One observes that the eigenvectors are orthogonal. In case of $\mathcal{CP}$ violation with conservation of $\mathcal{CPT}$ one has in vacuum

$$\cos\beta = 0,\; \alpha \neq 0 \tag{242}$$

and in this case both eigenvectors are *not* orthogonal $< V_1|V_2 > \propto 2\text{Re}\,\alpha$.

5.3 Regeneration

Here, we discuss the remarkable phenomenon of regeneration. When we start with a pure K^0-state, then after a certain time we obtain a practically pure K_2^0-state (see Fig. 35). This state contains equals amounts of K^0- and $\overline{K}^0$-mesons,

$$|K_2^0 >= \frac{1}{\sqrt{2}}(|K^0 > +|\overline{K}^0 >). \tag{243}$$

In case we scatter this beam into matter, then for both states *different* strong interaction effects will take place. The $\overline{K}^0$ can for example induce the reaction

$$\overline{K}^0 + p \to \Lambda + \pi^+. \tag{244}$$

Due to strangeness conservation this reaction cannot take place for the K^0. This has, again according to the superposition principle, as consequence that after the regenerator we do not have a pure K_2^0 beam anymore. The elastic scattering amplitudes, f and $\overline{f}$,

[34] A complex 2×2 matrix can be easily diagonalized with the following representation:

$$M - i\frac{\Gamma}{2} = D1 + \vec{E}\cdot\vec{\sigma} = D\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + E\begin{pmatrix} \cos\beta & \sin\beta e^{-i\alpha} \\ \sin\beta e^{i\alpha} & -\cos\beta \end{pmatrix}, \tag{238}$$

with D and $\vec{E}$ a complex scalar and complex vector. One has $E_x = E\sin\beta\cos\alpha$, $E_y = E\sin\beta\sin\alpha$ and $E_z = E\cos\beta$. The matrices $\vec{\sigma}$ are the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{239}$$

The eigenvectors are

$$V_1 = \begin{pmatrix} 1 \\ \tan\frac{\beta}{2}e^{i\alpha} \end{pmatrix} \quad\text{and}\quad V_2 = \begin{pmatrix} 1 \\ -\cot\frac{\beta}{2}e^{i\alpha} \end{pmatrix} \tag{240}$$

and the corresponding eigenvalues are $D \pm E$.

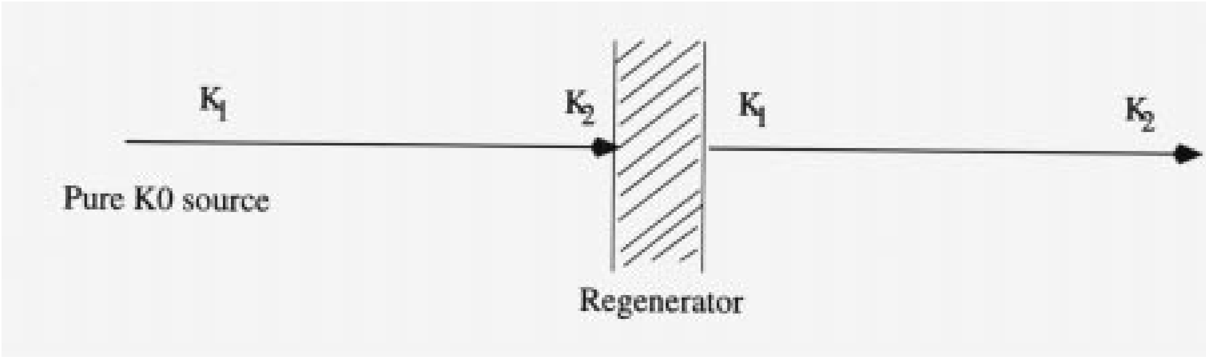

Figure 35: *Schematic representation of the phenomenon of regeneration of K_S-mesons from a beam of K_L-mesons.*

for the reactions $K^0 p$ and $\overline{K}^0 p$ are different. This results in an additional phase factor, $\phi_{\rm reg} = k(n-1)l$ from elastic scattering for kaons moving in the forward direction. Here, l is the distance the particle travels in the medium, while n is the index of refraction and k the wave number of the incoming particles. The distance l is related to the eigentime interval as $l = \tau v/\sqrt{1-v^2}$. The index of refraction is given by

$$n = 1 + \frac{2\pi N}{k^2} f(0), \tag{245}$$

with N the density of scattering centers, and $f(0)$ the complex elastic scattering amplitude. The optical theorem related $f(0)$ to the total cross section as

$$\sigma_{\rm total} = \frac{4\pi}{k} {\rm Im}\ f(0). \tag{246}$$

Since K^0 and $\overline{K}^0$ have different total cross sections, the corresponding indices of refraction will differ, and we find for their phases

$$\phi_{\rm reg} = \frac{2\pi N \tau v}{k\sqrt{1-v^2}} f(0). \tag{247}$$

We will take this additional phase into account in the mass matrix and denote the amplitude for elastic scattering of K^0 with f and for $\overline{K}^0$ with $\overline{f}$. We find for a neutral kaon beam in matter

$$i\frac{\partial \psi}{\partial \tau} = \begin{pmatrix} \overline{m} - i\frac{\overline{\Gamma}}{2} - \frac{\phi_{\rm reg}}{\tau} & \Delta m - i\frac{\Delta\Gamma}{2} \\ \Delta m - i\frac{\Delta\Gamma}{2} & \overline{m} - i\frac{\overline{\Gamma}}{2} - \frac{\overline{\phi}_{\rm reg}}{\tau} \end{pmatrix} \psi. \tag{248}$$

From Eq. (238) we find that

$$D = \overline{m} - i\frac{\overline{\Gamma}}{2} - \frac{\overline{\phi}_{\rm reg} - \phi_{\rm reg}}{2\tau}, \quad E = \Delta m - i\frac{\Delta\Gamma}{2}, \quad \cos\beta = \frac{\overline{\phi}_{\rm reg} - \phi_{\rm reg}}{(2\Delta m - i\Delta\Gamma)\tau}, \tag{249}$$

with $|\cos\beta| \ll 1$ and $\sin\beta \approx 1$.

From the relations $\tan(\beta/2) = (1-\cos\beta)/\sin\beta$ and $\cot(\beta/2) = (1+\cos\beta)/\sin\beta$ it follows that the eigenvectors are proportional to

$$\begin{pmatrix} 1 \\ 1-\cos\beta \end{pmatrix}, \quad \begin{pmatrix} 1-\cos\beta \\ -1 \end{pmatrix}, \quad \text{or to} \quad \begin{pmatrix} 1+r_{\rm reg} \\ 1-r_{\rm reg} \end{pmatrix}, \quad \begin{pmatrix} 1-r_{\rm reg} \\ -1-r_{\rm reg} \end{pmatrix}, \tag{250}$$

with $r_{\text{reg}} = \cos\beta/2$. We can write these states as

$$|K_1^{0\prime}> = |K_1^0> + r_{\text{reg}}|K_2^0>, \quad \text{and} \quad |\overline{K}_2^{0\prime}> = -r_{\text{reg}}|K_1^0> + |K_2^0>, \tag{251}$$

where r_{reg} is a small number, typically in the order of 10^{-3}, given by

$$r_{\text{reg}} = \frac{\cos\beta}{2} = \frac{\overline{\phi}_{\text{reg}} - \phi_{\text{reg}}}{2(2\Delta m - i\Delta\Gamma)\tau}. \tag{252}$$

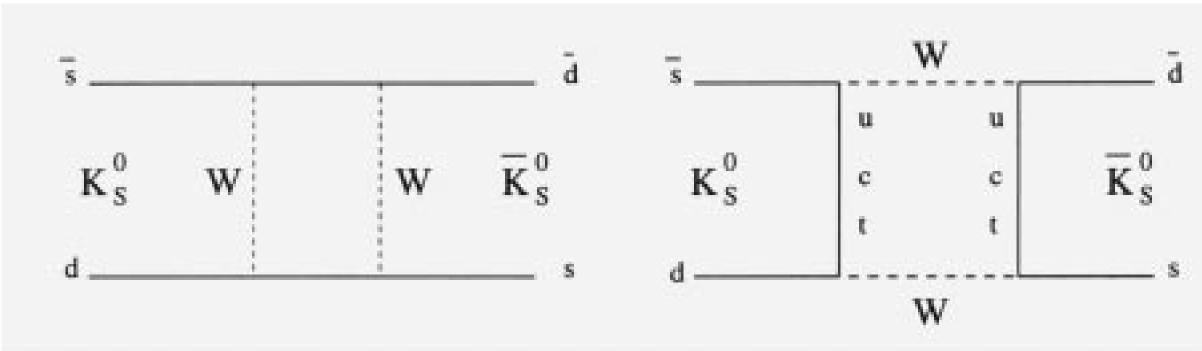

Figure 36: *Mixing of $K^0 \leftrightarrow \overline{K}^0$ due to the weak interaction. Note that the strangeness changes by two units.*

When a neutral kaon beam travels over a large distance, then it only will consist of K_2^0's. When the K_2^0's travel through matter, their propagation needs to be analyzed as function of the eigenstates in that medium. The K_2^0 is almost completely $K_2^{0\prime}$, but with a small component $K_1^{0\prime}$. These two components will develop different phases when traveling through matter. After leaving the medium the states need to be analyzed again in terms of K_1^0 and K_2^0. This reintroduces a component K_1^0 of the order r_{reg}, where the amplitude of regenerated K_1^0 is proportional to r_{reg}. Thus, the amount of regenerated K_1^0 depends on the mass difference. We expect that now again two-pion decay will take place. Indeed it is possible that such phenomena, which one calls regeneration, can be experimentally observed.

Why do these remarkable phenomena only occur in the K^0-system and not for example with neutrons? The reason for this is the fact that due to second-order weak interaction effects a mixing of $K^0 \leftrightarrow \overline{K}^0$ occurs (see Fig. 36). Consequently the strangeness changes by two units, $\Delta S = 2$. A similar transition from a neutron to an antineutron $n \leftrightarrow \overline{n}$ is excluded due to conservation of baryon number.

5.4 CP Violation with Neutral Kaons

In 1964 Christenson, Cronin, Fitch and Turlay [6] demonstrated in an experiment at the Alternating Gradient Synchrotron (AGS) at Brookhaven that also the long-lived neutral kaon can decay into two pions with a branching ratio of about 2×10^{-3}.

The experiment of Christenson *et al.* is schematically represented in Fig. 37. The apparatus was a two-arm spectrometer and each arm had a magnet for momentum determination, scintillators to trigger on charged particles, a Čerenkov to discriminate between $\pi^{\pm}$ and $e^{\pm}$, and spark chambers to visualize the tracks of charged particles.

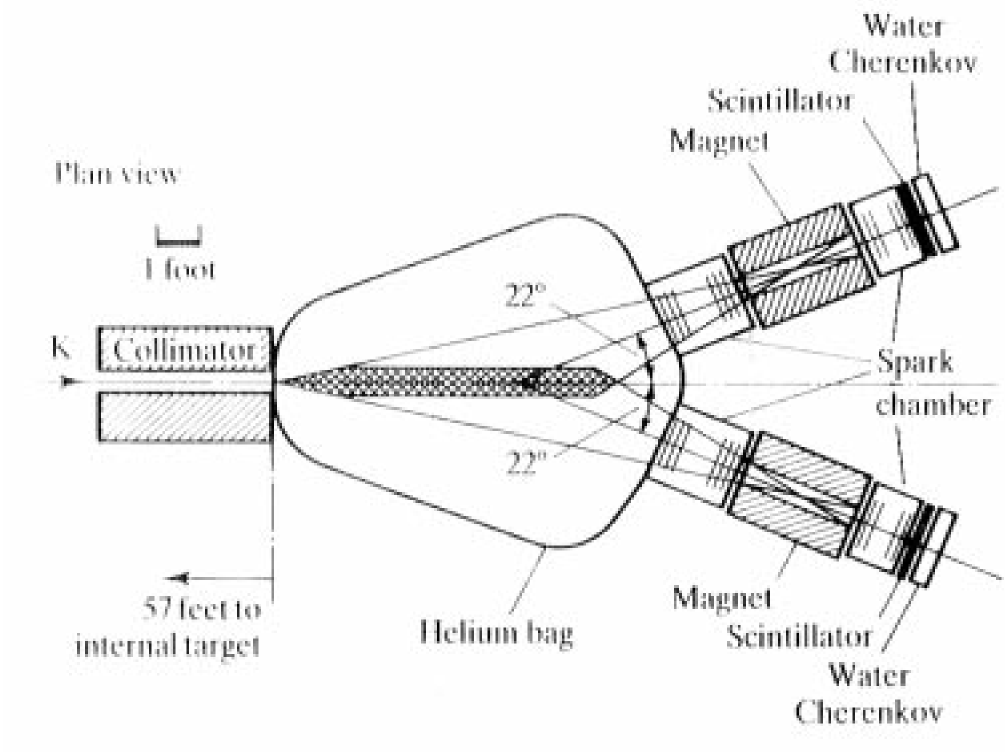

Figure 37: *Schematic representation of the experimental set-up used by Christensor, Cronin, Fitch and Turlay to demonstrate $\mathcal{CP}$-violation in K_L-decay.*

In the decay of $K_L^0 \to \pi^+\pi^-$ the invariant mass M of both pions is

$$Mc^2 = (E_1 + E_2)^2 - c^2(\vec{p}_1 + \vec{p}_2)^2 = M_K = 498 \text{ MeV/c}^2, \tag{253}$$

while the three-particle decay $K_L^0 \to \pi^+\pi^-\pi^0$, the invariant mass is smaller

$$280 \text{ MeV/c}^2 < M < 363 \text{ MeV/c}^2. \tag{254}$$

In addition in the two-particle decay the vector sum of both momenta $\vec{p}_1 + \vec{p}_2$ needs to correspond with the beam direction; in the three-particle decay this is not required.

Fig. 38 shows the angular distribution in the extreme forward direction for two oppositely charged pions from the decay $K_2^0 \to \pi^+\pi^-$. The pronounced forward peak for the central mass range corresponding to two-body decay provides a small but convincing signature for the $\mathcal{CP}$ violating decay. Shortly thereafter, the experiment was repeated at several laboratories and the results were confirmed.

We will now change notation. The reason is that Eq. (209) *defines* the states K_1^0 en K_2^0 as eigenstates of $\mathcal{CP}$. However, the measurements of Christenson *et al*. show that the long-lived neutral kaon is *not* an eigenstate of $\mathcal{CP}$. It is custom to maintain the notation K_1^0 en K_2^0 for the eigenstates of $\mathcal{CP}$, and to indicate the real particles with K_S^0 (short-lived neutral kaon) and K_L^0 (long-lived neutral kaon). The K_S^0 and K_L^0 represent the physical decay states and have unique but differing life times.

In the mass and decay matrix that governs the time evolution of the $K^0 - \overline{K}^0$ system, it is possible to realize $\mathcal{CP}$ violation in case the non-diagonal elements Δm and $\Delta\Gamma$ are

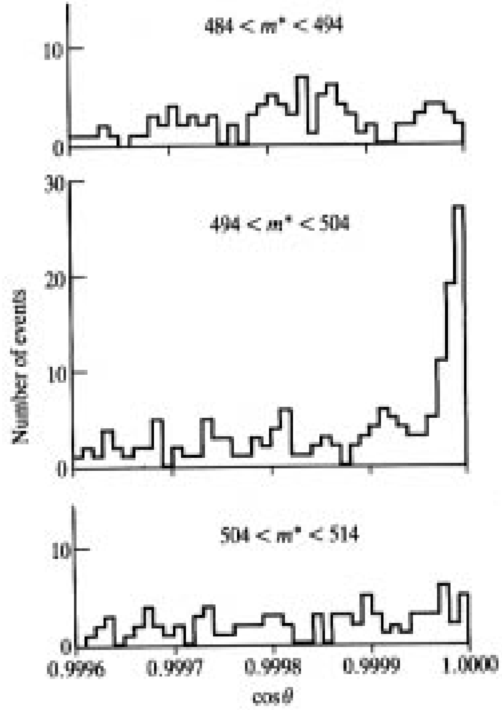

Figure 38: *Angular distribution of pions with opposite charge from the decay $K_2^0 \to \pi^+\pi^-$ versus $\cos\theta$ for three mass ranges.*

complex (both $\overline{m}$ and $\overline{\Gamma}$ remain real).

$$i\frac{\partial\psi}{\partial\tau} = \begin{pmatrix} \overline{m} - i\frac{\overline{\Gamma}}{2} & \Delta m - i\frac{\Delta\Gamma}{2} \\ \Delta m^* - i\frac{\Delta\Gamma^*}{2} & \overline{m} - i\frac{\overline{\Gamma}}{2} \end{pmatrix} \psi = \left\{ D \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + E \begin{pmatrix} \cos\beta & \sin\beta e^{-i\alpha} \\ \sin\beta e^{i\alpha} & -\cos\beta \end{pmatrix} \right\} \psi. \tag{255}$$

For $\cos\beta = 0$ and $\alpha \neq 0$ we find the solution

$$D = \overline{m} - i\frac{\overline{\Gamma}}{2} \quad \text{and} \quad E = \text{Re } \Delta m - i\text{Re } \frac{\Delta\Gamma}{2}. \tag{256}$$

In case we take the difference of the non-diagonal elements over their sum, we obtain an expression for α

$$\frac{e^{-i\alpha} - e^{i\alpha}}{e^{-i\alpha} + e^{i\alpha}} = i\text{tan}\alpha = \frac{i\text{Im } \Delta m + \text{Im } \frac{\Delta\Gamma}{2}}{\text{Re } \Delta m - i\text{Re } \frac{\Delta\Gamma}{2}}. \tag{257}$$

According to tradition we introduce the variables ϵ, p and q through the relation $q/p = (1-\epsilon)/(1+\epsilon) = e^{i\alpha}$. The eigenvectors are

$$\begin{pmatrix} p \\ q \end{pmatrix} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \begin{pmatrix} 1+\epsilon \\ 1-\epsilon \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} p \\ -q \end{pmatrix} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \begin{pmatrix} 1+\epsilon \\ -(1-\epsilon) \end{pmatrix}. \tag{258}$$

From Eq. (257) it follows that

$$-i\tan\alpha = \frac{\frac{1+\epsilon}{1-\epsilon} - \frac{1-\epsilon}{1+\epsilon}}{\frac{1+\epsilon}{1-\epsilon} + \frac{1-\epsilon}{1+\epsilon}} = \frac{(1+\epsilon)^2 - (1-\epsilon)^2}{(1+\epsilon)^2 + (1-\epsilon)^2} = \frac{2\epsilon}{1+\epsilon^2}. \tag{259}$$

In the kaon system both ϵ and α are small quantities, and we have

$$\epsilon \approx -\frac{i\alpha}{2} \approx -\frac{1}{2}\frac{i\text{Im }\Delta m + \text{Im }\frac{\Delta\Gamma}{2}}{\text{Re }\Delta m - i\text{Re }\frac{\Delta\Gamma}{2}}. \tag{260}$$

Thus, when $\mathcal{CP}$ is violated, but $[\mathcal{CPT}, \mathbf{H}] = 0$ for each interaction because of a corresponding violation of $\mathcal{T}$, then the deviations of K_S^0 and K_L^0 from K_1^0 and K_2^0 can be expressed as

$$|K_S^0> = \frac{(1+\epsilon)|K^0> + (1-\epsilon)|\overline{K}^0>}{\sqrt{2(1+|\epsilon|^2)}} \text{ and } |K_L^0> = \frac{(1+\epsilon)|K^0> - (1-\epsilon)|\overline{K}^0>}{\sqrt{2(1+|\epsilon|^2)}}, \tag{261}$$

where the superposition of the states is changed compared to that given by Eq. (209) by using the parameter ϵ, that characterizes the *mixing* of both states. Instead of equation (261) we also can write the states as

$$\begin{aligned} |K_S^0> &= \tfrac{|K_1^0> + \epsilon|K_2^0>}{\sqrt{1+|\epsilon|^2}} \approx |K_1^0> + \epsilon|K_2^0> \\ |K_L^0> &= \tfrac{|K_2^0> + \epsilon|K_1^0>}{\sqrt{1+|\epsilon|^2}} \approx |K_2^0> + \epsilon|K_1^0>, \end{aligned} \tag{262}$$

where on the right side of the equation the normalizations are correct to order ϵ. The existence of Re ϵ implies that the states $|K_S^0>$ and $|\overline{K}_L^0>$ are not orthogonal

$$< K_S^0|K_L^0> = \frac{(1+\epsilon^*)(1+\epsilon) - (1-\epsilon^*)(1-\epsilon)}{\sqrt{2(1+|\epsilon|^2}} \approx \epsilon + \epsilon^* = 2\text{Re }\epsilon. \tag{263}$$

The non-diagonal Δm corresponds to virtual $K^0 - \overline{K}^0$ transitions. The difference $\Delta\Gamma$ arises from real transitions and is dominated by the $\pi\pi$ state with isospin $I = 0$ (see next section). With the Wu-Yang choice [3] for the free phase between the $|K^0>$ and $|\overline{K}^0>$ the $I = 0$ amplitude is real and also $\Delta\Gamma$ is almost real.

