Introducing modelling into the Solar Cooking Box experiment

Introducing computer modelling into project based learning; does modelling support and improve the understanding of the experiment?

[redacted]

Introduction

For several years, project based learning (PBL) has been a key element of the VU lab course education. In the form of experiments, students are encouraged to explore and understand the physical principles that underlay several processes. At the same time, students are taught to obtain a scientific attitude towards handling those principles to obtain results from their setups. PBL has been described as a very powerful tool to get students to become internally motivated to acquire knowledge [1]. L. Bot claims that students of the current generation (2005 and after) have *"less patience and intellectual effort"* due to the virtual influences that have been around them from a young age [2]. Bot suggests *"a dose of reality"* may be a suitable remedy to get the students to become motivated to absorb the required knowledge. Hence, PBL has become an important concept in educational literature of the last decade.

The PBL method relies strongly on the students internal motivation and is therefore dependent on the problem that has to be solved; it has to be both interesting and manageable within the set time frame. Students must take charge of the project on their own and are made responsible for planning and delivering. When working with PBL, careful monitoring of the students' progress is necessary to avoid students from getting stuck and experience an excessive workload [3].

The subject of this work is to introduce a new element into the existing Solar cooking box (SCB) experiment that is a part of the obligatory experimental lab course of the first year physics curriculum at the VU. The new element will be the development of a computer model to simulate the circumstances of the box. It is intended as an addition to the regular experiment and is supposed to support the students in their understanding of the SCB. The benefits of simulating problems using computer code have been described for a variety of scientific fields [3], [7], [8]. In these works it is stated that the use of computer code creates a necessity for abstract thinking and understanding the fundamental concepts of the problem that is simulated. Besides these beneficial effects, it is also expected that the students motivation will be enhanced by using computational methods [3]. developing a model vs using a model

By letting the students gather knowledge and apply it at the same time, there is an explicit interaction between knowing and doing, modelling and experimenting. The modelling can help the experimenting because it can predict how certain experimental features will behave. The experiment can help in making the model more realistic and as a check whether the important features are modelled correctly.

Project Goal

The goal of this work is to investigate how the computer model can best be fitted into the project and to evaluate its educational value. The main question is; does the simulation add to the knowledge and understanding of the students? A secondary goal is to form a set of recommendations and assignments to improve the project for next year. The setup of this research is mostly observatory. No hypothesis was formed before the beginning the project. Instead, a strategy was thought out on how to best guide the students towards an understood and completed model. This case study is therefore a description of the way the students handled the instructions and materials that were handed to them, and how this fits into certain educational views.

A choice has been made to let the students begin from the basics, to maximise the understanding of the theory. The experiment is performed in two groups of three/four students from both the UvA and the VU. Of each of the two groups, one student was asked to take charge of the modelling. After a short theoretical introduction, the groups were instructed how the SCB works and were then left to think of a research question. Afterwards, some modelling deadlines were set. The results and progress were monitored and at the end of the project, an evaluation was held.

Report outline

This work consists of a general introduction on some educational views on modelling and simulating. In the following section, some important educational views and concepts are treated, which will be used to put the observations and findings of the students progress in perspective. The outcome of the observation and a concluding questionnaire is stated in the *Students comments and experiences* section. From analysing these remarks, a few conclusions and recommendations are made. A very important returning question is: is it possible to incorporate modelling into the SCB experiment without compromising too much on basic concepts of either one of the elements?

Educational views on modelling

There are numerous works on teaching through computational methods. Most of these works give ground rules or basic requirements on how to best proceed when implementing a computational physics course. The module that was added to the SCB experiment is not a complete course. Therefore, the views that are expressed in these works only provide a framework in which we can place the observations, but do not apply as stringent rules.

Modelling Competencies

In the work of O. van Buuren [5], five modelling competencies are distinguished. These competencies are seen by van Buuren as basic requirements for success of the modelling learning path. In our course, these are not only requirements, but also educational goals. The competencies regard the use and understanding of:

- Computer environment
- Graphs as means for interpreting outcomes
- Variables and formulas for analysing the context and reducing it to a manageable problem
- Relation elements of model to variables
- Evaluation processes regarding models, output and experiments

The chosen programming environment for the modelling of the solar cooking box model is Mathematica. It was chosen because the students were assumed to have experience with the program; a mandatory course in Mathematica is a part of the first year curriculum at the VU. At the UvA this is not the case, though pairing of VU and UvA students was a foreseen solution to this problem. The supervisor had taken several courses about programming in Mathematica and could therefore sufficiently tutor the students on this aspect of the modelling.

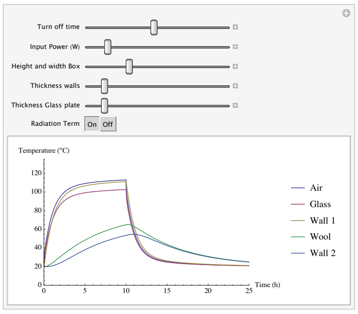

Figure 1: Example of a SCB simulation with manipulate option, as shown in Mathematica.

Mathematica is quite powerful in solving equations in an analytic way, which allows for the specific task of showing the students that it is possible to let a computer solve their differential equations and get immediate results. Because it works without directly switching to numerical solutions, the interpretation of the outcome is still available. The environment also allows for quick visualisations of data and formulas. Combined with the Manipulate option, it provides an ideal base for understanding the influence of the various parameters. This plays into the concept of 'direct gratification', which is also posted as an important concept in the work of C. Mias [3], in which the author suggests visual and interactive courses provide a greater sense of accomplishment than abstract theories. 'Modern' students should therefore be more motivated to work on the problem.

Graphs as means of interpreting outputs naturally follow from the project and program. The output of the experiment and the output of the model are both in graph format, which allows for the second competency to be satisfied. This competency is very important, since it ties directly into the main benefit of modelling: getting a better grip on the problem by understanding the abstract basics. This abstractness is very prominent in graphs, since they are by nature an abstract representation of a complicated system.

The third competency, *'Variables and formulas for analysing the context and reducing it to a manageable problem'* is strongly connected with using graphs, since Mathematica allows for easy manipulation of single variables and analysing the outcome.

To satisfy the fourth competencty, the derivation of a simple version of the SCB model is provided and explained. Expanding and understanding the model will be the students' responsibility, however help is offered when the modelling fails due to lack of mathematica skills. It is stressed continuously that the model is an approximation and that the students have to think on how to improve upon it.

Something that is also stressed from the outset of the project is to look at the model and the experimental situation and compare the output values. Students are also encouraged to look up literature on the different variables and constants that come into play, to get a feel for their magnitude and influence. Understanding the inherent limitations of models and the experiment is a specific learning goal in this experiment. The last competency, *'Evaluation processes regarding models, output and experiments'* is therefore not a basic skill students already posses, nor can it be taught within the first week.

The modelling cycle, displayed in figure 2, shows an overview of the path students are expected to take whilst programming simulations. The indicated process provides what could almost be seen as 'ground rules' for modelling [5], [6]. The cycle assumes a start from a *'real world situation'*, which has to be simplified and mathematised. The following feedback-loop provides a realistic model that can be used to approximate and predict the real world situation. This cycle also shows the necessity for abstract thinking and using formulas, as can be seen in the descriptions of steps 1 through 4.

From *van Buuren et al.* [5] we can also derive guidelines for the modelling learning sequences.

