ORIGINAL ARTICLE

Pseudo Forward Ray-Tracing: A New Method for Surface Validation in Cornea Topography

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ABSTRACT

Purpose. A pseudo forward ray-tracing (PFRT) algorithm is developed to evaluate surface reconstruction in corneal topography. The method can be applied to topographers where one-to-one correspondence between mire and image points can be established.

Methods. The PFRT algorithm was applied on a corneal topographer designed and constructed at the VU University Medical Center, Amsterdam, The Netherlands. Performance of the algorithm was evaluated using artificial test surfaces and two sample eyes. The residual output of the PFRT algorithm is displayed as pixel displacements of actual feature points on the corneal image. Displacement of 1 pixel indicates submicrometer corneal height accuracy.

Results. PFRT residual increases with complexity of the measured surface. Using Zernike radial order 6, the mean residual for the artificial surfaces is subpixel. The mean residual for the regular cornea and the irregular cornea is 1.16 and 2.94 respectively. To some extent, increasing the Zernike radial order improves the accuracy. The improvement from order 6 to 20 is factor 2.3 for the irregular cornea. Using the residuals to further improve the accuracy brought local changes as high as 0.28 D in some areas of the reconstructed corneal power map.

Conclusion. PFRT can be used to evaluate how close a reconstructed corneal surface is to the actual one. The residue information obtained from this algorithm can be displayed simultaneously with the corneal image. This provides accurate information about the corneal shape that is useful for application in laser refractive surgery. (Optom Vis Sci 2007;84:E915–E923)

Key Words: corneal topography, ray-tracing algorithm, on-screen corneal surface diagnostics, VU topographer

he anterior corneal surface contributes to about 70% of the refractive power of the eye. Thus, measurement of the corneal shape is becoming a common procedure in ophthalmic practice. As early as the 19th century, the French Ophthalmologist Ferdinand Cuignet studied images reflected off the anterior corneal surface, calling the technique keratoscopie (keratoscopy).¹ This technique allowed him to study the corneal image and interpreted image distortions as an indication of abnormal cornea topography. In keratoscopy there are two fundamental elements: (1) the use of a reflection source and (2) the derivation of the corneal shape from the reflection. The source is used to illuminate the cornea and information from the reflection is used to reconstruct the corneal shape. Reflection comes in two forms: diffuse and specular. Methods that involve diffuse reflection include moire fringes, rasterstereography, and Fourier Transform Profilometry.² In these methods, fluores-

cein solvent is used. This has two disadvantages: first, adding solvents to the eye is inconvenient for patients and second, the reconstructed corneal surface does not represent the cornea in its natural condition. In this regard, methods that use specular reflection are better alternatives.

One of the most commonly used specular reflection source is the Placido ring pattern introduced by Antonio Placido in 1880.³ His keratoscopy target consisted of a disk with alternating black and white rings. The disk had a hole in its center through which the observer could inspect the subject's cornea. This target is still in use today for qualitative inspection. But nowadays, the reflected image is captured by a camera and computer algorithms process this information to reconstruct the corneal shape. However, this procedure is not without problems. When reconstructing the corneal surface, numerical algorithms used in commercially available Placido disk topographers neglect skew ray reflections.^{4–6} This



FIGURE 1.

Schematic diagram that shows the effect of skew ray ambiguity in determining source points in Placido disk topography. A color version of this figure is available online at www.optvissci.com.

leads to inaccuracy in reconstructing corneal surfaces that are not rotationally symmetric.^{7,8}

In Placido disk topography, the corneal shape is reconstructed using reflection principles that relate an image point A (captured from a video camera or charge-coupled device, CCD) to its mire point A1 as shown in Fig. 1. It is assumed that the reflection occurs in a meridian plane. However, this assumption is valid only if the corneal surface is rotation-symmetric. For nonrotation-symmetric surfaces, skew ray reflections can occur. For instance, rays coming from points A2, A3, and A4 in Fig. 1 are skew rays. This means that in Placido-based topography, there is ambiguity in the relationship between mire points and image points especially because the cornea is not a rotationally-symmetric surface.