From Eq. (260) we have

$$\epsilon = -\frac{i}{2}\frac{\text{Im }\Delta m + \text{Im }\frac{\Delta\Gamma}{2}}{\text{Re }\Delta m - i\text{Re }\frac{\Delta\Gamma}{2}} \approx -\frac{i}{2}\frac{\text{Im }\Delta m}{\text{Re }\Delta m - i\text{Re }\frac{\Delta\Gamma}{2}}, \tag{264}$$

which holds when Im $\Delta\Gamma \ll$ Im Δm. In good approximation one has that $m_S = m_1$, $m_L = m_2$, $\Gamma_S = \Gamma_1$, $\Gamma_L = \Gamma_2$ with $\Delta m_I \ll \Delta m_R$. From this it follows that the phase angle of ϵ is equal to

$$\arg\,\epsilon \approx \arctan\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L} \approx 43.7° = \phi_{\text{SW}}. \tag{265}$$

Here, the subscript 'SW' refers to the superweak model of Wolfenstein that attributes $\mathcal{CP}$ violation to the term Δm.

5.5 Isospin Analysis

Next, we will consider the decay $K_L^0 \to \pi\pi$ in some more detail. Bose symmetry restricts the two pions to symmetric isospin states, $I = 0$ and $I = 2$. In case the pions would not experience final-state interaction effects (FSI), then $\mathcal{CPT}$ would give a simple relation between the K^0 and $\overline{K}^0$ amplitudes,

$$\begin{aligned} &< (2\pi)_{I=0;\ \text{no FSI}}|H_{\text{weak}}|K^0 >= A_0 \quad \text{and} \\ &< (2\pi)_{I=0;\ \text{no FSI}}|H_{\text{weak}}|\overline{K}^0 >= A_0^*, \end{aligned} \tag{266}$$

with similar relations for the $I = 2$ states. We can write for the $\pi\pi$ states

$$\begin{aligned} |(2\pi)_{I=2} >^s &= \tfrac{1}{\sqrt{6}} \left(\sqrt{2}|\pi_1^+\pi_2^- >^s + 2|\pi_1^0\pi_2^0 >^s\right), \\ |(2\pi)_{I=0} >^s &= \tfrac{1}{\sqrt{3}} \left(\sqrt{2}|\pi_1^+\pi_2^- >^s - |\pi_1^0\pi_2^0 >^s\right), \end{aligned} \tag{267}$$

where the superscript s indicates that we assume that there are no strong-interaction effects between the pions. Thus we can write the stationary final states as

$$\begin{aligned} < \pi^+\pi^-|^s &= \tfrac{1}{\sqrt{3}} \left(< (2\pi)_{I=2}|^s + \sqrt{2} < (2\pi)_{I=0}|^s\right), \\ < \pi^0\pi^0|^s &= \tfrac{1}{\sqrt{3}} \left(\sqrt{2} < (2\pi)_{I=2}|^s - < (2\pi)_{I=0}|^s\right). \end{aligned} \tag{268}$$

In reality the final states are not 'stationary' states, but final states where the pions interact strongly. Consequently, the pions develop a strong-interaction phase[35]. Thus each amplitude is multiplied with $\exp(i\delta_I)$, where I denotes isospin $I = 0$ or 2.

$$\begin{aligned} < \pi^+\pi^-|^s &= \tfrac{1}{\sqrt{3}} \left(e^{i\delta_2} < (2\pi)_{I=2}|^s + \sqrt{2}e^{i\delta_0} < (2\pi)_{I=0}|^s\right) \\ < \pi^0\pi^0|^s &= \tfrac{1}{\sqrt{3}} \left(\sqrt{2}e^{i\delta_2} < (2\pi)_{I=2}|^s - e^{i\delta_0} < (2\pi)_{I=0}|^s\right). \end{aligned} \tag{269}$$

With Eq. (261) we find for the $K^0 \to \pi\pi$ amplitudes

$$\begin{aligned} < \pi^+\pi^-|H_{\text{weak}}|K_L^0 > &= \sqrt{\tfrac{2}{3}}e^{i\delta_2} \left(\epsilon\text{Re } A_2 + i\text{Im } A_2\right) + \sqrt{\tfrac{4}{3}}e^{i\delta_0} \left(\epsilon\text{Re } A_0 + i\text{Im } A_0\right) \\ < \pi^0\pi^0|H_{\text{weak}}|K_L^0 > &= \sqrt{\tfrac{4}{3}}e^{i\delta_2} \left(\epsilon\text{Re } A_2 - i\text{Im } A_2\right) + \sqrt{\tfrac{2}{3}}e^{i\delta_0} \left(\epsilon\text{Re } A_0 + i\text{Im } A_0\right) \\ < \pi^+\pi^-|H_{\text{weak}}|K_S^0 > &= \sqrt{\tfrac{2}{3}}e^{i\delta_2} \left(\text{Re } A_2 + i\epsilon\text{Im } A_2\right) + \sqrt{\tfrac{4}{3}}e^{i\delta_0} \left(\text{Re } A_0 + i\epsilon\text{Im } A_0\right) \\ < \pi^0\pi^0|H_{\text{weak}}|K_S^0 > &= \sqrt{\tfrac{4}{3}}e^{i\delta_2} \left(\text{Re } A_2 - i\epsilon\text{Im } A_2\right) + \sqrt{\tfrac{2}{3}}e^{i\delta_0} \left(\text{Re } A_0 + i\epsilon\text{Im } A_0\right) \end{aligned} \tag{270}$$

We can simplify these reactions by noting that K_S^0 decay proceeds much faster than charged kaon decay, $\Gamma_{K_S^0 \to \pi^+\pi^-}/\Gamma_{K^+ \to \pi^+\pi^0} \approx 450$ and thus $|A_0| \gg |A_2|$. Furthermore we have

$$\frac{\Gamma(K_S^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^0\pi^0)} = 2.19 \pm 0.02 \quad \text{and} \quad \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_L^0 \to \pi^0\pi^0)} = 2.23 \pm 0.09, \tag{271}$$

[35]Since the pions are present only in the final state, they will obtain about half of the usual strong-interaction phase.

which is in good agreement with the assumption that the decay proceeds through the $I = 0$ channel. Finally we note that the phase of the K^0 state is a matter of agreement. Following T.T. Wu and C.N. Yang [3] we make the choice that A_0 is real. Thus we can drop the Im A_0 terms, and in addition terms of order $\epsilon A_2/A_0$. In order to facilitate the comparison of $\mathcal{CP}$ violating $K_L^0 \to 2\pi$ amplitudes to the $\mathcal{CP}$ conserving $K_S^0 \to 2\pi$ amplitudes, we define the following ratios,

$$\begin{aligned} \eta_{+-} &= \frac{<\pi^+\pi^-|H_{\text{weak}}|K_L^0>}{<\pi^+\pi^-|H_{\text{weak}}|K_S^0>} = \epsilon + \epsilon', \\ \eta_{00} &= \frac{<\pi^0\pi 0-|H_{\text{weak}}|K_L^0>}{<\pi^0\pi^0|H_{\text{weak}}|K_S^0>} = \epsilon - 2\epsilon', \end{aligned} \tag{272}$$

with

$$\epsilon' = \frac{i}{\sqrt{2}} \frac{\text{Im } A_2}{A_0} e^{i(\delta_2 - \delta_0)}. \tag{273}$$

With the Wu-Yang definition, the quantity ϵ measures the violation of $\mathcal{CP}$ due to kaon state mixing (see Fig. 36), whereas ϵ' measures $\mathcal{CP}$ violation in the decay, *i.e.* direct $\mathcal{CP}$ violation[36]. The phase of ϵ' is denoted $\phi_{\epsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2}$. Pion scattering experiments[45] have determined $\delta_2 - \delta_0 = -43^\circ \pm 6^\circ$ and thus $\phi_{\epsilon'} = 47^\circ \pm 6^\circ$.

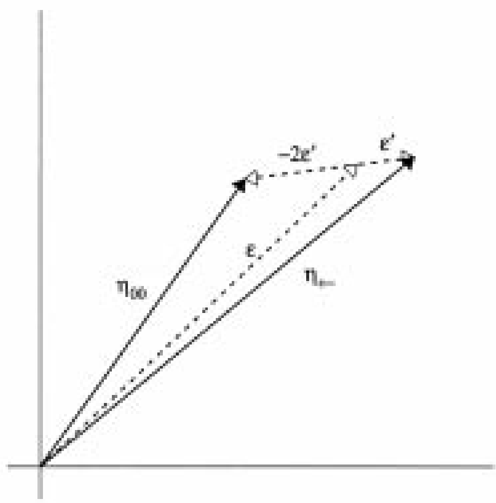

Figure 39: *Schematic representation of the various complex amplitudes that play a role in the description of CP violation in the decay of kaons.*

Fig. 39 gives a schematic representation of the various complex amplitudes that play a role in the description of CP violation in the decay of kaons. If only the asymmetric $K^0 - \overline{K^0}$ mixing in the mass matrix contributes to the $\mathcal{CP}$ violating amplitudes, then $\eta_{+-} = \eta_{00} = \epsilon$. Fig. 40 shows a so-called penguin diagram that is expected to play a role in direct $\mathcal{CP}$ violation. If instead of a gluon, a γ of Z is exchanged, then one speaks of an electromagnetic penguin amplitude.

Measurements of ϵ and ϵ' are important to distinguish between various models of $\mathcal{CP}$ violation. Fig 41 shows that the experimental accuracy improves over time. For example

[36]One observes that direct $\mathcal{CP}$ violation arises in case there is more than one decay channel, with different strong interaction phases, and non-zero imaginary amplitudes. We will encounter this again in the decay of B mesons.

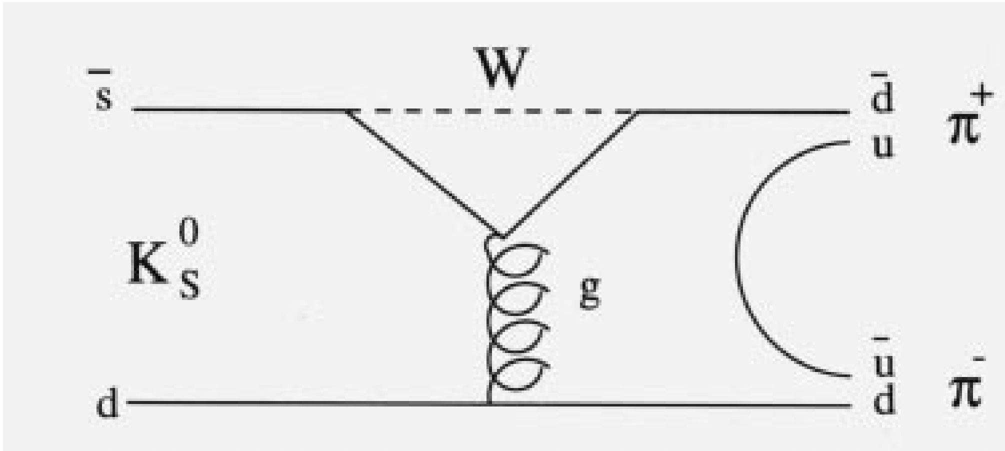

Figure 40: *Penguin amplitude that plays a role in direct CP violation in the decay of kaons. Note that the strangeness changes by one unit.*

in the superweak model of Wolfenstein $\mathcal{CP}$ violation in the decay $K_L^0 \to 2\pi$ arises through a new $\Delta S = 2$ superweak interaction which couples $K_L^0 \to \overline{K}_S^0$.

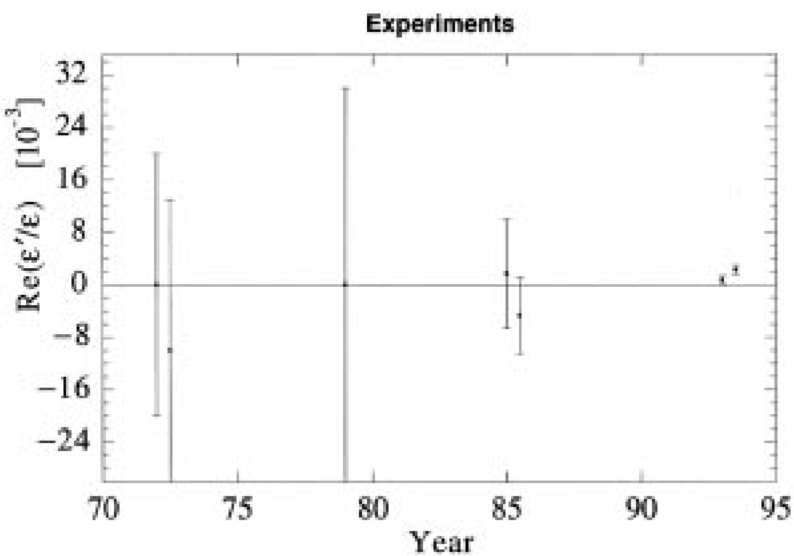

Figure 41: *Results of the experiments that measure the possible contribution of direct CP violating contributions in the decay of kaons. It is seen that the experimental accuracy dramatically improves over time.*

Subsequently, the K_S^0 decays through the normal weak interaction. Given the small mass difference Δm, the superweak coupling is required to be only of the order 10^{-10} of the normal weak coupling to explain the observed state mixing $\mathcal{CP}$ violation. The superweak model predicts $\epsilon' = 0$, whereas the standard model predicts $\epsilon' \neq 0$.

Measurement of the branching ratio, B.R., for $K_L^0 \to \pi^+\pi^-$ yields

$$|\eta_{+-}|^2 = \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} = \frac{\text{B.R. } (K_L^0 \to \pi^+\pi^-)\Gamma\ (K_L^0 \to \text{all})}{\text{B.R. } (K_S^0 \to \pi^+\pi^-)\Gamma\ (K_L^0 \to \text{all})} \approx ((2.285 \pm 0.019) \times 10^{-3})^2. \tag{274}$$

The corresponding measurement for neutral pions has also been performed and yields

$$|\eta_{00}|^2 = \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)} = \frac{\text{B.R. } (K_L^0 \to \pi^0\pi^0)\Gamma\ (K_L^0 \to \text{all})}{\text{B.R. } (K_S^0 \to \pi^0\pi^0)\Gamma\ (K_L^0 \to \text{all})} \approx ((2.275 \pm 0.019) \times 10^{-3})^2. \tag{275}$$

In addition, one has $|\eta_{00}/\eta_{+-}| = 0.9956 \pm 0.0023$[37].

Measurement of the phases of η_{+-} and η_{00} requires the observation of interference between $K_L^0 \to \pi\pi$ and $K_S^0 \to \pi\pi$. This can be accomplished by starting with a pure K^0 beam, or by generating a small amount of K_S^0 in a K_L^0 beam. In the latter case one first observes the fast decaying K_S^0 component. At the end one only observes the $\mathcal{CP}$ violating K_L^0 decay. In the intermediate region the contributions of K_S^0 and K_L^0 have similar magnitude and interference effects can be measured (see Fig. 33).

The interference pattern can be understood from the following. Suppose we start with a pure K^0 beam. We then have

$$|K^0> = \frac{|K_S^0> + |K_L^0>}{\sqrt{2}(1+\epsilon)}. \tag{277}$$

The time evolution can be written as

$$|\psi(\tau)> \frac{1}{\sqrt{2}(1+\epsilon)} \left(|K_S^0> e^{-\Gamma_S\tau/2 - im_S\tau} + |K_L^0> e^{-\Gamma_L\tau/2 - im_L\tau}\right). \tag{278}$$

With Eq. (272) the decay amplitude to $\pi^+\pi^-$ can be written as

$$<\pi^+\pi^-|H_{\text{weak}}|\psi(\tau)> = \frac{<\pi^+\pi^-|H_{\text{weak}}|K_S^0>}{\sqrt{2}(1+\epsilon)} e^{-i\overline{m}\tau} \left[e^{-\Gamma_S\tau/2 - i\Delta m\tau} + \eta_{+-}e^{-\Gamma_L\tau/2 + i\Delta m\tau}\right]. \tag{279}$$

With $\eta_{+-} = |\eta_{+-}|e^{\phi_{+-}}$ the observed intensity is

$$I_{\pi^+\pi^-}(\tau) = I_{\pi^+\pi^-}(0) \left[e^{-\Gamma_S\tau} + |\eta_{+-}|^2 e^{-\Gamma_L\tau} + 2|\eta_{+-}|^2 e^{-\overline{\Gamma}\tau}\cos(\tau\Delta m + \phi_{+-})\right]. \tag{280}$$

From the interference pattern shown in Fig. 33 both $|\eta_{+-}|$, Δm and ϕ_{+-} can be determined. In the same manner ϕ_{00} was determined from a measurement of $\pi^0\pi^0$ decay. The current values [2] are $\phi_{+-} = (43.5 \pm 0.6)^\circ$ and $\phi_{00} = (43.4 \pm 1.0)^\circ$. In addition, one has $\phi_{00} - \phi_{+-} = (-0.1 \pm 0.8)^\circ$.

[37]Note that several uncertainties can be minimized by forming the ratio

$$\frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \approx 1 - 6\text{Re}\,\frac{\epsilon'}{\epsilon} \approx 1 - 6\frac{\epsilon'}{\epsilon}, \tag{276}$$

where ϵ'/ϵ is almost real, because the phases of ϵ' and ϵ are almost equal. The most recent result is $\epsilon' - \epsilon = (1.5 \pm 0.8) \times 10^{-3}$ [2]. However, it should be noted that the CERN (Na31) and Fermilab (E731) experiments yield inconsistent results.

5.6 CP Violation in Semileptonic Decay

Violation of $\mathcal{CP}$ is also demonstrated in the semileptonic decay of neutral kaons,

$$\begin{array}{ll} K^0 \to l^+\nu_l\pi^- & (\Delta S = \Delta Q) \\ \overline{K}^0 \to l^-\overline{\nu}_l\pi^+ & (\Delta S = -\Delta Q), \end{array} \tag{281}$$

where the lepton l is an electron or a muon (these decays are termed K_{e3} and $\mathrm{K}_{\mu 3}$ decays, respectively). The final states transform in each other under $\mathcal{CP}$. We thus expect that violation of $\mathcal{CP}$ leads to a small charge asymmetry, defined as

$$\delta = \frac{\Gamma(K_L^0 \to \pi^-\mu^+\nu) + \Gamma(K_L^0 \to \pi^+\mu^-\overline{\nu})}{\Gamma(K_L^0 \to \pi^-\mu^+\nu) - \Gamma(K_L^0 \to \pi^+\mu^-\overline{\nu})} = \frac{|<K_L^0|K^0>|^2 - |<K_L^0|\overline{K}^0>|^2}{|<K_L^0|K^0>|^2 + |<K_L^0|\overline{K}^0>|^2} = 2\mathrm{Re}\ \epsilon. \tag{282}$$

Contrary to the decay $K_L^0 \to \pi\pi$, the semileptonic process is allowed, even without violation of $\mathcal{CP}$. Only the small difference between the two allowed decay rates is due to $\mathcal{CP}$ violation. Furthermore, this asymmetry will be a function of time and exibit interference effects between the K_L^0 and K_S^0 states. This is clearly observable in Fig. 42.