- For situations that have to be modelled, but which are new to students, experiments must precede the modelling. This order of events is necessary, otherwise students can not become acquainted with the events and phenomena that have to be modelled.
- Data from measurements is used for evaluation of the model.
- The basic mathematical concepts and physics concepts have to be introduced before the experimenting and modelling begins.

kom je op terug?

These guidelines are not all met in the modelling part of the SCB experiment. The experimenting and modelling are processes that both start on day 1 and are carried out simultaneously. Though this may seem illogical from the standpoint of the guidelines, there are several reasons this can not be changed; the students have a limited amount of time in which they have to finish both the experiment and the model. By synchronous work on the experiment and the model (which is mostly comprised of understanding the theory), the students understanding of the SCB is expected to increase faster. The original though of the modelling was that it would enhance the students creativity in the experiment, as it would increase their understanding and curiosity. It can be argued that, purely for the development of the model, this approach is not optimal. It should therefore be remembered that the construction of the model is not the main focus of the practicum, but rather a supportive element in the SCB experiment.

The second guideline, *'data from measurements is used for evaluation of the model'*, is used in the construction of the model and is therefore clearly met. As the model is built and expanded, data is produced by the experiment and the students were instructed to contemplate on how to

still, also developing a model in stead of using a model (simulate)

best expand the model, after comparing it to the output of the experiment. What factors are missing? Is there something else going on? How would could this best me described mathematically? These are typical questions that go back to the very core of what physics is; a mathematical description of observable phenomena.
The third guideline, introducing theory before working with the experiment, is not completely met due to time constraints and the general setup of the practicum. Having said this, in the presentation at the start of the course, the students are introduced to the basic principles and concepts. These are left to be further explored by the students themselves afterwards.
Students are not only expected to complete the experiment and the model, but are also judged on how they handle the gap between the knowledge they have at the beginning of the course and the knowledge they require to bring the experiment to a successful end. They are made responsible for bridging this gap as well. This is a key part in the PBL approach that is implemented in the course.

Difference between secondary school and undergraduate?

Modelling versus Simulating

There are various ways of teaching physics through computer techniques. The goal of this project was to implement the construction of a virtual model that would help the students in their understanding of the underlying physics. However, modelling and simulating a system are two distinct and different things.
In a simulation that represents a physical situation or object, a student may tweak variables and change parameters, but he or she can not change the essence of the simulation; the mathematical elements that form its basis. A student's interaction with the simulation has an exploratory character. [9] This means that simulations can be used to analyse the relations between different variables and relations that are built into the model: they have a very qualitative character. In modelling, the underlying basic mathematics and principles have to be understood as well, since it is the students own responsibility to get them operational. A student will have full control over a model and can improve and redesign it until the results are satisfactory. The downside to modelling is that it usually requires more time than simulating. Students can also lose themselves in modelling, because a model can be accurate enough, but never complete. Careful time management is therefore required, as stated by C. Mias [3].

Project development and observations

Before the project started, a model of the SCB was was built in Mathematica, to see which steps the students would have to take and what kind of end result could be expected. This model can be found in Appendix 2, figure 1 is an example output of that program. The program uses coupled differential equations to describe the energy exchange between the various layers of the box and plots the temperatures as a function of time. Almost all the input parameters are manipulable, creating a highly flexible simulation. The differential equations are comprised of source- and leak-terms. These represent in- and outgoing heat, and are the most important element of the abstract understanding the students have to acquire.

Opening presentation and first assignment

After an instruction on the use of the experimental setup, the students were presented with the basic principles of the SCB; its use and purpose, the different forms of energy/heat exchange, the simple conduction approach and the derivation and solution of the simplified differential equation (the presentation can be found in Appendix 1). Armed with these basic principles, they were left to brainstorm about a suitable research question.
The presentation provided the students with the gist of the theoretical model; source terms and loss terms, these were explicitly pointed out. The terms are explained in Appendix 3, as well as in the presentation. The modelers were then instructed to make a simple Mathematica graph of the provided solution of the differential equation. This was done to offer some direction, so the students would not be stuck at the beginning of the course.
The students were given handouts that further explained the theory and were then instructed to perform a few simple assignments.

Students comments and experiences

The students were asked specific questions about the modelling and how they felt it fitted into the experiment and were also asked for general feedback. There were two groups, both with one modeller.
The students did not agree among each other when asked about the added value of the simulation. The students who had been working on the models were mostly positive, they found that the model had contributed to their understanding of the problem and the limitations of modelling in general. The workload was found to be high.
The students that had not been working on the model were less positive about the simulations contribution to the experiment. The models power to predict certain behaviour remained absent due to the fact that these were not finished in the first week. The "experimentalists" found that the modelling task took away manpower from the experiment, seemingly without a useful end result. The students that hád been working on the model had different thoughts about this: they indicated that their mathematical insight of the problem was better than that of their fellow students. Specifically, working with the source- and leak terms in the model had helped them understand the processes that occur inside the box. One student also mentioned that by working on the model, he was compelled to find literature about the theory.
Both groups agreed on the fact that the modelling helped them in their understanding of the theory, when it came to writing a report and making a poster. The students were also satisfied with the amount of supervision on the modelling and experiment. The handout was considered to be helpful. The modelling students indicated a possible improvement of the modelling: adding some simple mathematica assignments to the course. The modelling students felt that starting from scratch with the model is the best for understanding the model completely. They advised not to change this.
One of the groups (group 2) experienced communication difficulties. Though undoubtedly caused mainly by the personal difficulties between the group members, it may also partly have been caused by the setup of the experiment. The students were frustrated by the fact that the promised advantages of the simulation did not present themselves

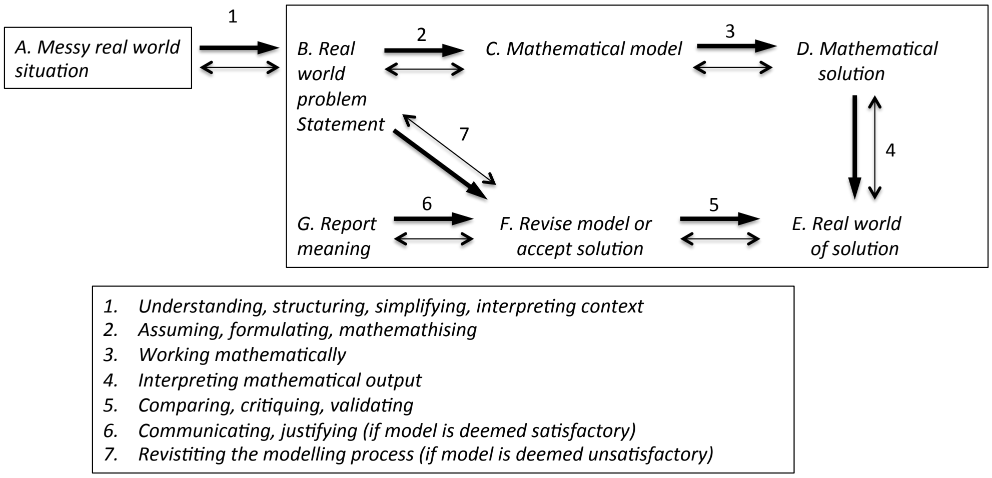

Figure 2: The modelling cycle [5], originally from Galbraith and Stillman [6].

in time. On the other hand, the modelling student was frustrated by the lack of understanding and support that was received from the other group members. They did not seem to be willing to make time to think about the difference between the models outcome and the experimental outcome, they merely discarded the model as being too inaccurate and incomplete. The modelling students also had difficulty with the *absolute* differences between model and experiment.
The group 2 students mentioned that it would probably have benefited them if the theory had been treated before starting the experimental work. This would have given all of them the chance to gain understanding of the model, instead of leaving the responsibility of figuring out the theory to one person. They also indicated that more group meetings about the model would have benefited them. They did not initiate these meetings themselves in the beginning of the experiment because they did not know how much work the modelling would be.
The modelling student of group 2 also indicated that the gap between the simple fit that was used to analyse the experimental data and the model she built, was too big.

