A one dimensional (1D) Oceans model (Hans Spoelder[1] and Egbert Boeker)

Description based on a paper by Dr Aad van Ulden (Royal Netherlands Meteorological Institute) whose permission to adapt and use his text is gratefully acknowledged[2].

1. Introduction

Radiative forcing by increase of the concentration of greenhouse gases will heat up the top layer of the oceans, which is called the mixed layer. The added heat is transported by the big currents of the oceans; on the surface these currents run to the North and South Poles, where they go to the bottom by a process called downwelling (EP3, Fig. 3.17). The bottom current separates off in many different places and disperses to the surface slowly with an average velocity $w \approx 4$ [m yr^{-1}].

The overall effect is that in time the complete ocean will heat up by human induced radiation forcing. In this paper we describe two effects: advection of heat with the ocean currents described here, and heat diffusion between layers of different temperature (EP3, Sect. 4.1). This diffusion has to take into account the turbulent motion of sea water and will be described by a parameter for turbulent diffusion k which will be much bigger than that for water at rest.

In reality there are three oceans which play a role: the Pacific, the Atlantic and the Indian Ocean. We will ignore this difference and keep in mind only one big ocean with downwelling at one location and upwelling everywhere else. In this paper we describe a one-dimensional (1D) approximation to the oceans and in a second paper a two dimensional (2D) approximation.

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[1] Hans Spoelder died while working on this project, April 1, 2002

[2] The model described here is a summary by van Ulden of the equations used by A. P. van Ulden and R. van Dorland in their calculations (see Aad P. van Ulden and Rob van Dorland, Natural Variability of Global Mean Temperatures: Contributions from solar irradiance changes, volcanic eruptions and El Nino, Proc. 1st Solar and Space Euroconference, *'The Solar Cycle and Terrestrial Climate'*, Santa Cruz de Tenerife, Tenerife, Spain, 25-29 September 2000, (ESA SP-463, December 2000). The text after eq. (--) is added by E. Boeker in order to clarify the numerical procedure. References to 'Environmental Physics, Third Edition' are given within parentheses as (EP3, pp--). Van Ulden and Dorland based their model on: M. I. Hoffert, A. J. Callegari and C-T Hsieh, The Role of Deep Sea Heat Storage in the Secular Response to Climate Forcing, *J Geophysical research* 85 (1980) C11, 6667-6679

2. The Model

The model is summarized in Fig. 1. On top of the ocean one will notice the mixed layer, which is in strong turbulence. Therefore the extra heat ΔT_m is distributed uniformly over the depth h_m of this layer. We assume that the ocean currents and their temperature distribution are not influenced by the