Vos et al. introduced a different topographer (the VU topographer) to deal with the skew ray problem.⁹ Instead of the ring stimulus, a color-coded pattern is used (Fig. 2). The stimulus pattern is designed so that the reflection pattern will appear as square objects if the corneal surface is spherical in form. In this system, the crossing points in the square pattern are chosen as starting information for the surface reconstruction algorithm. In principle, because the corresponding crossing points in the stimulator pattern



FIGURE 2.

Schematic diagram that shows one-to-one correspondence between image crossings and stimulator crossings of the VU topographer. There is no skew ray ambiguity in this system. are known due to the color coding, skew ray ambiguity is eliminated. Recently, Sicam and van der Heijde¹⁰ have validated the accurate performance of this topographer in reconstructing the corneal surface, the nonrotationally-symmetric features in particular. Thus, the process of improving the quality and usefulness of corneal surface reconstruction has been continued going on for almost 200 years. However, up until now little attention has been given to developing surface evaluation techniques that can be displayed on screen simultaneously with the corneal image. Advances in technology have now made this possible and the design of the VU topographer is particularly suited for this. Exploiting this feature could potentially be useful for application in laser refractive surgery. A technique to evaluate the correctness of surface reconstruction algorithms was developed by Halstead et al. and at that time this was applied to a Placido-based topography system.^{11,12} They evaluated a reconstructed corneal surface in two ways. First, the surface normals of the reconstructed surface were calculated and compared with the angle bisector between incident and reflected rays for each pair of source point and image point. For the correct surface these two vectors should be identical. Second, they mapped feature points from the image onto the source ring pattern and compared this with the corresponding feature points on the source ring pattern. A minimum error here corresponds to the correct surface. Because this technique requires feature points, this can only be applied to topographers where one-to-one correspondence between mire points and image points can be established. For Placido-based topographers, this can be implemented when the stimulus pattern is modified as a checkerboard pattern.⁸ However, there are two important aspects not covered in the latter method. First, the method is not validated using real eye data and second, there is no display of residual information on the captured image itself. The interactive visualization of the cornea is limited only to displaying residual of the surface normals. Furthermore, this technique has never been used by commercial instruments.

In this study, a pseudo forward ray-tracing (PFRT) algorithm is developed to evaluate surface reconstruction in cornea topography. Given the stimulator points and the reconstructed corneal surface, "forward" ray-tracing to the corresponding image points is achieved using a special procedure. The method was applied using the VU topographer, which has the advantage of providing oneto-one correspondence between stimulator points and image points. Using various test surfaces and real eye data as input, the procedure yielded detailed residual information for every feature point. This information can then be used to provide a quantitative statement on how close the reconstructed surface is to the actual one. Furthermore, it is possible to use the output of this residual calculation as input for an optimization procedure to obtain an even better corneal surface reconstruction. A special feature of this procedure is the availability of residue information that can be displayed on screen along with the captured image of the cornea. The next three sections will elucidate this procedure.

METHODS The Algorithm

The stimulator pattern of the VU topographer is designed so that the reflection pattern will appear as square objects if the corneal surface is spherical in form. For a typical corneal reflection, each image crossing point (depicted as a red dot in Fig. 2) will be surrounded by four quadrilateral objects: two of them are black and the other two are colored. The two black quadrilaterals are aligned opposite to each other in the diagonal direction and the other two colored quadrilaterals are aligned in the complementary diagonal direction. Because of this property, suitable filter functions can be used in the image processing to determine the location of the crossing points.¹³ Each colored quadrilateral is either cyan or yellow, which makes it possible to implement a pseudo random binary array encoding that will give a global description of the color pattern.¹³⁻¹⁵ This global description enables the unique assignment of each crossing point captured by the CCD camera to its corresponding source point. In this manner, one-to-one correspondence between stimulator source points and image points is established. There are topographers available in the market today that can also achieve this one-to-one correspondence relationship. One such topographer uses a black and white dart-board mire pattern.¹⁶ Because the mire pattern is black and white, a three camera system is used to achieve the one-to-one correspondence relationship. However, a multicamera system is more complex than a one-camera system. It could be a way to determine corneal height more accurately but could also be a source of error propagation. The monocamera system of the VU topographer is simpler and less prone to numerical errors. Nevertheless, corneal topographers that provide one-to-one correspondence between stimulator/mire source points and image points allow for development of a corneal surface reconstruction algorithm that does not neglect skew ray reflections.