Evaluation and Recommendations

The progress of the students was closely monitored for this work. There were several assignments that were provided during the first two weeks, in order to help speed along the process of modelling the SCB. Furthermore, a number of group meetings were held over the course of 3 weeks and personal guidance was offered on a daily basis. During the first week, both groups were instructed on how to start up their experiment, set and work towards an end goal. The observed progress of the students in these tasks and obstacles herein were logged and analysed during and after the course.

Communication

Despite the fact that the differences between the model and the experiment provide ample possibility to have interesting discussions, the students did not engage in these discussions much by themselves. It would therefore seem logical to add another modelling competency to the list, specifically for this type of project: group communication. Though not critically important for the success of the model itself, communication is essential in transferring knowledge and understanding of concepts that the model brings with it. Lack of communication between students was one of the observed weaknesses of the performed implementation of the modelling in the SCB experiment, by both the students and the supervisors.
Though this problem can not be blamed solely on the way the modelling was implemented into the experiment, a solution could be to implement a number of discussion sessions about the model and the theory. Also, group sessions specifically about the progress of the model seem to be necessary. The students that were not involved in the modelling, were not very receptive to discussing the discrepancies between the model and the experiment by themselves. However, discussing these issues seem to be a vital part, if not the most important part, of the perceived problems with the simulation. The group communication must therefore be monitored and encouraged more than before, to prevent the modeller and the experimenters from working completely separate and loose the advantage of shared knowledge.

Presenting the modelling

The focus of the modelling could be slightly altered by focusing more on the phenomenological aspect, instead of the quantitative results. The shape of the graphs and information that can be derived from them, are more important than obtaining the exact same values as the experimental setup from the simulation. This is once more reflected in one of the statements above, in which a student expresses to find the discrepancy between model and reality hard to accept. This is a very important statement, from which we can learn how to make the students value the model more without changing a lot; focusing on the trends and relations between the different parameters instead of focusing on trying to get as close to the "real" situation.
As previously stated, the modelling competencies are not

only requirements for successful programming, but also goals of the course. This seems to cause a sort of chicken and egg situation, in which it is unclear what the students should already have mastered and what they should learn. Since it is very clear that most of them have never worked explicitly with this sort of educational setup, the focus will be on learning and getting the students to put the competencies in practice, without explicitly telling them to do so.

The first day

As mentioned before, the first day was largely dedicated to introductory elements such as the presentation of the basic principles of the SCB and and introduction to the physical experiment and how to use it. There were some coördination problems that arose, which can be solved fairly easily.

First of all, the preparation the students need, have to be communicated in advance. The students must download and install Mathematica before starting the SCB experiment. There are also possibilities to use Mathematica on a computer that is present at the practicum, but this makes it impossible to work on the model at home. In the first week, this is expected to be necessary.

The presentation in its current form seems to suffice for instructing the students in the basic concepts of the SCB and providing the mathematical ground for the model. In the presentation, the group is instructed for the first time to divide tasks among themselves. The importance of coöperation and frequent group meetings could be stressed at this point, as a first measure to prevent the group communication to become an obstacle in the experiment. This measure would refer to the new competency that was mentioned before.

On the first day, there is rather a lot to take in for the students; they are introduced to an experiment, the way they are to operate it, the physical and mathematical principles and they have been given specific tasks within their group, a working form which has not been implemented like that before in the curriculum. For this reason, the modeler may experience difficulty in starting work on the model very energetically. For this reason, the first assignment should not be too large.

When analysing the first day by using the modelling cycle by Galbraith and Stillman [6], the students were set on track by introducing them to *A. the messy real world situation*. They were left to think about how to handle the setup and come up with a research question that would be manageable within the 4 week time frame. The presentation also offered them with the basic mathematical concepts and solutions, which means that also parts of *C. Mathematical model* and *D. Mathematical solution* are offered.

Assignments

The first set of assignments took the students longer to complete than expected. In the first group, this could be attributed to inexperience with Mathematica. In the second group, this was also a factor, but the time schedule of the modeler was also a possible cause of the problem; she had another course next to the full time practicum, which caused her to be absent sometimes. This can not be blamed on the setup of the course, but it should have been picked up before asking the group to distribute tasks and influencing the result so the modeler is not often indisposed.

The most time consuming step was grasping the abstraction of the model in an already quite abstract environment as Mathematica. Assignments could help with this. The students were less well prepared for the course than anticipated. It was not the purpose of the project to teach them to use Mathematica, but it turned out to be necessary. This cost nearly a week, which was impractical. If it is unsafe to assume the students are familiar with Mathematica, the addition to the project should be questioned.

The content of the assignments was assumed to be easy; the assignment was almost a copy-paste from the introductory presentation. The students did not seem to take the assignment as simple; instead they started expanding the model. They wanted to understand every aspect of the model from the start, which is understandable but not practical. The task was to input the provided formula for a simple SCB into a Mathematica *'manipulate'* and get a feeling for the behaviour of the model. It would seem that a clear explanation of the assignment would solve this problem and save a lot of time.

The above described observation ties into the ordering of the modelling path, which was applied wrongly in the assignment. Students should have been introduced to the parameters in more detail before trying to work with them. This connects to step 1 in the modelling path, *'Understanding, structuring, simplifying, interpreting context'* [5]. This step was purposefully skipped in favour of getting the students acquainted with the graphic output and simple model faster, but this approach seemed to backfire immediately. It would seem that an introductory assignment that deals with the meaning of the parameters might benefit the progress at the beginning of the course.

After observing this misstep, the modelling students were instructed to investigate the role of the various parameters in the model and discuss them with their fellow group members. This turned out to progress slowly as well, since the students worked on the model in parallel to researching the parameters. The expanding of the model had a higher priority due to the group pressure to finish the model fast and possibly because it was more interesting and 'fun' to do. The *'direct gratification'* concept of Mias [3] seems to apply. This provides another reason to improve the assignments on the investigation of the various parameters.

are the model of Galbraith and the constraints of van Buuren useful?

Time schedule

Setting and monitoring a time schedule could help the students finish their product in time. However, the project requires them to make their own planning. Letting the students come up with their own research question is an important part of the PBL setup, the students are made responsible for setting goals and planning the project. The planning of the modelling is a part of this.

Looking at the simulating of the SCB experiment and the modelling cycle in figure 2 [6], the situation forces a start from both point A and D. The modelling has to start simultaneously with the reviewing of the theory, otherwise it's power to predict the behaviour of the system will be lost, because the simulation will not be finished in time. This is achieved by providing the students with a quick introduction of the theory. This might prohibit students from taking the optimal path through the modelling cycle, because some of the key simplifications have been performed for them. However, the experiment as a whole does benefit from this, due to the foreseen faster completion of the model.