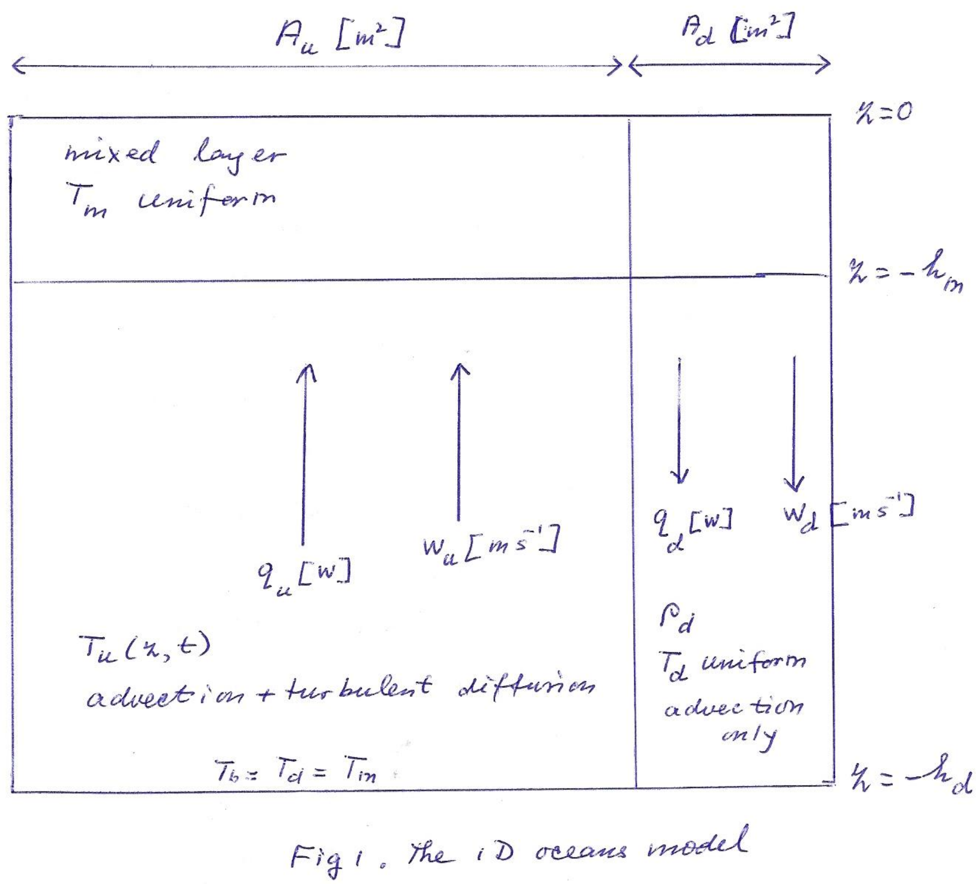


Figure 1. The 1D oceans model with the downwelling on the right and the upwelling on the left. The notation of the model is indicated. With a shortcut the downwelling will disappear from the equations which leaves only a description of the upwelling with one vertical coordinate z.

radiative forcing and the extra heat. Therefore, in order to simplify the notation we will omit the symbol Δ everywhere in figures and equations.

Conservation of mass requires that the downwelling mass should be equal to the upwelling mass [kg s^{-1}]

$$\rho_d A_d w_d = \rho_u A_u w_u \quad (1)$$

For simplicity we take the downwelling velocity w_d and the upwelling velocity w_u both positive; below we also take the heat fluxes q_d and q_u [W] both as positive.

We assume that the downwelling goes so fast that the temperature T_d in the downwelling column is uniform everywhere. Because this temperature originates from the mixed layer it is assumed that the downwelling temperature equals that in the mixed layer: $T_d = T_m$. Finally, as perhaps the strongest assumption it is taken that the downwelling water distributes quickly and evenly over the bottom of the ocean. This gives for the temperature T_b at the bottom $T_b = T_d = T_m$. This is indicated in the figure as well.

The heat flux down van be written as

$$q_d = \rho_d A_d w_d c T_d \quad \text{[W]} \quad (2)$$

This is the product of the downward mass [kg s^{-1}], the heat capacity of sea water *c* [J kg^{-1}K^{-1}] and the temperature increase T_d [K]. The product indeed is in [J s^{-1}=W]. Because of eq. (1) this also can be written as

$$q_d = \rho_u A_u w_u c T_d \quad \text{[W]} \quad (3)$$

In the upwelling region the heat current is less simple as the temperature increase T_u is a function of position. From EP3 (4.16) we have

$$\rho_u c \frac{\partial T_u}{\partial t} = -\operatorname{div} q''$$

Multiplication by A_u and using the fact that the heat current density is a function of z only leads to

$$\rho_u c A_u \frac{\partial T_u}{\partial t} = -A_u \frac{\partial q''}{\partial z} \quad (4)$$

The heat current density q'' can be written as the sum of two terms. The first one originates from EP3 (4.3) $q'' = -k \operatorname{grad} T_u$. In our case we use a little different definition of the thermal conductivity in order to simplify our equations.

Therefore we write $q'' = -kc\rho_u \,\mathrm{grad}\, T_u = -kc\rho_u \partial T_u / \partial z$. We again used that T_u is a function of z only. The other contribution is the advection of heat to the flow. Its representation is given by the first part of EP3 (7.5). These two contributions result in

$$q'' = c\rho_u\{-k\frac{\partial T_u}{\partial z} + w_u T_u\} \quad [\mathrm{W\ m^{-2}}] \quad (5)$$

De dimensions of the advection term $c\rho_u w_u T_u$ are readily checked : [J kg^{-1}K^{-1}]×[kg m^{-3}]×[m s^{-1}]×[K]=[J s^{-1}m^{-2}=W m^{-2}].

Equations (4) and (5) may be combined and divided by A_u. This gives

$$\frac{\partial T_u}{\partial t} = -\frac{\partial}{\partial z}\{-k\frac{\partial T_u}{\partial z} + w_u T_u\} \quad (6)$$

This equation holds everywhere, except where there are extra sources or sinks of heat such as in the mixed layer.

Boundary conditions

1. At the bottom of the mixed layer $z = -h_m$:

$$T_d = T_m \quad (7)$$

The temperature T_u of the upwelling water at this location must be a little lower in the physical situation of a slowly heating mixed layer, as the water entering from below has not heated up yet. Note: Van Ulden takes $T_d = T_m = T_u$ at the bottom of the mixed layer.

