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# Set-up

The questionnaire given to the Autumn 2017 class of ECS100 had the following questions, rated from "Strongly disagree" to "Strongly agree" for the questions in Table 1, namely the "quizz opinion questions", and a single box for the questions in Table 2, namely the "education background questions", followed by additional space for positive and negative critique.

in my opinion, the quizzes
were relevant to the course homework
were relevant to the course material
made me think more about the mathematics of the course
helped me understand the mathematics of the course
made me think more about the meaning of the physics in the course
helped me understand the physics concepts of the course
were clear
were fun
were too long
were too difficult
should remain a part of the course
Table 1: quizz opinions

I have taken some form of the following:

High school basic maths

High school scientific maths

High school physics

University level physics

University level calculus

Table 2: education background

# Analysis

32 responses were recorded. From the opinion questions, the responses were translated to "grades": 1 for "strongly disagree", 2 for "disagree", etc. For the background questions, a cross was translated to a count of 1. Two additional parameters were determined, "sum maths", defined as the sum of the counts on mathematics background (with a maximum of 2), and similarly for the "sum physics" (with a maximum of 2). The "sum maths" only goes to 2 because either "high school basic maths" (with a weight of 0.5) or "high school scientific maths" (with a weight of 1) is counted, since some student crossed both, while others crossed only for the second.



For the opinion questions, the positive ratio, neutral ratio, negative ratio, and average grade were calculated. The positive ratio was calculated as the count of positive answers (4 or 5) divided by the total answer count. The neutral and negative ratios were calculated similarly for neutral answers (3), and negative answers (1 or 2) respectively. The average grade was calculated as the mean grade for the answers.

Finally, the opinion question responses were self-correlated, and the opinion and background question responses cross-correlated. The pearson correlation test was used to find the strength of the correlations (not the magnitude of the covariance). For some statistical rigorousness, only results that rejected the hypothesis of 0 correlation at a 95% significance level are shown and discussed here.

# Results

All students from the sample had taken maths in high school. About 75% had taken scientific maths, and 65%, calculus at university, see Fig. 1. About 65% of students had taken physics in high school, but only 40% at university. This means that about 35% of students were likely to encounter new maths concepts in the class, while 60% were likely to encounter new physics concepts, assuming that the course presented university level maths and physics.



Fig. 1: education background of students.

Opinions of the quizzes given in the class were generally favorable. On average, students tended to agree that the quizzes should remain part of the course, that they were relevant to the course material, and made them think about and understand mathematical and physical concepts of the class. However, students neither agreed nor disagreed on whether the quizzes were fun, too long, or too difficult. See Fig. 2, for the average grades given to the quiz questions.

Some opinions were controversial, as indicated in Fig. 3. Almost half of the students were neutral or disagreed that the quizzes helped them understand the mathematics of the course, while 35% were neutral on whether the quizzes helped them understand the physics of the





1 = Strongly disagree, 5 = Strongly agree

Quizz opinions

Fig. 2: opinion of students on the quizzes. The error bars show the standard deviation of the responses.



Fig. 3: ratios of the opinion of students on the quizzes.

course. 80% of students agreed that the quizzes made them think more about the mathematics of the course, while 15% disagreed that the same was true for the physics. The most controversial topics however, were on the fun, length, and difficulty of the quizzes. An equal 30% of student agreed and disagreed that the quizzes were fun, 40% undecided. 20% of students agreed that the quizzes were too long, while 40% disagreed, 45% undecided. Finally, nearly 30% of students agreed that the quizzes were too difficult, while little over 30% disagreed, 40% undecided. Looking at the correlations of the opinions can help shed light on which students disagree with each other.



Correlation of education background and opinion on quizzes

Fig. 4: cross correlation of education background and opinion on quizzes. The green/orange colors relate to mathematics, blue/red to physics. All correlations shown have a 2-sigma significance.

ground of the students and their opinions of

The cross correlations between the education background of the students and their opinions of the quizzes can be seen in Fig. 4. Unsurprisingly, the perceived difficulty of the quizzes were inversely correlated with previous education in mathematics and physics. However, the correlation was stronger for education in physics than for education in maths. This suggests that the quizzes challenged the mathematics and physics knowledge of the students, but physics to a larger degree.

A better physics education background was correlated with a stronger agreement that the quizzes were relevant to the course material and homework, helped with understanding physics concepts, were fun, and should remain part of the course. A better mathematics background was correlated with a stronger agreement that the quizzes helped understanding





both the mathematics and physics of the course. This suggests that students that had a better mathematics and physics background found the quizzes more useful in general.

Finally, to understand what opinions occurred together, the self-correlation matrix was calculated, seen in Fig. 5. Finding the quizzes not too difficult, fun, clear, helping with understanding physics and mathematics, and relevant to course materials and homework was correlated with wanting the quizzes to remain part of the course. Interestingly, the quiz lengths were not significantly correlated to whether the quizzes should remain part of the course. Another interesting result was the inverse correlation between clarity of the quizzes and them being too long. This suggests that students that found the quizzes clear did not find them too long to the same degree.

# Conclusion

1/3 of students in the course had not taken university level calculus, and nearly 2/3 had not taken university level physics. However, the opinions on the relevance and difficulty of the quizzes were mostly correlated with background in physics, indicating that a physics background played at least as big a role as a background in mathematics in solving the quizzes.

Students were favorable towards the quizzes, with nearly no students disagreeing that they should remain part of the course, and helped with understanding the concepts of the course. However, this positive feedback came to a larger degree from students that had a stronger background in mathematics and physics. It would seem that students in general think the quizzes are useful, but more probing is needed to figure out whether they actually help the students with understanding of the concepts.