The presentation was received well, handouts were distributed to further explain the derivation and to provide background information. The first assignment did not go as smoothly as planned, due to an unexpected lack of Mathematica knowledge and experience on the students side. This inhibited the planned swift start of the project by several days.
What could be done to improve the progress of the project, is checking the time schedule more often and be stricter on adapting it. It could be a standard part of the meetings with the supervisors, which would keep both the students' and the supervisors' attention on the progress and the planning. For the modelling, deadlines were set in advance. However, these were not met. This also calls for stricter monitoring, but also less stringent deadlines. This may sound contradictory, but the set deadlines were apparently not realistic, so the students may need more time to complete the assignments. The monitoring of those deadlines then becomes more important, because the time schedule is tight.
These measures seem easy enough, but are possibly hard to unite with the purpose and philosophy of PBL, since the students must be made to feel that the project is in their own hands and the planning is their own. A patronising attitude will discourage students to work on the project. This should be taken into account when monitoring. The importance of monitoring was also described by Mias [3], who stated that PBL requires carefull monitoring of the students progress is necessary to avoid students from getting stuck and experience an excessive workload.
For the modelling, the time schedule should specify the relevant deadlines for assignments.

Conclusion

Refer to the 'research questions'
enthusiasm

The Solar Cooking Box experiment was expanded with a modelling part. This lead to an updated presentation and an improved handout. (see Appendix 1 and 3)
From the evaluation of the course and discussions with students, a few important constraints and requirements can be formulated.

- Only if the required programmer knowledge is present beforehand, will the modelling be a viable part of the project. The project will not benefit from adding a course in programming, since it will take too much time.
- Paying extra attention to group communication is vital when adding a programming module. One of the group members will physically work on a different problem than the rest of the group. To ensure the experience and gained knowledge are transferred, regular group meetings must be held, both on progress in modelling and experimenting.
- To prevent frustration about slow progress and discrepancies between the model and the experimental outcomes, the focus of the modelling should be shifted more to the phenomenological aspect, instead of the quantitative results. It must deal with trends and relations, not absolute numbers.
- The students should be given assignments about the meaning and relations between the various parameters of the SCB, before they begin experimenting and modelling. This will improve their understanding and speed up the modelling process.
- The students must make a time schedule, in which they have to set deadlines for certain stages of the experiment and the modelling. Supervisors have to keep the students on track or ensure delays do not lead to frustration.

Taking all the above requirements into account, adding a modelling part to the SCB project could definitely be of educational value. The advantages of the abstract programming and the need to understand the underlying theory clearly showed during the students explanations and discussions. The modelling could add to the project, and does not need to interfere with experiment.

something about developing vs using/simulating

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Appendix 1 - Solar Cooking Box Opening Presentation

Project fysica van leven en energie

De Solar Cooking Box (SCB)

1

Belang van de SCB



- Meer dan 50 landen
- Afrikaanse landen zoals Ethiopië, Somalië, Eritrea
- India, Afghanistan en vluchtelingenkampen

2

Belang van de SCB

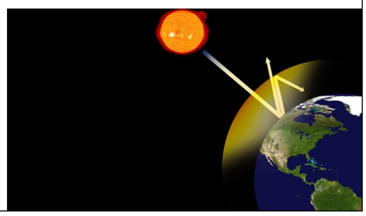


- Duurzaam product
- Ontbossingsprobleem
- Elektriciteit in Afrikaanse dorpen

3

Broeikas effect

- Werking van de SCB berust op het broeikas effect
- Korte golflengtes → Transmissie
- Lange golflengtes → Absorptie

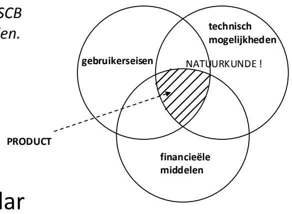


Productontwikkeling

- Gebruikerseisen:

De SCB moet gebruikt kunnen worden in geïsoleerde dorpen in Afrikaanse landen. De SCB moet twee maaltijden per dag kunnen bereiden.

- Lijst van gebruikerseisen opstellen
- Bespreken met andere groepjes
- Interview met Stichting Solar Cooking



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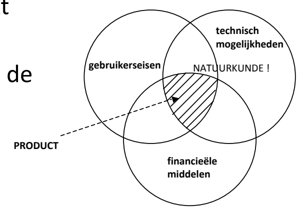
Productontwikkeling

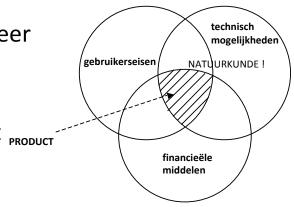
- Financiële mogelijkheden:
- De materialen die gebruikt worden
- Weeg de voordelen tegen de kosten af



6

Productontwikkeling

- Kies een aspect dat je wilt onderzoeken
- Probeer niet alles in een keer te onderzoeken
- Geef een volledige beschrijving van dit aspect



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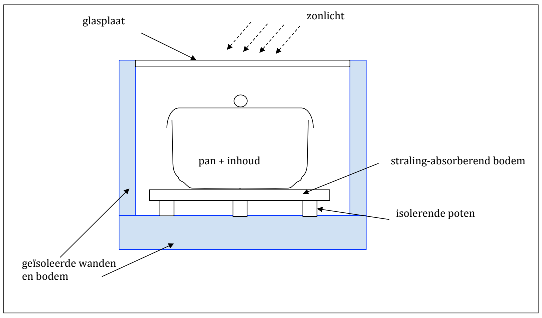
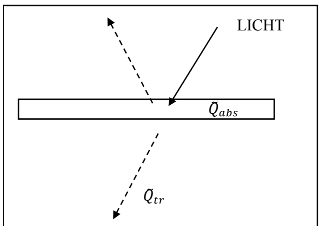
Simpel ontwerp van de SCB



8

Natuurkunde van de SCB



figuur 3: drie processen als licht op een glasplaat valt

Het zonlicht vertegenwoordigt een zekere energie/warmtestroom Q_{in} (J/s)

Op grond van de wet van behoud van energie geldt:

$Q_{in} = Q_{abs} + Q_{tr} + Q_r$ *(1)*

De transmissiecoëfficiënt T is gedefinieerd als

$T = \frac{\bar{Q}_t}{\bar{Q}_{in}}$ *(2)*

T is i.h.a. golflengte afhankelijk.

