A New Instrument to Measure the Shape of the Cornea Based on Pseudorandom Color Coding

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Abstract—We have developed a prototype instrument providing a new way to measure the shape of the cornea. Our approach exploits properties of pseudorandom binary arrays (PRBA's). Encoded in a colored pattern (stimulus) that is mirrored to the eye, a large PRBA allows for unique identification of unambiguously characterized positions both in stimulus and reflected image. This is used to come to an integral reconstruction of the cornea. It is demonstrated that the new technique contributes to a very robust measurement method.

Index Terms—Biomedical signal analysis, image processing, optical measurements, parameter estimation, pseudonoise processes.

I. INTRODUCTION

THE anterior surface (cornea) of the human eye is responsible for 84% of the refraction of light and is thus of eminent importance for good sight. Ophthalmologists have recognized the role of the cornea in the refraction process. A modern technique to improve a patient's sight is by adjusting the corneal surface. The rapid development of this keratorefractive surgery induced the need for accurate methods to evaluate the shape of the cornea.

To measure the shape of the cornea in the standard way, a cylindrically symmetrical ringpattern (stimulus) is mirrored to the eye. Analysis of the recorded reflection (Purkinje image) allows for reconstruction of the corneal surface. Available instruments employing this technique are known not to work ideally [1]. In pathological cases with severely irregular or badly reflecting corneas, automatic recognition of the ringpattern often fails as the mires become extremely distorted or vague. Moreover, as it is impossible to identify positions uniquely along each ring it has to be assumed that reflection occurs in meridional planes to come to a description of the corneal shape per meridian only.

We have developed a prototype-stimulus that exploits the virtues of pseudorandom binary arrays (PRBA's), which, when encoded in a colored stimulus pattern, contribute to a very

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 $\begin{array}{c}
0 & 1 \\
0 & 0 \\
1 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\end{array}$

Fig. 1. Example of a PRBA: each subarray of 2×2 or larger contains a unique bitpattern.

robust measurement technique with uniquely characterized points. In our approach, it is not necessary to make the aforementioned meridional assumption and an integral surface reconstruction of the cornea is possible.

Details of our design are discussed in Section II. In Section III, some experimental aspects of the stimulus are presented and the accuracy and robustness of our approach is studied. Section IV describes the reconstruction of the corneal surface.

II. METHOD

The essence of our novel technique lies in the use of uniquely characterized positions both on the stimulus and in the recorded reflection. In this way a one-to-one correspondence between points on the stimulus and on the registering device is established which can be used to reconstruct the corneal shape. Implementation of this design is facilitated by properties of PRBA's.

PRBA's $(A[i, j]|i < n, j < m, A[i, j] \in \{0, 1\})$ are the two-dimensional (2–D) analogy of pseudorandom binary sequences $(S[i]|i < n, S[i] \in \{0, 1\})$. Primitive polynomials with coefficients 0 or 1 provide the feedback path of shift registers that are used to generate these maximum length sequences. PRBA's exist in various sizes and have many useful distinguishing features. The key virtue we use is the "window"-property: each bitpattern seen through a prescribed window of k-by-l (or larger) bits sliding over the array is unique. Conversely, each k-by-l (or larger) subpart of the PRBA uniquely defines a position in the PRBA. Fig. 1 shows an example of a PRBA (with n = 3, m = 5, k = 2, l = 2). For a more formal treatment of PRBA's we refer to [2].

We have opted to encode a large PRBA (n = 65, m = 63, k = 6, l = 2) through a representation of the 0's and the 1's by two bright colors. Currently cyan and yellow are used to encode the bits as experiments have shown that these colors provide optimal recognition. This was investigated by mirroring patches of color to a blue and a brown eye and monitoring the response of the camera [4]. A third color (black in our approach) is necessary to provide distinction between adjacent bits of the same value. The assembled pattern presents



Fig. 2. Example of our encoding: two colors are used to represent the bits of the PRBA of Fig. 1 (the 0's and 1's are added for clarity).

a checkerboard pattern when mirrored to a sphere of radius 8 mm (average shape of the human eye). Fig. 2 demonstrates our encoding of the PRBA of Fig. 1. Each position where two black squares meet ("crossing") can be uniquely labeled with the positional indices of the adjacent bits in the encoded PRBA.