2. At the bottom of the ocean $z = -h_d$ we have

$$q_u = q_d \quad (8)$$

because of conservation of heat. We also call to mind the relation for the temperatures which we discussed earlier $T_b = T_d = T_m$

The mixed layer

The heat budget of the mixed layer consists of three parts: the upwelling from below, the downwelling at the poles and the interaction with the radiation from and to the atmosphere. We discuss them separately, starting with the downwelling which is the simplest.

The downwelling is by advection only and described by eq. (3):

$$q_d = \rho_u A_u w_u c T_d = \rho_u A_u w_u c T_m \quad (9)$$

The upwelling is described by $A_u q''$ which by eq. (5) gives

$$q_u = A_u q'' = A_u c \rho_u \{-k \frac{\partial T_u}{\partial z} + w_u T_u\}_{z=-h_m} \quad (10)$$

These two terms together give

$$\begin{aligned} q_u - q_d &= A_u c \rho_u \{-k \frac{\partial T_u}{\partial z} + w_u T_u\}_{z=-h_m} - \rho_u A_u w_u c T_m = \\ & A_u c \rho_u \{\left(-k \frac{\partial T_u}{\partial z}\right)_{z=-h_m} + w_u (T_{u,z=-h_m} - T_m)\} \end{aligned} \quad (11)$$

The radiative interaction between atmosphere and ocean is described by

$$q_r = A\Delta F - A \frac{T_m}{S_{eq}} \quad (12)$$

Here $A = A_u + A_d + A_{land}$, the total surface area of the world. We assume that the heat from the total globe will end up in the oceans, plus a small contribution from the atmosphere which we will take into account by giving an extra thickness to the mixed layer later on.

The radiative forcing ΔF [W m^{-2}] is the extra flux coming down from the extra greenhouse gases. Because of the temperature increase T_m extra radiation is leaving the earth, in first order proportional to T_m. If the temperature increase after some time is such that $T_m = S_{eq}\Delta F$ the incoming and outgoing radiation balance again and $q_r = 0$. Before that time $q_r > 0$. We define the heat capacity of the mixed layer by

$$C_m = c \rho_m A_m h_m \text{ [J K}^{-1}\text{]} \quad (13)$$

This expression is close to the left-hand side of EP3 (4.16); we find

$$C_m \frac{dT_m}{dt} = q_u - q_d + q_r \quad (14)$$

It is wise to check the signs of these terms, starting with $q_u - q_d$. Suppose that the layer below the mixed layer is cooler than the mixed one. Then the first term of eq. (11) is negative, which gives a heat flow downwards, as we expect. The consequence is a negative derivative dT_m / dt representing cooling of the mixed layer. The second term is negative as well, because the temperature difference is negative, again giving a cooling.

The last term in eq. (14) will be positive until equilibrium is reached again. This gives a positive contribution to the derivative dT_m / dt as it should.

We now substitute (11), (12) and (13) in (14) and divide both sides by $c\rho_m A_m$. This gives

$$h_m \frac{dT_m}{dt} = \frac{A_u}{A_m}\left(-k\frac{\partial T_u}{\partial z}\right)_{z=-h_m} + \frac{A_u}{A_m} w_u (T_{u,z=-h_m} - T_m) + \frac{A}{A_m c\rho_m}\Delta F - \frac{A}{A_m c\rho_m}\frac{T_m}{S_{eq}} \quad (15)$$

We approximate $A_u = A_m$ and introduce a parameter

$$c_w = c\rho_m A_m / A \quad (16)$$

Eq. (15) for the mixed layer then can be simplified to

$$h_m \frac{dT_m}{dt} = \left(-k\frac{\partial T_u}{\partial z}\right)_{z=-h_m} + w_u (T_{u,z=-h_m} - T_m) + \frac{\Delta F}{c_w} - \frac{T_m}{c_w S_{eq}} \quad (17)$$

The basic equations of the model are (5) for the general layers and (17) for the mixed layer, with the boundary conditions indicated in (7) and (8). All equations only refer to the upwelling region and the mixed layer. We therefore have one dimension only, the vertical one. This may be considered as a column of sea water with an enormous area A_u and a vertical coordinate z which runs from $z = -h_d$ at the bottom upwards to $z = 0$ at the ocean surface.