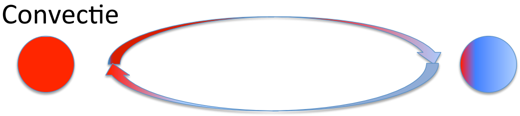
9

Warmtetransport

- Geleiding

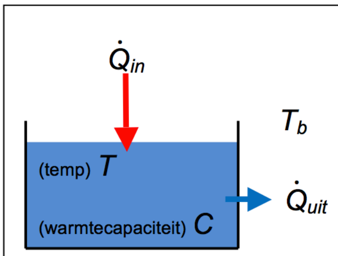
- Straling

- Convectie



10

Theorie



figuur 1: opwarm model

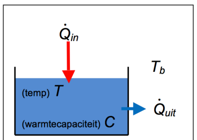
$$\Delta T = \frac{\dot{Q}_{in}}{C}$$

11

Theorie

$$\frac{dQ_{uit}}{dt} = \dot{Q}_{uit} = k\frac{S}{l}(T - T_b).$$

$$\dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{uit} = \dot{Q}_{in} - k(T - T_b).$$



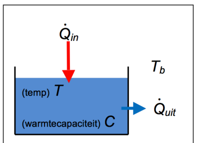
figuur 1: opwarm model

12

Theorie

$$\dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{uit} = \dot{Q}_{in} - k(T - T_b).$$

$$\lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} = \frac{dT}{dt} = \frac{\dot{Q}_{net}}{C} = \frac{\dot{Q}_{in} - k(T - T_b)}{C}.$$



figuur 1: opwarm model

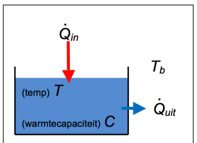
13

Theorie

$$\lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} = \frac{dT}{dt} = \frac{\dot{Q}_{net}}{C} = \frac{\dot{Q}_{in} - k(T - T_b)}{C}.$$

$$\frac{dT}{dt} = \frac{\dot{Q}_{in}}{C} + \frac{kT_b}{C} - \frac{k}{C}T = A - BT = -B(-\frac{A}{B} + T) = -BT^*;$$

$$A = \frac{\dot{Q}_{in} + kT_b}{C}, \qquad B = \frac{k}{C} \qquad en \qquad T^* = -\frac{A}{B} + T.$$

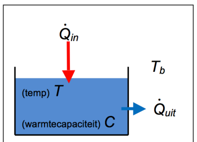


figuur 1: opwarm model

14

Theorie

$$\frac{dT^*}{dt} = -BT^*$$



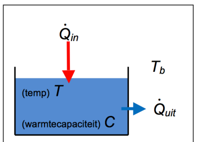
figuur 1: opwarm model

15

Theorie

$$\frac{dT^*}{dt} = -BT^*$$

$$T^* = ce^{-Bt}.$$



figuur 1: opwarm model

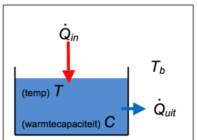
16

Theorie

$$T^* = -\frac{A}{B} + T. \qquad\qquad T^* = ce^{-Bt}.$$

$$T^*(0) = T(0) - \frac{A}{B} = T_b - \frac{A}{B} = ce^0 = c$$

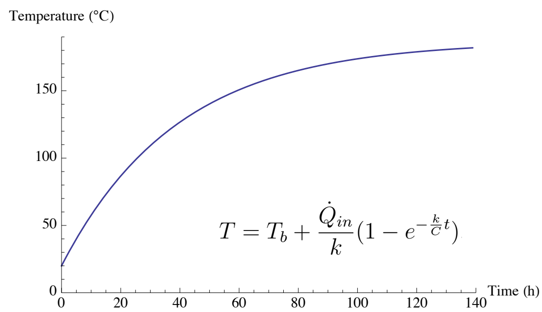
$$T^* = (T_b - \frac{A}{B})e^{-Bt} \qquad en \qquad T = T_b + \frac{\dot{Q}_{in}}{k}(1 - e^{-\frac{k}{C}t}).$$



figuur 1: opwarm model

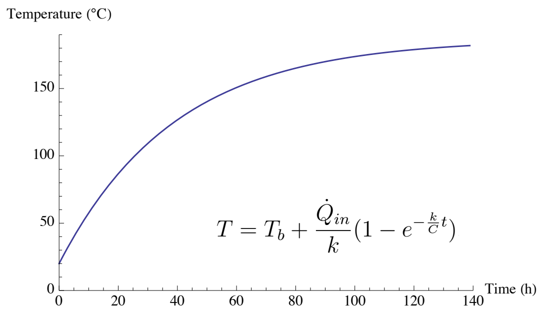
17

Theorie



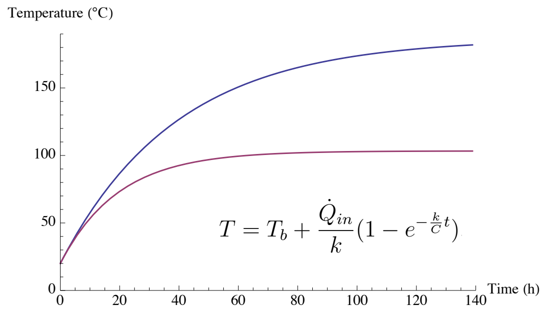
18

Theorie



18

Theorie



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Modelleren

- Verwacht wordt dat jullie zowel modelleren als experimenteren
- Enige uitleg over modelleren volgt later
- Omgang met Mathematica?

20

Groepsproces

- Verdeel de rollen in het groepje:
  - Voorzitter
  - Notulist
  - Modelleren
  - Experimenteren
  - (Poster)

21

Groepsproces

- Voorzitter: Zorgt dat het groepsproces goed verloopt en lost eventuele problemen op. Houdt overzicht en verduidelijkt problemen.
- Notulist: Houdt de verslaglegging <u>goed</u> en <u>secuur</u> bij, houdt de planning bij en maakt afspraken.

22

Groepsproces

- Planning is uiterst belangrijk
- Zorg dat je tijdig de verschillende resultaten kan vergelijken (modelleren en experimenteren)

23

Succes!!