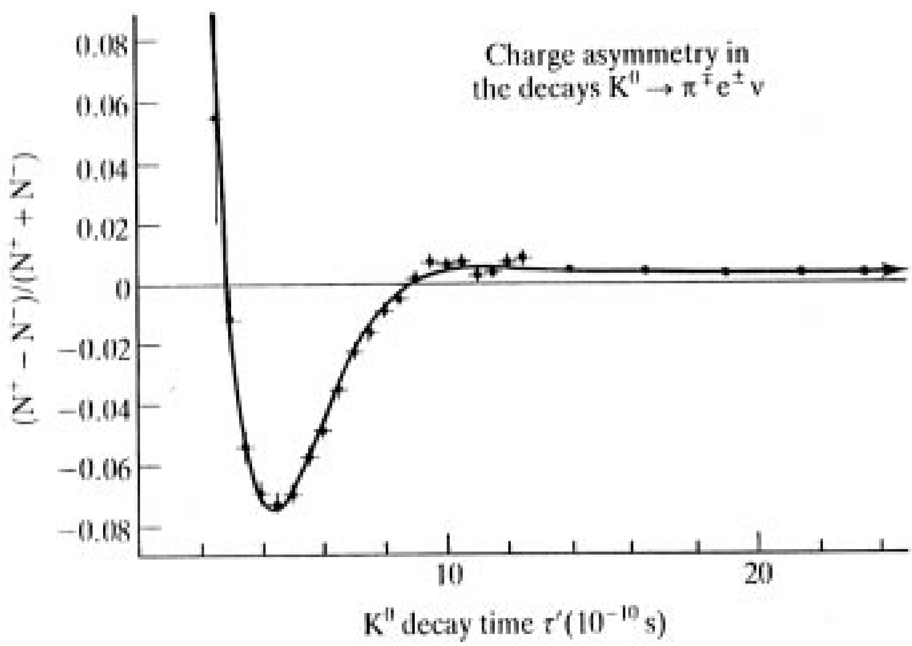

Figure 42: *Charge asymmetry for semileptonic decay of neutral kaons (from Ref. [4]).*

We see that after a certain time a net charge asymmetry persists, given by $\delta = (0.327 \pm 0.012)\%$. This experiment allows for an *absolute* definition of the sign of electrical charge.

At this moment we do not understand the reason for this small violation of $\mathcal{CP}$-invariance (or with the help of the $\mathcal{CPT}$-theorem: the time reversal invariance). Is all this only the consequence of a, more or less trivial, free phase in the Cabibbo-Kobayashi-Maskawa matrix[38], or is all this an announcement of new physics effects?

[38] In that case the Standard Model predicts a small positive value for ϵ'/ϵ.

Important is the small $\mathcal{CP}$-violation, because we assume that in the first phase of the formation of our universe (before $t_{\text{absolute}} < 10^{-4}$ s, and $kT > 1$ GeV) baryon-antibaryon pairs were created and annihilated in equilibrium with photons (after a period of cooling that was dominated by annihilation). The mechanism of $\mathcal{CP}$-violation, in combination with violation of conservation of baryon number, can give an explanation of the fact that today we life in a universe where we find predominantly matter (baryons) and no antimatter.

5.7 CPT Theorem and the Neutral Kaon System

5.7.1 Generalized Formalism

We will now introduce a more general formalism that will allow us to test $\mathcal{CPT}$ invariance. The variable $\overline{\delta}$ will denote the $\mathcal{CPT}$ violating mixing parameter. We can write

$$\begin{aligned} |K_S^0> &= \tfrac{1}{\sqrt{2}} \begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} \\ 1-\overline{\epsilon}-\overline{\delta} \end{pmatrix} = K_1^0 + (\overline{\epsilon}+\overline{\delta})K_2^0 \\ |K_L^0> &= \tfrac{1}{\sqrt{2}} \begin{pmatrix} 1+\overline{\epsilon}-\overline{\delta} \\ -1+\overline{\epsilon}-\overline{\delta} \end{pmatrix} = K_2^0 + (\overline{\epsilon}-\overline{\delta})K_1^0. \end{aligned} \tag{283}$$

Here, both $\overline{\epsilon}$ and $\overline{\delta}$ are small quantities. The $|K_S^0>$ and $|K_L^0>$ are eigenstates with eigenvalues $M_S \equiv m_S - i\Gamma_S/2$ and $M_L \equiv m_L - i\Gamma_L/2$ with $m_L > m_S$ and $\Gamma_S > \Gamma_L$. We then obtain

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} \\ 1-\overline{\epsilon}-\overline{\delta} \end{pmatrix} = M_S \begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} \\ 1-\overline{\epsilon}-\overline{\delta} \end{pmatrix} \tag{284}$$

and

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} 1+\overline{\epsilon}-\overline{\delta} \\ -1+\overline{\epsilon}-\overline{\delta} \end{pmatrix} = M_L \begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} \\ -1+\overline{\epsilon}-\overline{\delta} \end{pmatrix}. \tag{285}$$

These last two equations can be written as

$$\begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} & 1-\overline{\epsilon}-\overline{\delta} \\ 1+\overline{\epsilon}-\overline{\delta} & -1+\overline{\epsilon}-\overline{\delta} \end{pmatrix} \begin{pmatrix} H_{11} \\ H_{12} \end{pmatrix} = \begin{pmatrix} M_S(1+\overline{\epsilon}+\overline{\delta}) \\ M_L(1+\overline{\epsilon}-\overline{\delta}) \end{pmatrix} \tag{286}$$

and

$$\begin{pmatrix} 1+\overline{\epsilon}+\overline{\delta} & 1-\overline{\epsilon}-\overline{\delta} \\ 1+\overline{\epsilon}-\overline{\delta} & -1+\overline{\epsilon}-\overline{\delta} \end{pmatrix} \begin{pmatrix} H_{21} \\ H_{22} \end{pmatrix} = \begin{pmatrix} M_S(1-\overline{\epsilon}-\overline{\delta}) \\ M_L(-1+\overline{\epsilon}-\overline{\delta}) \end{pmatrix} \tag{287}$$

and these can be solved for H_{11}, H_{12}, H_{21} and H_{22}. The result is

$$\begin{aligned} H_{11} &= (M_S+M_L)/2 + \overline{\delta}(M_S-M_L) \\ H_{22} &= (M_S+M_L)/2 - \overline{\delta}(M_S-M_L) \\ H_{12} &= (M_S+M_L)(\tfrac{1}{2}+\overline{\epsilon}) \\ H_{21} &= (M_S+M_L)(\tfrac{1}{2}-\overline{\epsilon}) \end{aligned} \tag{288}$$

From this we can obtain $\overline{\epsilon}$ and $\overline{\delta}$. We find

$$\begin{aligned} H_{12} - H_{21} &= 2\overline{\epsilon}(M_S-M_L) \\ H_{11} - H_{22} &= 2\overline{\delta}(M_S-M_L). \end{aligned} \tag{289}$$

Note that all quantities in Eq. (289) are complex. We have $H = M - i\Gamma/2$, with M and Γ hermitian matrices. Thus $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$. From this we find

$$\begin{aligned} H_{12} - H_{21} &= 2i\mathrm{Im}\ M_{12} + \mathrm{Im}\ \Gamma_{12} \\ H_{11} - H_{22} &= M_{11} - M_{22} - \tfrac{i}{2}(\Gamma_{11} - \Gamma_{22}) \end{aligned} \tag{290}$$

In addition we have for the superweak phase angle the expression given by Eq. (265) and for the mass difference

$$M_S - M_L = \frac{e^{-\phi_{\mathrm{SW}}}}{i}\sqrt{(m_L - m_S)^2 + \frac{1}{4}(\Gamma_S - \Gamma_L)^2}. \tag{291}$$

Consequently, we can write for the mixing parameters

$$\begin{aligned} \overline{\epsilon} &= \frac{H_{12}-H_{21}}{2(M_S-M_L)} = \frac{i\mathrm{Im}\ M_{12}+\mathrm{Im}\ \Gamma_{12}/2}{(M_S-M_L)} = \frac{-\mathrm{Im}\ M_{12}+i\mathrm{Im}\ \Gamma_{12}/2}{\sqrt{(m_L-m_S)^2+\frac{1}{4}(\Gamma_S-\Gamma_L)^2}} \\ &\approx \frac{-\mathrm{Im}\ M_{12}+i\mathrm{Im}\ \Gamma_{12}/2}{(m_L-m_S)\sqrt{2}} e^{i\phi_{\mathrm{SW}}} \end{aligned} \tag{292}$$

and

$$\begin{aligned} \overline{\delta} &= \frac{H_{11}-H_{22}}{2(M_S-M_L)} = \frac{M_{11}-M_{22}-\frac{i}{2}(\Gamma_{11}-\Gamma_{22})}{2(M_S-M_L)} \\ &\approx \frac{M_{11}-M_{22}-\frac{i}{2}(\Gamma_{11}-\Gamma_{22})}{2(m_L-m_S)\sqrt{2}}\, ie^{i\phi_{\mathrm{SW}}}. \end{aligned} \tag{293}$$

Here we used the approximation that $(\Gamma_S - \Gamma_L)/2 \approx m_L - m_S$.

In the generalized representation we can calculate the overlap between the $|K_S^0>$ and $|K_L^0>$ states. In lowest order in $\overline{\epsilon}$ and $\overline{\delta}$ we find

$$< K_L^0|K_S^0 >= 2\mathrm{Re}\ \overline{\epsilon} + 2i\mathrm{Im}\ \overline{\delta}. \tag{294}$$

In addition, we can derive

$$\delta = 2\mathrm{Re}\ \overline{\epsilon} - 2i\mathrm{Re}\ \overline{\delta}. \tag{295}$$

5.7.2 CP Symmetry

The operation $\mathcal{CP}$ interchanges K^0 and $\overline{K}^0$ (see Eq. (208)). With these states as basis the Pauli matrix σ_1 acts as the $\mathcal{CP}$ operator, $|K^0 >= \sigma_1|\overline{K}^0 >$, or more specifically

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{296}$$

The hamiltonian transforms as $H \to \sigma_1 H \sigma_1$ and as function of its matrix elements we have

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \Longrightarrow \ \mathrm{CP} \ \Longrightarrow \begin{pmatrix} H_{22} & H_{21} \\ H_{12} & H_{11} \end{pmatrix} \tag{297}$$

Thus when $\mathcal{CP}$ is conserved we have $H_{12} = H_{21}$ and $H_{11} = H_{22}$. In addition, both M and Γ are hermitian matrices, and we find

$$M_{12} \equiv M_{21}^* = M_{21} \equiv M_{12}^* \quad \text{and} \quad \Gamma_{12} \equiv \Gamma_{21}^* = \Gamma_{21} \equiv \Gamma_{12}^* \tag{298}$$

and M_{12} and Γ_{12} are necessarily real when $\mathcal{CP}$ is conserved.

5.7.3 Time Reversal Invariance and CPT Invariance

Here we address the case where $\mathcal{CP}$ is violated[39]. We need to distinguish between a simultaneous violation of $\mathcal{T}$ with conservation of $\mathcal{CPT}$, and conservation of $\mathcal{T}$ with violation of $\mathcal{CPT}$. We consider the transition amplitude

$$< f|e^{-iH(t)t}|i > . \tag{299}$$

This amplitude describes the time evolution of an initial state $|i>$ at time $t = 0$ to a final state $|f>$ at time t. After a time reversal operation the state $|f>$ starts at time t and evolves to state $|i>$ with a time difference that is now $-t$ and with a hamiltonian in the time moves backwards. The corresponding amplitude is

$$< i|e^{-iH(-t)(-t)}|f >=< f^*|e^{-i\tilde{H}(-t)(-t)}|i^* > . \tag{300}$$

The matrix $\tilde{H}$ represents the transposed of H. When $\mathcal{T}$ is conserved then the last matrix element has to equal to Eq. (299), where states and operators have been transformed with the $\mathcal{T}$ operator,

$$< fT^{-1}|Te^{-iH(t)(t)}T^{-1}|Ti > . \tag{301}$$

We conclude that the manner in which the $\mathcal{T}$ operator acts on states and the hamiltonian is given by

$$T|i >= |i^* > \quad \text{and} \quad THT^{-1} = \tilde{H}. \tag{302}$$

The hamiltonian transforms as $H \to \tilde{H}$. Conservation of $\mathcal{T}$ symmetry and hermiticity implies that $H = H^\dagger$. Consequently its matrix elements are real and $\overline{\epsilon} = 0$, since $H_{21} = H_{12}$.

Under a $\mathcal{CPT}$ operation the hamiltonian transforms as $H \to \sigma_1 \tilde{H} \sigma_1$. Assuming $\mathcal{CPT}$ invariance we have for the matrix elements $H_{11} = H_{22}$. Consequently we have in this case $\overline{\delta} = 0$.

5.7.4 Isospin Amplitudes

We will now briefly discuss the isospin states in the two-pion decay channel in the generalized formalism. The decay amplitude for the kaon can be written as

$$A(K \to 2\pi, I) = (A_I + B_I)e^{i\delta_I}, \tag{303}$$

where δ_I is the rescattering phase of the two pions. Here A_I are $\mathcal{CPT}$ invariant amplitudes, while B_I are non-$\mathcal{CPT}$ invariant amplitudes. For the $\overline{K}$ we have the corresponding parametrization

$$A(\overline{K} \to 2\pi, I) = (A_I^* - B_I^*)e^{i\delta_I}. \tag{304}$$

When $\mathcal{CP}$ is conserved, then the K^0 and $\overline{K}^0$ amplitudes are equal. When $\mathcal{CPT}$ is conserved, the amplitudes are related through complex conjugation (in which the rescattering phase in unaltered).

A similar parametrization can be formulated for the semileptonic decay amplitudes,

$$A(K \to \pi^- l^+ \nu_l) = (a + b), \tag{305}$$

[39] When $\mathcal{CP}$ is violated, it is not possible that both $\overline{\epsilon}$ and $\overline{\delta}$ vanish.

with $b \ll a$ and a the $\mathcal{CP}$ violating and $\mathcal{CPT}$ conserving amplitude, and with b the $\mathcal{CPT}$ violating amplitude. For the $\overline{K}$ we have the corresponding parametrization

$$A(\overline{K} \to \pi^+ l^- \overline{\nu}_l) = (a^* - b^*), \tag{306}$$

with identical demands concerning $\mathcal{CP}$, $\mathcal{T}$ and $\mathcal{CPT}$ conservation as was the case for A and B.

Again it is possible to work out the $\mathcal{CP}$ and $\mathcal{CPT}$ violating parameters and the experimental quantities ϵ, ϵ' and δ.

- In case of $\mathcal{CP}$ violation and $\mathcal{CPT}$ conservation one has $\overline{\delta} = 0$ and B $(b) = 0$. With the choice Im $A_0 = 0$ we find

$$\begin{aligned} \epsilon &= \overline{\epsilon} \\ \epsilon' &= \tfrac{1}{\sqrt{2}} \tfrac{\text{Im } A_2}{A_0} e^{i(\delta_2 - \delta_0 + \frac{\pi}{2})}, \\ \delta &= 2\text{Re } \epsilon \end{aligned} \tag{307}$$

- In case we allow for $\mathcal{CPT}$ violation we find

$$\begin{aligned} \epsilon &= \overline{\epsilon} - \overline{\delta} + i \tfrac{\text{Im } A_0}{\text{Re } A_0} + \tfrac{\text{Re } B_0}{\text{Re } A_0} \\ \epsilon' &= \tfrac{e^{i(\delta_2 - \delta_0 + \frac{\pi}{2})}}{\sqrt{2}} \left(\tfrac{\text{Im } A_2}{\text{Re } A_0} - i \tfrac{\text{Re } B_2}{\text{Re } A_2} \right), \\ \delta &= 2\text{Re } \overline{\epsilon} - 2\text{Re } \overline{\delta} + 2 \tfrac{\text{Re } b}{\text{Re } a}, \end{aligned} \tag{308}$$

  where the amplitudes B (or b) and the parameter $\overline{\delta}$ parametrize the violation of $\mathcal{CPT}$ in the decay and mixing, respectively. Furthermore, one has Im $a \ll$ Re a and Im $A \ll$ Re A.

From pion scattering we know that $\delta_2 - \delta_0 + \frac{\pi}{2} \approx 44°$, while the phase $\phi_{\eta_{+-}}^{\pi^+\pi^-} = (43.7 \pm 0.6)°$. Recently, also the difference in phase between the $\mathcal{CP}$ violating decay to $\pi^0\pi^0$ and $\pi^+\pi^-$ has been accurately determined [30] and amounts to $(0.30 \pm 0.88)°$. This implies that the phase of ϵ' is almost equal to that of ϵ, in agreement with $\mathcal{CPT}$ conservation. Furthermore, we know from unitarity constrains that $\mathcal{CP}$ violation is associated with $\mathcal{T}$ violation. More quantitative tests do not give any indication for $\mathcal{CPT}$ violation.

5.8 Particle Mixing and the Standard Model

Why do these remarkable phenomena only occur in the K^0-system and not for example with neutrons? The reason for this is the fact that due to second-order weak interaction effects a mixing of $K^0 \leftrightarrow \overline{K}^0$ occurs (see Fig. 43). Consequently the strangeness changes by two units, $\Delta S = 2$. A similar transition from a neutron to an antineutron $n \leftrightarrow \overline{n}$ is excluded due to conservation of baryon number.

First we will discuss the mass difference Δm in terms of the Standard Model. We have the following relation

$$\Delta m = m_{\text{S}} - m_{\text{L}} = < K_S^0 | H | K_S^0 > - < K_L^0 | H | K_L^0 >, \tag{309}$$

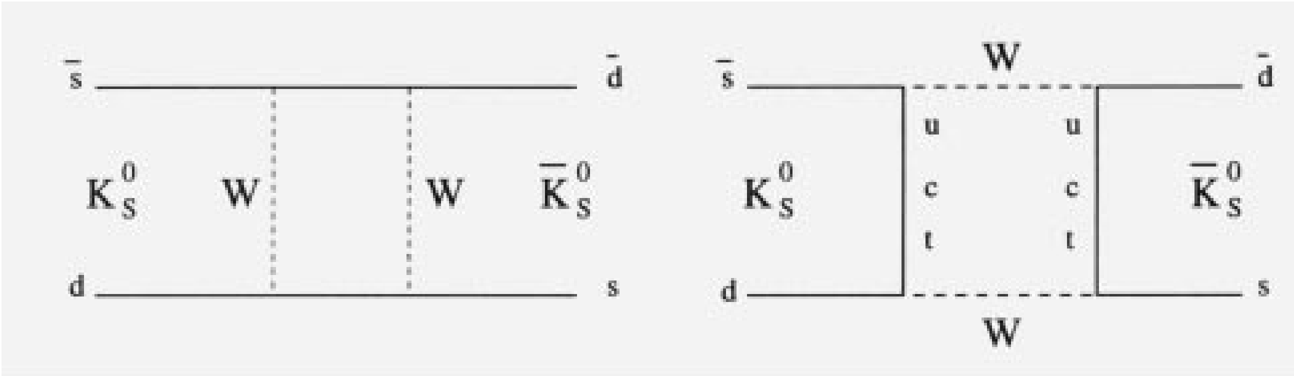

Figure 43: *Mixing of* $K^0 \leftrightarrow \overline{K}^0$ *due to the weak interaction. Note that the strangeness changes by two units.*

which can be written as[40]

$$\Delta m = 2\text{Re} < K^0|H|\overline{K^0} > -4\text{Im}\epsilon \ \text{Im} < K^0|H|\overline{K^0} > . \tag{311}$$

Next, we calculate the non-diagonal element of the mass matrix $< K^0|H|\overline{K^0} >$ with the box diagrams shown in Fig. 36. The amplitude for the box diagram is obtained by applying the Feynman rules which state that

- each fermion propagator is represented by

$$iS_{\text{F}}(k) = \frac{i}{\gamma^\mu k_\mu - m + i\epsilon} = i\frac{\gamma^\mu k_\mu + m}{k^2 - m^2 + i\epsilon},$$

  with m the mass of the fermion (quarks in our case);

- expressions for spin-1 propagators depend both on the theory and on the gauge. In general we have the 't Hooft propagator represented by

$$iD_{\text{F}}(k)^{\mu\nu} = i\frac{-g^{\mu\nu} + k^\mu k^\nu (1-\zeta)(k^2 - \zeta m_W^2)^{-1}}{k^2 - m_W^2},$$

  where ζ is an arbitrary parameter first proposed by 't Hooft in 1971. The choice $\zeta = 1$ corresponds to 't Hooft-Feynman gauge, $\zeta = 0$ Landau gauge, and $\zeta = \infty$ unitary gauge. In our case we take $\zeta \to \infty$ and obtain the usual form of the propagator of massive vector bosons, namely

$$iD_{\text{F}}(k)^{\mu\nu} = i\frac{-g^{\mu\nu} + k^\mu k^\nu / m_W^2}{k^2 - m_W^2};$$

[40]We can derive the result by using the representation given in Eq. (261). One then has

$$\begin{aligned} \Delta m = \ & \tfrac{1}{\sqrt{2}}\left[(1+\epsilon^*) < K^0| + (1-\epsilon^*) < \overline{K^0}|\right] \tfrac{1}{\sqrt{2}}\left[(1+\epsilon)H|K^0 + (1-\epsilon)H|\overline{K^0} >\right] \\ & -\tfrac{1}{\sqrt{2}}\left[(1+\epsilon^*) < K^0| - (1-\epsilon^*) < \overline{K^0}|\right] \tfrac{1}{\sqrt{2}}\left[(1+\epsilon)H|K^0 - (1-\epsilon)H|\overline{K^0} >\right] \\ = \ & (1 + (\epsilon^* - \epsilon) - \epsilon\epsilon^*) < K|H|\overline{K^0} > + (1 - (\epsilon^*\epsilon) - \epsilon^*\epsilon) < K^0|H|\overline{K^0} > . \end{aligned} \tag{310}$$

We then use the relations $\epsilon - \epsilon^* = 2i\text{Im}\epsilon$ and $\epsilon^*\epsilon = (\text{Re}\epsilon)^2 + (\text{Im}\epsilon)^2$. In addition, we neglect all terms quadratic in ϵ and obtain the required result.