Surface Reconstruction

Some parameters used in the surface reconstruction algorithm of the VU topographer need to be predetermined by calibration; these are: (1) locations of crossings on stimulator, (2) location of corneal apex, (3) magnification factor relating corneal Purkinje image to CCD image. Because the focal plane of the lens-camera system is fixed, the location of the corneal apex is dependent on the corneal curvature. The relationship between corneal apex and corneal curvature was determined by calibration using several polymethyl methacrylate (PMMA) spheres of varying radii. Once this relationship is available, an initial value for the mean curvature of the corneal surface for every corneal measurement provides the needed information to calculate the location of the corneal apex. The details of the surface reconstruction method are well documented in literature.¹⁷ Nevertheless, the algorithm will be briefly described here.

The detected crossings (DCs) on the CCD image are traced to the points on the stimulator via backward ray-tracing. In this procedure, a ray is traced back from a point on the CCD through the nodal point of the lens to the corneal surface (Fig. 3). The intersection point on the corneal surface can be calculated because the surface is represented by an analytical function. In this case, the corneal height is represented by a function composed of Zernike polynomials:

$$Z_{c} = \sum_{i=1}^{K} C_{n}^{m} Z_{n}^{m} (x_{c}, y_{c})$$
(1)



FIGURE 3.

Schematic diagram of ray-tracing procedure showing relevant points. SC, stimulator crossing; DC, detected crossing.

where the Z_n^m 's are Zernike polynomials and the C_n^m 's are the corresponding Zernike coefficients. To be consistent with the orthogonal property of Zernike polynomials, the dimension units of every length parameter is scaled accordingly so that the corneal zone covers a unit circle. Each intersection point on the cornea is determined by the following three equations¹⁷:

$$Z_{c} = \sum_{i=1}^{K} C_{n}^{m} \left[Z_{n}^{m} \left(-u, -v \right) - Z_{n}^{m} \left(0, 0 \right) \right] \cdot \left[1 + \frac{u^{2} + v^{2}}{OA \cdot R} \right]$$
(2)

$$x_c = -u - \frac{u}{OA} z_c \tag{3}$$

$$y_c = -v - \frac{v}{OA} y_c \tag{4}$$

here, R is the radius of curvature of the surface that is determined from the Purkinje image, OA is the distance from the nodal point to the apex.

Because the surface is unknown, the initial Zernike coefficients are set equal to zero, which describes a flat surface. Once the intersection point on the surface is determined, the vector $\overrightarrow{AB}_{i \leftrightarrow r}$ that is an angle bisector between incident (*i*) and reflected (*r*) ray is calculated (Fig. 4). This procedure is then repeated for every DC. Each calculated angle bisector should have a direction equal to the normal vector \overrightarrow{n} on the surface. Initially this is not the case and thus, using the criterion expressed in Eq. 5, a least-square fitting routine is employed to determine a better model for the surface





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FIGURE 5.

Schematic diagram showing the pseudo forward ray-tracing method. Pixels in the neighborhood of each crossing point in the CCD plane are traced back to the stimulator source. The pixel location that would produce the closest projection to the stimulator source is the location of the residual crossing (RC).

until the error e is minimized and the optimal corneal shape is found.

$$\overrightarrow{n} \times \overrightarrow{AB}_{i \Leftrightarrow r} = \overrightarrow{e}$$
(5)

PFRT

To verify the correctness of the surface reconstruction procedure, a PFRT routine is employed. This routine traces a ray from each stimulator crossing (SC) and reflects these rays via the derived corneal shape to the CCD plane. The intersection points on the CCD plane are defined as the locations of the residual crossings (RC). The distance of the RC compared with the initially DC is called the residual of the derived corneal shape compared with the true shape.