III. PATTERN RECOGNITION

A. Implementation

In uniquely identifying the crossings, the first step is to localize them in the input image. This is done with a matched filter approach [3]. Locations are accepted as crossings if the response to the matched filter exceeds a preset threshold value. By adjusting the threshold falsepositive responses are eliminated. Secondly, the colors of the adjacent colored squares must be determined. To do this, the RGB values of the pixels of the colored squares are aggregated, leaving out the pixels of the outermost two layers to diminish the effect of blurring. The RGB-values are clustered with the K-means clustering algorithm into two groups representing the 0's and 1's [5]. If the average color of a square is closer to the center of the 0-cluster than to the other it is identified as a 0, otherwise as a 1. Thus we come to a tentative assignment of bit values.

Subsequently, a graph is constituted by linking crossings to their respective neighboring crossings. Here, we take into account that with two neighboring crossings the configuration of colored and black squares inverts. This process can be conceived as reconstructing the checkerboard pattern. From the graph a binary "subarray" is recovered (possibly not the entire PRBA is "recognized" as not all the crossings might be identified). Matching of the subarray against the "mother" PRBA leads to a unique identification of the crossings.

B. Precision in the Localization of Pattern Elements

The precision of our matched filter approach for the localization of the crossings was tested in the following experiment.

Artificial images were generated holding a checkerboard of 11×11 squares representing our design. In each simulated image the location of the crossings (100 in total) was exactly known. The distance between neighboring crossings was set to a prefixed number of pixels (blocksize). The RGB-color of the

black, cyan, and yellow squares were taken as (0, 0, 0), (0, 255, 255), and (255, 255, 0), respectively. To introduce disturbance uncorrelated uniformly distributed noise of preset standard deviation (SD) was added to the individual color channels separately. Subsequently we related the average distance between the real and detected location (and corresponding standard deviation) to the blocksize and the standard deviation of the applied noise.

The results, an average distance of 1.0 ± 0.5 pixel, demonstrate a robust response under various amounts of noise and with different blocksizes. The procedure fails *only* with huge amounts of noise (SD 200) and with small blocksize (six pixels) as the average distance plus twice the standard deviation exceeds half the blocksize. A precision of about 1 pixel in the localization of pattern elements was demonstrated under very noisy circumstances (SD 100) and with small pattern elements (six pixels).

C. Accuracy in the Identification of Pattern Elements

To determine the accuracy of our *identification* procedure we devised four stimulus patterns containing our encoding. The average blocksize registered by our camera when mirroring these stimuli to a sphere of 8 mm radius was 7×7 , 9×9 , 11×11 , or 13×13 pixels.

The percentage erroneously identified blocks when mirroring these patterns to a blue eye were found to be 19.2, 12.1, 7.5, and 8.6% (with continuously growing blocksizes). The corresponding values for a brown eye were found to be 21.7, 16.7, 16.3, and 11.7%.

From these figures it can be concluded that in the process of recovering the pattern from the input image a significant number of bits will be wrongly interpreted. Thus it is necessary to evaluate to what extent (despite misinterpretations) it is still possible to come to a unique identification of the subpattern (and thus to come to a complete fault correction).

D. Relocation Experiments

To investigate the robustness of the PRBA property of unique characterization of subarrays we conducted the following experiment.

From the "master" PRBA (the one which was used during the course of our project with n = 63, m = 65, k = 6, and l = 2) all rectangular subarrays were taken of all sizes larger than 6×2 . In each individual subarray bits were inverted with an a priority taken probability (noise level). Subsequently, the master PRBA (M) was searched for the original position of each subarray (S) by sliding the subarray over the PRBA (relocation). At each position (x, y) the Hamming distance was recorded (the number of different bit values)

$$H = \sum_{i=1}^{k} \sum_{j=1}^{l} |S(i, j) - M(x+i, y+j)|$$
(1)

(with k and l the height and the width of the subarray). Correct relocation was admitted if a subarray had an absolute minimum in Hamming distance at its original position. Fig. 3 illustrates this process for a specific subarray containing 1550 bits at noise level 17.5%.



Fig. 3. Normalized Hamming-distance for all possible relocations of a 1550 bit subarray with 17.5% noise; the average normalized Hamming distance is 0.5, at the position standing out it is 0.175.



Fig. 4. Relocation properties of a PRBA: a search was undertaken to the original position of all subarrays of all sizes (relocation). Each curve shows the fraction correctly relocated subarrays of a given size at a specific noise level.

Fig. 4 summarizes the results of the simulations. Along the horizontal axis is the size of the subarray (height times width), vertically the fraction of subarrays of a given size that could be correctly relocated. Each curve corresponds to a different noise level. From this figure it can be concluded that when no noise is applied all subarrays of all sizes will be correctly relocated. Conversely, the original position of not even one subarray is correctly identified when approximately 50% of the bits are inverted. At a noise level of 20% (about the worst case in our application, see Section III-C) a rectangular subarray has to include minimally 150 bits to allow for correct relocation. Experiments with different random seeds showed identical profiles.