- for the external quark lines we use the spinors u and v;
- furthermore, at each vertex we have the weak coupling constant g, the parity violating $V-A$ coupling given by $i\gamma^\mu(1-\gamma^5)$, and the appropriate element of the Cabibbo-Kobayashi-Maskawa matrix.

The amplitude for the box diagram with the intermediate u-quark is given by

$$\begin{aligned}\mathcal{M} = \; & i\left(\tfrac{-ig_w}{2\sqrt{2}}\right)^4 (V_{us}^* V_{ud})^2 \int \tfrac{d^4k}{(2\pi)^2}\left(i\tfrac{-g^{\lambda\sigma}+k^\lambda k^\sigma/m_W^2}{k^2-m_W^2}\right)\left(i\tfrac{-g^{\alpha\rho}+k^\alpha k^\rho/m_W^2}{k^2-m_W^2}\right) \\ & \left[\bar{u}_s\gamma_\lambda(1-\gamma^5)\tfrac{\not{k}+m_u}{k^2-m_u^2}\gamma_\rho(1-\gamma^5)u_d\right]\left[\bar{v}_s\gamma_\alpha(1-\gamma^5)\tfrac{\not{k}+m_u}{k^2-m_u^2}\gamma_\sigma(1-\gamma^5)v_d\right].\end{aligned} \tag{312}$$

The calculation of this matrix element is rather tedious. One neglects the terms $k^\lambda k^\sigma/m_W^2$ and $k^\alpha k^\rho/m_W^2$ in the W propagators. Furthermore, one neglects terms proportional to the mass of the u quark and finds

$$\mathcal{M} \approx \frac{-ig_w^4}{(4\pi)^4}(V_{us}^* V_{ud})\, T^{\mu\nu}\int \frac{d^4k}{(2\pi)^2}\frac{k_\mu k_\nu}{(k^2-m_u^2)^2(k^2-m_W^2)^2}, \tag{313}$$

with

$$T^{\mu\nu} = \bar{u}_s\gamma_\lambda\gamma^\mu\gamma_\rho(1-\gamma^5)u_d\bar{v}_s\gamma_\rho\gamma^\nu\gamma_\lambda(1-\gamma^5)v_d. \tag{314}$$

We neglect the quark momenta in the meson with the result that only a single momentum k circulates in the box diagram. The integral

$$I_{\mu\nu} = \int \frac{d^4k}{(2\pi)^2}\frac{k_\mu k_\nu}{(k^2-m_u^2)^2(k^2-m_W^2)^2} \tag{315}$$

becomes, when terms of order m_u^2/m_W^2 are ignored,

$$I_{\mu\nu} = g_{\mu\nu}/64\pi^2 i m_W^2. \tag{316}$$

The matrix element can now be obtained, carrying out a Fierz transformation, as

$$\begin{aligned}\mathcal{M} = \; & \Sigma_n' \tfrac{<\overline{K^0}|H|n><n|H|K^0>}{m_{K^0}-E_n} \\ \approx \; & \tfrac{-ig_w^4}{(4\pi)^4}(V_{us}^* V_{ud})\left[<\overline{K^0}|\overline{S}\gamma_\lambda(1-\gamma_5)D|0><0|\overline{S}\gamma^\lambda(1-\gamma_5)D|k^0>\right],\end{aligned} \tag{317}$$

where D and $\overline{S}$ are operators and the vacuum-insertion approximation corresponds to inserting a factor $1=\Sigma_n'|n><n| \approx |0><0|$. The factors in square brackets may be related to the kaon decay constant f_K, which appears in the matrix element for K_{l2} decay. We find

$$\mathcal{M} = \frac{-ig_w^4}{(4\pi)^4}(V_{us}^* V_{ud})\, f_K^2 m_K^2 (2E_{K^0}2E_{\overline{K^0}})^{-1/2} \tag{318}$$

which yields for $E_{K^0} = E_{\overline{K^0}} = m_K$,

$$\Delta m = m_L - m_S \approx \frac{G_F^2}{4\pi^2} m_W^2 \cos^2\theta_C \sin^2\theta_C f_K^2 m_K. \tag{319}$$

This formula gives much too large a value of Δm if $m_W \approx 80$ GeV/c^2 is used.

Next, we take also exchange of c and t quarks into account. Since $m_u \ll m_c$ and $m_u \ll m_t$ one can neglect the terms proportial to m_u. After these simplifications one can write the matrix element as

$$\mathcal{M} = \frac{G_F^2}{2} \int \frac{d^4k}{(2\pi)^2} \Sigma_{i,j} a_i a_j \bar{u}_d \gamma_\lambda (1-\gamma_5) \frac{\not k + m_i}{k^2 - m_i^2} \gamma^\lambda (1-\gamma_5) u_s \bar{v}_d \gamma_\sigma (1-\gamma_5) \frac{\not k + m_j}{k^2 - m_j^2} \gamma^\sigma (1-\gamma_5) v_s, \tag{320}$$

where $i, j = u, c, t$ and where $a_{i,j}$ are the CKM matrix elements for transitions of the form $s \to i \to d$, given by

$$\begin{aligned} a_1 =& \ +c_1 s_1 c_3 \\ a_2 =& \ -s_1 c_2 (c_1 c_2 c_3 - s_2 s_3 e^{i\delta}) \\ a_3 =& \ -s_1 s_2 (c_1 s_2 c_3 + c_2 s_3 e^{i\delta}). \end{aligned} \tag{321}$$

To obtain ϵ, we separate the integral ino real and imaginary parts and evaluate them by means of contour integration, assuming $m_{c,t}^2 \gg m_u^2 \approx 0$. Thus we find

$$\mathrm{Im} M_{12} / \mathrm{Re} M_{12} = 2 s_2 c_2 s_3 \sin\delta P(\theta_2, \eta), \tag{322}$$

where $\eta = m_c^2/m_t^2$ and

$$P(\theta_2, \eta) = \frac{s_2^2(1 + [\eta/(1-\eta)] \ln\eta) - c_2^2(\eta - [\eta/(1-\eta)] \ln\eta)}{c_1 c_3 (c_2^4 \eta + s_2^2 - 2 s_2^2 c_2^2 [\eta/(1-\eta)] \ln\eta)}. \tag{323}$$

This model yields $\mathcal{CP}$ violation with approximately the right magnitude.

In the derivation one can simplify things by demanding that the CKM matrix is unitary and thus

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0. \tag{324}$$

We then obtain for the integral

$$\int \frac{d^4k}{(2\pi)^2} k_\mu k_\nu \left(V_{cs}^* V_{cd} \left[\frac{1}{k^2 - m_c^2} - \frac{1}{k^2 - m_u^2} \right] + V_{ts}^* V_{td} \left[\frac{1}{k^2 - m_t^2} - \frac{1}{k^2 - m_u^2} \right] \right)^2. \tag{325}$$

This integral rapidly converges for high values of k^2, because the propagators have pairwise opposite signs. After integration we are left with one term that is proportional to m_c^2 and one term that is proportional to m_t^2. Furthermore, the integral is proportional to $g_{\mu\nu}$.

The tensor term $T^{\mu\nu}$ also contains the transition of K^0 to the d and $\bar{s}$ quarks, that is described by the kaon form factor. There are various uncertainties left, such as the vacuum-insertion approximation $|0><0|$ in the middle of the expression. This leads to a factor $B = 1$, while other estimates lead to values between 0.5 and 1. In addition, one has to include both a gluon correction η in the results and a color factor 1/3. Combining all these ingredients we arrive to the value

$$\Delta m_{\mathrm{K}^0} \approx \frac{1}{12\pi^2} G_F^2 m_c^2 |(V_{cd})^2 (V_{cs}^*)^2| \eta B f_{\mathrm{K}}^2 m_{\mathrm{K}}. \tag{326}$$

Note that Δm is a complex quantity and that we only give its magnitude here. It is the imaginary part of Δm that leads to $\mathcal{CP}$ violation. For the imaginary part we have to include a small correction for the t-quark. Winstein and Wolfenstein find the following approximation for the parameter ϵ,

$$\epsilon \approx 3.4 \times 10^{-3} A^2 \eta B \left[1 + 1.3 A^2 (1-\rho) \left(\frac{m_t}{m_W} \right)^{1.6} \right]. \tag{327}$$

Here, A and ρ are CKM parameters in the Wolfenstein parametrization of the CKM matrix. The parameter ρ is predicted to vary between -0.5 and 0.5. The QCD parameter η is estimated to be 0.85.

It is possible to calculate $B^0 \leftrightarrow \overline{B^0}$ in a similar fashion using the box diagram. However, there is one significant difference. For the K^0 system the absolute value of the box diagram, which determines Δm, is proportional to the CKM factor

$$\Delta m \propto (V_{cs}^* V_{cd})^2 = \lambda^2, \tag{328}$$

while the imaginary part is proportional to $A^2\lambda^6\eta$. Consequently,

$$\frac{\mathrm{Im}\Delta m}{\mathrm{Re}\Delta m} \approx A^2\lambda^4\eta. \tag{329}$$

In contrast, in the B^0 case, independent of the intermediate quarks in the box diagram (u, c or t), there is always a factor λ^6. Given the dominance of the t quark, we have now in the expression for Δm the factor

$$(V_{tb}^* V_{td})^2 = (1-\rho-i\eta)^2 A^2 \lambda^6. \tag{330}$$

Therefore, it follows that the $\mathcal{CP}$ violating term equals

$$\frac{\mathrm{Im}\Delta m}{|\Delta m|} = \frac{1}{2}\sin 2\beta = \frac{\eta(1-\rho)}{(1-\rho)^2+\eta^2}, \tag{331}$$

where β is the phase of V_{td}. Consequently, $\mathcal{CP}$ violation in the mass matrix of the B^0 system is not suppressed by a factor λ, while in case of the K^0 system there is a suppression by a factor λ^4 ($= 2.3 \times 10^{-3}$).

6 The B-meson system

A Lagrangian is $\mathcal{CP}$ conserving whenever all the coupling and mass terms in the Lagrangian can be made real by an appropriate set of field redefinitions. Within the Standard Model, the most general theory with only two quark generations and a single Higgs multiplet is of this type. However, when we add a third quark generation then the most general quark mass matrix allows for $\mathcal{CP}$ violation. The three-generation Standard Model with a single Higgs multiplet has only a single non zero phase. It appears in the matrix that relates weak eigenstates to mass eigenstates, commonly known as the CKM matrix. This matrix must be unitary, a constraint that provides relationships between its elements. With relatively few further assumptions this translates into specific predictions for the relationships between the parameters measured in different B-decay processes. This makes the B decays an ideal tool to probe for physics beyond the Standard Model; theories with other types of $\mathcal{CP}$-violating parameters generally predict different relationships. This section gives a brief description of the theoretical background of B-physics with respect to $\mathcal{CP}$ violation.

6.1 CP Violation in the Standard Model

The charged current interaction can be written in the mass eigenbasis, where the matrices V_{qL} define the transformation from the eigenstates to the mass eigenstates [31]:

$$-L_w = \frac{g}{\sqrt{2}} \left[\overline{u}_L \overline{c}_L \overline{t}_L\right] \gamma^{\mu} V_{uL} V_{dL}^{\dagger} \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} W_{\mu} + h.c., \tag{332}$$

where $h.c.$ is the hermitic conjugate. The object $V_{uL}V_{dL}^{\dagger}$ is the mixing matrix for three quark generations. It is a 3 by 3 unitary matrix which contains three real parameters and six phases. The number of phases can be reduced to one physically meaningful phase δ. The three-generation Standard Model predicts $\mathcal{CP}$ violation unless $\delta = 0$.

The mixing matrix with three angles and one phase is called the Cabbibo-Kobayashi-Maskawa (CKM) matrix. It relates weak and mass eigenstates and its elements V_{ij}, multiplied by $g/\sqrt{2}$, give the coupling of the weak charged current. The unitary CKM matrix can be written as follows

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}. \tag{333}$$

Each element corresponds to the amplitude of a transition of a (u, c, t) quark to a (d, s, b) quark or the transformation from a weak interaction eigenstate to a strong interaction eigenstate. The unitarity of the CKM matrix ($VV^* = 1$) leads to six relations such as

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{334}$$

The unitarity triangle is a geometrical representation of this relation in the complex plane; the three complex quantities form a triangle, as shown in Fig. 44. Only two of the six relations give distinguishable triangles with measurable angles.

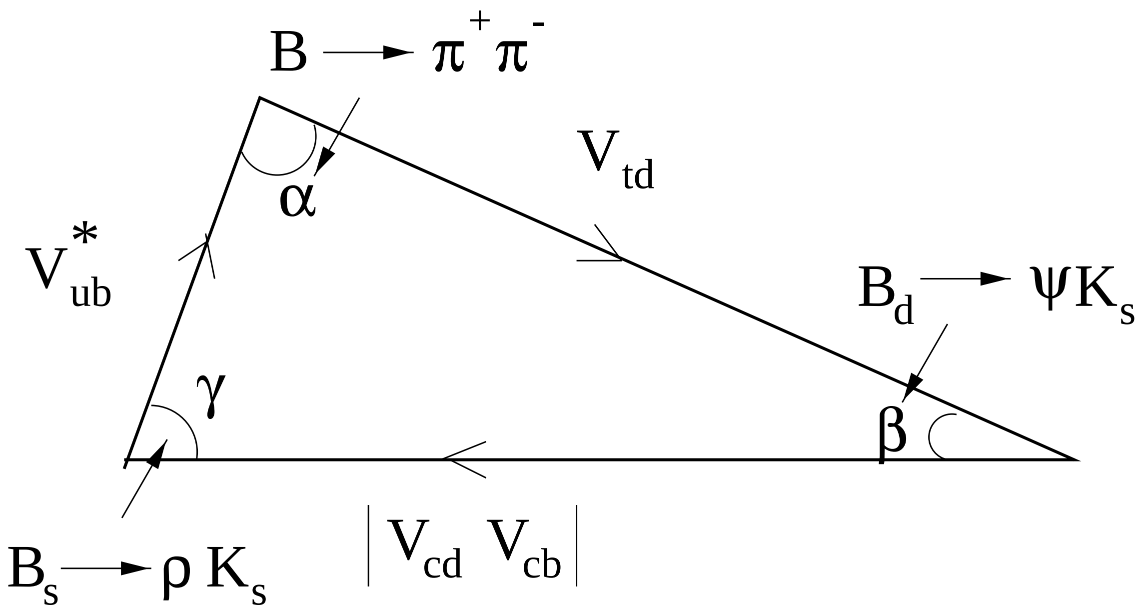

Figure 44: *The unitarity triangle is a representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ud}V_{ub}^*$, $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$.*

The Standard Model predictions for the $\mathcal{CP}$ asymmetries in neutral B decays into certain $\mathcal{CP}$ eigenstates are fully determined by the values of the three angles of the unitarity triangle, α, β and γ. Their measurement will test these Standard Model predictions and consequently provide a probe for physics beyond the Standard Model.

$\mathcal{CP}$ violation is put in the Standard Model for empirical reasons, by allowing the Yukawa coupling of the Higgs to the quarks to be complex. In the case of three or more generations, this results in a non-trivial (imaginary) phase in the CKM matrix which, in turn, gives rise to $\mathcal{CP}$ violating observables in weak interactions. $\mathcal{CP}$ violating observables in the decays of B mesons are expected to provide highly constraining information. The two most common neutral meson systems useful for measuring $\mathcal{CP}$ violation are: the neutral K system where all relevant phases are small, while in the neutral B_d and B_s systems the two mass eigenstates have similar lifetimes. In the B systems it is the mass difference that dominates the physics. $\mathcal{CP}$ violation in the D system is expected to be small and uncertain, dominated by long distance effects. Mixing between neutral B^0 and $\overline{B}^0$ does not require $\mathcal{CP}$ violation but depends on the existence of common eigenstates to which both mesons can decay. B^0 has many decay channels with $\mathcal{CP} = -\infty$ and $\mathcal{CP} = +\infty$. Therefore the two eigenstates have the same lifetimes.

We distinguish between three types of $\mathcal{CP}$ violation [32]:

- $\mathcal{CP}$ violation in decay or direct $\mathcal{CP}$ violation. It results from the interference among various decay amplitudes that lead to the final state. This is interference between tree and penguin diagrams when they have different weak phases, or, in channels where there are no tree contributions, it can also arise because of different weak phases of different penguin contributions. $\mathcal{CP}$ asymmetries in charged meson decays are of this type. Here $|\overline{A}_f/A_f| \neq 1$ indicates $\mathcal{CP}$ violation (see 6.1.3).

- $\mathcal{CP}$ violation in mixing or indirect $\mathcal{CP}$ violation. It results from the mass eigenstates being different from the $\mathcal{CP}$ eigenstates. Or more precisely from complex mixing effects that will also give a nonvanishing lifetime difference for the two B mass eigenstates. $\mathcal{CP}$ asymmetries in semileptonic decays are of this type. Here $|q/p| \neq 1$ indicates $\mathcal{CP}$ violation (see 6.1.3).

- $\mathcal{CP}$ violation in the interference of mixing and direct decay. $\mathcal{CP}$ asymmetries in neutral meson decays into $\mathcal{CP}$ eigenstates are of this type. For $\mathcal{CP}$ violation the quantity $\kappa \neq 1$, see next sections.

LHCb will mainly measure $\mathcal{CP}$ violation of the type kind. The importance of $\mathcal{CP}$ violation in neutral meson decays lies in the possibility of theoretical interpretation relatively free of hadronic uncertainties. The two $\mathcal{CP}$ violating parameters which have been experimentally measured in the K-system, ϵ and ϵ'/ϵ, belong to this class of $\mathcal{CP}$ violation.

6.1.1 CP Violation in neutral B decays

The time evolution of a mixed neutral B-meson mixing state $a|B^0> +b|\overline{B}^0>$ is given by the time dependent Schrödinger equation [31, 32, 33, 34]

$$i\frac{\partial}{\partial\tau}\begin{bmatrix} B^0 \\ \overline{B}^0 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} B^0 \\ \overline{B}^0 \end{bmatrix} = \begin{bmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{bmatrix}\begin{bmatrix} B^0 \\ \overline{B}^0 \end{bmatrix}. \quad (335)$$

$\mathbf{M}$ and $\mathbf{\Gamma}$ are Hermitic matrices. $\mathcal{CPT}$ invariance guarantees $H_{11} = H_{22}$. The anti-hermitic part ($i\Gamma$) describes the exponential decay of the meson system, while the hermitic part (M) is called the mass matrix. $\mathbf{H}$ contains nonvanishing off-diagonal matrix elements due to weak interactions, which do not conserve flavour, therefore the two meson states are coupled through virtual intermediate channels (described by coupled Schrödinger equations, see above). The non-diagonal terms are important in the discussion of $\mathcal{CP}$ violation.