Students thought the quizzes were too long across the board, so the format of the quizzes should match their length in the future.

# Quiz #1

## **ECS100**

November 16, 2017

### Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate! 9 ~ 10, <sup>10</sup>/<sub>11</sub> ~ 1, etc...
  Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

**Useful** Equations

$$Q = c_p m \Delta T \tag{1}$$

#### 1. Cooking pasta

You are making pasta. You get 1 liter of water from the tap at room temperature, and heat it on your 1500W stove. It takes 13 minutes before the water starts bubbling.

- The specific heat capacity of water is about 4 J/g·K
- Room temperature is 25°C.

a. Calculate the amount of energy the stove had to transform.

b. Calculate the amount of energy necessary to get the water to boil.

c. How efficient is it to boil water in this way, i.e., what percentage of the energy went to waste in this process?

#### **2.** $CO_2$ in the classroom

The concentration of  $CO_2$  in the atmosphere is currently around 400 parts per million (ppm).

- 1 unified atomic mass unit (u)  $\simeq 2 \times 10^{-27}$ kg
- 1 oxygen atom weighs 16 u. 1 carbon atom weighs 12 u.
- The number of atoms per unit volume of air at STP (number density, n) is  $25 \times 10^{24} m^{-3}$  (we found this using the ideal gas law and known values for temperature and pressure of the atmosphere in lecture!)
- a. Roughly estimate the volume of this classroom, in  $m^3$ .
- b. Calculate the number of  $CO_2$  molecules in this classroom.
- c. Calculate the mass of a single  $CO_2$  molecule, in kg.
- d. Estimate the mass in grams of  $CO_2$  in this classroom.

Fuel releases energy by undergoing a chemical reaction in which the fuel molecules change into different molecules. Here is one such reaction, showing complete combustion of ethanol (drinking alcohol) :

$$C_2H_5OH + 3O_2 \rightarrow 2CO_2 + 3H_2O \tag{2}$$

Fuel that you would use to power your car is made up of many different types of molecules. The standard fuel in the Netherlands is E10, which means gasoline with 10% ethanol. E10 produces right under 3 kg of  $CO_2$  for every 1 kg combusted fuel<sup>1</sup>.

e. Calculate the amount of grams of E10 you would need to combust in this classroom to double the air's concentration of  $CO_2$ .

<sup>&</sup>lt;sup>1</sup>Bonus question: Are you surprised that fuel produces a larger mass of  $CO_2$  than what is combusted? Look at the equation for ethanol combustion to make sense of this. How many  $CO_2$  molecules are produced by combusting one ethanol molecule? What are their respective masses? Now calculate how many grams of ethanol are necessary to produce 100g of  $CO_2$  under complete combustion. Where does this mass come from?

 ${
m Quiz}$  #1 (soln)

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November 16, 2017

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- Room temperature is 25°C.
- a. Calculate the amount of energy the stove had to transform.
- b. Calculate the amount of energy necessary to get the water to boil.

c. How efficient is it to boil water in this way, i.e., what percentage of the energy went to waste in this process?

#### Solution:

a. The stove works at 1500W, which is 1500J/s. It operates for 13 minutes, which is  $13min \cdot 60 \frac{s}{min} \simeq 800s$ . At 1500J for every of those seconds, we get  $1500 \frac{J}{s} \cdot 800s = 1200000J =$  $1.2 \times 10^6 J = 1.2 M J$ 

b. The question is asking for the amount of heat produced given a change in temperature. The relation between the two is given above as  $Q = c_p m \Delta T$ . Water boils at 100°C, so the change in temperature is  $\Delta T = 100C - 25C = 75C$ . 1L of water weighs 1kg. The heat becomes  $Q = 4 J/g \cdot C \cdot 1000g \cdot 75C = 300000 \frac{J \cdot g \cdot C}{g \cdot C} = 3 \times 10^5 J = 300 kJ$ 

c. We used about 1.2MJ = 1200kJ, but the water only absorbed 300kJ of this energy.  $\frac{300kJ}{1200kJ} = \frac{3}{12} = 0.25 = \underline{25\%}$ 

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$$C_2H_5OH + 3O_2 \rightarrow 2CO_2 + 3H_2O \tag{2}$$

Fuel that you would use to power your car is made up of many different types of molecules. The standard fuel in the Netherlands is E10, which means gasoline with 10% ethanol. E10 produces right under 3 kg of  $CO_2$  for every 1 kg combusted fuel<sup>1</sup>.

e. Calculate the amount of grams of E10 you would need to combust in this classroom to double the air's concentration of  $CO_2$ .

#### Solution:

a. Dimensions ~10m by 10m by 3m.  $V = length \cdot width \cdot height = 10m \cdot 10m \cdot 3m = 300m \cdot m \cdot m = 300m^3$ 

b. We have  $25 \times 10^{24}$  air molecules for every metre cubed of classroom, and we have 300 of those. That's  $25 \times 10^{24} m^{-3} \cdot 300 m^3 = 7500 \times 10^{24} \frac{m^3}{m_3} = 7.5 \times 10^{27}$  molecules. However, only 400 ppm of the classroom air is CO2. This means that out of one million molecules of air, 400 of those will turn out to be CO2:  $7.5 \times 10^{27} \cdot \frac{400}{10^6} = \frac{3000 \times 10^{27}}{10^6} = 3 \times 10^{24}$ 

c. One CO2 molecule is made up of one carbon atom and two oxygen atoms. That's  $12u + 2 \cdot 16u = 44u$ . One u is the same as  $2 \times 10^{-27} kg$ , so 44u is the same as  $44 \cdot 2 \times 10^{-27} kg \simeq 9 \times 10^{-26} kg$ 

d. We know how many molecules there are in the classroom, and we know how many each of those weigh.  $3 \times 10^{24}$  atoms that each have a mass of  $9 \times 10^{-26} kg$  gives a total mass of  $3 \times 10^{24} \cdot 9 \times 10^{-26} kg = 27 \frac{10^{24}}{10^{26}} kg \simeq 30 \times 10^{-2} kg = 0.3 kg = 300 g$ 

