Note that the symbol K is already predefined, so we use k

In[1]:=
```
?? K
```

K is a default generic name for a summation index in a symbolic sum.

$y = R_{P,ss}/R_T$

As derived, we must solve these two equations, for y, eliminating x :

In[2]:=
```
GKrule = Solve[{x + y == 1, (u x) / (J + x) == (v y) / (k + y)}, y, x]
```

··· Solve: Warning: x is not a valid domain specification. Assuming it is a variable to eliminate.

Out[2]= $$\left\{\left\{y \to \frac{u - k\,u - v - J\,v - \sqrt{4\,k\,u\,(u - v) + (-u + k\,u + v + J\,v)^2}}{2\,(u - v)}\right\}, \left\{y \to \frac{u - k\,u - v - J\,v + \sqrt{4\,k\,u\,(u - v) + (-u + k\,u + v + J\,v)^2}}{2\,(u - v)}\right\}\right\}$$

In[3]:=
```
GK1 = y /. GKrule[[1]]
```

Out[3]= $$\left(u - k\,u - v - J\,v - \sqrt{\left(4\,k\,u\,(u - v) + (-u + k\,u + v + J\,v)^2\right)}\right) \Big/ (2\,(u - v))$$

eq .7 in PNAS 1981, ref [30], see http://www.nat.vu.nl/~ivo/SB/dynamic_models/MM/GoldbeterP-NAS1981_78_ 6840.pdf
 is arrived at by division through *v* in numerator and denominator:

In[4]:=
```
GK1 /. v → 1 /. u → (u / v)
```

Out[4]= $$\frac{-1 - J - \sqrt{\left(1 + J - \frac{u}{v} + \frac{k\,u}{v}\right)^2 + \frac{4\,k\,u\,\left(-1+\frac{u}{v}\right)}{v}} + \frac{u}{v} - \frac{k\,u}{v}}{2\,\left(-1 + \frac{u}{v}\right)}$$

To derive the Goldbeter - Koshland function we follow the derivation in
http : // en.wikipedia.org/wiki/Goldbeter - Koshland_kinetics

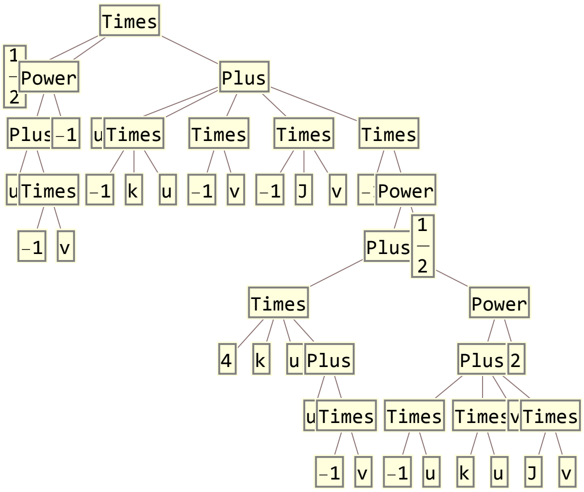
any Mathematica expression has an internal representation, which you can see using TreeForm or FullForm

In[5]:=
```
GK1 // FullForm
```

Out[5]//FullForm=
```
Times[Rational[1, 2], Power[Plus[u, Times[-1, v]], -1],
 Plus[u, Times[-1, k, u], Times[-1, v], Times[-1, J, v],
  Times[-1, Power[Plus[Times[4, k, u, Plus[u, Times[-1, v]]],
     Power[Plus[Times[-1, u], Times[k, u], v, Times[J, v]], 2]], Rational[1, 2]]]]]
```

In[6]:=
```
GK1 // TreeForm
```

Out[6]//TreeForm=



On the first level GK1 is a multiplication of three terms

In[7]:=
```
GK1[[1]]
```

Out[7]= $\frac{1}{2}$

In[8]:=
```
GK1[[2]]
```

Out[8]= $\frac{1}{u-v}$

In[9]:=
```
GK1[[3]]
```

Out[9]= $u-k\,u-v-J\,v-\sqrt{4\,k\,u\,(u-v)+(-u+k\,u+v+J\,v)^2}$

In[10]:=
```
GK2 = y /. GKrule[[2]]
```

Out[10]= $\frac{u-k\,u-v-J\,v+\sqrt{4\,k\,u\,(u-v)+(-u+k\,u+v+J\,v)^2}}{2\,(u-v)}$

Likewise, on the first level GK2 is a multiplication of three terms, and we need the third one for further simplification:

In[11]:=
```
GK2[[3]]
```

Out[11]= $u-k\,u-v-J\,v+\sqrt{4\,k\,u\,(u-v)+(-u+k\,u+v+J\,v)^2}$

In[12]:=
```
Simplify[GK1 GK2[[3]]] / GK2[[3]]
```

Out[12]= $-\frac{2\,k\,u}{u-k\,u-v-J\,v+\sqrt{4\,k\,u\,(u-v)+(-u+k\,u+v+J\,v)^2}}$

and this is the form we find in the Sniffers paper.

Exercise : prove eq.1.9 from the lecture notes

First study the functions.nb by executing the commands and reading the Help pages. Next define (using SetDelayed) the GK function with four arguments *u, v, J* and *k* using the above expression

In[13]:=

Now prove eq. 1.9 from the lecture notes using Simplify

In[14]:=