Forward ray-tracing from a stimulator point to the corneal surface is impossible because there are infinitely many rays emanating from this point. Thus, the algorithm for the PFRT actually employs backward ray-tracing procedures. To this end, a region around each DC on the CCD plane is considered (e.g., 11×11 pixel square region). Each pixel in this region is then traced back to the stimulator. The pixel with the closest projection to the SC is considered the pixel location of the RC. A visualization of this procedure is shown in Fig. 5. In this illustration, 5 pixel points on the CCD are shown: the DC (red) and 4 pixel points (blue, yellow, and two black) around the neighborhood of the DC. When traced back, the yellow line is found to have the closest projection on the SC. Thus, the yellow pixel is considered as the residual crossing.

Performance Testing

The PFRT method is applied to measurements of five different surfaces: (1) a PMMA^a spherical surface with 6.99 mm radius of curvature^b, (2) a PMMA^a spherical surface with 9.00 mm radius of curvature^b, (3) a PMMA^a toric surface with maximum axial radius of curvature of 8.02 mm and minimum axial radius of curvature of 7.05 mm^b (4) a human cornea, from the left eye of a 38-year-old man, with no known abnormality, and (5) a human cornea, from the left eye of a 61-year-old man, with subepithelial infiltrate.

TABLE 1.

Residue information and their corresponding units

Residue name	Unit
Residual	Pixel ^a
Angle residue	Arc minute
Stimulator residue	Millimeter

^aLength of 98.9 pixels on the CCD corresponds to 1.0 mm length in the Purkinje image plane. One pixel is equal to 0.01 mm spatial resolution at the corneal apex plane. Perpendicular to this plane, pixel accuracy corresponds to submicrometer corneal height accuracy.

Written consent was obtained from both subjects after they had been informed about the nature of the study, in accordance with the tenets of the Helsinki declaration.

PFRT Method and Residue Analysis. The PFRT method produces residual information in pixel^c units of the reconstructed surface on the CCD image itself. This residual is compared with two other residue forms: (1) angle residue—the angle between the angle bisectors and the normal vectors on the corneal surface (Fig. 4), and (2) the stimulator residue. The stimulator residue is determined by back ray-tracing from the DC to the corneal surface and then to the stimulator. The distance between the intersection point on the stimulator and the SC is designated as the stimulator residue. The three forms of residue information and their corresponding units are summarized in Table 1.

To produce an accurate description of the corneal surface, two things must happen. First, the location of the image crossings must be determined accurately. Second, the numerical reconstruction of the corneal surface must be consistent with the DCs. The output of the PFRT is an indicator whether the second procedure was implemented well. Thus, accuracy of the first procedure (crossing detection) is a prerequisite condition for PFRT to work. Nevertheless, previous results validate subpixel accuracy in detecting the location of image crossings.¹⁵ Because pixel accuracy corresponds to submicrometer corneal height accuracy, subpixel accuracy in the PFRT algorithm guarantees submicrometer accuracy in indicating the corneal height.

Use of PFRT Residue to Improve Surface Reconstruction. The residual information obtained from the PFRT method is used to improve the accuracy of the reconstructed surface. This is done by implementing a supplementary least-squares fitting routine using a modified version of Eq. 5 as criterion, this time using a weight function ($e^{-residual}$) multiplied to Eq. 5 to suppress the contribution of large residuals (mainly outliers) in the fitting procedure. Strictly speaking, the argument in the exponential weight function should be the angle residue because it is this residue which is directly related to the fitting procedure. However, as will be shown in the results, this argument can be replaced by the residual parameter because it is highly correlated with the angle residue.

^aPolymethyl methacrylate.

^bCurvature radii were obtained from Haag-Streit Ophthalmometer (HS, Bern, Switzerland) measurements.

^cLength of 98.9 pixels on the CCD corresponds to 1.0 mm length in the Purkinje image plane. One pixel is equal to 0.01 mm spatial resolution at the corneal apex plane. Perpendicular to this plane, pixel accuracy corresponds to submicrometer corneal height accuracy.

