Michelson interferometer - assistant manual

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1 Introduction

The Michelson interferometer experiment is a third year lab course. It is the students last opportunity to test and improve their experimental skills before engaging in real research during their Bachelor project. This experiment is significantly different than the experiments done in the first and second year both in length and complexity. In the first two years the experiments were short and were designed to focus on specific basic aspects of experimental physics e.g data acquisition, data analysis etc. The theoretical understanding of the physical phenomena was kept to a minimum and the experimental method was usually already derived. In this experiment the students are exposed to the whole experimental process from beginning to end. Although the outcome of the experiment is known in advance and students can look up the results online, the techniques used in this experiments are common to many experiments in the field laser physics. A Michelson interferometer can be found in many laser labs and Fourier techniques are standard tools for any atomic or molecular physicist. This point can serve as a motivation for students and show them that they are very close to doing real research.

A close examination of how this experiment was executed in the past few years has surfaced a few structural problems. The assistants of this experiment are PhD students and are therefore available for a maximum period of 4 years. The assistants come from different backgrounds and are not always familiar with the details of the setup. It should be remembered that PhD students are not trained teachers. As a result too much attention is paid to the experiment itself and not to the learning process. An example of a frequently occurring problem in this experiment is the alignment of the Sodium light through the interferometer. Finding the right alignment is a technically demanding exercise that usually takes quite a lot of time. In certain cases this alignment has taken more than two weeks. In these cases not enough time was left for proper data acquisition and analysis, so many of the learning goals of the experiment were not achieved.

This manual was written in order to solve the problems mentioned above, specifically by determining the learning goal and how to reach them. As a first step this manual provides practical information about the setup, especially were we expect students to have difficulties. The experimental method is fully derived and suggested methods for the data analysis are given. The manual is divided into chapter, each dedicated to a different phase of the experimental process. Each chapter starts by setting the educational goal of the specific phase, and continues with what the students need to do in order to achieve these goals. The last chapter includes a time table, keeping it will ensure that enough time is spend on each phase. It is high recommended not to deviate too much from this schedule.

2 Theory

Goal: The students will understand the theoretical foundation of spectroscopy and understand how it applies to Sodium.

The theory involved in this experiment is quite extensive. On the one hand we are dealing with atomic physics including energy levels of Sodium, fine structure splitting, dynamics of line broadening and types of line shapes. On the other hand we are dealing with optics and interferometry including wave properties, interference, interferometers geometry and Fourier transforms. It is absolutely not the purpose of this experiment to teach students atomic physics. It is, nevertheless, important to qualitatively understand the phenomenon we are investigating. In the following list we summarize the topics and the level of understanding expected from the students.

- Atomic physics: basic energy levels in atoms. The phenomenon of fine structure splitting (qualitative description). Natural line width and its origin. Doppler broadening and pressure broadening. In the first day of the experiment the assistant should give an introduction about atomic physics and spectroscopy. The introduction should be short and focus on elements which are relevant to this experiment. Due to time constraints it is better to keep the discussion of a qualitative level.
- Wave properties: simple relations, $\lambda f = c$, $\omega = 2\pi f$, $k = 2\pi/\lambda$, frequencies are additive but wavelengths are not.
- Interferometry: The derivation of the intensity as a function of frequency and path length difference. Students should be able to reproduce this derivation or at least understand it fully. This is a key to understanding why the interferogram consists of intensity oscillations and must appear in the final report. It is better make sure that this point is understood in the first week.
- Fourier transform: The derivation of the interferogram and the spectrum being a Fourier pair. The relation between time (space) and frequency (wave number) domains. There are two article attached to this manual that, together with appendix A , present all the necessary information about the relation between the interferogram and the spectrum. The derivation of this relation forms the basis of the experimental method which is treated in the second week of the experiment. Therefore, the students should read these articles in the first week.

3 Setup

Goal: The student will build the experimental setup, improve his technical skills, with emphasis on optical alignment and signal acquisition.

Building the setup and managing it is the time for students to show their technical skills and problem solving capacity. In this chapter the steps needed for correct assembly of the setup are specified along with suggestions how to evaluate the students work. It is unavoidable that some students will need more guidance in this phase while others progress independently. Regardless of amount of help needed it is important that the assistant follows the students progress on a daily basis. Even if the progress is satisfactory it is still important to discuss the steps taken and recapitulate the main issues. These core issues should also be documented in the log book.

Setting up the He-Ne laser:
We start by setting up the laser because it is much more simple than the alignment of the Sodium source. The same techniques will be implemented later for the Sodium spectral lamp. The He-Ne laser should be fixed to the optical table and directed into the interferometer using 2 mirrors. Note that the laser holders are usually quite unstable. A telescope should be built for the laser to increase the beam diameter. A typical magnification factor of 15 should be sufficient. The magnification of the beam will be particularly important for the simultaneous detection of the two sources.