<sup>&</sup>lt;sup>1</sup>Bonus question: Are you surprised that fuel produces a larger mass of  $CO_2$  than what is combusted? Look at the equation for ethanol combustion to make sense of this. How many  $CO_2$  molecules are produced by combusting one ethanol molecule? What are their respective masses? Now calculate how many grams of ethanol are necessary to produce 100g of  $CO_2$  under complete combustion. Where does this mass come from?

e. To double the amount of  $CO_2$  in this class, we need to add the same amount that is already there. Since there are 300 grams present, and 1kg fuel is required for every 3kg  $CO_2$  produced, we need to burn  $300g \cdot \frac{1000g}{3000g} = \underline{100g \text{ fuel}}$ .

# Quiz #2

#### **ECS100**

November 27, 2017

#### Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate! 9 ~ 10, <sup>10</sup>/<sub>11</sub> ~ 1, etc...
- Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

**Useful** Equations

$$PV = NkT \tag{1}$$

$$dU = dQ - dW \tag{2}$$

where  $dW \equiv PdV$  (from the definition of work), and dU = (#d.o.f./2)kNdT (from the equipartition theorem)

#### 1. Temperature in a vial

You have a temperature sensor inside a little vial that is capped by a piston on top, and has a little heat source which transfers heat energy to the fixed amount of air inside. The vial contains 100mL (0.1 litres) of air at 1 atm ( $10^5$  Pascals) and room temperature ( $27^{\circ}$ C). You can increase the temperature of the air in the vial in several different ways:

- 1. applying heat until the volume is doubled, while allowing the piston to move such that pressure inside remains at 1 atm
- 2. applying heat until the pressure is doubled, while keeping the piston position fixed and thus holding volume constant
- 3. quickly pressing the piston which reduces the volume to 50% of the original value

a. Each of these ways of increasing air temperature corresponds to one of the simplified thermodynamic processes that we studied. Please name and match them. *Hint: temperature is changing in each!* 

b. Find a formula for T(P, V).

c. Find a formula for the temperature change when I. only volume changes II. only pressure changes. *Hint: think about how to express the change in temperature mathematically. See* 

footnote if  $stuck^1$ 

d. Find a formula for dT in terms of T, V and dV.

e. Calculate the final temperature for each listed way of increasing the temperature. *Note:* there is more than one way of doing this. Using the results from c. and d. is not strictly necessary.

<sup>&</sup>lt;sup>1</sup>The change in temperature with respect to variable x is expressed mathematically as  $\frac{\partial T}{\partial x}$ 

 ${
m Quiz}\ \#2$  (soln)

#### **ECS100**

November 27, 2017

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footnote if stuck<sup>1</sup>

d. Find a formula for dT in terms of T, V and dV.

e. Calculate the final temperature for each listed way of increasing the temperature. *Note:* there is more than one way of doing this. Using the results from c. and d. is not strictly necessary.

#### Solutions:

a. 1. Pressure is constant  $\rightarrow$  isobaric. 2. Volume is constant  $\rightarrow$  isometric 3. Fast change in internal energy means heat does not have time to flow,  $dQ = 0 \rightarrow$  adiabatic.

b. 
$$T(P, V) = \frac{PV}{Nk}$$
  
c. I.  $\frac{\partial T}{\partial V} = \frac{\partial}{\partial V} \frac{PV}{Nk} = \frac{P}{Nk}$  II.  $\frac{\partial T}{\partial P} = \frac{\partial}{\partial P} \frac{PV}{Nk} = \frac{V}{Nk}$ 

d. Three translational and two rotational degrees of freedom in diatomic gas at room temperature.  $dQ = 0 \Rightarrow dU = (\#d.o.f./2)kNdT = -dW = -PdV \Rightarrow \frac{5}{2}kNdT = -PdV = -\frac{kNT}{V}dV \Rightarrow dT = -\frac{2T}{5V}dV$ 

e. We already know how much the temperature changes as another variable changes. We can now use separation of variables and integrate to get the total change of temperature dependent on various variables.

$$\begin{split} \text{I.} \ \frac{\partial T}{\partial V} &= \frac{P}{Nk} \Rightarrow \int_{T_0}^{T_1} \partial T = \int_{V_0}^{V_1} \frac{P}{Nk} \partial V \Rightarrow T_1 - T_0 = \frac{P}{Nk} (V_1 - V_0) \Rightarrow T_1 = T_0 + \frac{P}{Nk} V_0 = 2T_0 \\ \text{II.} \ \frac{\partial T}{\partial P} &= \frac{V}{Nk} \Rightarrow \int_{T_0}^{T_1} \partial T = \int_{P_0}^{P_1} \frac{V}{Nk} \partial P \Rightarrow T_1 - T_0 = \frac{V}{Nk} (P_1 - P_0) \Rightarrow T_1 = T_0 + \frac{V}{Nk} P_0 = 2T_0 \\ \text{III.} \ dT &= -\frac{2T}{5V} dV \Rightarrow \int_{T_0}^{T_1} \frac{1}{T} \partial T = \frac{-2}{5} \int_{V_0}^{V_1} \frac{1}{V} \partial V \Rightarrow \ln(T) \Big|_{T_0}^{T_1} = \frac{-2}{5} \ln(V) \Big|_{V_0}^{V_1} \Rightarrow \ln(T_1) - \ln(T_0) = \\ \frac{-2}{5} (\ln(V_1) - \ln(V_0)) \Rightarrow e^{\ln(T_1)} e^{-\ln(T_0)} = (e^{\ln(V_1)} e^{-\ln(V_0)})^{\frac{-2}{5}} \Rightarrow \frac{T_1}{T_0} = \sqrt[5]{V_0} \frac{V_0}{V_1}^2 \Rightarrow T_1 = T_0 \sqrt[5]{2}^2 \simeq \\ 1.3T_0 \end{split}$$