3. Numerical procedure

In order to solve the equations we have to make the problem discrete. We divide the vertical column in slices with height s_i as indicated in Fig. 2. The

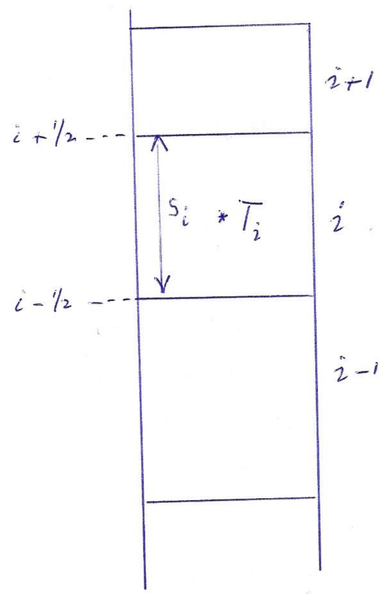


Figure 2. Definition of a slice i and its boundaries at $i+1/2$ and $i-1/2$.

upper boundary of the slice is indicated by $i+1/2$ and the lower boundary by $i-1/2$. We count slices from $i=1$ at the bottom to $i=N$ the top one below the mixed layer. The mixed layer itself is described by the index m or sometimes $N+1$.

Slice somewhere in between

We first take an arbitrary slice, not the bottom one, not the mixed layer, not the level just below: $i \neq 1, i \neq N, i \neq N+1$. Then equation (6) is applicable, there are no boundary conditions and we use the approximation

$$\frac{\partial f}{\partial z} \approx \frac{f(z(i+1/2)) - f(z(i-1/2))}{z(i+1/2)) - z(i-1/2)} = \frac{f(z(i+1/2)) - f(z(i-1/2))}{s_i} \quad (18)$$

Eq. (6) then gives in an obvious shorthand

$$\left.\frac{\partial T}{\partial t}\right|_i = -\frac{\left(wT - k\frac{\partial T}{\partial z}\right)_{i+1/2} - \left(wT - k\frac{\partial T}{\partial z}\right)_{i-1/2}}{s_i} \quad (19)$$

For the $z-$derivatives the same procedure is followed. We use

$$\left.\frac{\partial T}{\partial z}\right|_{i+1/2} = \frac{T_{i+1} - T_i}{s_i/2 + s_{i+1}/2} = \frac{2(T_{i+1} - T_i)}{s_i + s_{i+1}} \quad (20)$$

$$\left.\frac{\partial T}{\partial z}\right|_{i-1/2} = \frac{T_i - T_{i-1}}{s_i/2 + s_{i-1}/2} = \frac{2(T_i - T_{i-1})}{s_i + s_{i-1}} \quad (21)$$

$$T_{i+1/2} = \frac{s_i T_{i+1} + s_{i+1} T_i}{(s_i + s_{i+1})} \quad (22)$$

$$T_{i-1/2} = \frac{s_{i-1} T_i + s_i T_{i-1}}{(s_{i-1} + s_i)} \quad (23)$$

Bottom slice $(i = 1)$

For the bottom slice we cannot apply (21) as there is no slice below. We therefore write for the second term in the numerator of (19)

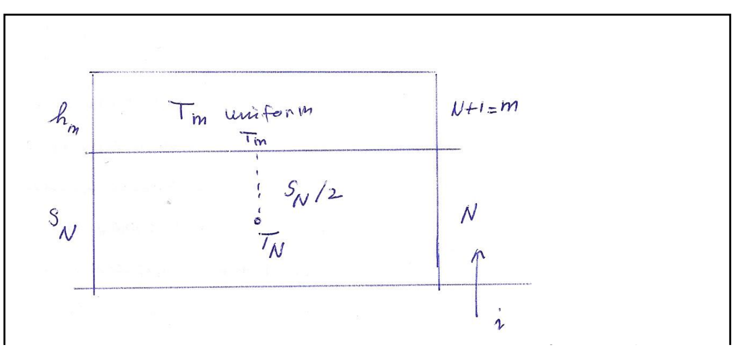
$$\left(wT - k\frac{\partial T}{\partial z}\right)_{i-1/2} = wT|_{i-1/2} = wT_m \quad (24)$$

Where we used the assumption that the temperature at the bottom equals the one in the mixed layer because of the fast downwelling. In the bottom slice we have

$$\left.\frac{\partial T}{\partial t}\right|_i = -\frac{\left(wT - k\frac{\partial T}{\partial z}\right)_{i+1/2} - wT_m}{s_i} \quad (25)$$

For the derivative at the top of the bottom slice we again use (20)