The mass matrix can be diagonalized by a similarity transformation. This transformation yields two eigenvectors; the two mass eigenstates B_H and B_L (heavy and light respectively) of $\mathbf{H}$ are resonances and not elementary particles

$$\begin{aligned} |B_L> &= p|B^0> +q|\overline{B}^0> \\ |B_H> &= p|B^0> -q|\overline{B}^0> . \end{aligned} \quad (336)$$

The mass eigenstates are not $\mathcal{CP}$ eigenstates, but mixtures of the two $\mathcal{CP}$-conjugate quark states. The mixing is due to box diagrams as shown in figure 45. With the eigenvectors given in Eq. (336) the coupled Schrödinger equations (see Eq. (335)) become two decoupled equations, with certain eigenvalues

$$\begin{aligned} i\tfrac{d}{dt}|B_L> &= M_L|B_L> \quad M_L \equiv m_L - \tfrac{i}{2}\gamma_L \\ i\tfrac{d}{dt}|B_H> &= M_H|B_H> \quad M_H \equiv m_H - \tfrac{i}{2}\gamma_H. \end{aligned} \quad (337)$$

Here $\Delta\gamma = \gamma_H - \gamma_L \ll \gamma$, since $\Delta\gamma$ is produced by channels with branching ratio of order 10^{-3} that contribute with alternating signs, and therefore one may safely set $\gamma_H = \gamma_L = \gamma$, for both the B_d and the B_s system [31]. The eigenvalues from Eq. (337) can be written as function of the matrix elements. One can extract information on these parameters by a theoretical analysis of the box diagram. M_{12} coincides with the real part of H_{12} ($M_{12} = \mathrm{Re}\{H_{12}\} = \mathrm{Re}\{< B^0|H|\overline{B}^0 >\}$) and $-\Gamma/2$ with its imaginary part. Because of the largest contribution arising from the top quark, one has [34, 35]

$$M_{12} \sim (V_{td}V_{tb}^*)^2 m_t^2. \tag{338}$$

In the computation of Γ_{12} only up or charm internal quarks are allowed in the box diagrams because the energy of the final state cannot exceed m_b

$$\Gamma_{12} \sim m_b^2(V_{cd}V_{cb}^* + V_{ud}V_{ub}^*)^2 = (V_{td}V_{tb}^*)^2 m_b^2. \tag{339}$$

This has the following consequences: $|\Gamma_{12}| \ll |M_{12}|$ since $m_t \gg m_b$ and have the same CKM phase for B-mixing $\Phi_M = \arg(M_{12}) = \arg(\Gamma_{12})$. Since $\Delta\gamma = \gamma_H - \gamma_L \simeq 0$ the states B_H and B_L have the same lifetime. Finally the so called phase factor, for example in the B_d system, can be written as

$$\frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} \simeq e^{-i2\Phi_M} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}. \tag{340}$$

So $|\Gamma_{12}| \ll |M_{12}|$ gives $|q/p| = 1$. From the mixing parameter x_d (or Δm_d), obtained by calculating the box diagram, the leptonic B decay constant f_b and the B_b parameter can be determined. $B_b \neq 1$ would signal deviations from the factorization approximation.

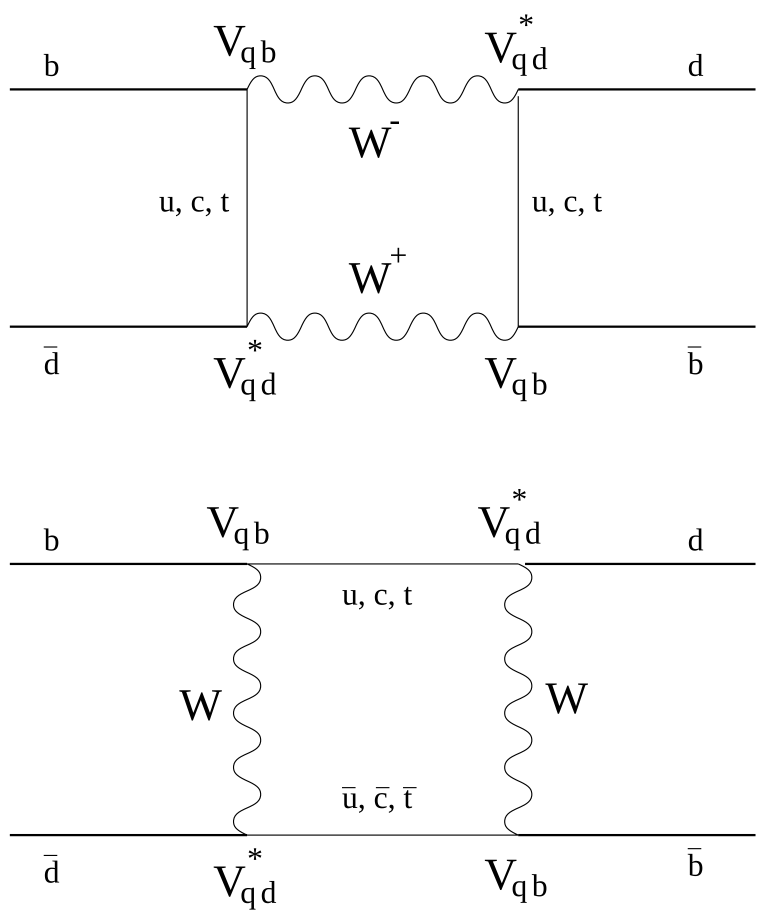

Figure 45: *Mixing diagrams for B-mesons. The corresponding CKM entries are shown.*

The eigenstates are often written

$$|B_{L,H}>=\frac{(1+\epsilon)|B^0>\pm(1-\epsilon)|\overline{B}^0>}{\sqrt{2(1+|\epsilon|^2)}}, \tag{341}$$

where $\epsilon=(p-q)/(p+q)$ and the plus sign belongs to 'L'. Since $M_L\neq M_H$, the time evolution of the mass eigenstates B_L and B_H (that are given by $\sim e^{-iM_jt}|B_j(0)>$) are different. Therefore one can have oscillations between B^0 and $\overline{B}^0$. Denoting by $|B^0_{\rm phys}(t)>$ ($|\overline{B}^0_{\rm phys}(t)>$) the state that at time $t=0$ is pure B^0 (respectively $\overline{B}^0$), we have

$$\begin{aligned}|B^0_{\rm phys}(t)>&=g_+(t)|B^0>+\left(\tfrac{q}{p}\right)g_-(t)|\overline{B}^0>\\ |\overline{B}^0_{\rm phys}(t)>&=\left(\tfrac{q}{p}\right)g_+(t)|B^0>-g_-(t)|\overline{B}^0>,\end{aligned} \tag{342}$$

where

$$\begin{aligned}g_+(t)&=e^{-\frac{t}{2}\gamma}e^{-imt}\cos\tfrac{\Delta mt}{2}\\ g_-(t)&=e^{-\frac{t}{2}\gamma}e^{-imt}i\sin\tfrac{\Delta mt}{2}.\end{aligned} \tag{343}$$

Here, we have defined $m\equiv(m_H+m_L)/2$ and $\Delta m=m_H-m_L$. The probability that an initial B^0 ($\overline{B}^0$) decays as a $\overline{B}^0$ (B^0) is simply

$$P(t)=\frac{1}{2}e^{-\gamma t}[1-\cos(\Delta mt)]. \tag{344}$$

Observation of the time dependency in the four combinations of initial and final state flavours enables the mass difference and a dilution factor to be determined. From Δm_q and γ the mixing parameter $x_q=\Delta m_q/\gamma_q$ ($q=d,s,b$), the number of oscillations per lifetime, can be derived.

In general the final states (f and their $\mathcal{CP}$ conjugate $\overline{f}$) of B meson decay are not $\mathcal{CP}$ eigenstates. There are four decay amplitudes for pure beauty eigenstates to be measured

$$\begin{aligned}M_f&=<f|H|B^0> \qquad \overline{M}_f=<f|H|\overline{B}^0>\\ M_{\overline{f}}&=<\overline{f}|H|B^0> \qquad \overline{M}_{\overline{f}}=\overline{f}|H|\overline{B}^0>.\end{aligned} \tag{345}$$

The amplitudes M_f can be factorized with the two phases and an absolute value (see also Eq. (357)), here two different phases appear. The 'weak' phases Φ are parameters in the Hamiltonian. They account for $\mathcal{CP}$ violation and have opposite signs in M_f and $\overline{M}_f$. The 'strong' phases δ appear in scattering and decay amplitudes, they do not account for $\mathcal{CP}$ violation and have equal signs in M_f and $\overline{M}_f$. From equations (339) and (340) the time-dependent rates are

$$\begin{aligned}\overset{(-)}{\Gamma}_f(t)&=Ae^{-\gamma t}[1\pm I(t)]\\ \overset{(-)}{\Gamma}_{\overline{f}}(t)&=\overline{A}e^{-\gamma t}[1\pm\overline{I}(t)].\end{aligned} \tag{346}$$

Here $A = \frac{1}{2}(|M_f|^2 + |\overline{M}_f|^2)$, $\overline{A} = \frac{1}{2}(|M_{\overline{f}}|^2 + |\overline{M}_{\overline{f}}|^2)$ and I ($\overline{I}$) are the $B_L - B_H$ interference terms due to mixing

$$\begin{aligned} I(t) &= \tfrac{1-|\kappa|^2}{1+|\kappa|^2}\cos(\Delta m t) + \tfrac{2\mathrm{Im}\kappa}{1+|\kappa|^2}\sin(\Delta m t) \qquad \kappa = \tfrac{q}{p}\,\tfrac{\overline{M}_f}{M_f} \\ \overline{I}(t) &= \tfrac{1-|\overline{\kappa}|^2}{1+|\overline{\kappa}|^2}\cos(\Delta m t) + \tfrac{2\mathrm{Im}\overline{\kappa}}{1+|\overline{\kappa}|^2}\sin(\Delta m t) \qquad \overline{\kappa} = \tfrac{q}{p}\,\tfrac{\overline{M}_{\overline{f}}}{M_{\overline{f}}}. \end{aligned} \tag{347}$$

Of special interest is the case, where only a single diagram and therefore a single CKM phase and hadronic matrix element contributes to the decay. In this case $|\overline{M}_{\overline{f}}| = |M_f|$, $|M_{\overline{f}}| = |\overline{M}_f|$ and with $|q/p| = 1$ this gives $|\kappa| = 1/|\overline{\kappa}|$.

6.1.2 Measurement of Relevant Parameters

The two unitarity conditions relevant for B-meson systems are

$$\begin{aligned} V_{tb}V_{ub}^* + V_{ts}V_{us}^* + V_{td}V_{ud}^* = 0 \\ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \end{aligned} \tag{348}$$

The angles of the triangles shown in Fig. 46 can be extracted either indirectly by measuring the lengths of the sides, or, within the Standard Model, directly from $\mathcal{CP}$ asymmetries. If the angles extracted by the two different methods disagree, then this would indicate new physics.

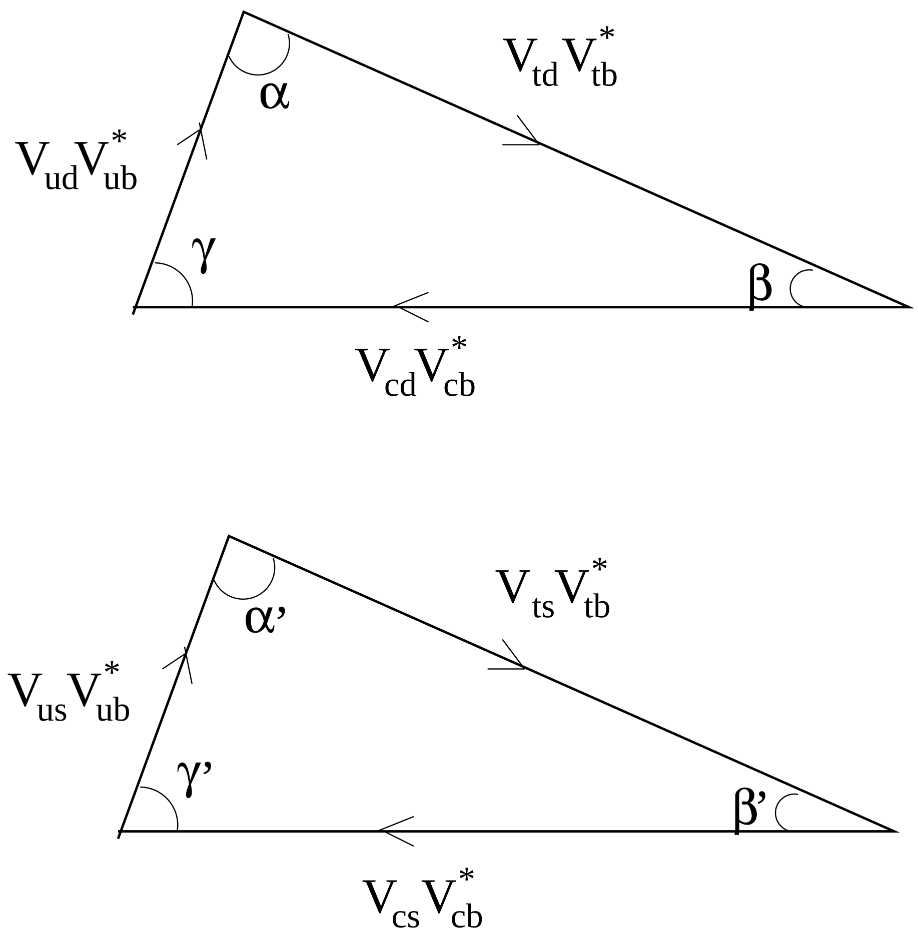

Figure 46: *Two unitarity triangles which provide the measurable angles in B-physics.*

LHCb is ideally suited to determine all the angles of the two unitarity triangles using high-statistics data. Table 2 indicates the decay modes used and the expected precision on the unitarity triangles, obtained after one year of data taking.

Another common parameterization of the CKM matrix due to Wolfenstein is [36]

$$V = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \tag{349}$$

Table 2: *Expected precision on the angles of the unitarity triangles obtained by the LHCb experiment in one year of data taking.*

Parameter	Decay Mode	σ [1 year]
$\beta+\gamma$	B_d^0 and $\overline{B}_d^0 \to \pi^+\pi^-$; no penguin	0.03
$(=\pi-\alpha)$	penguin/tree $= 0.20 \pm 0.02$	0.03 - 0.16
β	B_d^0 and $\overline{B}_d^0 \to \mathrm{J}/\psi K_\mathrm{S}$	0.01
$\gamma - 2\delta\gamma$	B_s^0 and $\overline{B}_s^0 \to D_s^\pm K^\mp$	0.05 - 0.28
γ	$B_d^0 \to \overline{D}^0 K^{*0}, D^0 K^{*0}, D_1 K^{*0}$ and $\overline{B}_d^0 \to \overline{D}^0 \overline{K}^{*0}, D^0 \overline{K}^{*0}, D_1 \overline{K}^{*0}$	0.07 - 0.31
$\delta\gamma$	B_s^0 and $\overline{B}_s^0 \to \mathrm{J}/\psi\phi$	0.01
x_s	B_s^0 and $\overline{B}_s^0 \to D_s^\pm \pi^\mp$	up to 90 (95 % CL)

with four real parameters A, λ, ρ and η. A consequence of unitarity in the Wolfenstein parameterization is $J = \eta A^2 \lambda^6$. 'J' has the geometrical interpretation of two times the area of the unitarity triangle; therefore all the triangles, for instance the triangles defined by Eq. (348), have the same area.

The parameters ρ and η are poorly determined. A major goal of the LHCb experiment is a precise measurement of these two quantities as a sensitive test of the Standard Model description of quark mixing. In conjunction with measurements already made, this can be accomplished by the measurement of $B_s^0 - \overline{B}_s^0$ mixing, $\mathcal{CP}$ violation in various decays of B mesons and rare decays of B mesons. The measurement of the B_s mixing parameter x_s, when combined with our knowledge of B_d mixing, provides a measurement of the CKM parameters $|V_{ts}/V_{td}|$ with relatively low theoretical uncertainties. So far, $|V_{ud}|$ and $|V_{us}|$ give a precise measurement of $\lambda \approx 0.22$. $|V_{cb}|$, and thus A, is determined from semileptonic B decays; $A \approx 0.84 \pm 0.06$. Three present measurements provide limited information on the remaining two variables, ρ and η:

- measurement of $|V_{ub}/V_{cb}|$ gives $\lambda\sqrt{\rho^2+\eta^2}$.
- $B_d^0 - \overline{B}_d^0$ mixing depends on the combination $|V_{td}V_{tb}^*|^2$ which gives the quantity $A\lambda^3\sqrt{(1-\rho)^2-\eta^2}$.
- $\mathcal{CP}$ violation parameter ϵ in K decays depends on the quantity $(1-\rho)\eta$.

The latter two suffer from theoretical uncertainties and a dependency on the inaccuracy in the measured top-quark mass. Resuming this gives three constraints on the CKM parameters, see Fig. 47. The above measurements are usually discussed with reference to the unitarity triangle. $\mathcal{CP}$ violation in the B-meson system is only possible if all the three angles are different from zero. The angles can be rewritten in terms of the Wolfenstein parameters

$$\tan = \frac{\eta}{\eta^2-\rho(1-\rho)}, \qquad \tan\beta = \frac{\eta}{1-\rho}, \qquad \tan\gamma = \frac{\eta}{\rho}. \tag{350}$$

The sign and the magnitude of these angles are meaningful and can be measured.

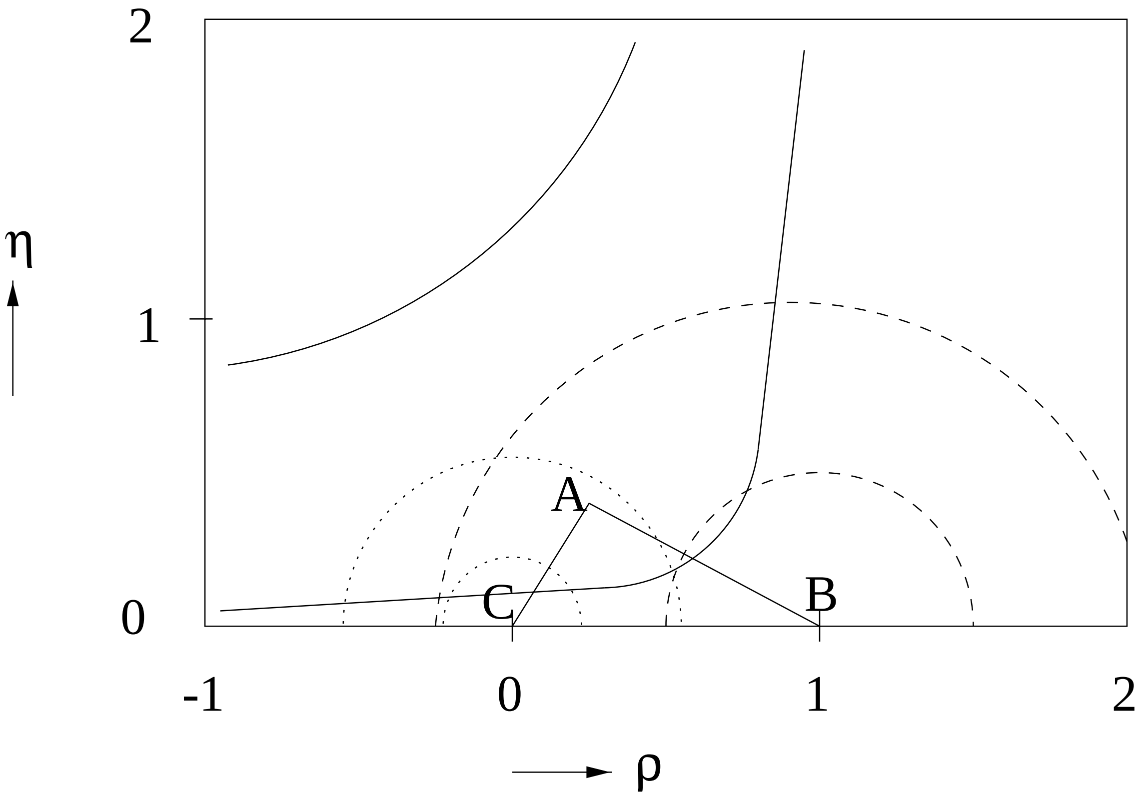

Figure 47: *Constraints in the (ρ,η) plane from ϵ_K measurement (solid curves), from $|V_{ub}/V_{cb}|$ (dotted curves) and from $B_d^0-\overline{B}_d^0$ mixing (dashed curves). Also the unitarity triangle is reported with coordinates A(ρ,η), B(1,0) and C(0,0).*

The LHCb experiment will determine the unitarity triangle completely, without using previous measurements. All angles, one side and the height of the triangle can be measured independently. The general strategy might look as follows [36, 37]:

- $B^0 \to J/\psi K_S^0$ and similar channels will obtain a clean and precise measurement of the angle β. Measurement of B_s mixing determines the side opposite to the angle γ. Now the whole unitarity triangle is determined.