RESULTS AND DISCUSSION

Fig. 6a, d, and g show photos captured by the VU topographer for three surfaces: a spherical surface of 9.00 mm radius, a regular cornea, and an irregular cornea. The DCs are also shown in the photos. A red circle outline is provided to show the 7.00 mm corneal zone where the surface reconstruction is evaluated. Because the subject fixates to the central point of the stimulator, the VU topographer measures the corneal shape along the line of sight.¹⁰ Fig. 6d is an example where there was no proper alignment to the line of sight when the photos were taken. Misalignment happens because of operator error and/or if the subject did not fixate properly. This makes the pupil not concentric with the red outline circle. Fig. 6g shows a better alignment when compared with Fig. 6d.

The corresponding reconstructed curvature maps (instantaneous radius of curvature)¹⁸ and height maps are also displayed. The height map shows the third and higher order Zernike shape feature of the surfaces. The color bar for the curvature map has the same color convention for the three surfaces, only the scaling was adjusted accordingly. The local power P expressed in diopters can be derived from the instantaneous radius of curvature r:

$$P = (n_{eff} - 1)/r \tag{6}$$

where $n_{eff} = 1.3375$ is the effective corneal refractive index. For the height map, a positive/negative value means a direction outward/inward the eye.

Robustness of the Surface Reconstruction Algorithm

The curvature map for the 9.00 mm spherical surface is relatively flat. The displayed diopter value of 37.5 demonstrates that the VU topographer was able to reconstruct the correct shape for this surface (refer to Eq. 6). There is a presence of a horizontal band (0.5 D deviation) in the superior part of the surface. This deviation in diopter is equivalent to 120 μ m instantaneous radius of curvature deviation. It can be shown that this amount is equivalent to 40 μ m axial radius of curvature deviation, which corresponds to cor-



FIGURE 6.

Photos obtained from the VU topographer for three surfaces: (a) 9.00 mm radius spherical surface, (d) regular cornea, and (g) irregular cornea. The detected crossings are also shown in the photo. The second and the third columns show the corresponding curvature and height map. (a), (d), (g): Red circle outline denotes 7.0 mm corneal zone. (b), (e), (h): Curvature map (diopter); colormap coding is the same for maps (b), (e), and (h), map (h) has an extended colormap range. (c), (f), (i): Third- and higher-order Zernike height map (mm); maps (c) and (f) have micrometer scale.

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neal height changes not exceeding 1 µm^d.¹⁸ This indicates how sensitive the instantaneous radius of curvature is with respect to small changes in corneal height. The largest nonrotation-symmetric Zernike coefficient for this surface is vertical coma (Z_3^{-1}) which is 0.2 µm. The corresponding height map shows that the nonrotationsymmetric higher-order shape features for this surface are negligible. This suggests that the accuracy in the manufacturing^e of this surface is good. It is notable that artifact formation can be observed from the curvature map and the height map. This happens because the actual surface is a continuous surface whereas the Zernike fitting requires that the surface be reconstructed within a unit circle. In this case, an artifact at the edges is produced which deviates from the actual shape. Increasing the corneal zone diameter for the surface reconstruction unravels the actual shape on these areas but produces new edge artifacts at a further peripheral distance from the corneal apex.

The curvature map for the regular cornea again reveals a map with no abrupt changes in diopter values. However, higher-order shape features are now present. This is shown more clearly in the height map, which displays a trefoil shape with 4.0 μ m amplitude. Although this feature is small relative to the overall shape of the cornea, the VU topographer was still able to capture this.

Serious deformations in the corneal surface can make it difficult to detect some of the crossings; this is especially the case for the irregular cornea. However, the crossings that are detected are still sufficient to reconstruct a corneal surface that closely resembles the actual surface. In this case, the corneal shape was reconstructed over a 7.00 mm corneal zone. The reconstructed curvature map and height map for this cornea show the irregularity especially on the edges of the area chosen for surface reconstruction. The central zone is reconstructed properly because a sufficient number of crossings are present in this region. The irregular features on the periphery could also be reconstructed. An interesting feature in this peripheral reconstruction is the blue region in the upper middle part of the curvature map which coincides with a depression (blue color) in the height map. This means that this region has a concave shape that causes a spreading in the reflection of the color pattern as seen in the Fig. 6g. Although there are no crossings detected in the central part of this region, the crossings detected on the edges of this region contain information such that the qualitative shape is still reconstructed. Further improvements can be done in the future so that the actual amplitude of this depression will be reconstructed accurately.