24

Appendix 2 - Example of SCB Mathematica Model

```
In[1]:= Manipulate[
  vars = {
    kWood → .1,
    kGlass → .84,
    kWool → 0.029,
    kAir → 0.023,

    CWater → 4.210 * 10^6,
    Cair -> 0.00121 * 10^6,
    CWool -> 1.656 * 10^6,
    CWood → 1.224 * 10^6,
    CGlass → 2.184 * 10^6,

    Qin → InputPower,
    Tout -> 20 + 273.15,
    σ → 5.67 * 10^-8,
    uren = timeoff;

    ApanesWood → hwBox,
    dpanesWood → tBox,
    ApaneGlass → hwBox,
    dpaneGlass → tGlass,

     eAir → 0,
    eWood → .82,
    eWool → .9};


   DiffTlucht = (D[Cair Tlucht[t], t] ==
       1/ApanesWood^3 (Qin - 5 (ApanesWood^2/dpanesWood) kWood ( Tlucht[t] - Twall1[t])) -
        1/ApanesWood^3 (ApaneGlass^2/dpaneGlass kGlass ( Tlucht[t] - TGlass[t])) +
       If[RadOnOff == "On", - (eAir * σ * 5 ApanesWood^2 (Tlucht[t]^4 - Twall1[t]^4)) -
          (eAir * σ * ApaneGlass^2 (Tlucht[t]^4 - TGlass[t]^4)), 0]) //. vars;

   DiffTGlass =
    D[CGlass TGlass[t], t] ==
      1/(ApanesWood^2 dpanesWood)
       (1/ApanesWood^3 (ApaneGlass^2/dpaneGlass) kGlass ( Tlucht[t] - TGlass[t]) -
         (ApanesWood^2/dpanesWood) kGlass ( TGlass[t] - Tout)) //. vars;

   DiffTwall1 =
```

```
(D[CWood Twall1[t], t] ==
    1/(5 ApanesWood^2 dpanesWood) (1/ApanesWood^3 5 (ApanesWood^2/dpanesWood) kWood
        (Tlucht[t] - Twall1[t]) - 5 (ApanesWood^2/dpanesWood) kWool (Twall1[t] - TWool[t])) +
     If[RadOnOff == "On", (eAir * σ * 5 ApanesWood^2 (Tlucht[t]^4 - Twall1[t]^4)) -
       (eWood * σ * ApanesWood^2 (Twall1[t]^4 - TWool[t]^4)), 0]) //. vars;
 DiffTWool =
  (D[CWool TWool[t], t] ==
    1/(5 ApanesWood^2 dpanesWood) (5 (ApanesWood^2/dpanesWood) kWool (Twall1[t] - TWool[t]) -
       5 (ApanesWood^2/dpanesWood) kWood (TWool[t] - Twall2[t])) +
     If[RadOnOff == "On", (eWood * σ * 5 ApanesWood^2 (Twall1[t]^4 - TWool[t]^4)) -
       (eWool * σ * ApanesWood^2 (TWool[t]^4 - Twall2[t]^4)), 0]) //. vars;
 DiffTwall2 =
  (D[CWood Twall2[t], t] ==
    1/(5 ApanesWood^2 dpanesWood) (5 (ApanesWood^2/dpanesWood) kWood (TWool[t] - Twall2[t]) -
       5 (ApanesWood^2/dpanesWood) kAir (Twall2[t] - Tout)) +
     If[RadOnOff == "On", (eWool * σ * ApanesWood^2 (TWool[t]^4 - Twall2[t]^4)) -
       (eWood * σ * 5 ApanesWood^2 (Twall2[t]^4 - Tout^4)), 0]) //. vars;
Tsol = NDSolve[{DiffTlucht, DiffTGlass, DiffTwall1, DiffTWool, DiffTwall2,
         TGlass[0] == Tout, Tlucht[0] == Tout, TWool[0] == Tout, Twall1[0] == Tout,
         Twall2[0] == Tout} //. vars, {Tlucht, TGlass, Twall1, TWool, Twall2},
      {t, 0, 3600 * uren}] // FullSimplify // Flatten;
 Evaluate[TluchtOff = Tlucht[3600 * uren] /. Tsol];
 Evaluate[TGlassOff = TGlass[3600 * uren] /. Tsol];
 Evaluate[Twall1Off = Twall1[3600 * uren] /. Tsol];
 Evaluate[TWoolOff = TWool[3600 * uren] /. Tsol];
 Evaluate[Twall2Off = Twall2[3600 * uren] /. Tsol];

Module[{Tlucht, TGlass, Twall1, TWool, Twall2},

  DiffTluchtNQ = (D[Cair Tlucht[t], t] ==
```

$$\frac{1}{\text{ApanesWood}^3}\left(-5\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kWood}\ (\text{Tlucht}[t]-\text{Twall1}[t])\right)-$$
$$\frac{1}{\text{ApanesWood}^3}\left(\frac{\text{ApaneGlass}^2}{\text{dpaneGlass}}\text{kGlass}\ (\text{Tlucht}[t]-\text{TGlass}[t])\right)-$$
$$\text{If}\Big[\text{RadOnOff}==\text{"On"},\ \left(\text{eAir}*\sigma*5\ \text{ApanesWood}^2\left(\text{Tlucht}[t]^4-\text{Twall1}[t]^4\right)\right)-$$
$$\left(\text{eAir}*\sigma*\text{ApaneGlass}^2\left(\text{Tlucht}[t]^4-\text{TGlass}[t]^4\right)\right),\ 0\Big]\Bigg)\ \text{//. vars;}$$

DiffTGlassNQ =
D[CGlass TGlass[t], t] ==

$$\frac{1}{\left(\text{ApanesWood}^2\ \text{dpanesWood}\right)}\left(\frac{1}{\text{ApanesWood}^3}\left(\frac{\text{ApaneGlass}^2}{\text{dpaneGlass}}\right)\text{kGlass}\ (\text{Tlucht}[t]-\text{TGlass}[t])-\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kGlass}\ (\text{TGlass}[t]-\text{Tout})\right)\ \text{//. vars;}$$

DiffTwall1NQ =

$$\Bigg(\text{D}[\text{CWood Twall1}[t],\ t]==\frac{1}{\left(5\ \text{ApanesWood}^2\ \text{dpanesWood}\right)}\left(\frac{1}{\text{ApanesWood}^3}5\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kWood}\ (\text{Tlucht}[t]-\text{Twall1}[t])-5\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kWool}\ (\text{Twall1}[t]-\text{TWool}[t])\right)+$$
$$\text{If}\Big[\text{RadOnOff}==\text{"On"},\ \left(\text{eAir}*\sigma*5\ \text{ApanesWood}^2\left(\text{Tlucht}[t]^4-\text{Twall1}[t]^4\right)\right)-\left(\text{eWood}*\sigma*\text{ApanesWood}^2\left(\text{Twall1}[t]^4-\text{TWool}[t]^4\right)\right),\ 0\Big]\Bigg)\ \text{//. vars;}$$

DiffTWoolNQ =

$$\Bigg(\text{D}[\text{CWool TWool}[t],\ t]==\frac{1}{\left(5\ \text{ApanesWood}^2\ \text{dpanesWood}\right)}\left(5\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kWool}\ (\text{Twall1}[t]-\text{TWool}[t])-5\left(\frac{\text{ApanesWood}^2}{\text{dpanesWood}}\right)\text{kWood}\ (\text{TWool}[t]-\text{Twall2}[t])\right)+$$
$$\text{If}\Big[\text{RadOnOff}==\text{"On"},\ \left(\text{eWood}*\sigma*5\ \text{ApanesWood}^2\left(\text{Twall1}[t]^4-\text{TWool}[t]^4\right)\right)-\left(\text{eWool}*\sigma*\text{ApanesWood}^2\left(\text{TWool}[t]^4-\text{Twall2}[t]^4\right)\right),\ 0\Big]\Bigg)\ \text{//. vars;}$$