  To understand how a measurement of B_s-mixing can improve the determination of the quantity $(1-\rho)^2+\eta^2$ we consider the mixing frequency ratio $\Delta m_d/\Delta m_s$, where the top-quark mass dependence is cancelled

$$(1-\rho)^2+\eta^2 = \left(\frac{\Delta m_d}{\Delta m_s}\right)\left(\frac{1}{\lambda^2}\right)\left(\frac{B_{B_s}}{B_{B_d}}\right)\left(\frac{f_{B_s}}{f_{B_d}}\right)^2. \tag{351}$$

So by measurement of the two mixing frequency parameters from B_s and B_d mixing it is possible to extract the above quantity. Failure to find B_s mixing frequencies in the theoretical allowed range will signal a failure of the Standard Model description of quark mixing in weak interactions.

- Check the internal consistency of the unitarity triangle with less clean or precise decay channels. Angle α may suffer from possible Penguin contribution. Angle γ is limited in precision because strong phases have to be experimentally determined together with the weak phase in fewer events. The triangle height using the small $\mathcal{CP}$ violating decay $B_s^0 \to J/\psi\phi$ is limited in statistical precision.

- Rare B decays can be studied yielding independent information on the CKM parameters.

As mentioned in section 6 there are three types of measurement. Here only $\mathcal{CP}$ asymmetries in neutral meson decay are considered.

- *Decays of B mesons to $\mathcal{CP}$ eigenstates.*

  Here we are interested in the decays of neutral B's into a $\mathcal{CP}$ eigenstate, which we denote by $f_{\mathcal{CP}}$. We define the amplitudes (compare Eq. (345)) for these processes as

$$A_f \equiv < f_{CP}|H|B^0 >, \qquad \overline{A}_f \equiv < f_{CP}|H|\overline{B}^0 > . \tag{352}$$

  Define further $\kappa \equiv (q/p)(\overline{A}_f/A_f)$. Now, for the B system, we should measure quantities of the form

$$a_{f_{CP}} \equiv \frac{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})}{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})}. \tag{353}$$

  This leads, using equation 342 and 343, to the following form for the time-dependent $\mathcal{CP}$ asymmetry

$$a_{f_{CP}} = \frac{1 - |\kappa|^2 \cos(\Delta m t) - 2\text{Im}\{\kappa\}\sin(\Delta M t)}{1 + |\kappa|^2}. \tag{354}$$

  The quantity $\text{Im}\{\kappa\}$ can be directly related to CKM matrix elements in the Standard Model. If the neutral B meson decays into a $\mathcal{CP}$ eigenstate ($|\overline{f}> = \pm CP|f>$), then $\kappa = \overline{\kappa}$ and one is left with only two independent decay rates. When in addition only a single diagram contributes to the decay, $|\kappa| = |\overline{\kappa}| = 1$ and the decay rates in equation 346 simplify to

$$\overset{(-)}{\Gamma}_f(t) = Ae^{-\gamma t}\left[1 \mp 2\text{Im}\{\kappa\}\sin(\Delta m t)\right]. \tag{355}$$

  Here $\kappa = e^{-2i(\Phi_M + \Phi_D)}$ with mixing phase Φ_M and the decay phase Φ_D, see also Eq. (358).

  For decay modes such that $|\kappa| = 1$ (the clean modes, no $\mathcal{CP}$ violation), the asymmetry given in expression (353) simplifies considerably

$$a_{f_{CP}}(|\kappa| = 1) = -\text{Im}\{\kappa\}\sin(\Delta m t). \tag{356}$$

The modes appropriate for measuring asymmetries of the type (356) are those dominated by a single weak phase. Likely candidates are $J/\psi K_S$, DD, $\pi\pi$, ϕK_S and others. More diagrams contribute to the channel $B^0 \to \pi\pi$. The asymmetry in lowest order governs the angle α. Penguins can also contribute and this makes interpretation of the measurements more difficult. Isospin analysis can in principle partially eliminate unknown hadronic contributions [32]. The channel $B_s^0 \to \rho K_S^0$ also suffers from hadronic contributions. The asymmetry measures γ with a very low branching fraction.

- *Decays of B_s mesons to non $\mathcal{CP}$ eigenstates.*

These decays can be used for a measurement of the angle γ. One measures independently the time dependence of the four possible decays ($B_s^0 \to f$). Only one tree diagram contributes, so again some simplifications can be made to equations (346) and (347). LHCb will use the decays $B_s^0 \to D_s K$ where the weak phase is given by $\phi_{f_D} = 2(\pi - \gamma)$.

6.1.3 Measurement of the Angles of the Unitarity Triangle

The measurement of the $\mathcal{CP}$ asymmetry, see Eq. (354), will determine Im$\{\kappa\}$ [31]. It depends on electroweak parameters only, without hadronic uncertainties. The amplitudes M_f can be factorized

$$A_f = \Sigma A_i e^{i\delta_i} e^{i\Phi_i} \quad \text{and} \quad \overline{A}_f = \Sigma A_i e^{i\delta_i} e^{-i\Phi_i}, \tag{357}$$

where A_i are real, Φ_i are weak CKM phases and δ_i are strong phases. Thus $|A_f/\overline{A}_f| = 1$ if all amplitudes that contribute to the direct decay have the same CKM phase, which is denoted by Φ_D: $|\overline{A}_f/A_f| = e^{-2i\Phi_D}$. For $\Gamma_{12} \ll M_{12}$, we have $q/p = \sqrt{M_{12}^*/M_{12}} = e^{-2i\Phi_M}$, where Φ_M is the CKM phase in the B-mixing. Thus

$$\kappa = e^{-2i(\Phi_M + \Phi_D)} \longrightarrow \text{Im}\{\kappa\} = -\text{sin}2(\Phi_M + \Phi_D). \tag{358}$$

Note that Im$\{\kappa\}$ is independent of phase convention and does not depend on any hadronic parameters. In what follows, we concentrate on those processes that, within the Standard Model, are dominated by amplitudes that have a single CKM phase.

For mixing in the B-system, see equations (338) and (339), the ratio q/p is given by the CKM phases

$$e^{-2i\Phi_M} = \left(\tfrac{q}{p}\right)_{B_d} \simeq 2 \text{ arg } (V_{td}V_{tb}^*) \qquad e^{-2i\Phi_M} = \left(\tfrac{q}{p}\right)_{B_s} \simeq 2 \text{ arg } (V_{ts}V_{tb}^*). \tag{359}$$

For decays via quark subprocesses $b \longrightarrow \overline{c}cd$, which are dominated by tree diagrams

$$e^{-2i\Phi_d} = \frac{\overline{A}_f}{A_f} = \frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}. \tag{360}$$

Thus for B_d decaying through $\overline{b} \longrightarrow \overline{c}c\overline{d}$

$$\text{Im}\kappa = \left(\frac{q}{p}\frac{\overline{A}_f}{A_f}\right) = \text{sin}\left[2 \text{ arg}\left(\frac{V_{cb}V_{cd}^*}{V_{tb^*V_{td}}}\right)\right]. \tag{361}$$

For decays with a single K_S (or K_L) in the final state, $K - \overline{K}$ mixing plays an essential role since $B^0 \rightarrow K^0$ and $\overline{B}^0 \rightarrow \overline{K}^0$. Interference is possible because $K - \overline{K}$ mixing, for these modes

$$\kappa = \left(\frac{q}{p}\right)\left(\frac{\overline{A}_f}{A_f}\right)\left(\frac{q}{p}\right)_K \quad \text{with} \quad \left(\frac{q}{p}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}. \tag{362}$$

Decay processes $b \longrightarrow \overline{s}sd$ and $b \longrightarrow \overline{s}ss$ are dominated by penguin diagrams. For these

$$\frac{\overline{A}_f}{A_f} = \frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}} \quad \text{respectively} \quad \frac{V_{tb}V_{ts}^*}{V_{tb}^*V_{ts}}. \tag{363}$$

Note that the sign of Im$\{\kappa\}$ depends on the $\mathcal{CP}$ transformation properties of the final state. The analysis above corresponds to $\mathcal{CP}$-even final states. For $\mathcal{CP}$-odd states, Im$\{\kappa\}$ has the opposite sign. In what follows, we specify Im$\{\kappa\}$ of $\mathcal{CP}$-even states, regardless of the $\mathcal{CP}$ assignments of specific hadronic modes discussed.

$\mathcal{CP}$ asymmetries in decays to $\mathcal{CP}$ eigenstates, $B^0 \to f_{CP}$, provide a way to measure the three angles of the unitary triangle (see Fig. 44) defined by

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ub}^*V_{ud}}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ub}^*V_{ud}}{V_{cd}V_{cb}^*}\right). \tag{364}$$

The angles (α', β', γ') of the second useful triangle can be defined in a similar way. Next three explicit examples will be given for asymmetries that measure the three angles α, β and γ.

Measuring sin(2β) in $B_d^0 \to J/\psi K_S$.

The mixing phase in the B_d system is given in Eq. (360). With a single kaon in the final state, one has to take into account the mixing phase in the K^0 system given in Eq. (363).

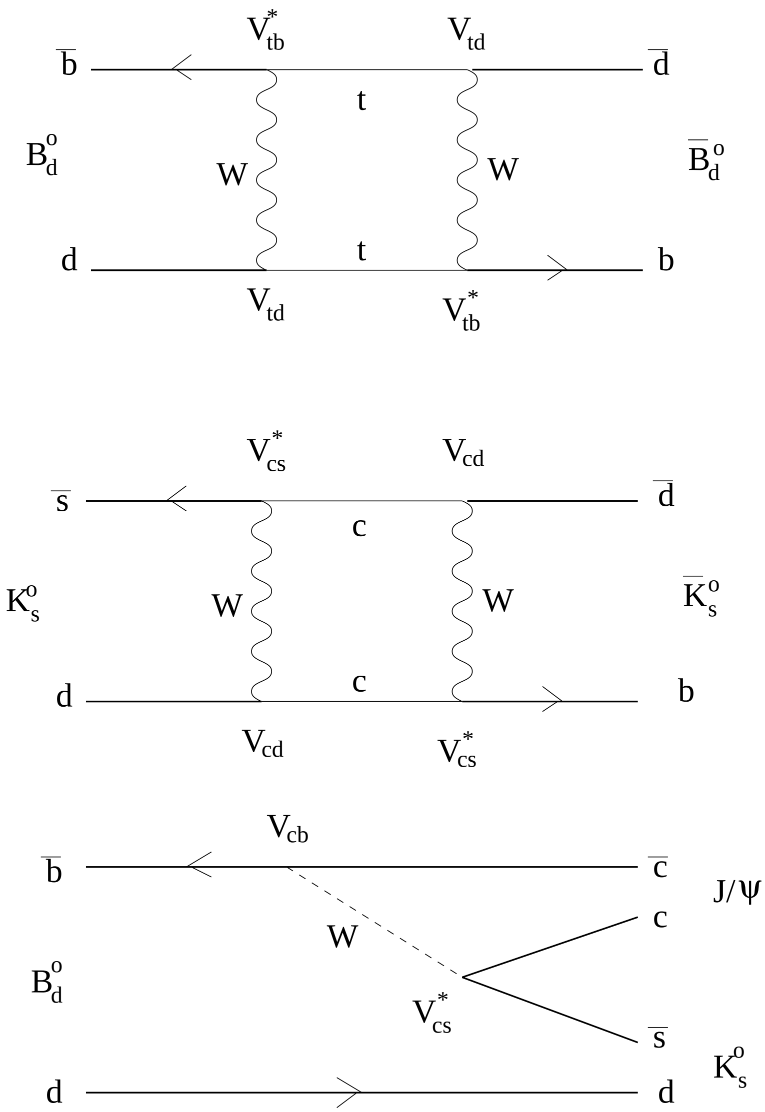

Figure 48: *B_d-mixing, K-mixing and tree diagrams contributing in the decay $B_d^0 \to J/\psi K_S$.*

The decay phase in the quark subprocess $b \to \bar{c}cs$ is $\overline{A}_f/A_f = (V_{cb}V_{cs}^*)/(V_{cs}V_{cb}^*)$. Thus

$$\kappa(B_d^0 \to J/\psi K_S) = \left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cs}V_{cb}^*}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cd}V_{cs}^*}\right) \to \mathrm{Im}\{\kappa\} = -\sin(2\beta). \tag{365}$$

(As $J/\psi K_S$ is a $\mathcal{CP} = -\infty$ state, there is an extra minus sign in the asymmetry that is suppressed here.) In addition there is a small penguin contribution to $b \to \bar{c}cs$. However, it depends on the CKM combination $V_{tb}V_{ts}^*$, which has to a very good approximation, the same phase (modulo π) as the tree diagram, which depends on $V_{cb}V_{cs}^*$. Hence, only one single weak phase contributes to the decay.

Measuring sin(2α) in $B_d^0 \to \pi\pi$.

Using q/p from Eq. (360) and $\overline{A}_f/A_f = (V_{ub}V_{ud}^*)/(V_{ud}V_{ub}^*)$ gives

$$\kappa(B_d^0 \to \pi\pi) = \left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right)\left(\frac{V_{ub}V_{ud}^*}{V_{ud}V_{ub}^*}\right) \to \mathrm{Im}\{\kappa\} = -\sin(2\alpha). \tag{366}$$

In this case, the penguin contribution is also expected to be small, but it depends on the CKM combination $V_{tb}V_{td}^*$, which has a phase different from that of the tree diagram. Uncertainties due to the penguin contribution can be eliminated using isospin analysis [31].

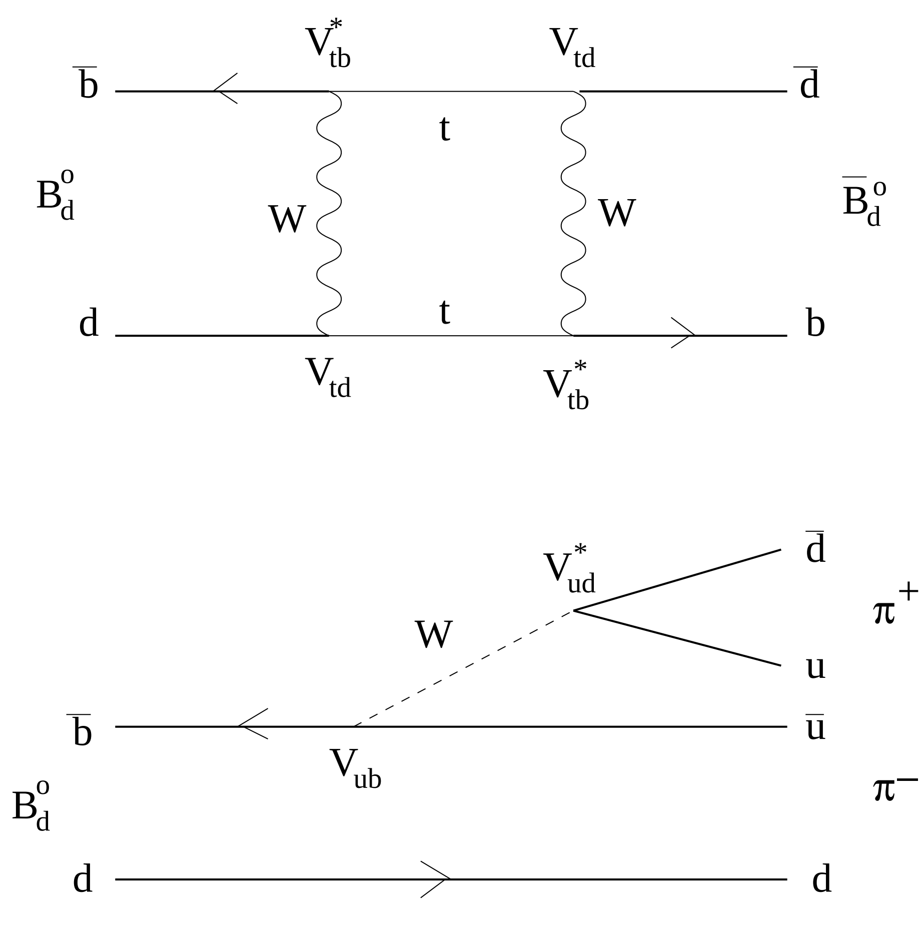

Figure 49: *B_d-mixing and tree diagrams contributing in the decay $B_d^0 \to \pi\pi$.*

Measuring sin(2γ) in $B_s^0 \to \rho K_S$.

The mixing phase in the B_s system is given in expression (360). Because of the final state K_S, the mixing phase for the K^0 system has to be taken into account. The quark subprocess is $b \to \overline{u}ud$ as in $B_d^0 \to \pi\pi$, thus

$$\kappa(B_s^0 \to \rho K_S) = \left(\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}\right)\left(\frac{V_{ub}V_{ud}^*}{V_{ud}V_{ub}^*}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cd}V_{cs}^*}\right) \to \mathrm{Im}\{\kappa\} = -\sin(2(\gamma+\beta')). \qquad (367)$$

In this case, the penguin contribution is also expected to be small.

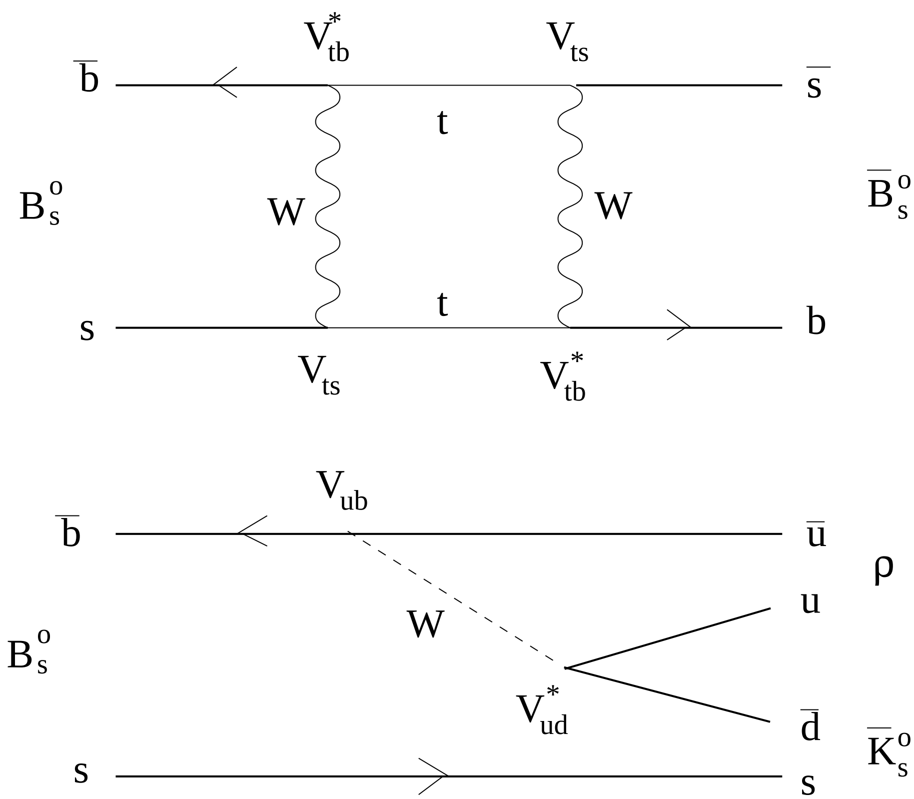

Figure 50: *B_s-mixing and tree diagrams contributing in the decay $B_s^0 \to \rho K_S$.*

7 Status of Experiments

7.1 The LHCb Experiment at CERN

The LHCb spectrometer, as shown in Fig. 51, is a dipole magnet configuration with a forward angular coverage of approximately 10 mrad to 300 (250) mrad in the bending (nonbending) plane. The silicon vertex detector is positioned inside the vacuum pipe for measurement of the B secondary vertices. The expected proper time resolution is about 0.043 ps. LHCb comprises a vertex detector system (including a pile-up veto counter), a tracking system (partially inside a dipole magnet), aerogel and gas RICH counters, an electromagnetic calorimeter with preshower detector, a hadron calorimeter and a muon detector. The polarity of the dipole magnetic field can be changed to reduce systematic errors in the $\mathcal{CP}$ violation measurements that could result from a left-right asymmetry of the detector. An iron shield upstream of the magnet reduces the stray field in the vicinity of the vertex detector and of the first RICH. The vertex detector system comprises a silicon vertex detector and a pile-up veto counter. The vertex detector has to provide precise information on the production and decay vertices of b-hadrons. The pile-up veto counter is used to suppress events containing multiple pp interactions in a single bunch-crossing, by counting the number of primary vertices. The tracking system, consisting of inner and outer tracker, provides efficient reconstruction and precise momentum measurement of charged tracks, track directions for ring reconstruction in the RICH, and information for the level1 and higher level triggers. The system comprises 11 stations between the vertex detector and the calorimeters. The RICH system has the task of identifying charged particles (K/p) over the momentum range 1-150 GeV/c, within an angular acceptance of 10-330 mrad. Particle identification is crucial to reduce background in selected final states and to provide an efficient kaon tag. K/p separation is used to eliminate backgrounds such as the or +K- background to the +p- mode. The RICH's give LHC-b the capability of separating the three modes.