<sup>&</sup>lt;sup>1</sup>The change in temperature with respect to variable x is expressed mathematically as  $\frac{\partial T}{\partial x}$ 

# Quiz #3

### **ECS100**

December 4th, 2017

Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate!  $9 \sim 10, \frac{10}{11} \sim 1, \text{ etc...}$
- Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

### 1. Windy Amsterdam

You are on your way to a course on energy science at AUC on your bike at 5m/s, when a gush of wind at a relative velocity of  $v_w = 15$ m/s hits you from the back and speeds you up! The gush of wind lasts for two seconds. Let's assume that you weigh 90kg with your bike and backpack, and cover an area  $A_w$  of  $\frac{3}{2}$  m<sup>2</sup>. At STP, the mass of air per cubic metres,  $\rho_{air}$  is about 1 kg/m<sup>3</sup>.

a. Make up a formula for the mass of air contained in a "block of wind" in terms of the air density,  $\rho_{air}$ , the speed of the wind,  $v_w$ , the area of the block of wind,  $A_w$ , and time, t.

b. If I have a block of wind that weighs 10kg when going at 15 m/s through 1 m<sup>2</sup>, how fast does that same mass go if the area is enlarged to 3 m<sup>2</sup>? Note that if the mass and air density remain unchanged,  $A \cdot v$  must indeed always remain the same.

c. Express the kinetic energy of this wind in terms of the same variables as in part a. Now calculate its power. *Hint: what is the relationship between power and energy?* 

d. In the case described in b., is the power before and the power after the same? That is, is the power conserved like the mass and density were? You can show this without actually calculating the number for the power.

e. Let's go back to you getting to class on your bike. The wind has a certain speed before it hits you, as it goes past you, and when it has gone past you. From b., you saw that  $A_0v_0 = A_1v_1 = A_2v_2$ , and in d., you found that power (and thus energy) is not conserved. Let's quantify this loss of energy. Find a formula for the difference in the kinetic of the energy of the wind before, and after pushing you for two seconds, and call it  $\Delta E$ .

f. In the ideal case, as seen in class, you get the highest energy from wind when  $v_1 = \frac{2}{3}v_0$ , and  $v_2 = \frac{1}{3}v_0$ . How much energy did the wind give you in two seconds assuming this ideal case? Now, how fast are you going?

g. Repeat f., but this time, we'll pick numbers that are more realistic for the highest efficiency wind turbines available today, where the efficiency gets close to 50%.  $v_1 = \frac{4}{5}v_0$ , and  $v_2 = \frac{16}{25}v_0$ 

 ${
m Quiz}\ \#3$  (soln)

#### **ECS100**

December 4th, 2017

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g. Repeat f., but this time, we'll pick numbers that are more realistic for the highest efficiency

wind turbines available today, where the efficiency gets close to 50%.  $v_1 = \frac{4}{5}v_0$ , and  $v_2 = \frac{16}{25}v_0$ 