Alignment of the interferometer:
5 mirrors are at our disposal for the alignment of the interferometer. Two mirrors before the interferometer, one mirror for each interferometer arm and the retroreflector. The following sequence is the correct way to align the interferometer. The two mirrors in front of the interferometer should be used first to direct the beam through the beam splitter and in the middle of the two interferometer mirrors. The retroreflector is then removed so we can spatially overlap the two beams. This is achieved by overlapping the beams on the two interferometer mirrors. The final step is to put the retroreflector back in place and align its mirror until circular fringes are visible when the beams are recombined in the beam splitter. It is highly likely that this process will need fine tuning before circular fringes are seen for different positions of the retroreflector.

This alignment procedure is very important because deviations will introduce systematic error which will be untraceable later. Furthermore, understanding these steps will make it easier to get the interferometer working again if something unexpected happens at later stages (which will happen in every optics experiment).

Measuring laser fringes:
Now that the interferometer is aligned for the laser we are ready to measure interference fringes. By positioning a DC photodetector at the end of the interferometer (it might be necessary to attenuate the laser intensity) and moving the retroreflector we are able to measure the laser fringes and thereby the laser frequency.

The signal from the photodetector is digitalized in a data acquisition card and is transfered to a PC for analysis. The analog signal can be amplified before the data acquisition card for better contrast. A LabView program that was written by the lab technician is available. In this computer program various digital filters can be applied to the signal. Time and frequency representations of the signal are also available.

remark: if the wrong laser power supply is used it can result in modulation of the intensity output of the laser. This problem is easy to trace by looking at the frequency domain graph of the signal for stationary retroreflector, it will show a very sharp peak at the resonance frequency. In the same way an unwanted 100Hz peak in the Sodium signal (the discharge lamp has an AC power supply) will be identified and filtered.

The alignment of the laser and the interferogram it produces are far more simple than the ones for the Sodium source. Therefore it is instructive to spent some time on the analysis of the laser interferogram which will be helpfull in the analysis of the Sodium interferogram. Important points for the analysis are (hopefully to be realized by the students):

- Due to instability of the motor the fringes will not be equidistant.
- The laser frequency is know with relatively high accuracy (about 10^{-6}), this can be used as a distance ruler. This also gives the upper bound of the accuracy of the experiment.
- The line width of the laser is relatively narrow, fringes are visible for large retroreflector displacement. How will this affect the procedure of finding Sodium fringes?
- The sampling frequency can be chosen by looking at the frequency domain graph of the signal.
- The average velocity of the retroreflector can be related to the current of the motor driver (don't forget that moving the retroreflector 1 cm changes the path length difference by 4 cm).

setting up the Sodium source:
The Sodium source should be mounted on the optical table, similar to the laser, using two mirrors to direct the light into the interferometer. The second mirror for the laser alignment needs to be replaced by a glass plate[1] in order to overlap the two beams. The Sodium discharge lamp is a spherical light source, therefore, a lens system is needed to collimate the light. The choice of lenses and their positions should be completely left to the students, in this process of trial and error the students should get as much light possible through the interferometer.

remark: due to the short coherence length of the Sodium source the fringes are visible only within a few mm from zero path length, even in this range no fringes will be seen if we happen to be in a minimum of a beat. Therefore it might be easier to align the interferometer for the Sodium source first and only then align the laser. Finding the Sodium fringes is, regardless of the method, a tedious job. It is important to make sure that this doesn't take too much time, therefore close guidance might be required.

Simultaneous detection of Sodium and laser fringes:
For simultaneous detection of the two light sources we need 2 photodetectors where each detector measures light from a single source. The laser can be used as a ruler only when the two sources follow the same path in the interferometer. There are two ways to solve this problem, the first one is to split the beam at the end of the interferometer and gather light from the same spot in the two beams using interference filters. The second method is to first calibrate two distinct points in the laser beam and then spatially blocking parts of the two beams such that each source only reaches one detector. The students can also

[1]in order to avoid interference from the two surfaces of the glass plate it is actually better to use a wedge

come up with other detection methods. Which method is eventually used is not important as long as the students justify their choice.

After the successful completion of the above mentioned steps an interferogram can be recorded. In principle the oscillating signal should present a steady sine wave on short time scale with low noise. This is sometimes not the case when the digital card is not configured properly or t noise is picked up by the cables. This problem should be solved before starting the actual measurements.

4 Experimental method

Goal: The student will decide what to measure within the capabilities of the setup and develop the equations that will be used for the data analysis.

The experimental method is the way to translate experimental data into the quantities we want to measure. The starting point is the general equation of the theory which is, in our case, the relation between the spectrum and the interferogram. In the situation of the Sodium doublet a compact equation can be found for the expected interferogram as explained in appendix A.