Figure 51: *The LHCb detector seen from above.*

The main purpose of the calorimeters is to provide identification of electrons (e/p discrimination) and hadrons for trigger and off-line analysis, with measurements of position and energy. The muon detector provides muon identification and level-0 trigger information. It consists of four stations embedded in an iron filter and a special station (M1) in front of the calorimeter.

8 PROBLEMS

8.1 Time Reversal

Problem 1. Wigner defined the operation of time reversal as

$$\mathcal{T} = \mathbf{U} \cdot \mathbf{J}, \tag{368}$$

where $\mathbf{U}$ is a unitary operator and $\mathbf{K}$ is an operator that complex conjugates all quantities it operates on.
(a) Show that an immediate consequence of Wigner's definition of time reversal is that a hamiltonian invariant to time must be real.
(b) Prove that another consequence of Wigner's definition of time reversal is the relation

$$< \mathcal{T}\psi_\alpha | \mathcal{T}\psi_\beta > = < \psi_\alpha | \psi_\beta >^* . \tag{369}$$

8.2 Charge Conjugation

Problem 2. For particles such as n, p, π^+, *etc.*, from which the corresponding antiparticles are distinguishable, the wave function Ψ must be complex.
(a) Prove this by using the current density of particle flow,

$$S = \frac{i\hbar}{2m}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*). \tag{370}$$

(b) Show that for self-conjugate particles such as γ and π^0 one must have $\Psi^* = \Psi$.

8.3 $\mathcal{CPT}$-Theorem

Problem 3. The $\mathcal{CPT}$-theorem states that the hamiltonians in a general class of quantum field theories are invariant to the combined operations of $\mathcal{CPT}$ even if they are not invariant to one or more of those operations. Prove that an important consequence of the $\mathcal{CPT}$-theom is that the lifetime of a particle is equal to that of an antiparticle. Consider the decays $A \to B$ and $\overline{A} \to \overline{B}$, where B represents the final state of many particles. Note that the lifetime is governed by the decay matrix element $< \Psi_b | H_{wk} | \Psi_a >$.

8.4 Cabibbo Angle

Estimate the Cabibbo angle θ_C by comparing the β decays $n \to pe^-\overline{\nu}_e$ and $\Lambda^0 \to pe^-\overline{\nu}_e$. The neutron ($\Lambda^0$) lifetime is $\tau_n = 900$ s ($\tau_\Lambda = 2.63 \times 10^{-10}$ s), branching fraction is $f_n = 1$ ($\tau_\Lambda = 8.3 \times 10^{-4}$), and the energy release is $E_n = 1.3$ MeV ($E_\Lambda = 177.3$ MeV). Assume that the phase space factors are approximately $E^5/30$, where E is the energy release in MeV.

8.5 Unitary Matrix

Problem 2. Show that an $n \times n$ unitary matrix has n^2 real parameters, and that

$$\mathbf{U} = e^{-i\alpha} \begin{pmatrix} \cos\theta_C e^{i\beta} & \sin\theta_C e^{i\gamma} \\ -\sin\theta_C e^{-i\gamma} & \cos\theta_C e^{-i\beta} \end{pmatrix} \tag{371}$$

is the most general form of a 2×2 unitary matrix.

8.6 Quark Phases

Problem 3. The most general form of (d, s) mixing is

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \mathbf{U} \begin{pmatrix} d \\ s \end{pmatrix}, \tag{372}$$

where $\mathbf{U}$ is an arbitrary 2×2 unitary matrix, $\mathbf{U}^\dagger \mathbf{U} = \mathbf{1}$. Show that this can be reduced to the form (186) by adjusting the arbitrary phases of the quark states s, s' and d.

8.7 Exploiting Unitarity

Problem 4. Show that if Eq. (186) were exact one can derive the remaining CKM matrix elements by exploiting the unitarity of the CKM matrix.

8.8 D-meson Decay

Problem 5. Estimate the relative rates for the following htree decay modes of the D^0 $(c\overline{u})$ meson: $D^0 \to K^-\pi^+$, $\pi^-\pi^+$, $K^+\pi^-$.

Classify the following semileptonic decays of the $D^+(1869) = c\overline{d}$ meson as Cabibbo-allowed, Cabibbo-suppressed, or forbidden in lowest-order weak interactions, by finding selection rules for the changes in strangeness, charm and electrical charge in such decays:

(a) $D^+ \to K^- + \pi^+ + e^+ + \nu_e$
(b) $D^+ \to K^+ + \pi^- + e^+ + \nu_e$
(c) $D^+ \to \pi^+ + \pi^+ + e^- + \overline{\nu}_e$
(d) $D^+ \to \pi^+ + \pi^- + e^+ + \nu_e$

8.9 Mass and Decay Matrix

Problem 6. Determine the eigenvalues and normalized eigenvectors of the mass matrix

$$\begin{pmatrix} M - \frac{i\Gamma}{2} & M_{12} - \frac{i\Gamma_{12}}{2} \\ \\ M_{12}^* - \frac{i\Gamma_{12}^*}{2} & M - \frac{i\Gamma}{2} \end{pmatrix} \tag{373}$$

8.10 Kaon Intensities

Problem 7. (a) Determine the intensities of K^0 and $\overline{K}^0$ as function of time.
(b) Derive Eq. (221) for the ratio of the number of leptons of the 'wrong sign' with respect to the number with the 'correct sign' for $K^0 \leftrightarrow \overline{K}^0$ oscillations. Give the expression for r in case $\Gamma_1 \gg \Gamma_2$.
(c) For $B^0 \leftrightarrow \overline{B}^0$ oscillations a good approximation is $\Gamma_1 \approx \Gamma_2$. Give the expression for r as function of $x = \Delta m / \Gamma$.

8.11 Optical Theorem

Problem 8. Describe an incoming particle with a plane wave $Ae^{i(kz-\omega t)}$, with $\omega = \sqrt{k^2+m^2}$, that enters perpendicular to a thin plate with thickness l. Determine the phase shift of the wave as it travels through the material. Derive the relation between the index of refraction n and the forward scattering amplitude,

$$n = 1 + \frac{2\pi N}{k^2} f(0). \tag{374}$$

Note that on the one hand a plane wave in the material can be described by a travel constant nk. This leads to a phase shift. On the other hand one can integrate over the coherent elastic scattering in the forward direction. The scattered wave is proportional to $Af(\theta)e^{i(kr-\omega t)}/r$, with r the distance between the scattering center in the plate to a point after the plate. One can make the approximation $f(\theta) = f(0)$.

8.12 Neutral Kaon States in Matter

Problem 9. (a) Verify Eq. (252) for the eigenstates of the neutral kaon system in matter.

(b) Explain how the value of $\Delta m/\Gamma$ is obtained in the experiment of F. Muller *et al*. [4].

8.13 Regeneration Parameter

Problem 10. Estimate the value of the regeneration parameter in Beryllium for a momentum of 1,100 MeV. This corresponds to the circumstances of the original $\mathcal{CP}$ violation experiment. Estimate f and $\overline{f}$ by using the optical theorem and data from K^+p and K^-p total cross sections.

8.14 Isospin Analysis

Problem 11. Derive Eq. (270).

9 APPENDIX B: SOLUTIONS

9.1 Cabibbo angle.

Problem 1. Estimate the Cabibbo angles θ_C by comparing the β decays $n \to pe^-\overline{\nu}_e$ and $\Lambda^0 \to pe^-\overline{\nu}_e$. The neutron ($\Lambda^0$) lifetime is $\tau_n = 900$ s ($\tau_\Lambda = 2.63 \times 10^{-10}$ s), branching fraction is $f_n = 1$ ($\tau_\Lambda = 8.3 \times 10^{-4}$), and the energy release is $E_n = 1.3$ MeV ($E_\Lambda = 177.3$ MeV). Assume that the phase space factors are approximately $E^5/30$, where E is the energy release in MeV.

Solution: At the quark level the neutron decay corresponds to $d \to ue^-\overline{\nu}_e$ which has a Cabibbo factor cosθ_C. For the Λ^0 we have $s \to ue^-\overline{\nu}_e$ which has a Cabibbo factor sinθ_C. Thus we have

$$\begin{aligned} \Gamma(n \to pe^-\overline{\nu}_e) = \tfrac{f_n}{\tau_n} \propto \tfrac{\cos\theta_C E_n^5}{30}, \\ \Gamma(\Lambda^0 \to pe^-\overline{\nu}_e) = \tfrac{f_\Lambda}{\tau_\Lambda} \propto \tfrac{\sin\theta_C E_n^5}{30}. \end{aligned} \tag{375}$$

From the ratio we find

$$\tan\theta_C{}^2 \frac{f_\Lambda \tau_n}{f_n \tau_\Lambda}\left(\frac{E_n}{E_\Lambda}\right)^5 = 0.060 \tag{376}$$

and we have that $\theta_C = 13.8°$.

9.2 Unitary Matrix

Problem 2. Show that an $n \times n$ unitary matrix has n^2 real parameters, and that

$$\mathbf{U} = e^{-i\alpha}\begin{pmatrix} \cos\theta_C e^{i\beta} & \sin\theta_C e^{i\gamma} \\ -\sin\theta_C e^{-i\gamma} & \cos\theta_C e^{-i\beta} \end{pmatrix} \tag{377}$$

is the most general form of a 2×2 unitary matrix.

Solution: An arbitrary complex $n \times n$ matrix $\mathbf{U}$ has $2n^2$ real parameters. The matrix $\mathbf{F} \equiv \mathbf{U}^\dagger\mathbf{U}$ is Hermitian by construction, so $F_{ij} = F_{ji}^*$ and it has n^2 real parameters. Hence, the condition $\mathbf{U}^\dagger\mathbf{U} = \mathbf{1}$ imposes n^2 conditions on $\mathbf{U}$, leaving n^2 real parameters undetermined. Since Eq. (377) has $n^2 = 4$ real parameters and satisfies $\mathbf{U}^\dagger\mathbf{U} = \mathbf{1}$, it is the most general 2×2 unitary matrix.

9.3 Quark Phases

Problem 3. The most general form of (d, s) mixing is

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \mathbf{U}\begin{pmatrix} d \\ s \end{pmatrix}, \tag{378}$$

where $\mathbf{U}$ is an arbitrary 2×2 unitary matrix, $\mathbf{U}^\dagger\mathbf{U} = \mathbf{1}$. Show that this can be reduced to the form (186) by adjusting the arbitrary phases of the quark states s, s' and d.

Solution: Substituting Eq. (378) into Eq. (377) gives

$$\begin{aligned} d' = e^{-i\alpha}(\cos\theta_C e^{i\beta} d + \sin\theta_C e^{i\gamma} s) \\ s' = e^{-i\alpha}(-\sin\theta_C e^{-i\gamma} d + \cos\theta_C e^{-i\beta} s), \end{aligned} \tag{379}$$

which can be written as

$$\begin{aligned} e^{i(\alpha-\beta)}d' &= \cos\theta_C d + \sin\theta_C (e^{i(\gamma-\beta)}s) \\ e^{i(\alpha+\gamma)}s' &= -\sin\theta_C d + \cos\theta_C (e^{i(\gamma-\beta)}s). \end{aligned} \tag{380}$$

Redefining the phases of the quark states by

$$e^{i(\alpha-\beta)}d' \to d', \quad e^{i(\alpha+\gamma)}s' \to s', \quad e^{i(\gamma-\beta)}s \to s \tag{381}$$

gives the required result.

9.4 Exploiting Unitarity

Problem 4. Show that if Eq. (186) were exact one can derive the remaining CKM matrix elements by exploiting the unitarity of the CKM matrix.

Solution: If Eq. (186) is exact then

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C & V_{ub} \\ -\sin\theta_C & \cos\theta_C & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{382}$$

The remaining elements are then determined by exploiting the fact that the sum of the squared moduli of the elements of any row or column of a unitary matrix must be unity (this follows directly from $\mathbf{U}\mathbf{U}^\dagger = \mathbf{1}$ and $\mathbf{U}^\dagger\mathbf{U} = \mathbf{1}$, respectively). For example

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = \cos\theta_C{}^2 + \sin\theta_C{}^2 + |V_{ub}|^2 = 1, \tag{383}$$

implying $V_{ub} = 0$, etc.

9.5 D-meson Decay

Problem 5. Estimate the relative rates for the following htree decay modes of the D^0 ($c\overline{u}$) meson: $D^0 \to K^-\pi^+$, $\pi^-\pi^+$, $K^+\pi^-$.

Classify the following semileptonic decays of the $D^+(1869) = c\overline{d}$ meson as Cabibbo-allowed, Cabibbo-suppressed, or forbidden in lowest-order weak interactions, by finding selection rules for the changes in strangeness, charm and electrical charge in such decays:

(a) $D^+ \to K^- + \pi^+ + e^+ + \nu_e$
(b) $D^+ \to K^+ + \pi^- + e^+ + \nu_e$
(c) $D^+ \to \pi^+ + \pi^+ + e^- + \overline{\nu}_e$
(d) $D^+ \to \pi^+ + \pi^- + e^+ + \nu_e$

Solution: The amplitudes for the decay of the D^0 meson for the channels $D^0 \to K^-\pi^+$, $\pi^-\pi^+$, $K^+\pi^-$ are respectively $\cos\theta_C{}^2$, $\sin\theta_C \cos\theta_C$, and $\sin\theta_C{}^2$.

The Cabibbo-allowed decays involve the csW vertex, giving rise to the selection rule $\Delta C = \Delta S = \Delta Q = \pm 1$. Cabibbo-suppressed decays involve the cdW vertex, giving rise to the selection rule $\Delta C \Delta Q = \pm 1$, $\Delta S = 0$. Using these rules one sees that the decays are (a) Cabibbo-allowed, (b) forbidden, (c) forbidden, and (d) Cabibbo-suppressed.

9.6 Mass and Decay Matrix

Problem 6. Determine the eigenvalues and normalized eigenvectors of the mass matrix

$$\begin{pmatrix} M - \frac{i\Gamma}{2} & M_{12} - \frac{i\Gamma_{12}}{2} \\ & \\ M_{12}^* - \frac{i\Gamma_{12}^*}{2} & M - \frac{i\Gamma}{2} \end{pmatrix} \tag{384}$$

Solution: We have $M - \frac{i}{2}\Gamma = D \cdot I + \vec{E} \cdot \vec{\sigma}$ with $\vec{\sigma}$ the Pauli matrices. Thus

$$\vec{E} \cdot \sigma = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \cdot \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_x \end{pmatrix} \cdot \begin{pmatrix} E_z & E_x - iE_y \\ E_x + iE_y & -E_z \end{pmatrix} = E \begin{pmatrix} \cos\beta & \sin\beta e^{-i\alpha} \\ \sin\beta e^{i\alpha} & -\cos\beta \end{pmatrix} \tag{385}$$

and we find

$$D \cdot I + \vec{E} \cdot \vec{\sigma} = \begin{pmatrix} D + E\cos\beta & E\sin\beta e^{-i\alpha} \\ E\sin\beta e^{i\alpha} & D - \cos\beta \end{pmatrix}. \tag{386}$$

The eigenvalues are found from

$$\det\left([D \cdot I + \vec{E} \cdot \vec{\sigma}] - \lambda I\right) = 0 \tag{387}$$

We find

$$\begin{array}{l} (D + E\cos\beta - \lambda)(D - E\cos\beta - \lambda) - E^2\sin^2\beta = 0 \\ D^2 - 2\lambda D + \lambda^2 - E^2 = 0 \\ (\lambda - D)^2 = E^2 \quad \text{or} \quad \lambda - D = \pm E \end{array} \tag{388}$$

We find for the eigenvalues $\lambda_1 = D + E$ and $\lambda_2 = D - E$.

The eigenvectors that correspond to these eigenvalues are found as follows.

$$\left([D \cdot I + \vec{E} \cdot \vec{\sigma}] - \lambda_n I\right) |\vec{x} >= 0. \tag{389}$$

This gives for the first eigenvalue, λ_1,

$$\begin{pmatrix} E(\cos\beta - 1) & E\sin\beta e^{-i\alpha} \\ E\sin\beta e^{i\alpha} & -E(\cos\beta + 1) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0. \tag{390}$$

Thus we find

$$\begin{array}{l} (\cos\beta - 1)x_1 + \sin\beta e^{-i\alpha} x_2 = 0 \\ \sin\beta e^{i\alpha} x_1 - (\cos\beta + 1)x_2 = 0 \end{array} \tag{391}$$

We assume $x_1 = 1$ and find using the geometrical relations $1 - \cos\beta = 2\sin^2\frac{\beta}{2}$, $1 + \cos\beta = 2\cos^2\frac{\beta}{2}$ and $\sin\beta = 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}$ for x_2 the following

$$\begin{array}{l} \cos\beta - 1 + \sin\beta e^{-i\alpha} = 0 \\ x_2 = \frac{1 - \cos\beta}{\sin\beta} e^{i\alpha} = \tan\frac{\beta}{2} e^{i\alpha}. \end{array} \tag{392}$$

For the second eigenvalue, λ_2, we find

$$\begin{pmatrix} E(\cos\beta + 1) & E\sin\beta e^{-i\alpha} \\ E\sin\beta e^{i\alpha} & E(1 - \cos\beta) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0. \tag{393}$$

Thus we find

$$\begin{aligned}(\cos\beta+1)y_1+\sin\beta e^{-i\alpha}y_2=0\\ \sin\beta e^{i\alpha}y_1+(1-\cos\beta)y_2=0\end{aligned} \tag{394}$$

We assume $y_1 = 1$ and find for y_2 the following

$$y_2=-\frac{\cos\beta+1}{\sin\beta}e^{i\alpha}=-\cot\frac{\beta}{2}e^{i\alpha}. \tag{395}$$

We find for the eigenvectors

$$V_1=\begin{pmatrix}1\\ \tan\frac{\beta}{2}e^{i\alpha}\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}1\\ -\cot\frac{\beta}{2}e^{i\alpha}\end{pmatrix}. \tag{396}$$

9.7 Kaon Intensities

Problem 7. (a) Determine the intensities of K^0 and $\overline{K}^0$ as function of time.
(b) Derive Eq. (221) for the ratio of the number of leptons of the 'wrong sign' with respect to the number with the 'correct sign' for $K^0 \leftrightarrow \overline{K}^0$ oscillations. Give the expression for r in case $\Gamma_1 \gg \Gamma_2$.
(c) For $B^0 \leftrightarrow \overline{B}^0$ oscillations a good approximation is $\Gamma_1 \approx \Gamma_2$. Give the expression for r as function of $x = \Delta m/\Gamma$.

Solution: (a) The intensity for K^0 after decay for a certain time τ is

$$I(K^0)=\frac{[a_1(\tau)+a_2(\tau)]}{\sqrt{2}}\cdot\frac{[a_1^*(\tau)+a_2^*(\tau)]}{\sqrt{2}}, \tag{397}$$

with as K_1 amplitude

$$a_1(\tau)=\frac{1}{\sqrt{2}}e^{-im_1\tau-\frac{1}{2}\Gamma_1\tau} \tag{398}$$

and for the K_2 amplitude

$$a_2(\tau)=\frac{1}{\sqrt{2}}e^{-im_2\tau-\frac{1}{2}\Gamma_2\tau}. \tag{399}$$

We find

$$I(K^0)=\frac{1}{4}\left[e^{-\Gamma_1\tau}+e^{-\Gamma_2\tau}+2e^{-\frac{1}{2}(\Gamma_1+\Gamma_2)\tau}\cos\Delta m\tau\right], \tag{400}$$

with $\Delta m = |m_1 - m_2|$.

The intensity for $\overline{K}^0$ is

$$I(\overline{K}^0)=\frac{[a_1(\tau)-a_2(\tau)]}{\sqrt{2}}\cdot\frac{[a_1^*(\tau)-a_2^*(\tau)]}{\sqrt{2}}, \tag{401}$$

and we find

$$I(\overline{K}^0)=\frac{1}{4}\left[e^{-\Gamma_1\tau}+e^{-\Gamma_2\tau}-2e^{-\frac{1}{2}(\Gamma_1+\Gamma_2)\tau}\cos\Delta m\tau\right], \tag{402}$$

with $\Delta m = |m_1 - m_2|$.