Solution: a.  $m_{air} = \rho(\text{kg/m}^3)A(\text{m}^2) \cdot v(\text{m/s}) \cdot t(\text{s}) = \rho Avt \text{ kg}$ b.  $A_0v_0 = A_1v_1 \Rightarrow v_1 = \frac{A_0v_0}{A_1} = \frac{1\cdot15}{3} = 5 \text{ m/s}$ c.  $E = \frac{1}{2}m_{air}v^2 = \frac{1}{2} \cdot \rho Avt \cdot v^2 = \frac{1}{2}\rho Atv^3$ . Power  $= \frac{dE}{dt} = \frac{1}{2}\rho Av^3$ d.  $P_0 = \frac{1}{2}\rho 15^3 = \frac{1}{2}\rho 3375$ ,  $P_1 = \frac{1}{2}\rho \cdot 3 \cdot 5^3 = \frac{1}{2}\rho 375 =$ . Not the same. This means that although the density and mass of the block of air is conserved, its energy is not. e.  $\Delta E = E_{after} - E_{before} = \frac{1}{2}\rho A_2 tv_2^3 - \frac{1}{2}\rho A_0 tv_0^3 = \frac{1}{2}\rho t(A_2 v_2^3 - A_0 v_0^3) = \frac{1}{2}\rho t(A_1 v_1 v_2^2 - A_1 v_1 v_0^2) = \frac{1}{2}\rho tA_1v_1(v_2^2 - v_0^2)$ f.  $\Delta E = \frac{1}{2}\rho tA_1v_1(v_2^2 - v_0^2) = \frac{1}{2}1(\text{kg/m}^3) \cdot 2(\text{s}) \cdot \frac{3}{2}(\text{m}^2) \cdot \frac{2}{3}15(\text{m/s})((\frac{1}{3} \cdot 15\text{m/s})^2 - (15\text{m/s})^2) = 15(5^2 - 15^2)\text{J} = 3000 \text{ J}$   $E_{bike\_after} = E_{bike\_before} + \Delta E = \frac{1}{2}m_{bike}v_{before}^2 + \Delta E = \frac{90\text{kg}\cdot25\text{m}^2/\text{s}^2}{2} + 3000\text{J} \simeq 4000\text{J}$ . Now, re-arrange for the speed after:  $E_{bike\_after} = \frac{1}{2}m_{bike}v_{after}^2 \Rightarrow v_{after} = \sqrt{\frac{E_{bike\_after}}{\frac{1}{2}m_{bike}}} = \sqrt{\frac{4000\text{J}}{\frac{1}{2}90\text{kg}}} = \sqrt{\frac{800}{9}} \simeq 9 \text{ m/s}$ g.  $\Delta E = \frac{1}{2}\rho tA_1v_1(v_2^2 - v_0^2) = \frac{1}{2}1(\text{kg/m}^3) \cdot 2(\text{s}) \cdot \frac{3}{2}(\text{m}^2) \cdot \frac{4}{5}15(\text{m/s})((\frac{16}{25} \cdot 15\text{m/s})^2 - (15\text{m/s})^2) = \frac{6}{5}15(\frac{48}{5}^2 - 15^2)\text{J} \simeq 18(100 - 225) \text{ J} \simeq 2000 \text{ J}.$   $E_{bike\_after} = E_{bike\_before} + \Delta E = \frac{1}{2}m_{bike}v_{before}^2 + \Delta E = \frac{90\text{kg}\cdot25\text{m}^2/\text{s}^2}{2} + 2000\text{J} \simeq 3000\text{J}.$  Now, re-arrange for the speed after:  $E_{bike\_after} = \frac{1}{2}m_{bike}v_{after}^2 \Rightarrow v_{after} = \sqrt{\frac{E_{bike\_after}}{2}} - (\frac{5000\text{J}}{2} - (15\text{m/s})^2) = \frac{6}{5}15(\frac{48}{5}^2 - 15^2)\text{J} \simeq 18(100 - 225) \text{ J} \simeq 2000 \text{ J}.$   $E_{bike\_after} = E_{bike\_before} + \Delta E = \frac{1}{2}m_{bike}v_{before}^2 + \Delta E = \frac{90\text{kg}\cdot25\text{m}^2/\text{s}^2}{2} + 2000\text{J} \simeq 3000\text{J}.$  Now, re-arrange for the speed after:  $E_{bike\_after} = \frac{1}{2}m_{bike}v_{after}^2 \Rightarrow v_{after} = \sqrt{\frac{E_{bike\_after}}{\frac{1}{2}m_{bike}}} = \sqrt{\frac{\frac{3000}{2}}{\frac$ 

$$\sqrt{\frac{600}{9}} \simeq \frac{25}{3} \simeq 8 \text{ m/s}$$

# Quiz #4

#### **ECS100**

December 11th, 2017

Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate!  $9 \sim 10, \frac{10}{11} \sim 1, \text{ etc...}$
- Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

#### 1. Power line

You install a shed in the backyard of your house, which you would like to power with electricity. You draw a 150m power cable from your house to the shed to achieve this. Let's say that at a given moment, the total power dissipated on the way to and inside the shed amounts to 2000W

a) Try not to look at your notes for this part! Voltage is defined as energy per charge  $(\frac{J}{C})$  and current as charge per time  $(\frac{C}{s})$ . Find a formula for the power using these two quantities, i.e., " $P_{circuit} = \dots$ .". This formula tells you the power transferred through a circuit of a given voltage and current. In other words, this is the power that must go into the system to power it (batteries, power sources, etc.), and that must exit the system (heat in resistance, make a motor work, etc.).

b) Ohm's law dictates that for the voltage dropped across the resistive wires in a cable,  $V_{cable} = I \cdot R_{cable}$ . Using this formula and the one you found in a), express the power lost to the resistance in the cables of a circuit,  $P_{loss}$ , in terms of the given total power of a circuit,  $P_{circuit}$ , the voltage of this circuit,  $V_{circuit}$  and resistance only.

c) Now let's say you use a cheap 4 mm<sup>2</sup> aluminium cable that has a resistance per distance of  $8 \times 10^{-3} \frac{\Omega}{m}$ . Assume that the mains <sup>[1]</sup> electricity has a supplied voltage of  $V_{circuit} = 200$  V <sup>[2]</sup>. How much power is lost in the cable?

d) You now install transformers to increase the circuit voltage a hundred-fold, to 20 kV <sup>[3]</sup>. How much power is lost to the cable now?

e) Perhaps it might be worth it to look into more expensive cables that have a lower resistance. Redo the calculation of c) and d) using a 120 mm<sup>2</sup> copper cable that has a resistance

<sup>&</sup>lt;sup>1</sup>the power supply from your house

 $<sup>^2\</sup>mathrm{nevermind}$  that it's AC voltage - as the value is given in "root mean square voltage" which nicely gives the same results as if it had been a constant DC voltage

<sup>&</sup>lt;sup>3</sup>Having twenty thousand volts running across your backyard is not necessarily a good idea, I'm just trying to demonstrate a proof of concept here

per distance of  $0.15\times 10^{-3}\frac{\Omega}{\rm m}$ 

 ${
m Quiz}$  #4 (soln)

#### **ECS100**

December 11th, 2017

Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate! 9 ~ 10, <sup>10</sup>/<sub>11</sub> ~ 1, etc...
  Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