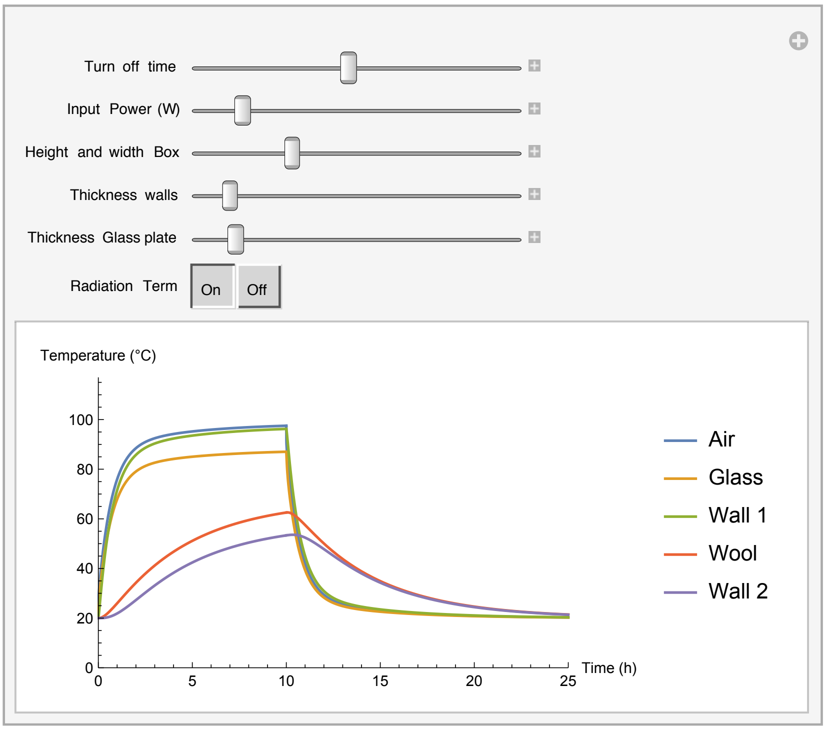
DiffTwall2NQ =

```
(D[CWood Twall2[t], t] ==
   1/(5 ApanesWood^2 dpanesWood) (5 (ApanesWood^2/dpanesWood) kWood (TWool[t] - Twall2[t]) -
      5 (ApanesWood^2/dpanesWood) kAir (Twall2[t] - Tout)) +
    If[RadOnOff == "On", (eWool * σ * ApanesWood^2 (TWool[t]^4 - Twall2[t]^4)) -
      (eWood * σ * 5 ApanesWood^2 (Twall2[t]^4 - Tout^4)), 0]) //. vars;
TsolNQ = NDSolve[{DiffTluchtNQ, DiffTGlassNQ, DiffTwall1NQ,
        DiffTWoolNQ, DiffTwall2NQ, TGlass[3600 uren] == TGlassOff,
        Tlucht[3600 uren] == TluchtOff, TWool[3600 uren] == TWoolOff,
        Twall1[3600 uren] == Twall1Off, Twall2[3600 uren] == Twall2Off} //. vars,
      {Tlucht, TGlass, Twall1, TWool, Twall2},
      {t, uren * 3600, 4 uren * 3600}] // FullSimplify // Flatten;
  Tlucht = Tlucht2;
  TGlass = TGlass2;
  Twall1 = Twall12;
  TWool = TWool2;
  Twall2 = Twall22;];

Show[
  Plot[
   {Evaluate[Tlucht[t * 3600] /. Tsol] - 273.15,
    Evaluate[TGlass[t * 3600] /. Tsol] - 273.15,
    Evaluate[Twall1[t * 3600] /. Tsol] - 273.15, Evaluate[TWool[t * 3600] /. Tsol] -
     273.15, Evaluate[Twall2[t * 3600] /. Tsol] - 273.15},
   {t, .001, uren}, AxesLabel → {"Time (h)", "Temperature (°C)"},
   PlotLegends → {"Air", "Glass", "Wall 1", "Wool", "Wall 2"},
   PlotRange → {{0, 2.5 uren}, {0, 1.2 (TluchtOff - 273.15)}}],

  Plot[
   {Evaluate[Tlucht2[t * 3600] /. TsolNQ] - 273.15,
    Evaluate[TGlass2[t * 3600] /. TsolNQ] - 273.15,
    Evaluate[Twall12[t * 3600] /. TsolNQ] - 273.15,
    Evaluate[TWool2[t * 3600] /. TsolNQ] - 273.15,
    Evaluate[Twall22[t * 3600] /. TsolNQ] - 273.15}, {t, uren, 2.5 uren},
   PlotRange → {{0, 30}, {0, 1.2 (TluchtOff - 273.15)}}]],
 {{timeoff, 10, "Turn off time"}, 1, 20, 0.001},
 {{InputPower, 120, "Input Power (W)"}, 0, 1000, 1},
 {{hwBox, 0.5, "Height and width Box"}, .1, 1.5, 0.1},
 {{tBox, .02, "Thickness walls"}, 0.001, .2, 0.001},
 {{tGlass, .02, "Thickness Glass plate"}, 0.001, .2, 0.001},
 {{RadOnOff, "On", "Radiation Term"}, {"On", "Off"}, ControlType → Setter}]
```

Out[1]=


Turn off time
Input Power (W)
Height and width Box
Thickness walls
Thickness Glass plate
Radiation Term
On
Off
Temperature (°C)
100
80
60
40
20
0
0
5
10
15
20
25
Time (h)
Air
Glass
Wall 1
Wool
Wall 2

Solar Cooking Box

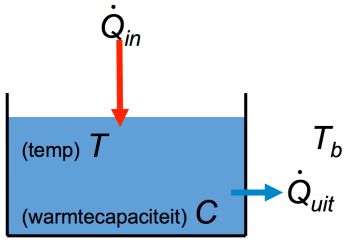
een wiskundige beschrijving in simpele benadering

najaar 2014

1 Opwarmen

Om het opwarmproces van een pan in een solar cooking box (SCB) te begrijpen is het volgende model een goed uitgangspunt:

Stel je neemt in eerste benadering aan dat de SCB een bak is met een zekere hoeveelheid vloeistof erin (zie figuur 1). De vloeistof heeft een warmtecapaciteit C. Deze constante is de hoeveelheid warmte die nodig is om de vloeistof 1°C te doen stijgen. Dit betekent dat als de hoeveelheid vloeistof Q aan warmte opneemt de temperatuur stijgt met $\frac{Q}{C}$.



Figuur 1: *model van een simpele Solar Cooking Box*

In de beginsituatie is de warmtebron (lamp of zon) nog niet aanwezig en dan is de SCB in thermisch evenwicht met de buitentemperatuur T_b. Dus is de temperatuur van de SCB $T = T_b$. Als vervolgens de warmtebron aangaat, dan nemen we aan dat de SCB opgewarmd wordt door een *constante* instroom van energie $\frac{dQ_{in}}{dt} = \dot{Q}_{in}$(J/s)

Door de warmte instroom zal de vloeistof opwarmen en krijgt een temperatuur T die hoger is dan de buitentemperatuur T_b. Als de temperatuur van de bak met vloeistof hoger is dan de omgeving zal er ook een afkoelingsproces plaatsvinden; er gaat warmte verloren. De afkoeling kan plaats vinden door verschillende processen: geleiding, convectie, straling, verdamping, etc.. Het meest eenvoudige model voor dit verlies is om aan te nemen dat de warmte-uitstroom evenredig is met het temperatuurverschil $(T - T_b)$:

$$\frac{dQ_{uit}}{dt} = \dot{Q}_{uit} = k\frac{S}{l}(T - T_b). \tag{1}$$

Hierbij is k de warmtegeleidingscoëfficiënt is van dit systeem; de voornaamste oorzaak van energieverlies. $\frac{S}{l}$ staat voor de ratio van het oppervlak S en de dikte van de wand l (het oppervlak waar warmte door weglekt). Om de formules overzichtelijk te houden, zal in de rest van deze beschrijving $\frac{S}{l}$ in de warmtegeleidingscoëfficiënt k getrokken worden. De netto warmtestroom $\dot{Q}_{net}$ (J/s) die het systeem in komt, is dan

$$\dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{uit} = \dot{Q}_{in} - k(T - T_b). \tag{2}$$

Als je deze vergelijking interpreteert, dan zie je dat de netto warmte-instroom steeds kleiner wordt naarmate de temperatuur stijgt. Het opwarmen zal dan ook steeds minder snel plaatsvinden. Dit kunnen we uitrekenen.

2 Oplossen van vergelijking 2

De verandering in temperatuur over de tijd is dus afhankelijk van de waarde van de temperatuur op tijdstip t. (Het is dus geen *constante* verandering) We schrijven dit proces daarom als een differentiaalvergelijking

$$\lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} = \frac{dT}{dt} = \frac{\dot{Q}_{net}}{C} = \frac{\dot{Q}_{in} - k(T - T_b)}{C}. \tag{3}$$

Dit is een eerste orde differentiaal vergelijking van T naar t. Daarin staat dat de verandering van de temperatuur afhankelijk is van de temperatuur zelf en van de warmtestroom.