In principle it is possible to record one long interferogram and apply a Fourier transform to it. All the degrees of freedom of the spectrum are found at once without making any assumptions. This method is, however, not recommended for this experiment. The error estimation in this method is complicated and not transparent. Furthermore, our goal can be determining one specific property of the spectrum so recording a full interferogram might not even be necessary. In the method explained below the interferogram is broken into segments such that we decouple the various degrees of freedom and find them separately. The educational added value is that error estimation of the different degrees of freedom requires different type of analysis. This property is what makes this experiment versatile from an educational point of view (and also what makes it difficult).

1. relative intensity:
   If we neglect for a moment the envelope function in equation 8 we see that the intensity at a top and bottom of a beat are $I_{max} = a_1 + a_2$ and $I_{min} = a_1 - a_2$ respectively. The ratio of the spectral strength between the lines is:

   $$\frac{a_1}{a_2} = \frac{I_{max} + I_{min}}{I_{max} - I_{min}} \tag{1}$$

   The variations due to the envelope function still has to be accounted for.

2. Average (carrier) frequency:
   Equation 8 (again neglecting the envelope function) consists of a sum of two terms, each of them demonstrates a fast oscillation modulated by a beat oscillation. The two terms are $\pi/2$ out of phase. For relative intensities $a_1/a_2 \approx 1$, which applies in this case, we can neglect the second term. The intensity of the Sodium source is then given by:

   $$I_s(x) = A_s cos(\overline{k}x)cos(\Delta kx) \tag{2}$$

The substantial difference between the beat and carrier frequencies (about 1000 carrier oscillations per beat) allows us to consider the two oscillations separately. The beat oscillation can be seen as a slow intensity variation and does not influence the phase of the carrier oscillation. The oscillations of the laser and the Sodium source will not present constant oscillations due to the nonuniform motion of the retroreflector. However, the ratio between the accumulated phases will be independent of the motion of the retroreflector. After the retroreflector has been displaced by an amount Δx the ratio of the accumulated phase will be inversely proportional to the wave numbers of the sources.

$$\overline{k}_{Na} = \frac{\phi_{Na}}{\phi_{laser}} k_{laser} \tag{3}$$

The advantage of this method is that the movement of the retroreflector does not affect the equation as long as we can resolve the fringes.

The simplest and most obvious way to find the total phase accumulation is to count the number of laser and Sodium peaks in a certain interval. The fractional phase at the beginning and end of the data set should be counted as well. A ready made counting algorithm (for example in Origin) can be used for this task. Ambitious students can try to find the phase of the signal using Fourier techniques. This method is elegant and accurate but takes more time to implement.

The accuracy of the measurement can be increased by analyzing longer data sets. However, the derivation of equation 2 is only valid in the vicinity of a beat maximum. The systematic error will be even more pronounced at a beat minimum where the phase undergoes a π jump. This issue has caused many problems in the past because students were not aware of it and did not understand why their results were inconsistent. It is important to make sure that the students are aware of this effect. In general, if the students understand the equations derived in this chapter and the approximations used to derive them, they should be ably to identify these systematic errors and correct for them.

3. Beat frequency:
   As opposed to the previous section where we neglected the slow beat oscillation now we are going to ignore the fast oscillation to only take the beat frequency into consideration. Equation 3 still applies with substituting the average sign with a difference sign.

$$\Delta k_{Na} = \frac{\phi_{Na}}{\phi_{laser}} k_{laser} \tag{4}$$

   Counting the accumulated phase needs a different approach than the one for the average wave number. Estimating a fractional phase of a beat oscillation is very inaccurate so it is advisable to take a data set which spans an integer number of beats and count the laser oscillations. In this way the only error to be considered is the number of laser oscillations. As this error will be larger than a whole laser oscillation there is no need to estimate the laser fractional phase. With this in mind we can write the beat wave number as follows:

$$\Delta k_{Na} = \frac{k_{laser}}{2N} \tag{5}$$

Where N is the number of laser oscillation per beat. The factor of 2 represents the fact that one beat is actually only half of the beat oscillation.

4. Line shape and width:
   In the derivation of the interferogram we have assumed that the line shape of the two spectral lines is identical. Using this assumption we can relate the envelope function of the interferogram to the line shape, as shown in appendix A. The line shape expected in this experiment can be a Lorentzian (pressure broadening), Gaussian (Doppler broadening) or a combination of the two (Voight profile). The proper analysis of Voight profiles goes far beyond the scope of this experiment and should only be pursued by extremely ambitious students. The line width can be determined by the relation $\Delta k \Delta x = k$ with $k = 0.142$ for a Lorentzian and $k = 0.44$ for a Gaussian. In principle, the linewidth can be determined by measuring Δx of the interferogram and filling in k for the right line shape. In practise, as mentioned above, the spectral lines are a combination of the two possible shapes thus the line width which makes it difficult to find the linewidth. The correct analysis would be to apply a Fourier transform on the envelope function and find the FWHM of the transformed data.