#### 1. Power line

You install a shed in the backyard of your house, which you would like to power with electricity. You draw a 150m power cable from your house to the shed to achieve this. Let's say that at a given moment, the total power dissipated on the way to and inside the shed amounts to 2000W

a) Try not to look at your notes for this part! Voltage is defined as energy per charge  $\left(\frac{J}{C}\right)$  and **current** as *charge per time*  $\left(\frac{C}{s}\right)$ . Find a formula for the **power** using these two quantities, i.e., " $P_{circuit} = \dots$ ". This formula tells you the power transferred through a circuit of a given voltage and current. In other words, this is the power that must go into the system to power it (batteries, power sources, etc.), and that must exit the system (heat in resistance, make a motor work, etc.).

b) Ohm's law dictates that for the voltage dropped across the resistive wires in a cable,  $V_{cable} = I \cdot R_{cable}$ . Using this formula and the one you found in a), express the power lost to the resistance in the cables of a circuit,  $P_{loss}$ , in terms of the given total power of a circuit,  $P_{circuit},$  the voltage of this circuit,  $V_{circuit}$  and resistance only.

c) Now let's say you use a cheap 4 mm<sup>2</sup> aluminium cable that has a resistance per distance of  $8 \times 10^{-3} \frac{\Omega}{m}$ . Assume that the mains <sup>[1]</sup> electricity has a supplied voltage of  $V_{circuit} = 200$  $V^{[2]}$ . How much power is lost in the cable?

d) You now install transformers to increase the circuit voltage a hundred-fold, to 20 kV  $^{[3]}$ . How much power is lost to the cable now?

e) Perhaps it might be worth it to look into more expensive cables that have a lower resistance. Redo the calculation of c) and d) using a 120 mm<sup>2</sup> copper cable that has a resistance per distance of  $0.15 \times 10^{-3} \frac{\Omega}{m}$ 

<sup>&</sup>lt;sup>1</sup>the power supply from your house

<sup>&</sup>lt;sup>2</sup>nevermind that it's AC voltage - as the value is given in "root mean square voltage" which nicely gives the same results as if it had been a constant DC voltage

<sup>&</sup>lt;sup>3</sup>Having twenty thousand volts running across your backyard is not necessarily a good idea, I'm just trying to demonstrate a proof of concept here

Solution:

a) Remember power is in units of Joules per second!

a) Remember power is in units of source per second  $I(\frac{J}{C}) \cdot V_{circuit}(\frac{C}{s}) = IV_{circuit}(\frac{J}{s}) = P_{circuit}(\frac{J}{s})$ b) Here, we have the total power of the circuit,  $P_{circuit} = I_{circuit}V_{circuit}$ , and the power lost in the cable,  $P_{cable} = I_{cable}V_{cable}$ . Note that the same amount of electrons must get through along the wire, so the current is the same anywhere on the circuit:  $I \equiv I_{cable} = I_{circuit} = \frac{P_{circuit}}{V_{circuit}}$ .

Our

$$\begin{aligned} P_{cable} &= IV_{cable} = I^2 R_{cable} = R_{cable} \left(\frac{P_{circuit}}{V_{circuit}}\right)^2 \\ \text{c) Note here that the electricity has to travel in the cable going to the shed, and back. Our total cable resistance is then  $R_{cable} = 2 \cdot 150 \text{m} \cdot 0.008 \frac{\Omega}{\text{m}} = 2.4\Omega$ . Next, the total power of the circuit is given to be  $P_{circuit} = 2000\text{W}$ , and the voltage  $V_{circuit} = 200\text{V}$ . We get  $P_{cable} = R_{cable} \left(\frac{P_{circuit}}{V_{circuit}}\right)^2 = 2.4\Omega \frac{2000W^2}{200V^2} = 240\text{W}^{[4]} \\ \text{d) } R_{cable} = 2 \cdot 150\text{m} \cdot 0.008 \frac{\Omega}{\text{m}} = 2.4\Omega, P_{circuit} = 2000\text{W}, V_{circuit} = 2000\text{V}. \\ P_{cable} = R_{cable} \left(\frac{P_{circuit}}{V_{circuit}}\right)^2 = 2.4\Omega \frac{2000W^2}{2000V^2} = 24\text{mW} \\ \text{e) } R_{cable} = 2 \cdot 150\text{m} \cdot 0.00015 \frac{\Omega}{\text{m}} = 0.045\Omega, P_{circuit} = 2000\text{W}, V_{circuit} = 200\text{V}. \end{aligned}$$$

$$P_{cable} = R_{cable} \left( \frac{V_{circuit}}{V_{circuit}} \right)^{2} = 0.045\Omega \frac{200V^{2}}{200V^{2}} = 4.5 \text{ W}$$
  
f)  $R_{cable} = 2 \cdot 150 \text{m} \cdot 0.00015 \frac{\Omega}{\text{m}} = 0.045\Omega, P_{circuit} = 2000 \text{W}, V_{circuit} = 2000 \text{V}$   
 $P_{cable} = R_{cable} \left( \frac{P_{circuit}}{V_{circuit}} \right)^{2} = 0.045\Omega \frac{2000W^{2}}{2000V^{2}} = 0.45 \text{mW}$ 

# Quiz #5

#### **ECS100**

December 11th, 2017

Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate! 9 ~ 10, <sup>10</sup>/<sub>11</sub> ~ 1, etc...
  Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

Useful Equations

$$E_{particle} = hf = \frac{hc}{\lambda} \tag{1}$$

#### 1. Photovoltaics

The purpose of this quiz is to make a mental photovoltaic cell from scratch to get a good understanding of how they work.

A crystal of silicon is doped with atoms of aluminium, which have three valence electrons in their outer shell.

a) Does the result become a p-type or n-type semi-conductor? Explain why.