Vergelijking (3) ziet er nog wat ingewikkeld uit, maar die kunnen we ook schrijven als:

$$\frac{dT}{dt} = \frac{\dot{Q}_{in}}{C} + \frac{kT_b}{C} - \frac{k}{C}T = A - BT = -B(-\frac{A}{B} + T) = -BT^*; \tag{4}$$

met

$$A = \frac{\dot{Q}_{in} + kT_b}{C}, \qquad B = \frac{k}{C} \qquad en \qquad T^* = -\frac{A}{B} + T. \tag{5}$$

Hierbij zijn A en B constanten gezien de aannames die hierboven zijn gemaakt.

Nu geldt dat $\frac{dT}{dt} = \frac{dT^*}{dt}$ (ga zelf na) en dat betekent dat we een oplossing moeten vinden voor de volgende nu simpele differentiaalvergelijking:

$$\frac{dT^*}{dt} = -BT^* \tag{6}$$

Zo hebben we de vergelijking die het temperatuurverloop van het op te warmen systeem in zijn meest eenvoudige vorm beschrijft.

Deze vergelijking heeft de oplossing $T^* = ce^{-Bt}$. Dat dit een oplossing is, kun je begrijpen door dit in te vullen in vergelijking (6).

3 Bepaling van de constanten c en B van de oplossing

Zoals hierboven aangegeven is op het moment dat de lamp aan gaat ($t = 0$) de temperatuur van de SCB gelijk aan T_b. Vullen we deze voorwaarde in dan vinden we een uitdrukking voor de temperatuur als functie van de tijd:

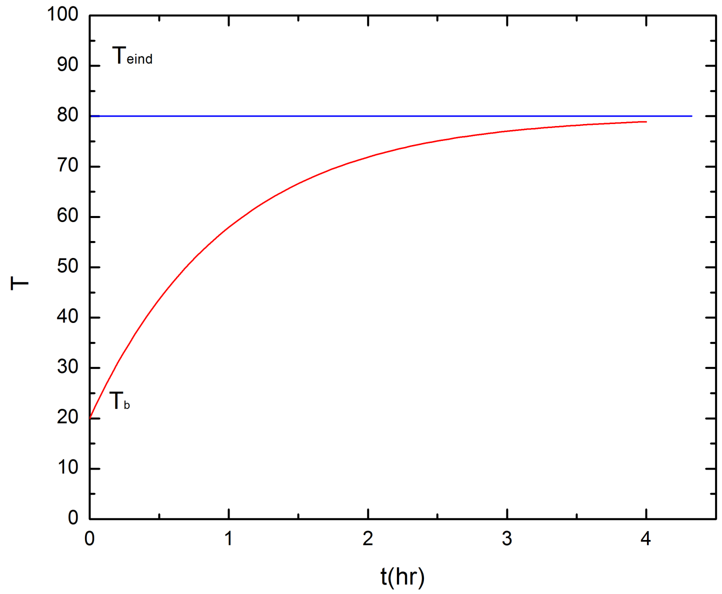
$$T^*(0) = T(0) - \frac{A}{B} = T_b - \frac{A}{B} = ce^0 = c \tag{7}$$

en dus geldt dat

$$T^* = (T_b - \frac{A}{B})e^{-Bt} \qquad en \qquad T = T_b + \frac{\dot{Q}_{in}}{k}(1 - e^{-\frac{k}{C}t}). \tag{8}$$

Het verloop van de temperatuur is dus een e-macht met een tijdconstante $\frac{k}{C}$. In figuur 2 is een typisch verloop geschetst van de temperatuur. Op tijdstip $t = 0$ is de temperatuur gelijk aan T_b. Op tijdstip $t = \infty$ is $T(\infty) = T_b + \frac{\dot{Q}_{in}}{k}$.

Dat betekent dat hoe groter de warmte-instroom $\dot{Q}_{in}$ is hoe hoger de eindtemperatuur, en hoe groter de warmteverliezen (k) hoe lager de eindtemperatuur.



Figuur 2: *een typische opwarmingscurve*

4 Afkoelen

Wat gebeurt er als de SCB is opgewarmd tot een zekere temperatuur T_A en de bron van verwarming wordt uitgeschakeld?

Dit houdt in dat $\dot{Q} = 0$ en daarmee gaat vergelijking (2) over in

$$\dot{Q}_{net} = -\dot{Q}_{uit} = -k(T - T_b). \tag{9}$$

en vergelijking(3) die de temperatuurverandering beschrijft wordt dan

$$\frac{dT}{dt} = \frac{-k(T - T_b)}{C}. \tag{10}$$

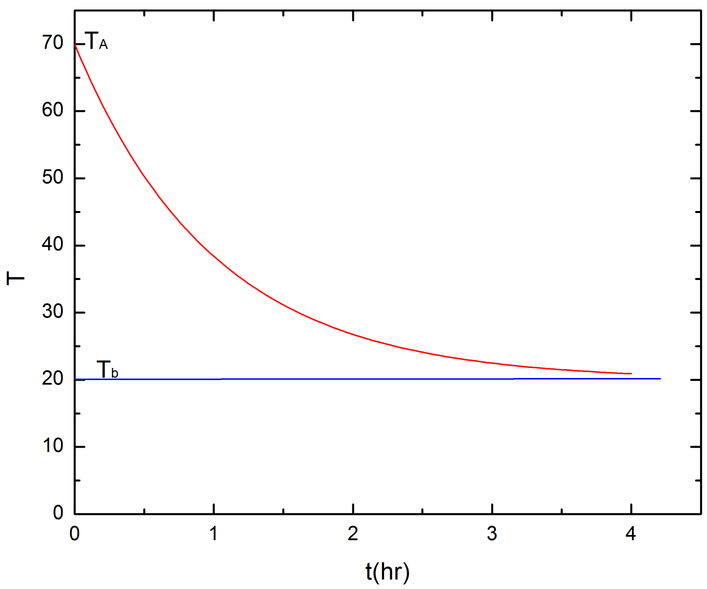
Ga nu over op een nieuwe variabele $T^{**} = (T - T_b)$, dan gaat vergelijking(10) over in (je weet dat $\frac{dT^{**}}{dt} = \frac{dT}{dt}$)

$$\frac{dT^{**}}{dt} = -\frac{k}{C}T^{**}. \tag{11}$$

Deze vergelijking heeft als oplossing $T^{**} = T - T_b = c_0 e^{-\frac{k}{C}t}$ dus $(T(t) = T_b + c_0 e^{-\frac{k}{C}t}$. Nu hadden we aangenomen dat bij het uitschakelen van de warmtebron (op $t = 0$) de temperatuur T_A was. Verwerken we deze voorwaarde dan wordt de oplossing

$$T(t) = T_b + (T_A - T_b)e^{-\frac{k}{C}t}. \tag{12}$$

Het verloop van de temperatuur T is ook nu weer een e-macht met dezelfde (!) tijdconstante k/C.



Figuur 3: *een typische afkoelingscurve*

Voor meer achtergrondinformatie over warmtestromen: zie hoofdstuk 19-10 van Physics for Scientists and Engineers, van Giancoli.