Top slice below mixed layer $(i = N)$



Here the difficulty obviously is what to take for the term at the top of this slice. The situation is sketched in Fig. 3. The mixed layer is homogeneous. Therefore instead of (20) we will take

$$\left.\frac{\partial T}{\partial z}\right|_{N+1/2} = \frac{T_m - T_N}{s_N / 2} \quad (26)$$

The advection term describes the heat which is leaving the top slice and entering the mixed layer. This temperature will be lower than T_m and should have the same temperature that applies to the water (10) entering the mixed layer. We call it again $T_{u,z=-h_m}$. For the top slice we find from (19)

$$\left.\frac{\partial T}{\partial t}\right|_{N} = -\frac{wT_{u,z=-h_m} - k\left(\frac{T_m - T_N}{s_N/2}\right) - \left(wT - k\frac{\partial T}{\partial z}\right)_{N-1/2}}{s_N} \quad (27)$$

The mixed layer

The mixed layer is described by (17). Its first term on the right again can be approximated by (26) as the turbulent diffusion downward from the mixed layer will be the diffusion entering the top slice. Eq. (17) then becomes

$$h_m \frac{dT_m}{dt} = -k\frac{T_m - T_N}{s_N/2} + w_u(T_{u,z=-h_m} - T_m) + \frac{\Delta F}{c_w} - \frac{T_m}{c_w S_{eq}} \quad (26)$$

The temperature $T_{u,z=-h_m}$

This temperature should be somewhat lower than that of the mixed layer. As an approximation we take the temperature at $z = -2h_m$. The simplest way to program this is to take $s_N = 2h_m$ and $T_{u,z=-h_m} = T_N$. When we would have may layers, say 100, it would be more appropriate to omit the condition $s_N = 2h_m$ and still take the temperature of the lower layer.

Parameters

We take the numerical values by Hoffert et. al. Summarized in table 1.

Table 1. Numerical parameters borrowed from Hoffert (note 1)			
$A_m / A = 0.65$	$\rho_m = 1030$ [kg m^{-3}]	$c = 4000$ [J kg^{-1}K^{-1}]	$c_w = c\rho_m A_m / A = 2.678 \times 10^6$ [J m^{-3}K^{-1}]
$w_u = 4$ [m yr^{-1}]	$k = 2000$ [m^{2}yr]	[yr]=3.158×10^7[s]	
$S_{eq} = 0.6$ [K W^{-1}m^2]		$h_m = 52.8$ [m]	$s_N = 100$ [m]

In climate studies the year [yr] is a more convenient unit than the second [s]. Look at eq. (17). With the parameters k and w_u expressed in years we are only left with the last two terms. The forcing is expressed in [W=J s^{-1}]. If we multiply this by [yr] from the table the unit of time is the year there as well. Finally, S_{eq} is expressed in [W^{-1}] and it occurs in the denominator. If we multiply the last term by [yr] as well everything will be expressed in years. For the other slices (19) makes it obvious that with k and w_u in the units of the table all is well there as well.

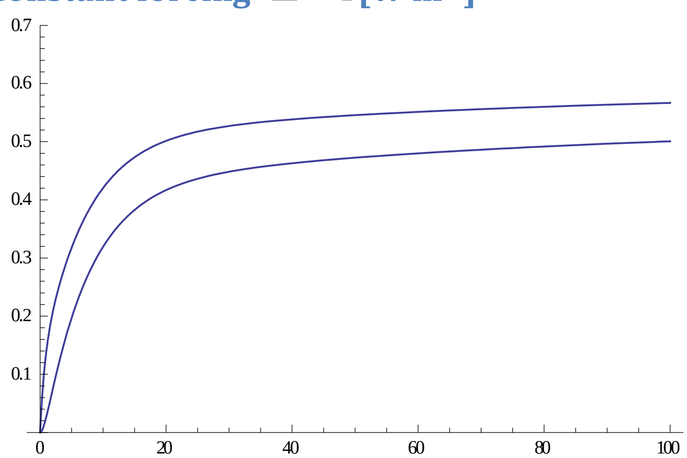
The thickness of the mixed layer is taken as 50 [m]. To take into account the heat capacity of the atmosphere and top soil 2.8 [m] is added. For the top layer, as discussed, twice the thickness of the mixed layer is taken.

4. Results

The simplest check on the equations and the computer code is to use a constant forcing, for example $\Delta F = 1$ [W m^{-2}]. All layers of the discrete set should monotonously rise their temperatures to the final temperature $T = S_{eq}$.