Accuracy of the Surface Reconstruction Algorithm

Fig. 7 shows the mean residual vs. Zernike radial order plot for different surfaces. The overall residual increases with complexity of the measured surface. The residuals were smallest for the artificial surfaces: the spherical surfaces (mean residual ≤ 0.70 pixel) and the toric surface (mean residual ≤ 0.81 pixel). The regular cornea has slightly higher residual compared with the artificial surfaces (mean residual ≤ 1.16 pixel). This indicates the effect of other higher order shape features. However, because these shape features are not as dominant when compared with the spherical and toric shape





Mean residual vs. Zernike radial order plot for different surfaces. The region below the gray dashed line indicates subpixel accuracy.

features, the effect on the residual is also relatively small. Whereas, for the irregular cornea, the effect of the higher-order shape features is larger, thus producing an increase in the mean value of the residual (at Zernike radial order 6 mean residual = 2.94 pixel).

The accuracy of the surface reconstruction improves when the Zernike radial order used to model the corneal surface is increased as demonstrated by the corresponding decrease in the magnitude of the residual (Fig. 7). The addition of more Zernike components enables better fitting of the local surface features. For the artificial surfaces, a lower radial order for the Zernike expansion (order 6) is sufficient to reconstruct the surface with subpixel accuracy. For the regular cornea, subpixel accuracy was observed only for Zernike radial order of 10 or higher. For the irregular cornea, order 20 is still not sufficient to produce subpixel accuracy for the surface reconstruction. Nevertheless, at this order the accuracy approaches pixel resolution, which is reasonable enough for clinical practice. This also indicates that to some extent the use of Zernike polynomials will produce accurate corneal surface reconstruction as long as a sufficient radial order is used. This finding is consistent with results from previous studies where it was demonstrated that corneal surface fitting tends to improve when more Zernike terms are added.19-21

Unbiased Surface Reconstruction Algorithm

Fig. 8 shows plots of residue on the CCD (residual) vs. three other parameters: angle residue, stimulator residue, and distance from the CCD center, obtained from the 9.00 mm radius spherical surface. The distance from center is expressed in mm, normalized accordingly with respect to the corneal apex plane. These results show that the residual is not correlated to the stimulator residue or the position of the crossings with respect to the optical center of the lens-camera system. This means that there is no bias in the accuracy of the surface reconstruction. The accuracy at the center of the cornea is practically comparable to that of the corneal periphery. There are two regions where outliers in the CCD residue vs. distance from center plot are found: the region between 1.00 and 2.00 mm zone and the peripheral edge region. The region between 1.00 and 2.00 mm zone corresponds to the neighborhood of the boundary between two stimulator sections. The first stimulator section is

^dEquations 5 to 7 of ref. 18.

^eManufactured by SUMIPRO BV, Almelo, The Netherlands.



FIGURE 8.

Plots of residual vs. three other parameters: angle residue, stimulator residue, and distance from center. Outliers on the third plot are enclosed by ovals. A color version of this figure is available online at www.optvissci.com.

a flat disk that is parallel to the CCD plane and the second section is that part of the cylindrical surface that is perpendicular to the CCD plane (the stimulator is shown in Fig. 3 as a bold dashed outline). The presence of the outliers in the vicinity of the boundary of these two sections suggests that the abrupt discontinuity between the two sections creates difficulties in the data processing. Similarly, the outliers at the peripheral edge region correspond to another form of discontinuity: a transition from an area with crossings to an area without.

Contrary to the relationship of the residual with the stimulator residue and the distance from the CCD center, the residual is highly correlated with the angle residue ($R^2 = 0.86$). This property can be put to good use. Considering that the angle residue is the minimizing criterion for the least-squares fitting procedure, this would mean that the residual can be used to further improve the accuracy of the surface reconstruction. The residual can substitute for the role of the angle residue as the argument in the exponential weight function used in the supplementary least-squares fitting routine to further improve the surface reconstruction.