You take a p-type and n-type semi-conductor, and make them touch. As free electrons and holes randomly move about in the materials, they will meet and cancel each other out in what is called the depletion layer.



b) Draw this situation, label the semi-conductors, the pn-junction, and the depletion layer. Indicate which regions are positive and negative. If an electron-hole pair is created in the depletion layer, in which direction does the electron go?

In semiconductors, electrons are only allowed to have certain energies. If they are in the lower energy region, they are said to be in the valence band, where they bind the atoms together. If they are in the higher energy region, they are said to be in the conduction band, where they are free to move in the material. The gap between the valence and conduction bands is called the band gap, which represents the minimum energy needed for an electron to go from the valence band to the conduction band.

c) Silicon has a band gap energy of about  $1 \text{ eV}^1$ . Continuing from b), we connect the right of the n-side and the left of the p-side to an external device which can store or use the voltage difference induced in the semiconductor. We'll call this whole contraption our *cell*. Remember that we can maximally extract the band gap energy of 1 eV per electron-hole pair created! To make sense of this, explain what happens for:

- i) an incoming photon with an energy of 0.5 eV
- ii) an incoming photon with an energy of 1.0 eV
- iii) an incoming photon with an energy of 2.0 eV

Remember that the efficiency of an energy source is defined as the ratio of the energy obtained and the energy available in the system,  $Eff \equiv \frac{E_{out}}{E_{in}}$ . There are many ways of defining exactly what this incoming and outgoing energy is, but let's say for now that  $E_{in}$  is the energy of a photon going through our cell, and  $E_{out}$  is the maximum amount of energy that we can get out of whatever this photon does in the cell.

d) Based on your answer in c), draw a graph showing the efficiency vs. photon energy. Label the band gap energy.

 $<sup>^{1}</sup>$ an eV, or electron-volt, is a unit of energy (like Joules, or Watt-hours) which represents the energy necessary to make a single electron cross a 1 Volt potential difference.

e) Repeat d), but this time make the x-axis the photon-wavelength.



You are appointed engineer in chief for a mission to an exotic planet! You will decide what type of material to use to build photovoltaic cells to power a research camp there. You have the choice between a semiconductor with a band gap energy of 1eV, and and another with a band gap energy of 2eV. Let's consider four cases, where the planet is lit by different light sources, shown in the graph above. The only difference between the sources is their *spectrum and intensity*, i.e., they shine for the same amount of time, same angle of incidence, etc. For each case, decide which material to use, and explain your choice:

a) A source that shines at constant intensity at every wavelength (green)

b) A source that shines at constant intensity at every wavelength above 1000 nm, and not at all below (pink)

c) A blackbody source that peaks at 800nm (red) d) A blackbody source that peaks at 400nm (blue)



#### **ECS100**

December 11th, 2017

Quiz rules:

- Your answers will not necessarily be exactly correct, but should be within 10 times or 1/10 of the correct answers; this is called an 'order of magnitude estimate'. Approximate! 9 ~ 10, <sup>10</sup>/<sub>11</sub> ~ 1, etc...
  Do not forget to include units in your answer. This actually helps you!
- No calculators, but we pick nice numbers for you.
- The aim here is not to nail this quiz, but to learn how to use maths in physics. It is supposed to be challenging!

**Useful** Equations

$$E_{particle} = hf = \frac{hc}{\lambda} \tag{1}$$

Where  $h \simeq 4 \times 10^{-15} \text{eV} \cdot \text{s}$  is the Planck's constant, and  $c \simeq 3 \times 10^8 \text{m/s}$  is the speed of light.

### 1. Photovoltaics

The purpose of this quiz is to make a mental photovoltaic cell from scratch to get a good understanding of how they work.

A crystal of silicon  $(_{14}Si)$  is doped with atoms of aluminium  $(_{13}Al)$ , which have three valence electrons in their outer shell.

a) Does the result become a p-type or n-type semi-conductor? Explain why.

You take a p-type and n-type semi-conductor, and make them touch. As free electrons and holes randomly move about in the materials, they will meet and cancel each other out in what is called the depletion layer.

b) Draw this situation, label the semi-conductors, the pn-junction, and the depletion layer. Indicate which regions are positive and negative. If an electron-hole pair is created in the depletion layer, in which direction does the electron go?

In semiconductors, electrons are only allowed to have certain energies. If they are in the lower energy region, they are said to be in the valence band, where they bind the atoms together. If they are in the higher energy region, they are said to be in the conduction band, where they are free to move in the material. The gap between the valence and conduction bands is called the band gap, which represents the minimum energy needed for an electron to go from the valence band to the conduction band.



c) Silicon has a band gap energy of about  $1 \text{ eV}^1$ . Continuing from b), we connect the right of the n-side and the left of the p-side to an external device which can store or use the voltage difference induced in the semiconductor. We'll call this whole contraption our *cell*. Remember that we can maximally extract the band gap energy of 1 eV per electron-hole pair created! To make sense of this, explain what happens for:

- i) an incoming photon with an energy of 0.5 eV
- ii) an incoming photon with an energy of  $1.0~{\rm eV}$
- iii) an incoming photon with an energy of 2.0 eV

Remember that the efficiency of an energy source is defined as the ratio of the energy obtained and the energy available in the system,  $Eff \equiv \frac{E_{out}}{E_{in}}$ . There are many ways of defining exactly what this incoming and outgoing energy is, but let's say for now that  $E_{in}$  iB2s the energy of a photon going through our cell, and  $E_{out}$  is the maximum amount of energy that we can get out of whatever this photon does in the cell.

d) Based on your answer in c), write the formula for Eff if  $E_{in} < E_{bandgap}$  and for Eff if  $E_{in} \leq E_{bandgap}$ . Draw a graph showing the efficiency vs. photon energy. Label the band gap energy.

e) Repeat d), but this time make the x-axis the photon-wavelength. What should you label instead of band gap energy now?