We use 10 layers. The height of the upper two is defined in table 1. The other 8 levels all have the same height.

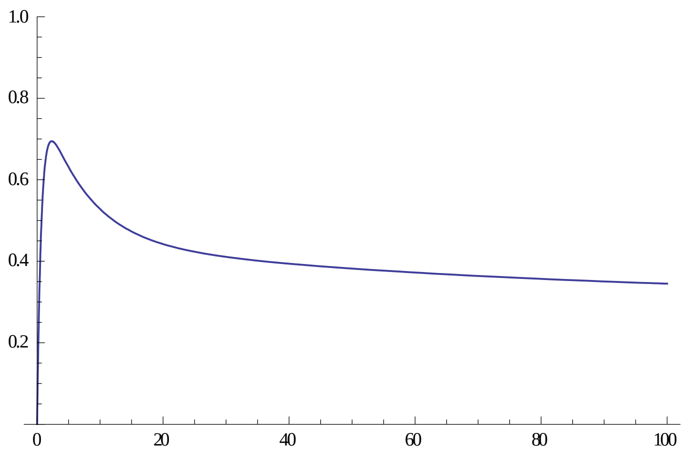
Constant forcing $\Delta F = 1$ [W m^{-2}]



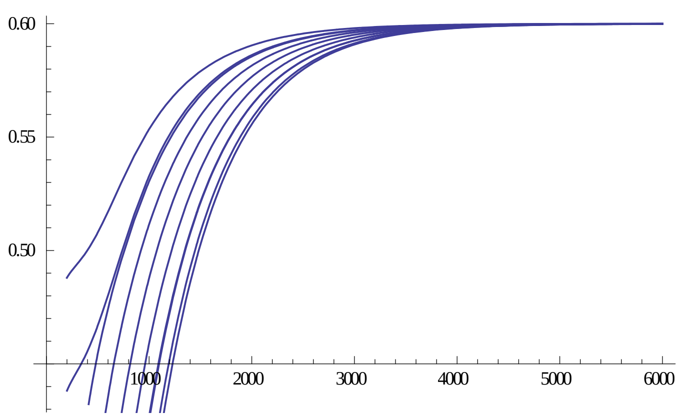
The top curve shows the temperature of the top layer [K] and the one below the temperature of layer 9, just underneath. Horizontal the time in years is indicated. It is clear that turbulent diffusion of heat must increase the temperature of the lower layer. That may be verified by plotting

$$c\rho_u k \left.\frac{\partial T}{\partial z}\right|_{z=-h_m} = c\rho_u k \frac{T_m - T_N}{s_N / 2}$$

and dividing by the length of the year. That gives the flux in [W m^{-2}]. This should be smaller than 1 [W m^{-2}] entering the ocean from above. We find the following plot

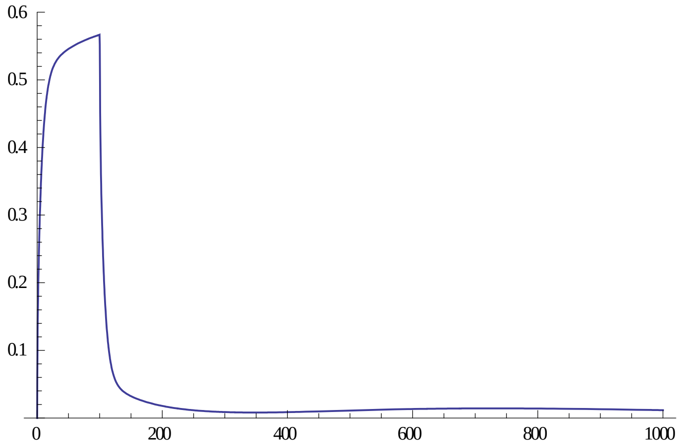


This indeed gives a considerable contribution. Finally we plot the temperature of all 10 layers for 6000 years:



They indeed all converge to the required 0.6 [K].

Block pulse



A block pulse of 1 [W m^{-2}] during 100 years is shown in this graph for the mixed layer. In the first 100 years the equilibrium of 0.6 [K] is not reached. Afterwards the heat disappears again by IR radiation. Interesting is the little hump after about 700 years. Apparently the heat from downwelling is reaching the ocean surface there again. Roughly it would take 4000/4 years for a complete cycle, but because of the turbulent diffusion it goes a little faster.

The hump is noticed better in the graph below, where the mixed layer and the one below it are displayed. The vertical coordinate is shown more in detail.