5 Data analysis

Goal: The students will apply proper data analysis, putting emphasis on error estimation, systematic and statistical errors.

Experience of past years shows that students have great difficulty with implementing proper data analysis. Analyzing student reports and talking to students have pointed out to the following problems. The experiments of the first and second year practicals are rather simple and have predicted results, it becomes common practice to estimate errors in a way that they fit within the expected value. Another problem is that the experimental setup and method of this experiment are more challenging than other experiment of the F-serie. As a result, by the time the setup is built and data is recorded not enough time is left for proper data analysis. Finally, it was found that the assistants do not actively participate in the data analysis. By the time that the report is being written it is too late to make substantial changes to the data analysis, so major mistakes are not corrected.

Because this is identified as a weak point it is highly recommended for the assistant to be more active during this phase. The students should be able to explain their method before they start the the actual analysis. At this point the assistant can decide whether intervention is needed. We give some general points that are relevant and should be a part of any specific method chosen by the students.

- Correct error estimation:
  It is often seen that when students get results that do not agree with what they expect they start increasing the statistical error until the error

bars fit within the expected value. For example, the estimation of the fractional phase at the beginning and end of a data set of the carrier oscillation can be estimated within 10% of an oscillation just by looking at the data. Any estimation larger then that will overestimate the statistical error. The discrepancy with the expected result usually comes from a systematic error. By overestimating the statistical error the systematic error are invisible and cannot be accounted for properly.

- Reproducibility:
  It has happened that due to time constraints the whole data analysis was based on one data set. Students should understand that basing their results on a single data set makes their conclusion less strong. It is highly recommended to use multiple measurements to show the reproducibility of the data. Even a single measurement can sometime be broken into smaller sets to show the consistency of the data.

- Identifying accuracy limitation:
  An important part of an experiment is to find what needs to be improved in order to do more accurate measurements in the future. This point can be explained with two examples. In the first one we consider equation 3. The accuracy of the Sodium carrier frequency depends on the sum of the relative accuracies of the laser frequency and the phases. It is obvious that the frequency uncertainty of the laser is much lower than how well we can measure the phases. The conclusion is that using a more accurate laser will not reduce the total error.

  In the second example we look at the equation of the two spectral lines, $k_{1,2} = \overline{k} \pm \Delta k$. The relative uncertainty of the carrier frequency is usually about 2 orders of magnitude lower than the beat frequency. However, the error in the wavelengths k_1 and k_2 depends on the absolute and not the relative uncertainty, therefore it will be dominated by the error of the carrier frequency. The separate contributions to the uncertainty of the results has to be understood and should be explained in the final report.

6 Time table

Week1: Theory and setup

- Introduction talk
- expectation from the student
- experimental sequence
- theory of spectroscopy
- Alignment of the laser and basic results for the laser
- setting up the Sodium source

Week 2: Setup and experimental method

- Searching for Sodium fringes

- Developing the relation between the interferogram and the spectrum
- Understanding the quantities we can measure
- Deciding what we want to measure (and why)
- Preliminary measurements
- determining analysis method (writing software if needed)

Week 3: Experimental method and measurements

- Finalizing the setup if needed (In case no Sodium fringes have been observed yet the assistant should help with the alignment procedure)
- Measurements
- First analysis (discusion about the analysis)
- Further measurements and analysis

Week 4: Measurements and data analysis

- last measurements
- Final analysis
- Report

A the interferogram of a doublet

The calculation of the interferogram is basically just a Fourier transform of the spectrum. The spectrum of two transitions can be written as a sum of two delta functions convoluted with a shape function:

$$G(k) = C(k) * [a_1\delta(k - k_1) + a_2\delta(k - k_2)] \tag{6}$$

where $C(k)$ is the normalized line shape a_1 and a_2 represent the individual line strengths. We assume that the line shape of the two transitions is the same which is acceptable for the Sodium doublet (why?).
When applying the Fourier transform to equation

$$I(x) = B(x)[a_1 cos(k_1 x) + a_2 cos(k_2 x)] \tag{7}$$

Where B(x) is the envelope function and x is the difference between the two arms.
Using trigonometric identities we get the final equation:

$$I(x) = B(x)[(a_1 + a_2)cos(\Delta kx)cos(\overline{k}x) + (a_1 - a_2)sin(\Delta kx)sin(\overline{k}x)] \tag{8}$$

Here we used the definitions $\overline{k} = \frac{k_1+k_2}{2}$ and $\Delta k = \frac{k_2-k_1}{2}$