(b) The ratio r_K is defined for semileptonic decay as

$$r_K=\frac{\int_0^\infty I(\overline{K}^0)d\tau}{\int_0^\infty I(K^0)d\tau}. \tag{403}$$

For the integral $\int_0^\infty I(\overline{K}^0)d\tau$ we find

$$\begin{aligned}
\int_0^\infty I(\overline{K}^0)d\tau &= \tfrac{1}{4}\left[\tfrac{1}{-\Gamma_1}e^{-\Gamma_1\tau} + \tfrac{1}{-\Gamma_2}e^{-\Gamma_2\tau} - 2e^{-\frac{1}{2}(\Gamma_1+\Gamma_2)\tau}\ \tfrac{(-\frac{1}{2}(\Gamma_1+\Gamma_2)\cos\Delta m\tau+\Delta m\sin\Delta m\tau)}{(\frac{1}{2}(\Gamma_1+\Gamma_2))^2+(\Delta m)^2}\right]_0^\infty \\
&= \tfrac{1}{4}\left[0 - \tfrac{1}{-\Gamma_1} - \tfrac{1}{-\Gamma_2} + 2\tfrac{[-\frac{1}{2}(\Gamma_1+\Gamma_2)]}{\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2}\right] \\
&= \tfrac{1}{4}\left[\tfrac{\Gamma_1+\Gamma_2}{\Gamma_1\Gamma_2} - \tfrac{\Gamma_1+\Gamma_2}{\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2}\right]
\end{aligned} \tag{404}$$

The same for $\int_0^\infty I(K^0)d\tau$ gives

$$\int_0^\infty I(K^0)d\tau = \frac{1}{4}\left[\frac{\Gamma_1+\Gamma_2}{\Gamma_1\Gamma_2} + \frac{\Gamma_1+\Gamma_2}{\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2}\right]. \tag{405}$$

Thus we find for r_K

$$\begin{aligned}
r_K &= \tfrac{(\Gamma_1+\Gamma_2)\left[\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2\right]-\Gamma_1\Gamma_2(\Gamma_1+\Gamma_2)}{(\Gamma_1+\Gamma_2)\left[\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2\right]+\Gamma_1\Gamma_2(\Gamma_1+\Gamma_2)} = \tfrac{\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2-\Gamma_1\Gamma_2}{\frac{1}{4}(\Gamma_1+\Gamma_2)^2+(\Delta m)^2+\Gamma_1\Gamma_2} \\
&= \tfrac{(\Gamma_1-\Gamma_2)^2+4(\Delta m)^2}{2(\Gamma_1+\Gamma_2)^2-(\Gamma_1-\Gamma_2)^2+4(\Delta m)^2}
\end{aligned} \tag{406}$$

For the neutral kaon system one has that $\Gamma_1 \gg \Gamma_2$ and we find

$$r_K \approx \frac{\Gamma_1^2+4(\Delta m)^2}{2\Gamma_1^2-\Gamma_1^2+4(\Delta m)^2} \approx 1. \tag{407}$$

(c) For the $B^0-\overline{B}^0$ system one has that $\Gamma_1 \approx \Gamma_2$ and we find

$$r \approx \frac{0+4(\Delta m)^2}{2(2\Gamma_1)^2-0+4(\Delta m)^2} = \frac{4(\Delta m)^2}{8\Gamma^2+4(\Delta m)^2} = \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{2+\left(\frac{\Delta m}{\Gamma}\right)^2} = \frac{x^2}{2+x^2}, \tag{408}$$

with $x = \Delta m/\Gamma$.

9.8 Optical Theorem

Problem 8. Describe an incoming particle with a plane wave $Ae^{i(kz-\omega t)}$, with $\omega = \sqrt{k^2+m^2}$, that enters perpendicular to a thin plate with thickness l. Determine the phase shift of the wave as it travels through the material. Derive the relation between the index of refraction n and the forward scattering amplitude,

$$n = 1 + \frac{2\pi N}{k^2}f(0). \tag{409}$$

Note that on the one hand a plane wave in the material can be described by a travel constant nk. This leads to a phase shift. On the other hand one can integrate over the coherent elastic scattering in the forward direction. The scattered wave is proportional to $Af(\theta)e^{i(kr-\omega t)}/r$, with r the distance between the scattering center in the plate to a point after the plate. One can make the approximation $f(\theta) = f(0)$.

Solution: We have an incoming particle with a plane wave $Ae^{i(kz-\omega t)}$, with $\omega = \sqrt{k^2+m^2}$. In the infinitely thin plane, the travel constant is nk, with n the index of refraction. We take the z direction along $\vec{k}$ of the wave. For the incoming wave we have $Ae^{ikz}e^{-i\omega t}$. For the wave after the thin plate we find $Ae^{i(k(z-l)+nkl)}e^{-i\omega t}$, where wave attenuation has been neglected. The phase shift amounts to $i(nkl-kl) = i(n-1)kl$. Thus we have

$$Ae^{ikz}e^{ilk(n-1)}e^{-i\omega t} \approx Ae^{ikz}\left[1+ikl(n-1)\right]e^{-i\omega t}. \tag{410}$$

The wave after the thin plate can also be written as the sum of the coherent incoming and scattered waves.

$$A\left(e^{ikz} + \int_0^\infty f(\theta)\frac{e^{ikr}}{r}\cdot N \cdot 2\pi l\rho \mathrm{d}\rho\right)e^{-i\omega t}, \tag{411}$$

with N the number of nuclei per volume, $2\pi l\rho\mathrm{d}\rho$ the volume, $f(\theta)$ the scattering amplitude. Here ρ is the distance along the plate, perpendicular to the wave. One has $\rho^2 = r^2 - z^2$. For $r = z \rightarrow f(\theta) = f(0)$ and for $r = \infty \rightarrow f(\theta) = 0$. We find

$$\begin{aligned} 2\pi Nl\int_0^\infty f(\theta)\tfrac{e^{ikr}}{r}\rho\mathrm{d}\rho &= 2\pi Nl\int_z^\infty f(\theta)e^{ikr}\mathrm{d}r = 2\pi Nlf(\theta)\tfrac{1}{ik}e^{ikr}|_z^\infty \\ &= 2\pi Nl\left[0-\tfrac{f(0)}{ik}e^{ikz}\right] = -2\pi Nl\tfrac{f(0)}{ik}e^{ikz}. \end{aligned} \tag{412}$$

We now combine Eq. (410) and Eq. (412) and find

$$\begin{aligned} Ae^{ikz}\left[1+ikl(n-1)\right]e^{-i\omega t} &\approx Ae^{ikz}\left[1-\tfrac{f(0)}{ik}2\pi Nl\right]e^{i\omega t} \\ ikl(n-1) &= -\tfrac{f(0)}{ik}2\pi Nl \\ (n-1) &= \tfrac{f(0)}{k^2}2\pi N \\ n &= 1+\tfrac{2\pi N}{k^2}f(0). \end{aligned} \tag{413}$$

9.9 Neutral Kaon States in Matter

Problem 9. (a) Verify Eq. (252) for the eigenstates of the neutral kaon system in matter.

(b) Explain how the value of $\Delta m/\Gamma$ is obtained in the experiment of F. Muller *et al.* [4].

Solution: The Schrödinger equation for the kaon system in matter can be written as

$$i\frac{\partial\psi}{\partial\tau} = (M-\frac{i}{2}\Gamma)\psi - \begin{pmatrix} \frac{\phi_{\rm reg}}{\tau} & 0 \\ 0 & \frac{\bar{\phi}_{\rm reg}}{\tau} \end{pmatrix}\psi, \tag{414}$$

with $\phi_{\rm reg} = \frac{2\pi N\tau v}{k\sqrt{1-v^2}}f(0)$. One observes that the Schrödinger equation for kaons in matter has an additional term compared to the case of vacuum. The eigenvectors are

$$|K_1^{0\prime}>= \begin{pmatrix} 1+r_{\rm reg} \\ 1-r_{\rm reg} \end{pmatrix} \quad \text{and} \quad |K_2^{0\prime}>= \begin{pmatrix} 1-r_{\rm reg} \\ -1-r_{\rm reg} \end{pmatrix}. \tag{415}$$

These states can be written as

$$
\begin{aligned}
|K_1^{0\prime}> &= |K_1^0> + r_{\text{reg}}|K_2^0> \\
&= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + r_{\text{reg}}\tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 + r_{\text{reg}} \\ 1 - r_{\text{reg}} \end{pmatrix}, \\
|K_2^{0\prime}> &= |K_2^0> - r_{\text{reg}}|K_1^0> \\
&= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} - r_{\text{reg}}\tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 - r_{\text{reg}} \\ -1 - r_{\text{reg}} \end{pmatrix}.
\end{aligned}
\tag{416}
$$

(b) The ratio of intensities of the 'transmitted' wave and the 'scattered' wave is a function of $\Delta m/\Gamma$. The 'transmitted' wave originates from regeneration of K_1^0 through so-called transmission regeneration by means of the thin plate. The 'scattered' wave originates from regeneration of K_1^0 from diffraction by the nuclei in the thin plate. The latter wave has a characteristic angular distribution corresponding to diffraction. The intensities of the K_1^0's generated by transmission and by diffraction is measured and from this $\Delta m/\Gamma$ can be calculated.

9.10 Regeneration Parameter

Problem 10. Estimate the value of the regeneration parameter in Beryllium for a momentum of 1,100 MeV. This corresponds to the circumstances of the original $\mathcal{CP}$ violation experiment. Estimate f and $\overline{f}$ by using the optical theorem and data from K^+p and K^-p total cross sections.

Solution: The regeneration parameter can be written as

$$
r_{\text{reg}} = \frac{1}{2}\frac{\overline{\phi}_{\text{reg}} - \phi_{\text{reg}}}{2\Delta m - i\Delta\Gamma}. \tag{417}
$$

The phase shifts in matter are given by

$$
\phi_{\text{reg}} = \frac{2\pi N\tau v}{k\sqrt{1-v^2}}f(0) \quad \text{and} \quad \overline{\phi}_{\text{reg}} = \frac{2\pi N\tau v}{k\sqrt{1-v^2}}\overline{f}(0). \tag{418}
$$

The optical theorem can be used to relate the forward elastic scattering amplitude to the imaginary part of the total cross section. We have

$$
\sigma_{\text{total}} = \frac{4\pi}{k}\text{Im } f(0) \rightarrow f(0) = \frac{k\sigma_{\text{total}}}{4\pi}, \tag{419}
$$

with a similar equation for $\overline{f}(0)$. From Ref. [2] we find for the kaon - proton scattering cross sections the values

$$
\begin{aligned}
K^+p: &\quad \sigma_{rmtotal} \approx 18 \text{ mb} \\
K^-p: &\quad \sigma_{rmtotal} \approx 55 \text{ mb}.
\end{aligned}
\tag{420}
$$

9.11 Isospin Analysis

Problem 11. Derive Eq. (270).

Solution: We can write down the following expressions for the $\pi\pi$ states,

$$
\begin{aligned}
|(2\pi)_{I=0}>^s &= \tfrac{1}{\sqrt{3}}\left(|\pi_1^+\pi_2^- >^s -|\pi_1^0\pi_2^0 >^s +|\pi_1^-\pi_2^+ >^s\right), \\
|(2\pi)_{I=2}>^s &= \tfrac{1}{\sqrt{6}}\left(|\pi_1^+\pi_2^- >^s +2|\pi_1^0\pi_2^0 >^s +|\pi_1^-\pi_2^+ >^s\right), \\
|\pi^+\pi^- >^s &= \tfrac{1}{\sqrt{2}}\left(|\pi_1^+\pi_2^- >^s +|\pi_1^-\pi_2^+ >^s\right).
\end{aligned}
\tag{421}
$$

where the superscript s indicates that we assume that there are no strong-interaction effects between the pions.

Thus we can write for the $\pi\pi$ states

$$
\begin{aligned}
|(2\pi)_{I=2}>^s &= \tfrac{1}{\sqrt{6}}\left(\sqrt{2}|\pi_1^+\pi_2^- >^s +2|\pi_1^0\pi_2^0 >^s\right) \\
|(2\pi)_{I=0}>^s &= \tfrac{1}{\sqrt{3}}\left(\sqrt{2}|\pi_1^+\pi_2^- >^s -|\pi_1^0\pi_2^0 >^s\right).
\end{aligned}
\tag{422}
$$

We multiply the latter equation by $\sqrt{2}$ and add up both expressions in Eq. (422). This gives

$$
< (2\pi)_{I=2}|^s + \sqrt{2} < (2\pi)_{I=0}|^s = \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2\sqrt{2}}{\sqrt{6}}\right) < \pi^+\pi^-|^s. \tag{423}
$$

The same procedure gives

$$
< (2\pi)_{I=2}|^s - < (2\pi)_{I=0}|^s = \frac{\sqrt{3}}{\sqrt{2}} < \pi^0\pi^0|^s. \tag{424}
$$

Next the strong-interaction phases for the pions is taken into account by multiplying each amplitude with $\exp(i\delta_I)$, where I denotes isospin $I = 0$ or 2. We find

$$
\begin{aligned}
< \pi^+\pi^-|^s &= \tfrac{1}{\sqrt{3}}\left(e^{i\delta_2} < (2\pi)_{I=2}|^s + \sqrt{2} < e^{i\delta_0}(2\pi)_{I=0}|^s\right) \\
< \pi^0\pi^0|^s &= \tfrac{1}{\sqrt{3}}\left(\sqrt{2}e^{i\delta_2} < (2\pi)_{I=2}|^s - < e^{i\delta_0}(2\pi)_{I=0}|^s\right).
\end{aligned}
\tag{425}
$$

The K^0 and $\overline{K}^0$ amplitudes are given by

$$
\begin{aligned}
&< (2\pi)_{I=0}|^s H_{\text{weak}}|K^0 >= A_0 \qquad < (2\pi)_{I=2}|^s H_{\text{weak}}|K^0 >= A_2 \\
&< (2\pi)_{I=0}|^s H_{\text{weak}}|\overline{K}^0 >= A_0^* \qquad < (2\pi)_{I=2}|^s H_{\text{weak}}|\overline{K}^0 >= A_2^*.
\end{aligned}
\tag{426}
$$

We write the eigenstates as

$$
|K_S^0 >= \frac{1}{\sqrt{2}}\left((1+\epsilon)|K^0 > +(1-\epsilon)|\overline{K}^0 >\right), \quad |K_L^0 >= \frac{1}{\sqrt{2}}\left((1+\epsilon)|K^0 > +(1-\epsilon)|\overline{K}^0 >\right). \tag{427}
$$

It is now possible to determine the $K^0 \to \pi\pi$ amplitudes. Here we use the relations $A + A^* = 2\text{Re } A$ and $A - A^* = 2i\text{Im } A$. We find

$$
\begin{aligned}
< \pi^+\pi^-|H_{\text{weak}}|K_L^0 > &= \tfrac{1}{\sqrt{6}}\left(e^{i\delta_2}\left[(1+\epsilon)A_2 - (1-\epsilon)A_2^*\right] + \sqrt{2}e^{i\delta_0}\left[(1+\epsilon)A_0 - (1-\epsilon)A_0^*\right]\right) \\
&= \tfrac{1}{\sqrt{6}}e^{i\delta_2}\left[(A_2 - A_2^*) + \epsilon(A_2 + A_2^*)\right] + \tfrac{\sqrt{2}}{\sqrt{6}}e^{i\delta_0}\left[(A_0 - A_0^*) + \epsilon(A_0 + A_0^*)\right] \\
&= \tfrac{2}{\sqrt{6}}e^{i\delta_2}\left[i\text{Im } A_2 + \epsilon\text{Re } A_2\right] + \tfrac{2}{\sqrt{3}}e^{i\delta_0}\left[i\text{Im } A_0 + \epsilon\text{Re } A_0\right].
\end{aligned}
\tag{428}
$$

The other expressions given in Eq. (**??**) are found in a similar manner.

References

[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[2] C. Caso *et al.*, *Review of Particle Physics*, The European Physics Journal **C3** (1998) 1. See also http://pdg.lbl.gov.

[3] T.T. Wu and C.N. Yang, Phys. Rev. Lett. **13**, 380 (1964).

[4] F. Muller *et al.*, Phys. Rev. Lett. **4**, 418 (1960); Phys. Rev. **124**, 1223 (19xx).

[5] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2**, 1285 (1970).

[6] Christenson, Cronin, Fitch and Turlay, Phys. Rev. Letters **13**, 138 (1964).

[7] H. Weyl, *The Theory of Groups and Quantum Mechanics*, Dover New York, 1950.

[8] F. Reines and C.L. Cowan, Physical Review **113** (1959) 273.

[9] L.B. Okun, Physics Letters **B239** (1990).

[10] X. Shi and D.N. Schramm, Physics Letters **B282** (1992) 305.

[11] Bellgardt *et al.*, Nuclear Physics **B299** (1988).

[12] Badetscher *et al.*, Nuclear Physics **A377** (1982) 106.

[13] T.D. Lee and C.N. Yang, Physical Review **104** (1956) 257.

[14] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hopes and R.P. Hudson, Physical Review **105** (1957) 1413.

[15] M. Goldhaber, L. Grodzins and A.W. Sunyar, Physical Review **109** (1958) 1015.

[16] S. Kystrin *et al.*, Physical Review Letters **58** (1987) 1616.

[17] B.M.K. Nefkens, G.A. Miller and I. Slaus, Comments on Nuclear and Particle Physics **20** (1992) 221.

[18] P.A.M. Dirac, Proc. Roy. Soc. (London) **A126** (1930) 360.

[19] C.D. Anderson, Physical Review **43** (1933) 491.

[20] R.P. Feynman, *Quantum Electrodynamics*, W.A. Benjamin, New York, 1961.

[21] A. Firestone *et al.*, Physical Review Letters **26** (1971) 410.

[22] A. Bohr and B. Mottelson, Nuclear Structure, Vol. I, Benjamin Inc., New York, 1969.

[23] I. Passchier *et al.*, Physical Review Letters **xxx** (2000) xxxx.

[24] W. Mampe, *Search for the Electric Dipole Moment of the Neutron*, Fundamnetal Symmetries in Nuclei and Particles, World Scientific, Singapore 1989.

[25] N.F. Ramsey, Ann. Rev. Nucl. Part. Sci. **32**, 211 (1982).

[26] P. Herzeg, Fundamental Symmetries in Nuclei and Particles, World Scientific Singapore 1989.

[27] M. Allet *et al.*, Physical Review Letters **68** (1992) 572.

[28] S. Gjesdal *et al.*, Phys. Lett. **B52**, 113 (1974).

[29] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[30] B. Schwingenheuer *et al.*, Phys. Rev. Lett. **74**, 4376 (1995).

[31] Y. Nir, H.R. Quinn, Annu. Rev. Nucl. Part. Sci. **42** (1992), 211.

[32] Y. Nir, CP Violation, SLAC-PUB-5874, WIS-92/67/Sep-PH, September 1992.

[33] D.C. Cheng, G.K. O'Neill, Elementary Particle Physics; an introduction, Addison-Wesley Publishing Company (1976), ISBN 0-201-05463-9.

[34] G. Nardulli, Nuclear Instruments and Methods in Physics Research **333** (1993), 3.

[35] F.C. Erne, Lectures on CP violation, Utrecht University (1998) not published.

[36] LHC-B Letter of intent, CERN/LHCC 95-5, LHCC/ I8, August 1995.

[37] LHCb Technical Proposal, CERN/LHCC 98-4, LHCC/P4, 20 February 1998.

[38] J.L. Rosner, Present and future aspects of CP violation, Lectures presented at the VIII J. A. Swieca Summer School, Rio de Janeiro (1995).

[39] F.J. Gilman, Y. Nir, Annu. Rev. Nucl. Part. Sci. **40** (1990), 213.

[40] E. Leader, E. Predazzi, An Introduction to gauge theories and modern particle physics; volume II, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (199).

[41] "CP-violation", G.C. Branco, L. Lavoura, and J. P. Silva, *Clarendon Press, Oxford* (1999).

[42] "Quantum field theory", L.H. Ryder, *Cambridge University Press* (1985).

[43] "Quantum field theory", C. Itzykson, J.-N. Zuber, *McGraw-Hill, New York* (1985).

[44] D.E. Groom *et al.*, *Eur. Phys. J.* **C15**, 1 (2000).

[45] W. Ochs, Münich Preprint MPI-Ph/Ph91-35. Newsletter, Nr. 3, 25 (1991).

[46] L. Wolfenstein, *Phys. Rev. lett.* **13**, 562 (1964).

[47] M. Bargiotti *et al.*, *Nuov. Cim.* **23 N3**,1 (2000).