E. Summary of the Identification Process

Under controlled, simulated circumstances we demonstrated a precision of one pixel in the *localization* of pattern elements, even under noisy circumstances $[10^{10} \log(Signal/Noise) =$ $10^{10} \log(255/100) = 4.06]$ and with small pattern elements (six by six pixels per square).



Fig. 5. General outline of the reconstruction: s is a point on the stimulus that is registered in c on the CCD; s' would result in registration in c when mirrored to an instance of our model function. The distance between all points s and s' is minimized by adjusting the parameters of the model function.

When the average size of pattern elements is 7×7 pixels approximately 20% of them is wrongly interpreted. These errors can be totally corrected if the pattern contains a rectangular area of at least 150 bits (see Fig. 4). As our prototype contains 4095 colored blocks (= 65×63 bits) a resolution that is even slightly better than that of commercially available instruments (8 pixels per ring) is feasible.

IV. RECONSTRUCTION

Our approach enables an integral reconstruction of the corneal shape whereas standard techniques lead only to a description of the cornea in crosscuts. To exploit the benefit of our method, we implemented a reconstruction algorithm that, as a first approximation, models the corneal shape as an ellipsoid with a fixed center and six degrees of freedom (representing the length of the three main axes and three rotation angles).

Suppose a point s on the stimulus is reflected by the cornea in a point c on the CCD-chip (see Fig. 5). Given an instance of our modelfunction T, it is possible to calculate a point s' on the stimulator that would be registered in c when mirrored to the surface described by T. (This can be done by simple backward raytracing.) Our objective is to minimize $||\mathbf{s}' - \mathbf{s}||^2$ for the crossings in our checkerboard pattern, coming down to nonlinear parameter estimation of the parameters of the modelfunction. For this aim standard parameter estimation procedures are available [7].

We have tested our reconstruction algorithm under artificial circumstances using a raytracing program (POVRAY [8]) as well as with calibrated steel balls and real eyes. In the raytracing environment a precision was attained of 0.01 mm in average mean curvature upon reconstruction of a sphere of 8 mm.

Table I collates the results of measurements of the radius of three steel balls with our prototype and with a Zeiss keratometer defining the gold standard. Specified are the maximal and minimal radius of curvature measured with the keratometer. The principal curvatures measured with our instrument are derived from the apex of the reconstructed ellipsoid. The table signifies the close resemblance of the results using fairly ideal test objects.

TABLE I THE MAXIMAL AND MINIMAL RADIUS OF CURVATURE OF THREE STEEL BALLS MEASURED WITH A KERATOMETER AND OUR PROTOTYPE

	kerato- meter		cornea- topograph	
object	Rmax	Rmin	Rmax	Rmin
1	6.52	6.52	6.49	6.47
2	7.05	7.05	7.01	7.01
3	8.01	8.01	7.99	7.99

 TABLE II

 The Curvature of the Corneal Apex of Three Eyes

 Measured with a Keratometer and Our Prototype

	kerato- meter		cornea- topograph	
subject	Rmax	Rmin	Rmax	Rmin
1	7.94	7.91	7.97	7.88
2	7.80	7.57	7.81	7.68
3	7.36	7.26	7.26	7.23

Table II holds the curvatures measured of the corneal apex of the right eye of three testpersons. In the experimental setup, the testperson's head is fixated by resting its chin in a head set (a commonly used device in ophthalmology). Alignment of the optical axis (line through the pupil center and the center of the retina) with the main axis of the instrument is enforced by asking the patient to look at the center of the pattern. Naturally, with this approach the tilting posture of the eye is not exactly reproducible. However, clearly the shape of the corneal surface itself is independent of this posture. Table II shows that also with nonideal objects the outcome of the two instruments is almost identical.

Currently research is directed toward adjusting the described reconstruction method taking into account local aberrations of the corneal shape as in [9].

V. CONCLUSIONS

Our newly developed instrument exploits properties of PRBA's to capture the corneal surface structure. Encoded in a colored stimulus pattern PRBA's allow for a unique identification of unambiguously characterized positions. Simulations demonstrate a high precision of our procedures in localizing pattern elements. Furthermore, despite a significant number of misinterpretations in the identification of pattern elements, using the characteristics of the encoded PRBA a complete fault correction is demonstrated. As we have introduced a unique characterization of points on the stimulus, integral reconstruction of the corneal shape becomes possible. Thus we have developed a new measurement technique providing a resolution that is at least as good as that of commercially available instruments.

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