FIGURE 9. Height difference map.

Indeed, when the routine is applied, we can see a reduction of the residual (0.08 \pm 0.03 pixel) in the surface reconstruction of the irregular cornea as shown in Fig. 7. The corresponding height difference map is shown in Fig. 9. The data for this map was obtained from measurements of the irregular cornea evaluated at Zernike radial order 14. The improvement in the accuracy brought about corneal height changes in local areas of the reconstructed surface by as high as 6.0 μ m. At the edge of the 7 mm corneal zone, this corresponds to local power changes of 0.28 D, which is greater than the generally accepted tolerance of 0.25 D in standard clinical practice. The residual is displayed with the corneal image as shown in Fig. 10a. Black crosses represent the DCs, red circles represent the RC after the first fitting procedure is applied, whereas the blue exes represent the RC after the supplementary least-squares fitting routine is applied. In general, the black crosses, red circles, and blue exes overlap. The blowup of the section inside the blue square in Fig. 10a as shown in Fig. 10b is an example of an area where the residual is minimal. In five crossing locations, the residual is subpixel and in the other four locations the initial residual (red circle) is of the order of 1 pixel. In one of these four locations (lower right corner of Fig. 10b), the effect of applying weights in the fitting procedure is to bring back the initial residual to a much more accurate location as seen by the overlapping of the black cross and the blue ex. However, there are certain regions with larger CCD residue. An example of this is a blowup of the section inside the red square in Fig. 10a as shown in Fig. 10b. A closer inspection of this area reveals that, in general, there is a migration of the blue exes moving from the position of the red circles closer to the position of the black crosses. This demonstrates that the PFRT produces residues that can be used in further optimizing the accuracy of the corneal surface reconstruction. However, the improvement in the accuracy of the surface reconstruction when applying weights in the least-squares fitting routine is less than the improvement when increasing the Zernike radial order. Nevertheless, applying weights to the least-squares fitting routine has the advantage that it requires less computational time when compared with the option of increasing the Zernike radial order.

It will be interesting to see the method introduced in this study applied to different types of corneal deformation, e.g., keratoconus, in addition to the one presented here. Work on this is still in progress. Another potential application is the availability of zooming to local areas of the cornea such as the example given in Fig. 10b, c. Because residual information is given, this will indicate accuracy information on the local level, which can be useful in







(b)



FIGURE 10.

(a) Schematic diagram showing crossing migration as a result of applying weights in the fitting procedure. Black crosses, detected crossings DC; red circles, initial residual crossings; blue exes, refined residual crossings. (b) Blowup of region indicated by the blue square outline. (c) Blowup of region indicated by the red square outline.

customized laser refractive surgery procedures. Recently, it was shown that topography-guided customized laser-assisted subepithelial keratectomy was able to provide improvements for both lower and higher aberrations of the eye.²² Although, a Placidobased topographer was used in the reported study, it can be anticipated that the use of a method which is skew ray error free could further improve results.

CONCLUSIONS

A new method, called PFRT, for evaluating the accuracy of the reconstructed corneal topography was demonstrated in this study. The method was applied using the VU topographer, an apparatus that uses a color coded pattern as a source for reflection on the cornea. Application of the algorithm using test surfaces and eye samples shows that to some extent the accuracy of surface reconstruction becomes better when the Zernike radial order is increased. Furthermore, it was demonstrated that the residue information obtained from this algorithm can be used to provide a weight function that can be applied in a new least-squares fitting routine. This produces a better corneal surface reconstruction with corresponding corneal local power difference of up to 0.28 D compared with the corneal power map obtained without using weights. This improvement in accuracy is significant when considering consequences in the quality of vision. Of further benefit is the special feature of this method that the residual information can be displayed simultaneously on screen along with the captured image of the cornea. These properties can facilitate the use of the PFRT method to provide accurate corneal shape information, which is important in customized laser refractive surgery.

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