You are appointed engineer in chief for a mission to an exotic planet! You will decide what type of material to use to build photovoltaic cells to power a research camp there. You have the choice between material B1, a semiconductor with a band gap energy of 1eV, and and material B2, with a band gap energy of 2eV. Let's consider four cases, where the planet

<sup>&</sup>lt;sup>1</sup>an eV, or electron-volt, is a unit of energy (like Joules, or Watt-hours) which represents the energy necessary to make a single electron cross a 1 Volt potential difference.



is lit by one of the different light sources shown in the graph above. The only difference between the sources is their *spectrum and intensity*, i.e., they shine for the same amount of time, same angle of incidence, etc.

f) For each case, decide which material to use, and explain your choice:

i) A source that shines at constant intensity at every wavelength between 800 and 1400 nm, and not at all otherwise (pink)

ii) A weaker source that shines at constant intensity at every wavelength between 100 and 800 nm, and not at all otherwise (green)

iii) A blackbody source that peaks at 800nm (red)

iv) A blackbody source that peaks at 400nm (blue)

# 1.1 Solution:

a) Aluminium has a proton and electron *less* than silicon (13 vs. 14). This means that the Al doped silicon will miss one electron, which is equivalent to a free electron hole, the positive charge carrier. The material which contains **p**ositive charge carriers are of the **p**-type.

b)



c)

i) Electrons in the valence band are forbidden to absorb this photon, so the photon simply passes through without interacting (question for forrest: could the photon excite an electron if the final energy keeps it in the valence band? or are all valence electrons at the same energy level?)

ii) The photon can excite an electron at the top of the valence band to exactly reach the conduction band. The electron and electron-hole go each their way which establishes a voltage difference across the semiconductor which can be exploited by the external device.

iii) The photon can excite an electron to jump from the valence band to the conduction band. The electron and electron-hole go each their way, but lose energy to heat until their energies have reached the bottom of the conduction band. The resulting voltage difference across the semiconductor is the same as in ii), and can be exploited by the external device as before.

d) If  $E_{in} < E_{bandgap}$ , we get no energy out, so  $E_{out} = 0$ , and Eff = 0. However if  $E_{in} \ge E_{bandgap}$ , we always get out the band gap energy, so  $E_{out} = E_{bandgap}$ , and  $Eff = \frac{E_{bandgap}}{E_{in}}$ . e) Let's repeat d), but substituting  $\frac{hc}{\lambda}$  for all E. If  $E_{in} = \frac{hc}{\lambda_{in}} < E_{bandgap} = \frac{hc}{\lambda_{bandgap}}$ , i.e.,  $\lambda_{bandgap} < \lambda_{in}$ , we get no energy out, so  $E_{out} = 0$ , and Eff = 0. However if  $E_{in} = \frac{hc}{\lambda_{in}} \ge E_{bandgap} = \frac{hc}{\lambda_{bandgap}}$ , i.e.,  $\lambda_{bandgap} \ge \lambda_{in}$ , we always get out the band gap energy, so  $E_{out} = E_{bandgap}$ , and  $Eff = \frac{E_{bandgap}}{E_{in}} = \frac{\frac{hc}{\lambda_{bandgap}}}{\frac{hc}{\lambda_{in}}} = \frac{\lambda_{in}}{\lambda_{bandgap}}$ . We now label something we could call the "band gap wavelength", the wavelength of a photon whose energy is the band gap energy.

f)



Figure 1: d) Up until the band gap energy, no energy is captured from the photons. At energies higher than the band gap energy, we can capture the band gap energy per photon, while the remaining energy of the photon is lost to heat.



Figure 2: e) Wavelength si inversely proportional to energy, so we can capture the band gap energy for the smallest wavelengths (highest energy photons) all the way down to the "band gap wavelength", after which we capture nothing for longer wavelengths (lower energy photons).

i) Let's first figure out the wavelength of 1eV and 2eV photons.  $\lambda_{1eV} = \frac{hc}{1eV} = \frac{4 \times 10^{-15} \text{eV} \cdot \text{s} \cdot 3 \times 10^8 \text{m/s}}{1eV} = 12 \cdot 10^{-7} \text{m} = 1200 \text{nm}$ . Dividing by 2 instead of 1 yields for 2 eV photons a wavelength of

 $\lambda_{2eV} = 600$  nm. From e), we know that we will only ever harvest photons with wavelength smaller than the "band gap wavelength". So if we use B3, no photon will ever be energetic enough to cause an electron-hole pair! Using B1, we can capture energy in the 1000 - 1200 nm band.

ii) Here, you have the same amount of photons of every energy in the 100 - 800 nm range. Using B3 instead of B1, you will harvest photons in the 100 - 600 nm instead of 100 - 800 nm. However, you will for every photon get 2eV of energy instead of 1eV! This means that B3 will yield  $\frac{2 \cdot (600-100)}{1 \cdot (800-100)} = \frac{10}{7} \simeq 1.4$  times the energy B1 would yield.

iii) Ideal blackbodies emit in every wavelengths, but at different intensities. Using B3 here would indeed harvest more energy per photon, but a very small amount of them. B1 would harvest much more than twice the amount of photons B3 would, so B1 is the safest choice here.

iv) In this case, using B1 would let us harvest photons of high energy and all the way down to 1200nm, covering the bulk of the blackbody. B3 would only let us harvest photons down to 600nm. However, B3 would yield twice the energy per photon that B1 would, and since you can see from the graph that B3 would harvest more than half the photons B1 would, B3 is the safest choice here.